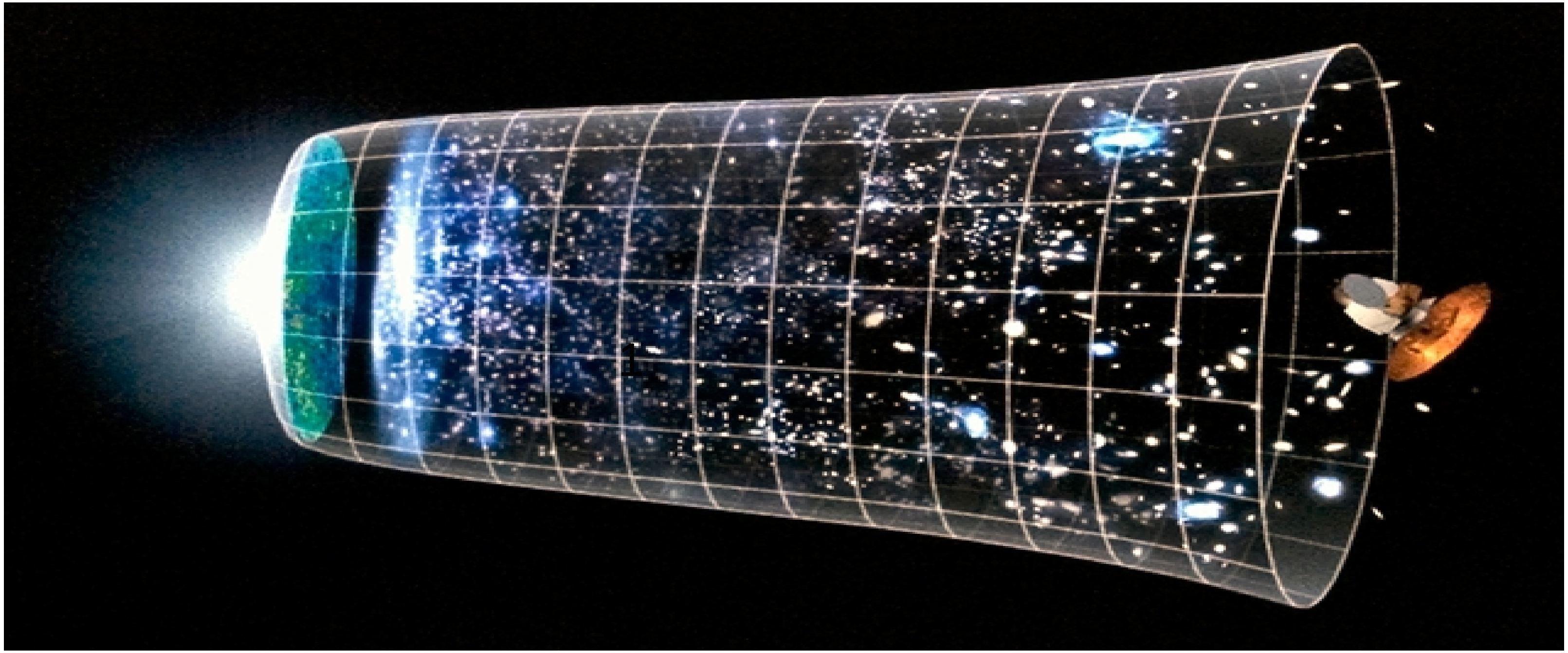
“Simply Inflation”



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Supervisors: Luigi Pilo and Sabino Matarrese

17th October 2019

Outline

- The role of the Weinberg Theorem (WT) in Cosmology,
- Single Field Inflation: WT validity

  - "ΔN formalism and conserved currents in Cosmology "

    (Matarrese, Pilo, Rollo)

- EFT Inflation: WT violation

  - "Adiabatic media Inflation" (Celoria, Comelli, Pilo, Rollo)

The Weinberg Theorem

- Hypothesis: - Whatever the Constituents of the Universe;

  - No entropy perturbations $\delta\sigma=0$;

  - Superhorizon scales $k/(a\,H) \longrightarrow 0$;

