

Gravitational-wave Tomography of the Moon: Constraining Lunar Structure with Calibrated Gravitational Waves

H. Yan & J. Harms

Phys. Rev. Lett. 137, 021409 (2026)

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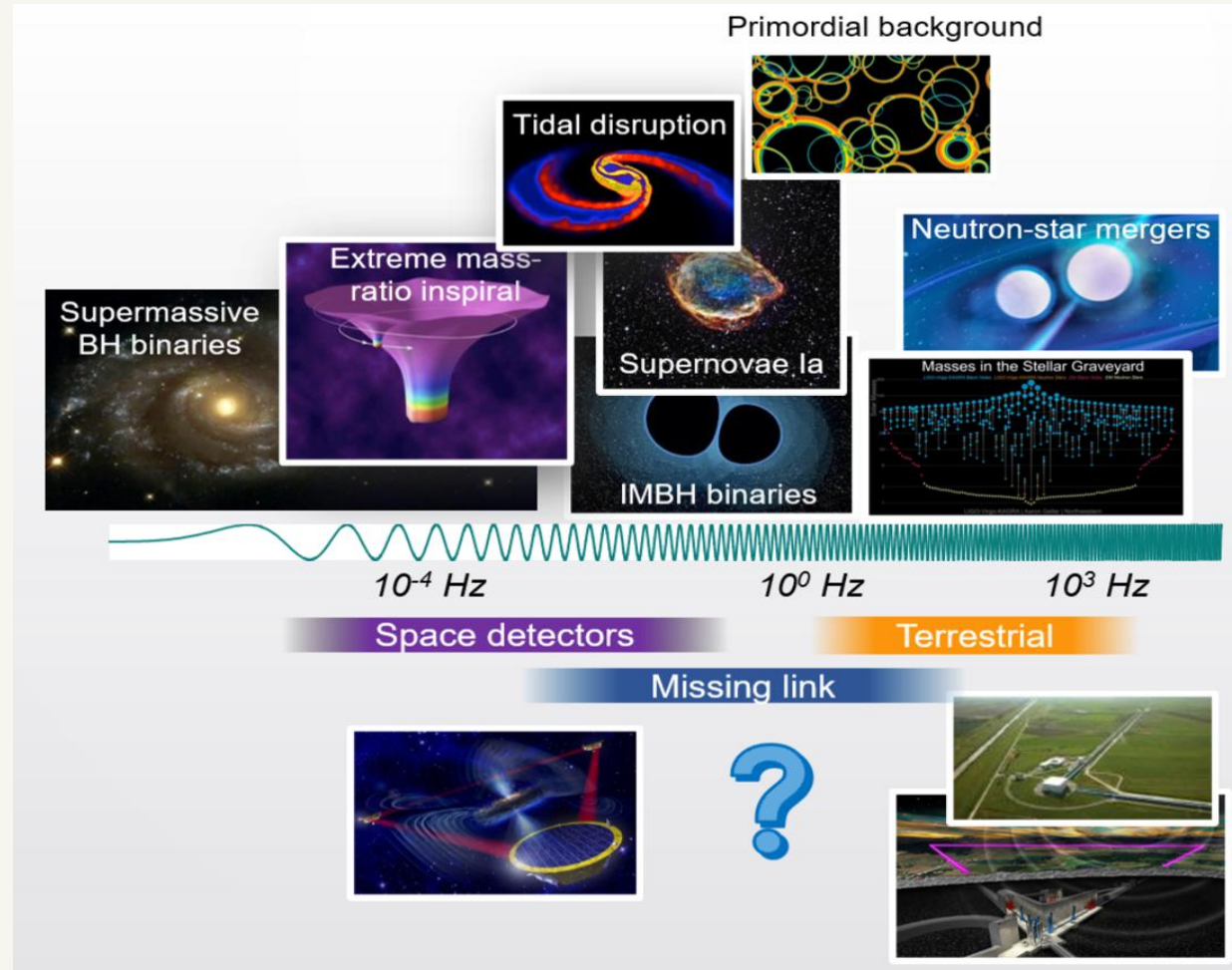
² Gran Sasso Science Institute

Supervisors: Xian CHEN¹, Jan HARMS²



If the incident gravitational wave is **known,
what can the Moon's vibration reveal?**

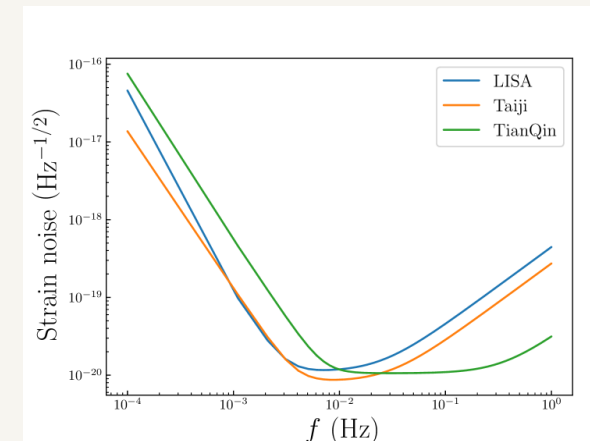
Multiband networks connect mHz to kHz GW detectors



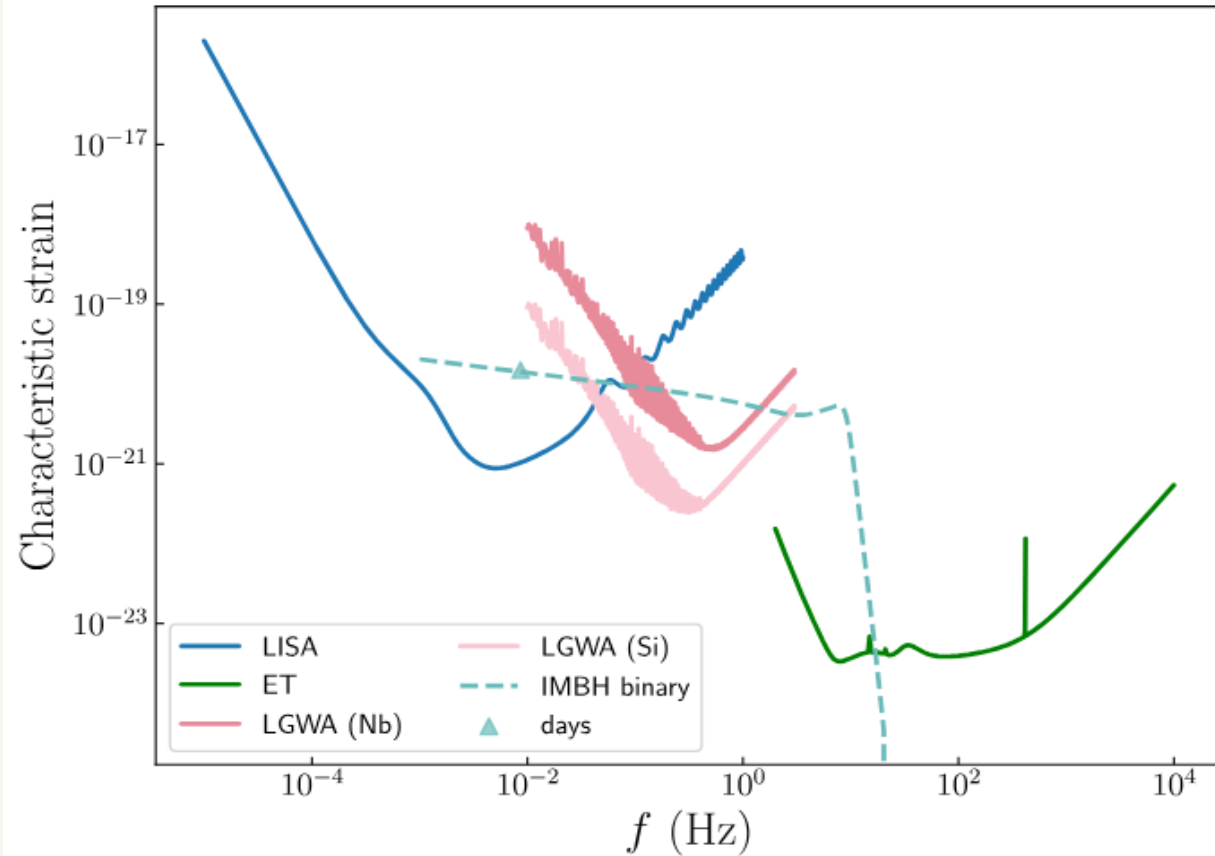
Decihertz is the missing link between space and ground detectors.

Three windows, one network

- mHz — LISA, TianQin, Taiji
early inspiral; years of warning
and precise sky position
- dHz — LGWA, LILA, DECIGO, ...
bridges the important gap
- Hz–kHz — ET, CE
late inspiral and merger



Space-borne
sensitivity curves;
Jin et al. 2026,
Fig. 1



1000+1000 $M_{\odot}$ IMBHB at $z = 1$, sweeps upward through the LISA, LGWA and ET bands; triangle marks ~ 10 days before merger.

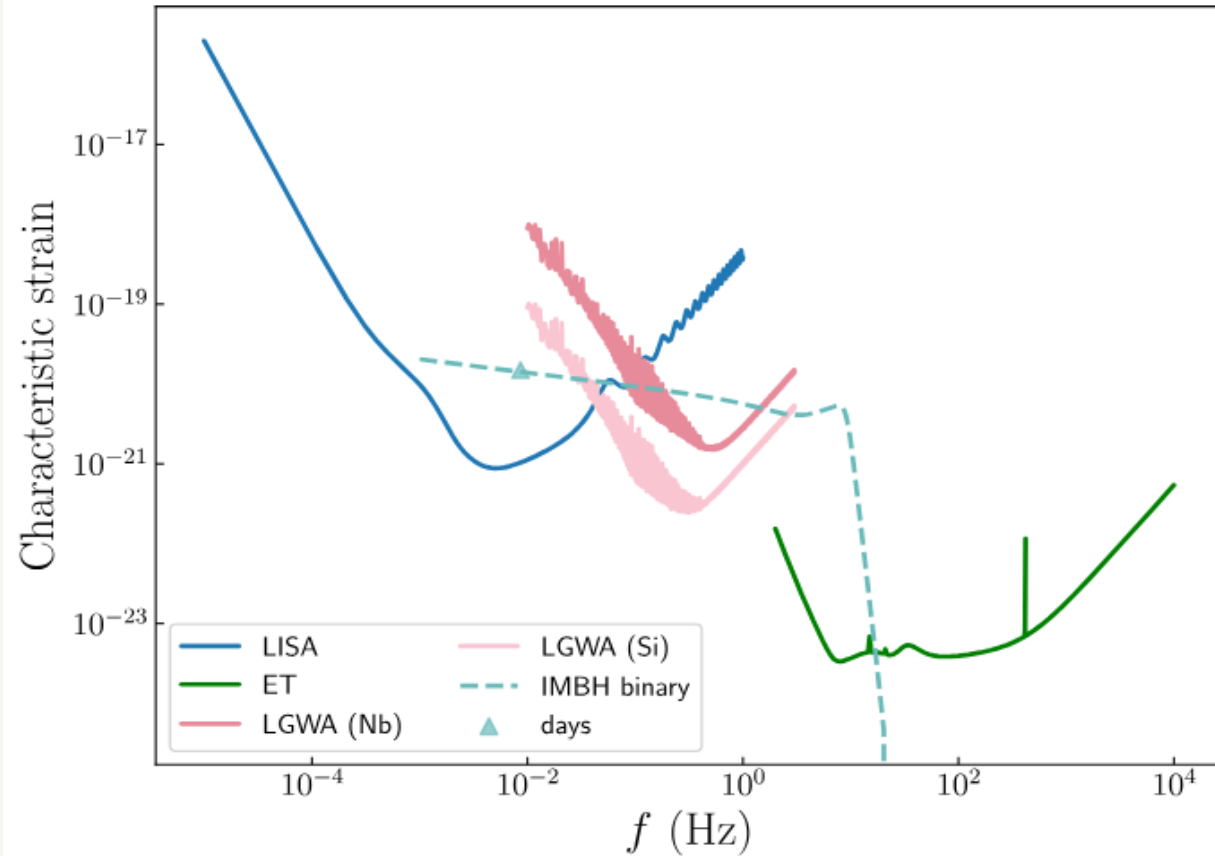
A network view of one source

- 1 *months before merger*
Space detectors (LISA / TianQin / Taiji) measure the early inspiral.
- 2 *days before merger*
LGWA tracks the decihertz evolution.
- 3 *final hours–minutes*
ET / CE measure the late inspiral and merger.

Joint estimation constrains source parameters θ : distance, sky position, polarization...

Source- and scenario-dependent; this work assumes a well-calibrated h .

Source: [Dong et al. 2025, Fig. 1, arXiv:2507.10165](#)



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Joint estimation constrains source parameters θ : distance, sky position, polarization...

Source- and scenario-dependent; this work assumes a well-calibrated h .

For suitable sources, $h(f, \theta)$ can be tightly constrained

Source: [Dong et al. 2025, Fig. 1, arXiv:2507.10165](#)



Weber's resonant bar

Sources: [APS Physics](#)