- THESIS: - ζ and *R* are equivalent at superhorizon scales, both characterized by a constant mode and a quickly decreasing one.

$$\zeta=-\psi-H\frac{\delta\rho}{\rho'};\qquad R=-\psi-H\,v.$$

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Single Field Inflation

$$S = M_{pl}^2 \int dx^4 \sqrt{-g}\, R + \int dx^4 \sqrt{-g} [\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + V(\phi)]$$

- Limits: $\epsilon = \frac{-\dot{H}}{H^2} \ll 1,$ $\eta = \frac{-\dot{\epsilon}}{H\epsilon} \ll 1,$

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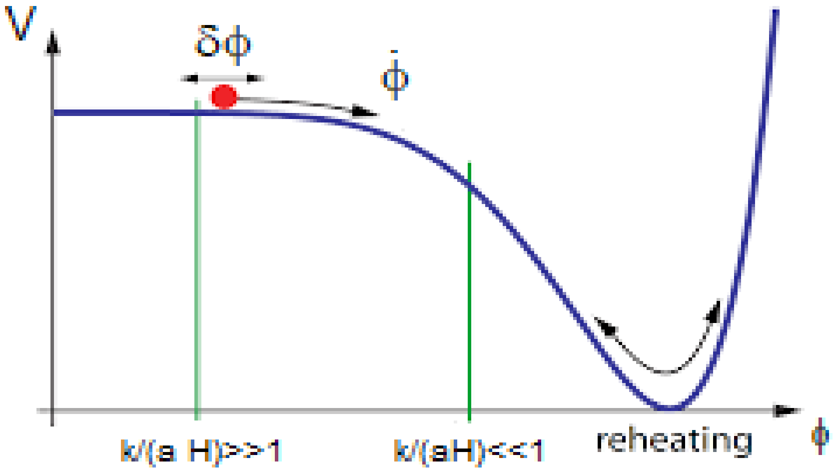

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- Correlation fun.: $<\zeta_k \zeta_{k_1}> = <R_k R_{k_1}> = (2\pi)^3 \delta^{(3)}(k+k_1) \boxed{\mathrm{P}(k)}$

$$<\zeta_k \zeta_{k_1} \zeta_{k_2}> = (2\pi)^3 \delta^{(3)}(k+k_1+k_2) \boxed{B(k, k_1, k_2)}$$

$$<\zeta_k \zeta_{k_1} \ldots \zeta_{k_n}>$$

Single Field Inflation

- Two points correlation function:

$$\Delta(k) = \frac{k^3}{2\pi^2} \mathrm{P}(k) = \frac{k^3}{2\pi^2} |\zeta(k)|^2 = \frac{3H^2}{8\pi^2 M_{pl}^2 \epsilon} k^{n_s - 1};$$

$$\frac{3H^2}{8\pi^2 M_{pl}^2 \epsilon} \rightarrow 2.5 x 10^{-9}$$

$$n_s = 1 - 2\epsilon - 2\eta = 0.96 \pm 0.014 .$$

Single Field Inflation

- Three points function: “Non-Gaussian features of primordial fluctuations in single field inflationary models” (Maldacena (2003))
- The squezeed limit: $k \ll k_1, k_2;\qquad \zeta = \zeta^{(1)} + \frac{3}{5} f_{NL} (\zeta^{(1)2} - <\zeta^{(1)2}>)$

- Maldacena consistency relation:

$$B(k, k_1, k_2) = \frac{6}{5} f_{NL} \mathrm{P}(k_1) \mathrm{P}(k_2);\qquad \boxed{f_{NL} = \frac{-5}{12}(n_s - 1);}$$

$$\underline{f_{NL} = 0.8 \pm 5}$$

Single Field Inflation

- Problem: $f_{NL} = \frac{-5}{12}(n_s - 1)$???