Weber's resonant bar

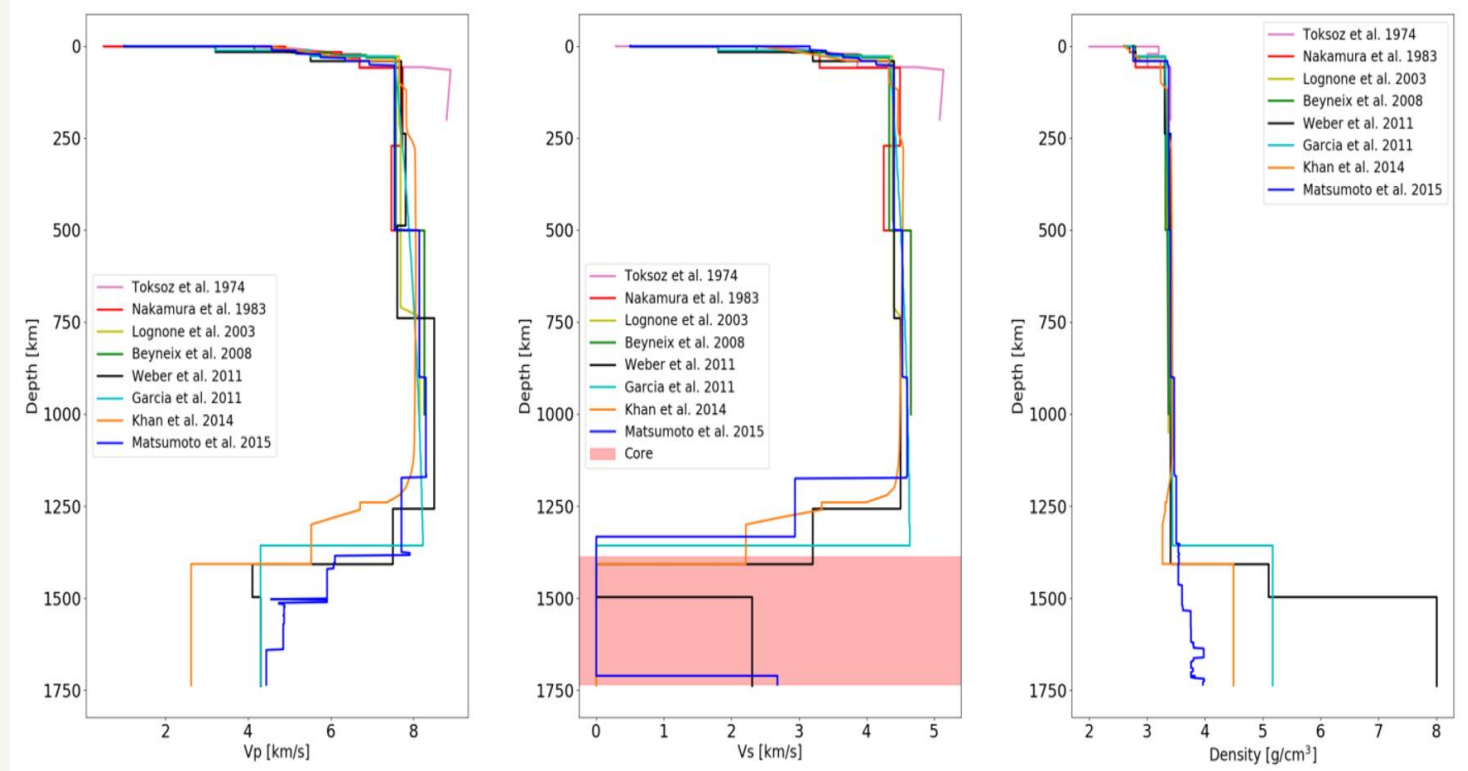
Difference: the Moon is a
NATURAL resonator

Sources: [APS Physics](#); photo by H. Yan



Weber's resonant bar

**Difference: the Moon is a
NATURAL resonator**



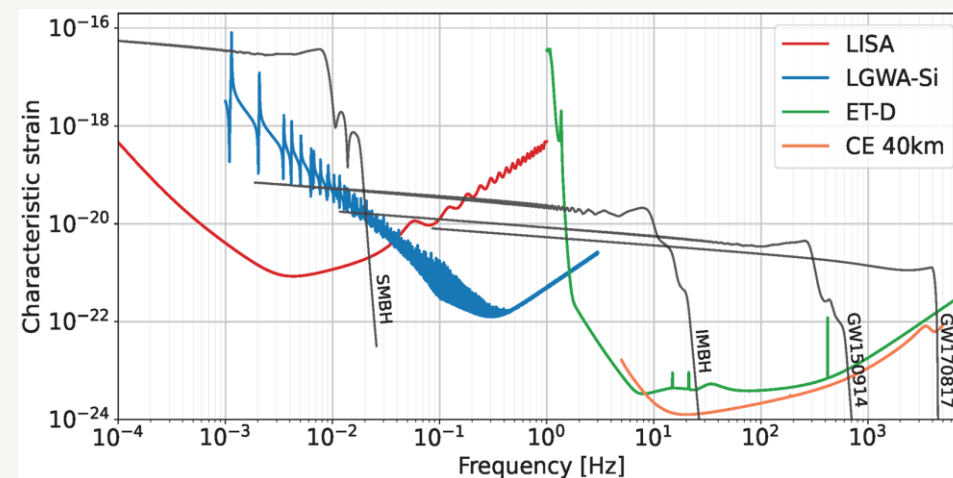
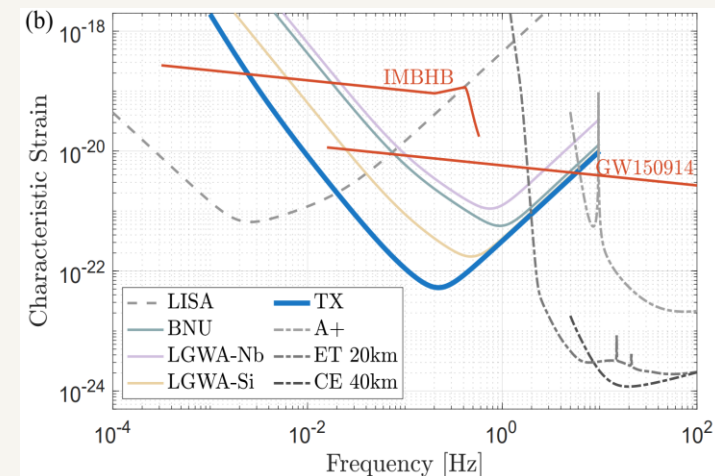
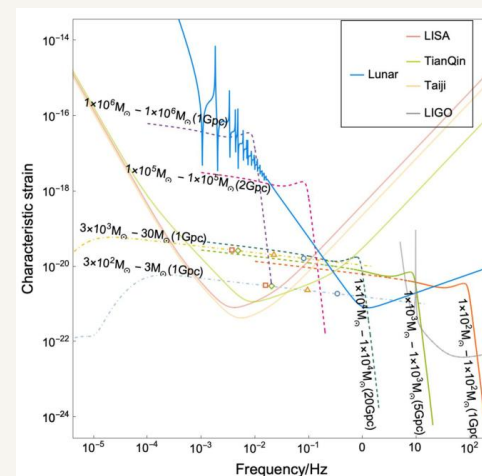
Eight published seismic models: largely consistent to ~1200 km depth, divergent in the deep mantle and core

LUNAR RESPONSE



Weber's resonant bar

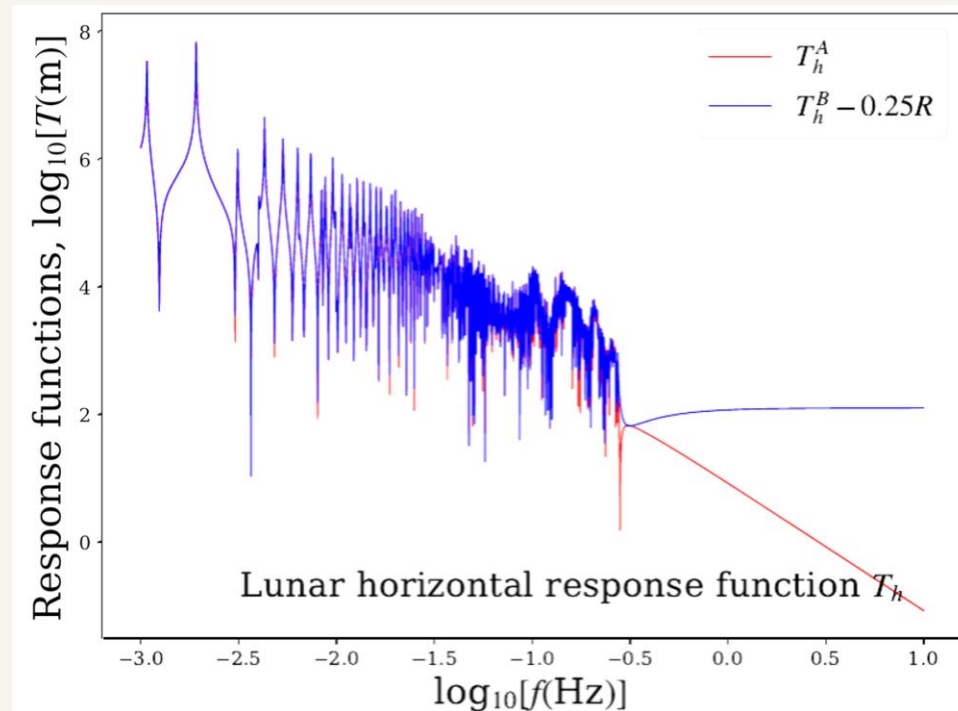
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Sensitivity curves for proposed
lunar GW instruments