  - "The Observed Squeezed Limit of Cosmological Three-Point Functions" (Pajer, Schmidt and Zaldarriaga (2013)): $f_{NL} = o(\epsilon^2, \eta^2, \epsilon\eta)$

- Our approach:

  - They do not use ζ but φ

  → "ΔN formalism and conserved currents in Cosmology " (Matarrese, Pilo, Rollo)

Single Field Inflation: Next step

- Work in progress: Necessary conditions to get MCR

-1) WT $\longrightarrow$ Maldacena Con. Relation;

$$<\zeta_L \zeta_{s_1} \zeta_{s_2}> = <\zeta_L <\zeta_{s_1} \zeta_{s_2}>_L>$$

-2) The Maldacena f_{NL} can be erased only by using “non-covariantly” defined quantities.

EFT: “Adiabatic Media Inflation”
(M.Celoria, D. Comelli, L. Pilo, R. Rollo)

- 4-Stuckelberg Field: $\Psi^A , \qquad A=1,2,3,4\,.$

$$\Psi^0=\psi^0(t)+\pi^0; \quad \Psi^i=x^i+\partial^i\pi_L+\pi_T^i\,.$$

- Action: $S=M_{pl}^2\int dx^4\sqrt{-g}\,R+\int dx^4\sqrt{-g}\,U(Y,\tau_1,\tau_2,\tau_3)$

$$Y=u^\mu\partial_\mu\Psi^0;$$
$$B^{ij}=g^{\mu\nu}\partial_\mu\Psi^i\partial_\nu\Psi^j;$$
$$\tau_n=Tr(B^n).$$

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- Solids and Fluids: $U(Y, \tau_1, \tau_2, \tau_3) \propto (\pi^{i\,\prime})^2+c_L^2(\partial_i \pi_L^i)^2+c_T^2(\partial_i \pi_T^i)^2$

Adiabatic M. Inflation: Parameters

- Barotropic parameter: $w = \frac{p}{\rho} = -1 + \frac{2}{3}\epsilon$
- Adiabatic sound speed: $c_s^2 = \frac{dp}{d\rho} = -1 + \frac{2}{3}\epsilon - \frac{\eta}{3}$
- Longitudinal sound speed: $c_L^2 = c_s^2 + \frac{4}{3}c_T^2$
- Entropy speed: c_b^2 such that $\Gamma = \delta p - c_s^2 \delta\rho = \frac{\psi^{0\,\prime}}{a^4}\left(c_b^2 - c_s^2\right)\delta\sigma$

Adiabatic M. Inflation: Regimes

$$\eta = const. \rightarrow \quad \epsilon = \frac{-\dot{H}}{H^2} = \epsilon_i(-H_i t)^{-\eta},$$

- Slow roll (SR): Solids with small ϵ and η,
- Super Slow roll (SSR): Fluids and Solids with η big and negative, negligible ϵ.
- W-Media (WM): Solids with η=0, ϵ=const. $\in (0,1)$,

$$w = -1 + \frac{2}{3}\epsilon \quad \in (-1, -1/3)$$

R and ζ solutions

- Curvature solution: $X = X^{(0)} + X^{(\delta\sigma)}$

- δσ=0 solutions:

  -We have Hankel-like solutions but: $\zeta^{(0)} - R^{(0)} \to 0$

  -In the limit -k/(a H) ⟶ ∞ : $\zeta^{(0)}, R^{(0)} \propto e^{i c_L k t}$ $\boxed{0 < c_L < 1}$

- Particular solution: $\zeta^{(\delta\sigma)} = R^{(\delta\sigma)} \propto \frac{\delta\sigma_k}{\epsilon_i}(-t)^{2+\eta+\frac{(1+3c_b^2)}{1+\epsilon_i}}$ $\boxed{\delta\sigma_k \sim \sqrt{\epsilon_i}}$

Power Spectrum and WT violation: "δσ=0 case"

- Features: -Presence of growing modes: → $\Delta(t)$

  -ζ and *R* are not equivalent: → $\Delta_R \neq \Delta_\zeta$

- *R* Power Spectrum:

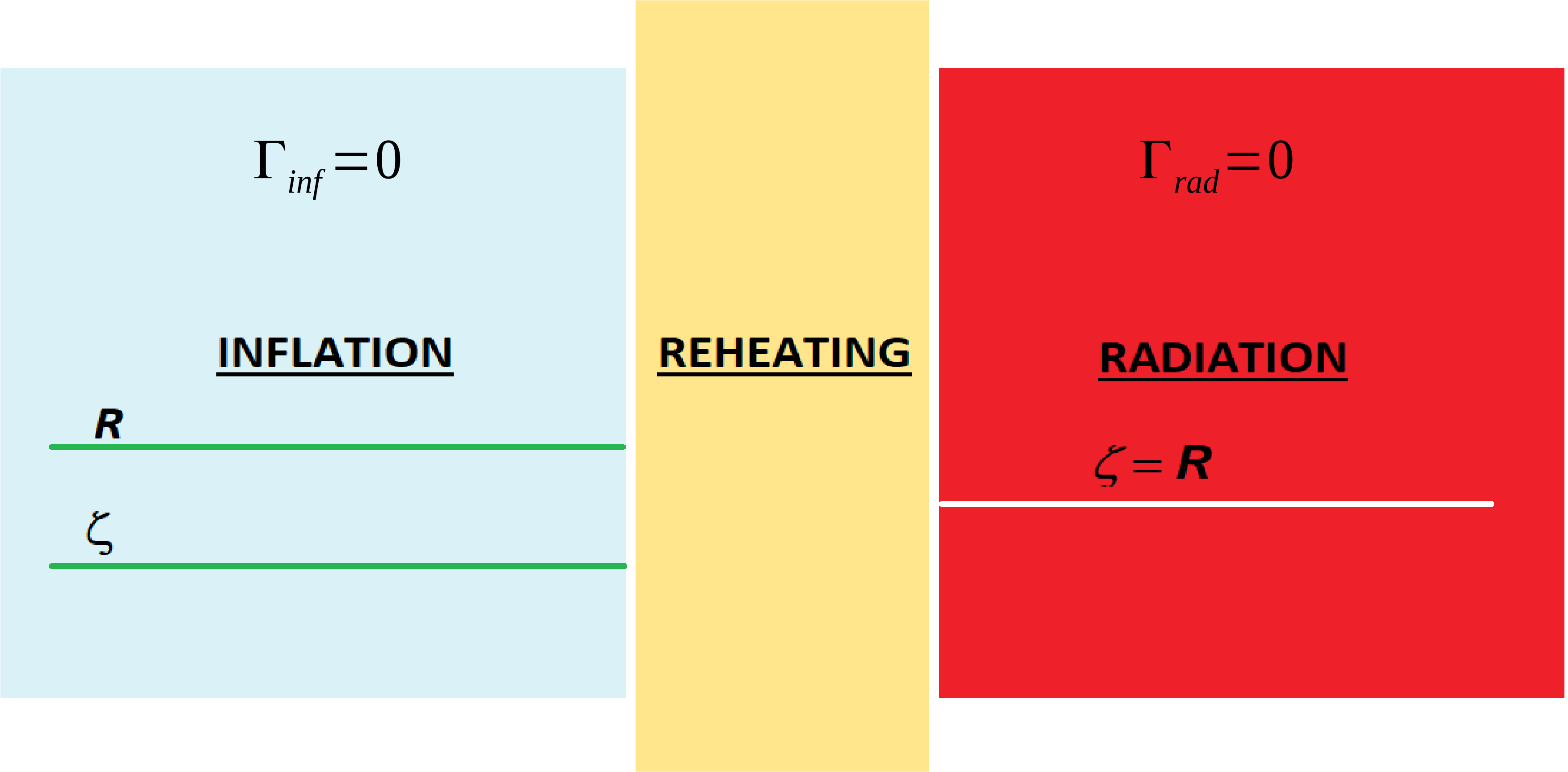
$$\Delta_R^{(0)} = \frac{k^3}{2\pi^2}|R(k)|^2 = Amp.(-t)^\alpha k^{n_s-1}, \quad Amp. \propto \frac{H_i^2}{M_{pl}^2 \epsilon_i}, \qquad n_s = 4-2\nu.$$

- ζ Power Spectrum:

  -SR and WM $\Delta_\zeta^{(0)} = \Delta_R^{(0)} X$

  -SSR $\Delta_\zeta^{(0)} = \Delta_R^{(0)} \frac{1}{36} \frac{(-kt)^4}{\beta^2(\nu-1)^2}$

From Inflation to RD



Instantaneus Reheating

- From the time-time Einstein Equation: 1.) $[\zeta]=0,$
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- Results: -SSR: $R_a=\zeta$, adiabatic RD;

  -SR: $R_a=\zeta+O(\epsilon_i)$, quasi-adiabatic RD;

  -WM: $R_a-\zeta=O(1)$ $\text{if } \epsilon\rightarrow 1\ldots$ Non adiabatic RD!