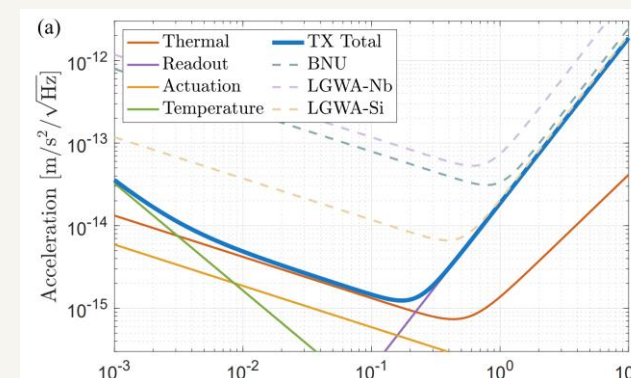
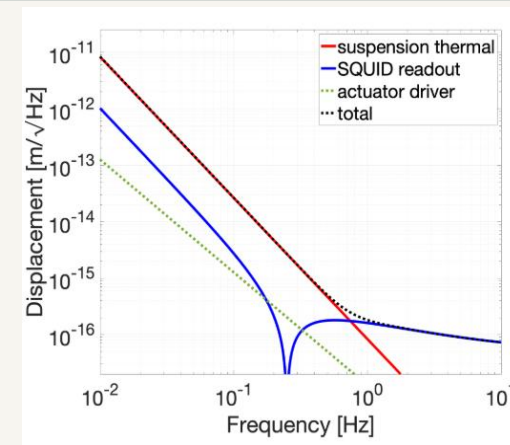
Sources: [APS Physics](#); photo by H. Yan; Li et al. 2023; Xia et al. 2026; Ajith et al. 2025

Sensitivity curves based on **assumed** lunar structure



Lunar transfer function T (response per strain)
based on normal-mode calculation, contributed by many
eigen-modes

Yan et al. 2024, Fig. 3(b)



noise
lunar response = strain sensitivity

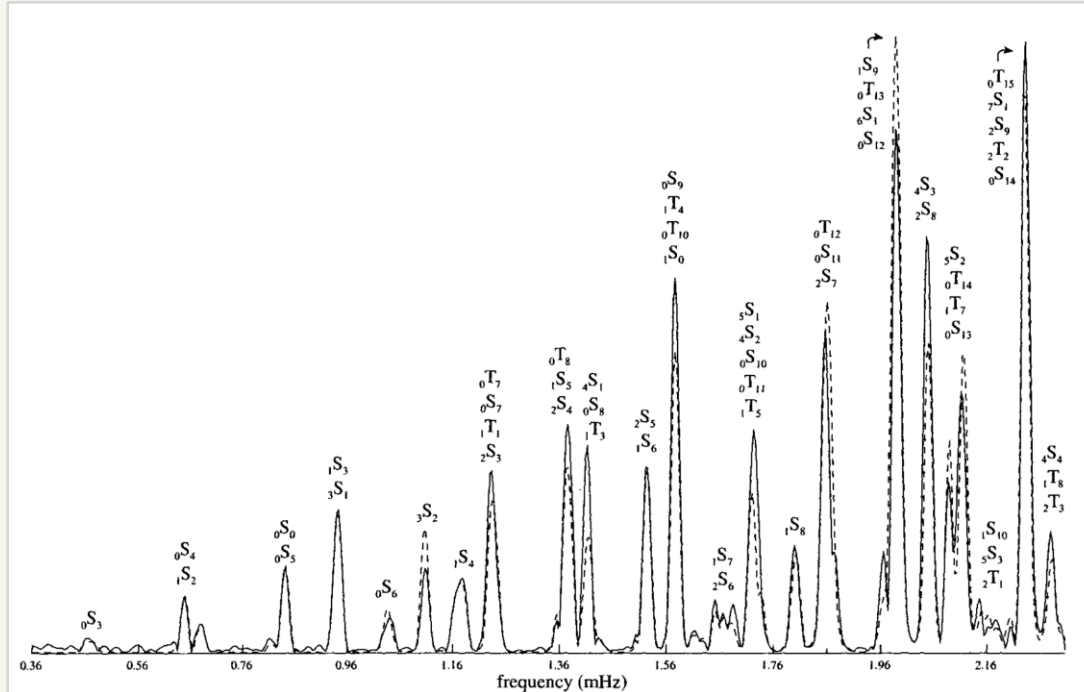
$$A/T = h$$

Sources: [Yan et al. 2024](#); Ajith et al. 2025; Xia et al. 2026

Our targets

- Improve our understanding of the **Moon's structure** and **sensitivity curve**
- Make use of **multiband GW detection**
- Open **new science cases for GW**

Planetary seismology: two complementary regimes



Earth · low-frequency eigenmodes

Moon · high-frequency propagating waves

Normal modes constrain global structure; travel times image heterogeneity.

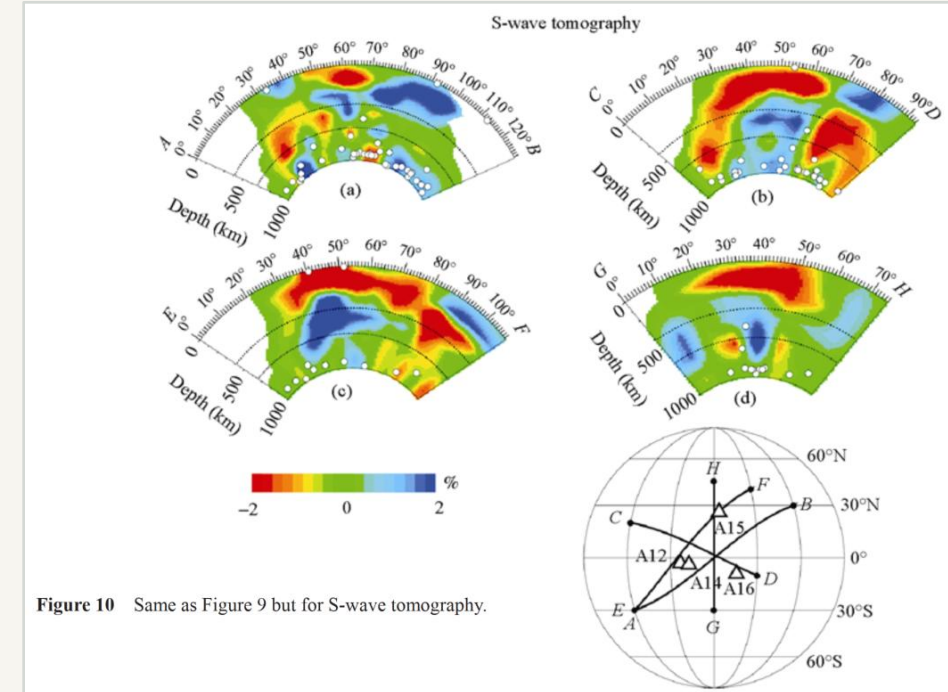
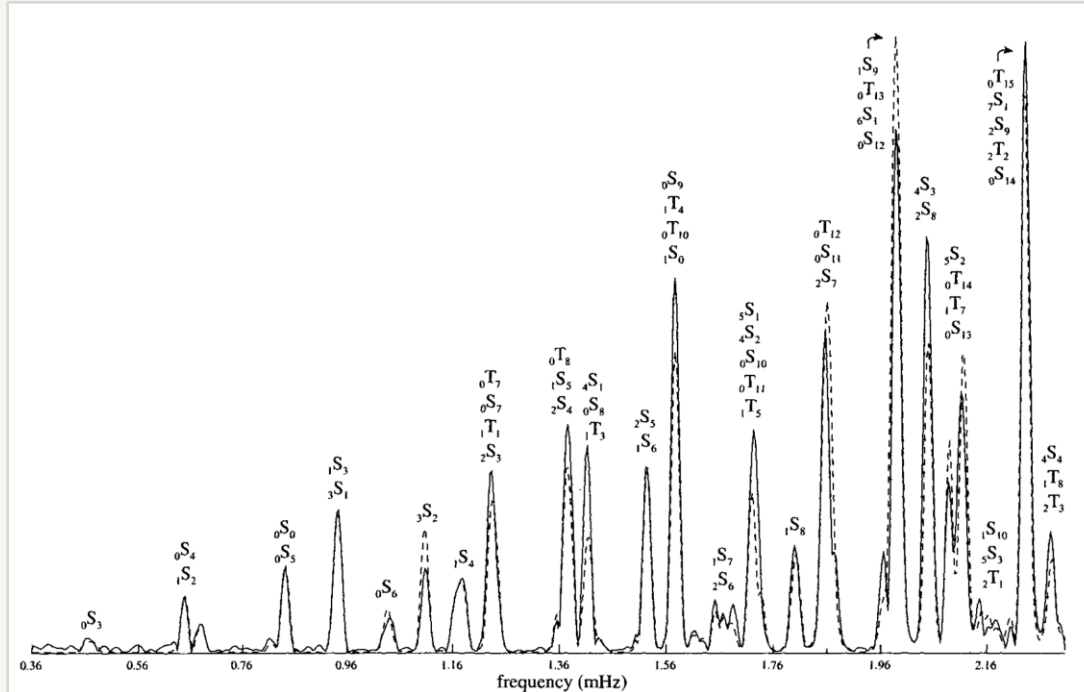


Figure 10 Same as Figure 9 but for S-wave tomography.

Planetary seismology: two complementary regimes



Earth · low-frequency eigenmodes

Moon · high-frequency propagating waves

Normal modes constrain global structure; travel times image heterogeneity.

In both cases, source amplitude uncertainty can limit structure information.

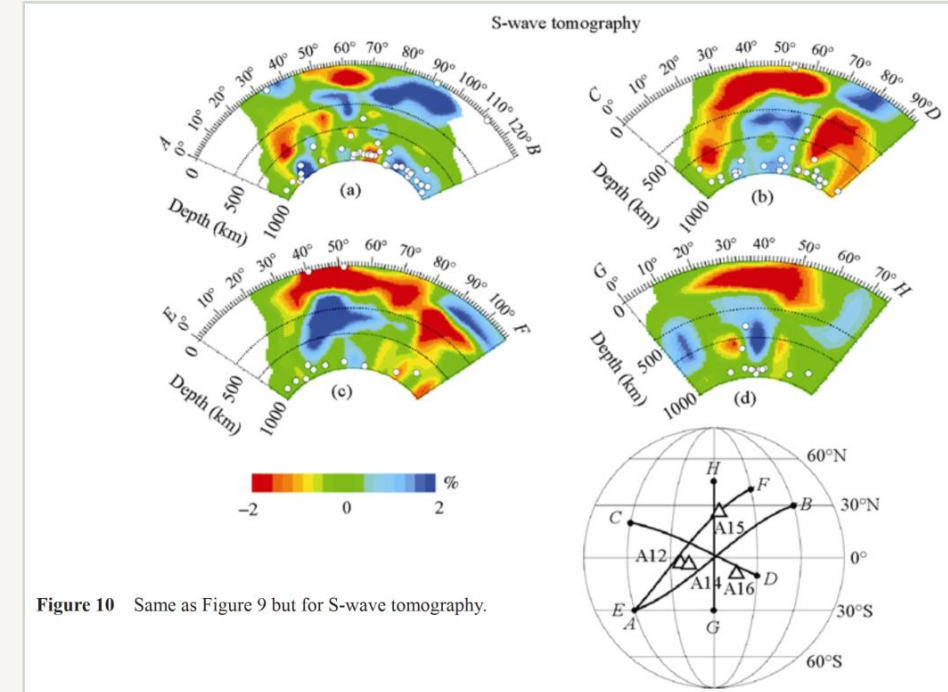
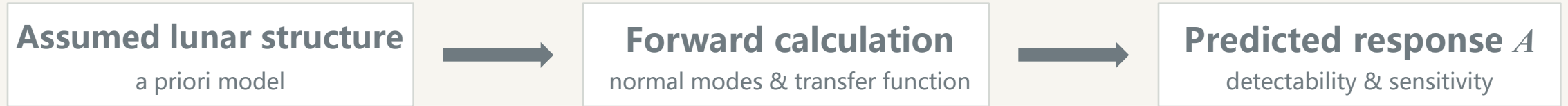


Figure 10 Same as Figure 9 but for S-wave tomography.

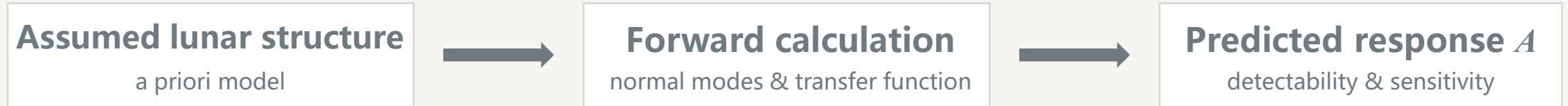
This work: response problem $\Rightarrow$ inverse structure problem

FORWARD — previous response studies



This work: response problem $\Rightarrow$ inverse structure problem

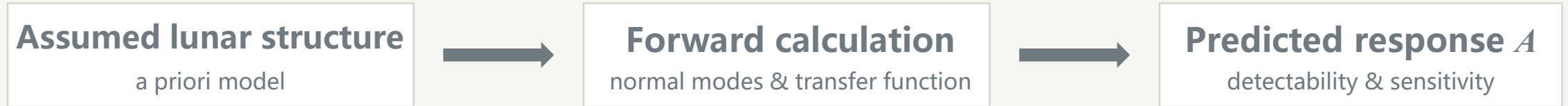
FORWARD — previous response studies



A calibrated h_0 reverses the information flow: $A / h_0 = T(\rho, \kappa, \mu, \dots)$