Examples

- SSR: “Fluid Inflation” $\Delta_R^{(0)} \propto k^{6+\eta} \Rightarrow \eta = -6, \quad c_s^2 = 1;$

$$\Delta_\zeta^{(0)} \propto k^{10+\eta} \Rightarrow \eta = -10, \quad c_s^2 = \frac{7}{3}.$$

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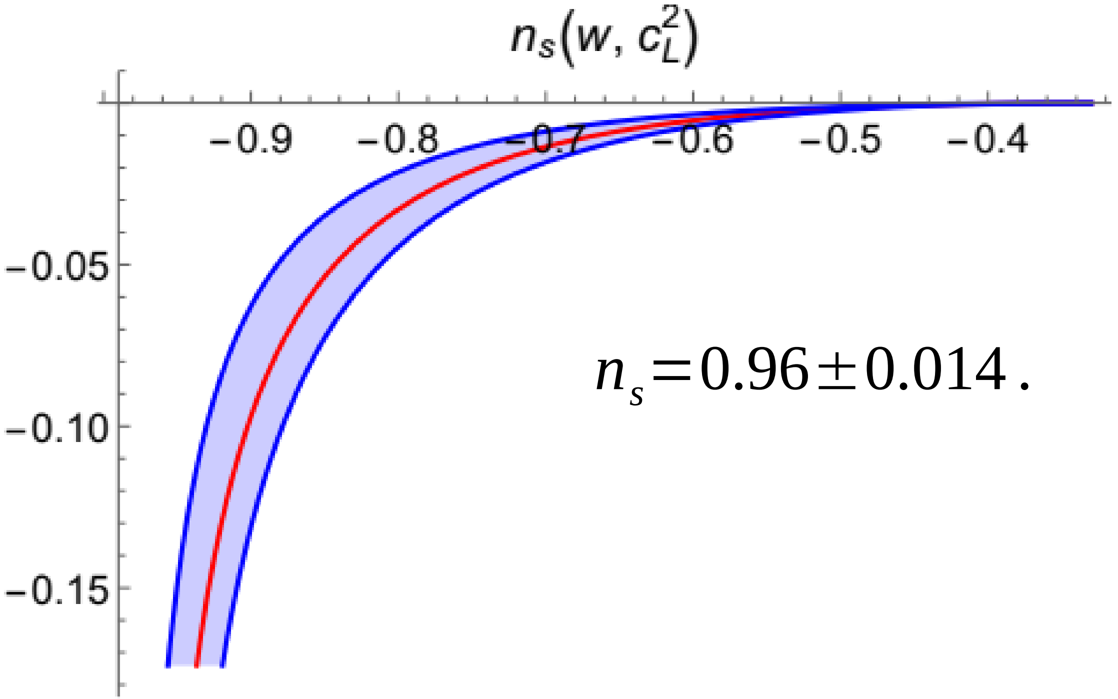
- SR: “Solid Inflation” $\nu = \frac{3}{2} + \epsilon_i + \frac{\eta}{2} - \frac{4}{3} c_T^2\, \epsilon_i\,; \quad \Delta_\zeta^{(0)} = \frac{1}{c_L^4} \Delta_{R^{(0)}} \sim \frac{k^{3-2\nu}}{\epsilon_i c_L^4}\,.$

Examples

- WM: $$\nu=\frac{3}{2}\sqrt{1-8c_L^2\frac{1+w}{(1+3w)^2}};\quad \Delta_{\xi}^{(0)}=\frac{(1+3w)^2}{4c_L^4}\Delta_{R^{(0)}}\sim\frac{(1+3w)^2}{4\epsilon_i c_L^4}k^{3-2\nu}.$$