This work: response problem $\Rightarrow$ inverse structure problem

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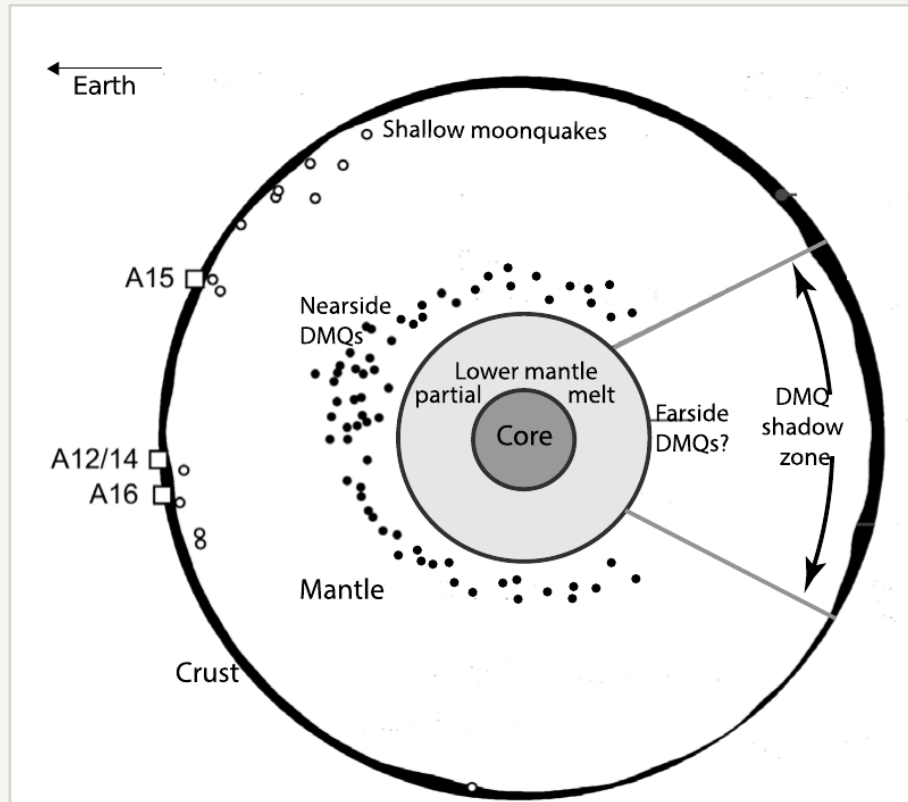
A calibrated h_0 reverses the information flow: $A / h_0 = T(\rho, \kappa, \mu, \dots)$

INVERSE — this work



Known input makes modal amplitude a structural observable.

Inverse target: a spherically symmetric 1D model



Geophysical view of the lunar interior: Apollo stations, moonquake zones, mantle, partial melt and core

Model parameters

$\rho(r)$ density

$\kappa(r)$ bulk modulus

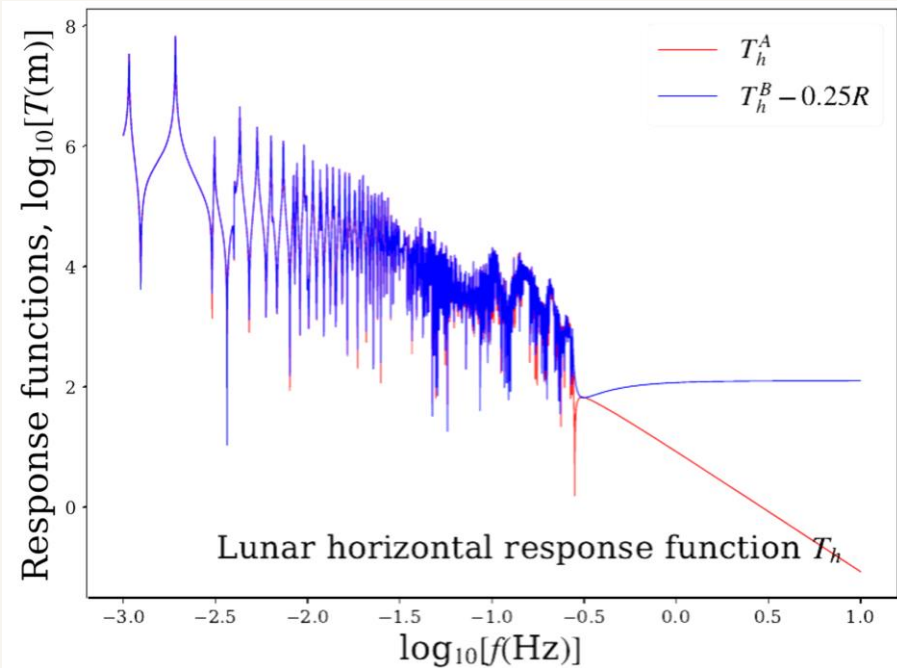
$\mu(r)$ shear modulus

d_i interface radii

Baseline: inner core, outer core, mantle, crust and surface layer

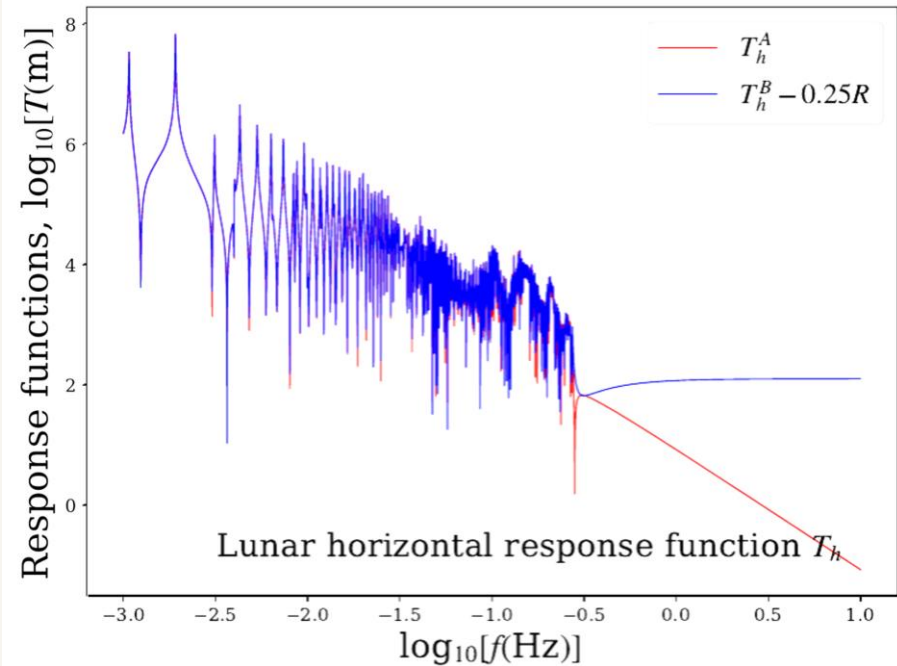
“Tomography” here means 1D radial parameter inference (not yet 3D imaging).

Normal modes provide three types of observables



Yan et al. 2024

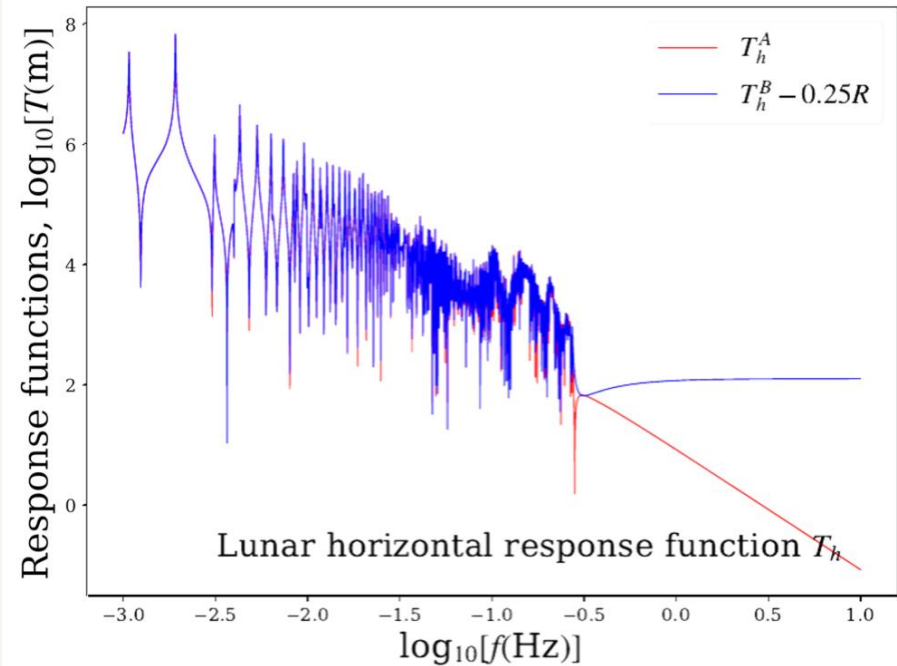
Normal modes provide three types of observables



Yan et al. 2024

$$\sum_n \frac{1}{\omega^2 - \omega_n^2 + \text{Dissipation}(Q_n)} \times A_n[h(\omega), U_n, V_n, I_n \dots]$$

Normal modes provide three types of observables



Yan et al. 2024

$$\sum_n \frac{1}{\omega^2 - \omega_n^2 + \text{Dissipation}(Q_n)} \times A_n[h(\omega), U_n, V_n, I_n \dots]$$

$$O_{1,n} = \omega_n^2$$

eigenfrequency

assume Q_n to be
independently measured

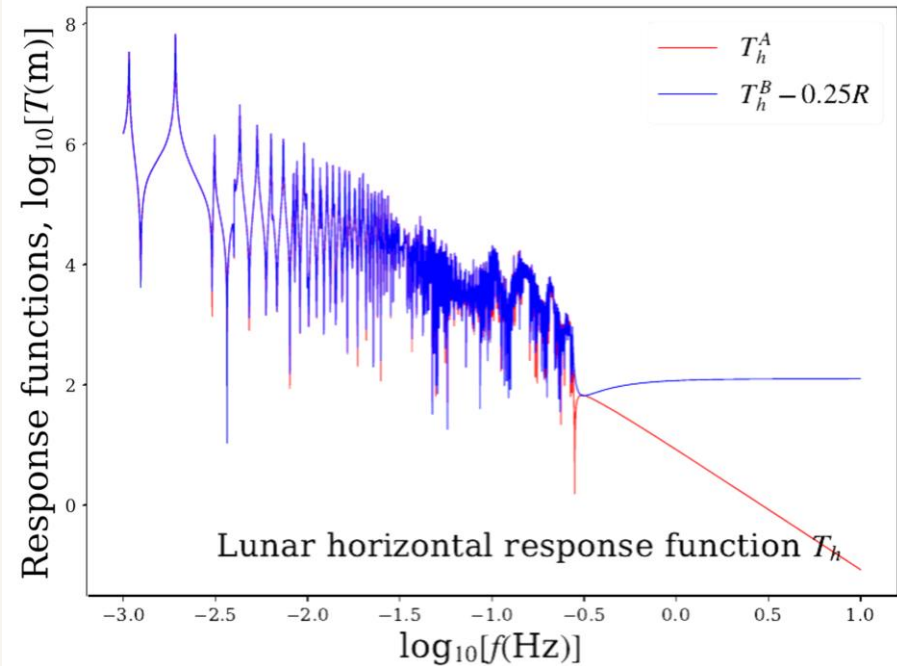
$$O_{2,n} = h_0 U_n(R) I_n \equiv A_n^r$$

radial surface amplitude

$$O_{3,n} = h_0 V_n(R) I_n \equiv A_n^h$$

horizontal surface amplitude

Normal modes provide three types of observables



Yan et al. 2024

$$\sum_n \frac{1}{\omega^2 - \omega_n^2 + \text{Dissipation}(Q_n)} \times A_n[h(\omega), U_n, V_n, I_n \dots]$$

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horizontal surface amplitude

Frequencies are accessible through geophysical measurements;
calibrated amplitudes require the multiband GW network.

Calibrated amplitudes are the new information channel

If h_0 is network-calibrated

$$A_n/h_0$$



$$U_n(R), V_n(R), I_n$$

**Amplitude constrains
eigenfunction shape and interior
coupling.**

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Linear perturbation regime

$$\delta A_n, \delta h_0$$



$$\delta U_n(R), \delta V_n(R), \delta I_n$$



$$\delta \rho(r), \delta \kappa(r), \delta \mu(r)$$

Need error propagation (Fisher information) +
perturbation formalism (Woodhouse kernel)

Fisher setup: an idealized but transparent high-SNR benchmark

$$\mathcal{F}_{ij} = \sum_n \sum_{\alpha=1}^3 \frac{1}{\sigma_{\alpha,n}^2} \frac{\partial O_{\alpha,n}}{\partial p_i} \frac{\partial O_{\alpha,n}}{\partial p_j} + (\mathbf{C}_p^{-1})_{ij}$$

$$\sigma_{p_i} = \sqrt{(\mathcal{F}^{-1})_{ii}}$$

TABLE I. Baseline lunar model. IC = inner core, OC = outer core, M = mantle, C = crust, S = surface

	$r(\text{km})$	$\rho(\text{g/cm}^3)$	$v_p(\text{km/s})$	$v_s(\text{km/s})$	Q_p	Q_s
IC	0-352	7.76	4.3	2.3	1000	1000
OC	353-480	3.4	8.2	3.2	675	300
M	481-1709	3.4	7.9	4.5	3400	1500
C	1709.1-1725	2.762	5.5	3.3	6750	6750
S	1725.1-1737.1	2.762	3.2	1.8	6750	6750

Baseline (prior) lunar model

Setup: Yan & Harms 2026

- 5-layer prior model; 18-layer posterior
- Flat priors
- Frequency relative error: $\frac{\sigma_\omega}{\omega} = 10^{-4}$; already achieved on Earth
- Amplitude relative error: $\frac{\sigma_A}{A} = 10^{-2}$, 100 times worse; corresponding to **SNR~100** for a calibrated LGWA source.
- First 150 modes (≤ 1.04 dHz) as observables

Fisher analysis estimates local parameter covariance; it is not a full low-SNR Bayesian inversion.