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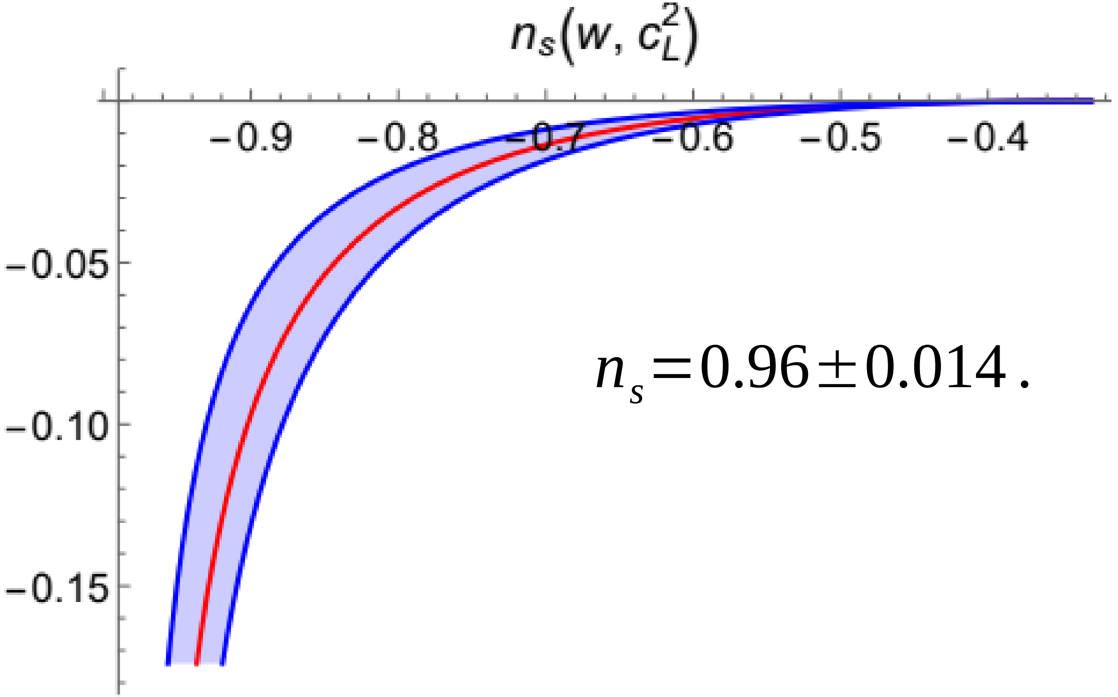

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Presence of Entropy

- Using the particular solutions: $\Delta_\zeta = \Delta_\zeta^{(0)} + \Delta^{(\delta\sigma)}$

$$\Delta^{(\delta\sigma)} \propto \frac{\delta\sigma_k^2}{\epsilon_i^2} k^3 (-t)^{4+2\eta+2\frac{(1+3c_b^2)}{1+\epsilon_i}}$$

- Regimes:

	$\Delta^{(\delta\sigma)} \ll \Delta_\zeta^{(0)}$;	$\Delta^{(\delta\sigma)} \sim \Delta_\zeta^{(0)}$;	$\Delta^{(\delta\sigma)} \gg \Delta_\zeta^{(0)}$;
-SSR:	$\frac{2}{3} < c_b^2 < 3$	$c_b^2 = \frac{2}{3}$	$c_b^2 < \frac{2}{3}$
-SR:	$-1 < c_b^2 < -1/3$	$c_b^2 = -1$	$c_b^2 < -1$
-WM:			$c_b^2 = w$

Works in progress

- Next papers: 1) Observability of the Maldacena Consistency relation. (Matarrese, Pilo, Rollo)
  2) Non Adiabatic Media Inflation: 2 DoF
  - Third order action: Non Gaussianities computation. (Celoria, Comelli, Pilo, Rollo)

- Interesting features: 1) -WT validity → MCR validity..
  2) -WT violation → MCR violation..
  -Good entropic contributions.