Linear perturbation and Woodhouse kernels (semi-analytical, verified by MINEOS)

$$\delta\omega_k^2 = \langle \mathbf{s}_{km} | \delta V - \omega_k^2 \delta T | \mathbf{s}_{km} \rangle \quad \langle \mathbf{s}_{k'm'} | \delta \mathbf{s}_{km} \rangle = \frac{\langle \mathbf{s}_{k'm'} | \delta V - \omega_k^2 \delta T | \mathbf{s}_{km} \rangle}{\omega_k^2 - \omega_{k'}^2}.$$

$$\langle \mathbf{s}_{km} | \delta V[\delta p] - \omega_k^2 \delta T[\delta p] | \mathbf{s}_{k'm'} \rangle = \delta_{ll'} \delta_{mm'} \sqrt{\frac{2l+1}{4\pi}} \\ \times \int_0^R r^2 \delta p(r) K_p^{n'n}(r) dr$$

$$p \in \{\rho, \kappa, \mu\} \quad K_p^{n'n} = V_p^{n'n} - \omega_k^2 T_p^{n'n}$$

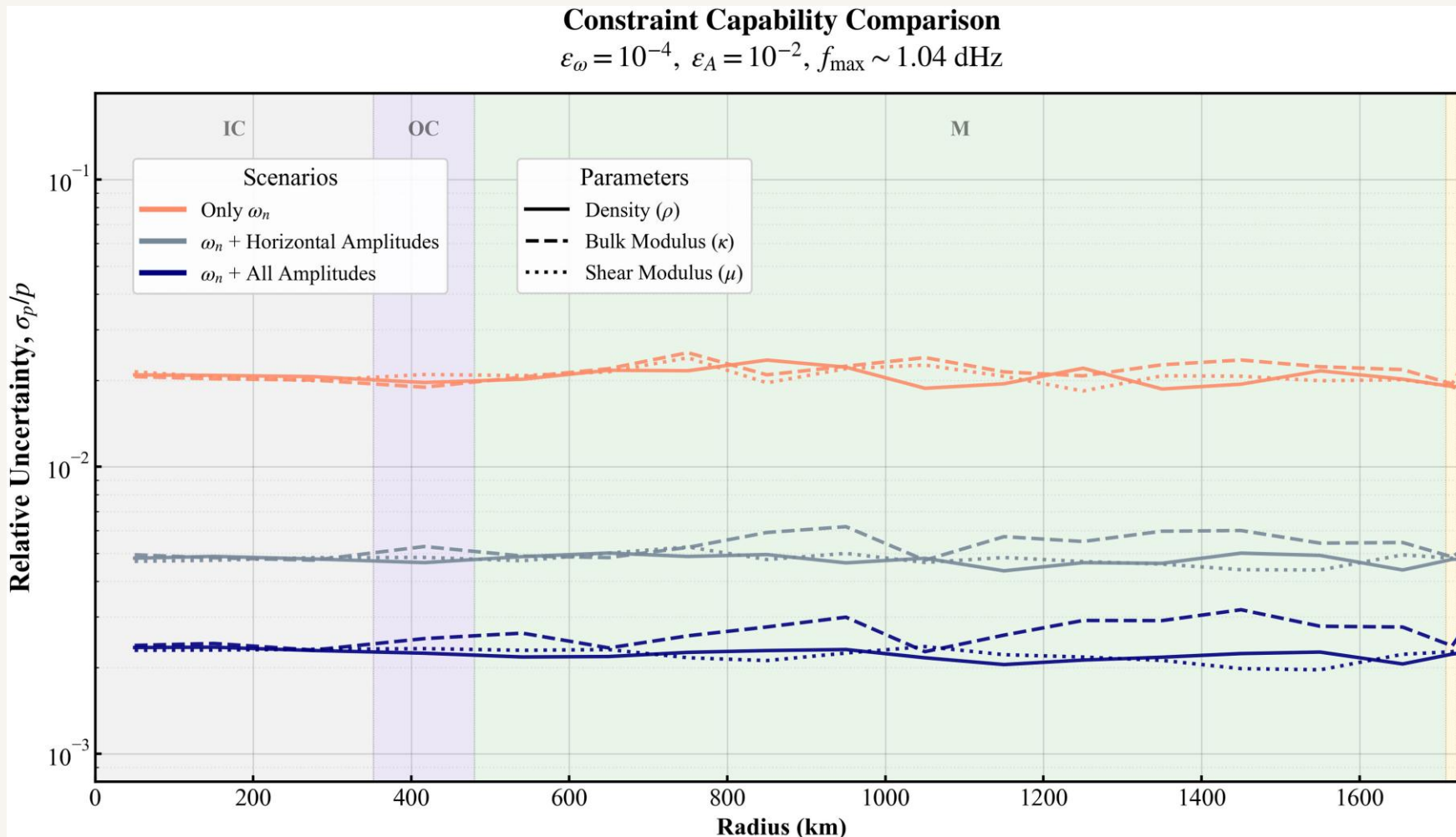
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$$p \in \{\rho, \kappa, \mu\} \quad K_p^{n'n} = V_p^{n'n} - \omega_k^2 T_p^{n'n}$$

Calibrated amplitudes improve structural constraints by $\sim 10\times$



Posterior uncertainty

Frequencies only

$$\sim 2 \times 10^{-2}$$

+ horizontal amplitudes

$$\sim 5 \times 10^{-3}$$

+ all amplitudes

$$\sim 2 \times 10^{-3}$$

Amplitude errors are **100 $\times$** larger, yet amplitudes dominate the information gain.

Source: Yan & Harms 2026, Fig. 2

Near-degeneracy can amplify amplitude sensitivity

First-order perturbation

Frequency shift: diagonal response

$$\delta\omega_k^2 = \langle s_{km} | \delta V - \omega_k^2 \delta T | s_{km} \rangle$$

Eigenfunction mixing: enhanced by small denominators

$$\langle s_{k'm'} | \delta s_{km} \rangle = \frac{\langle s_{k'm'} | \delta V - \omega_k^2 \delta T | s_{km} \rangle}{\omega_k^2 - \omega_{k'}^2}$$

When two modes lie close in frequency, the denominator $\omega_k^2 - \omega_{k'}^2$ becomes small, and mode interference can strongly change amplitudes.

Fisher information

Information is sensitivity per unit measurement error

$$\sigma_p^{-2} \sim \sum_o \frac{1}{\sigma_o^2} \left(\frac{\partial O}{\partial p} \right)^2$$

Larger sensitivity can compensate for larger error.

Fisher information + independent channel

Information is sensitivity per unit measurement error

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Larger sensitivity can compensate for larger error.

Amplitudes contain additional structural information

- Surface eigenfunctions U_n , V_n and the overlap integral I_n change with structure (intrinsically different from ω_n)
- Near-degenerate mode mixing can enhance the effect

Scope and open challenges

Assumed in this proof of concept

- 1D spherically symmetric lunar model
- High-SNR, linear regime
- Well-calibrated incident GW
- Frequency + radial / horizontal amplitude observables
- Perturbation formalism verified against MINEOS checks

Not yet included

- Q -factor measurement and perturbation
- Realistic calibration
- Precise SNR analysis for seismic GW detection
- Nonlinear full-Bayesian inversion beyond Fisher
- 3D GW tomography
- ...

Summary: multiband GW as a calibrated seismic probe

- 01 Calibrated GW.** Multiband GW networks might constrain the incident strain.
- 02 New structure observables.** Mode amplitudes become lunar structural observables, and improve typical parameter uncertainties (**$\sim 10\times$ gain**).
- 03 Calibrate LGWA sensitivity.** Better understanding of lunar structure leads to more realistic sensitivity curve.

What comes next

- Realistic network calibration
- Attenuation Q
- Nonlinear regime
- Lateral heterogeneity
- Joint inversion:
moonquakes, tides,
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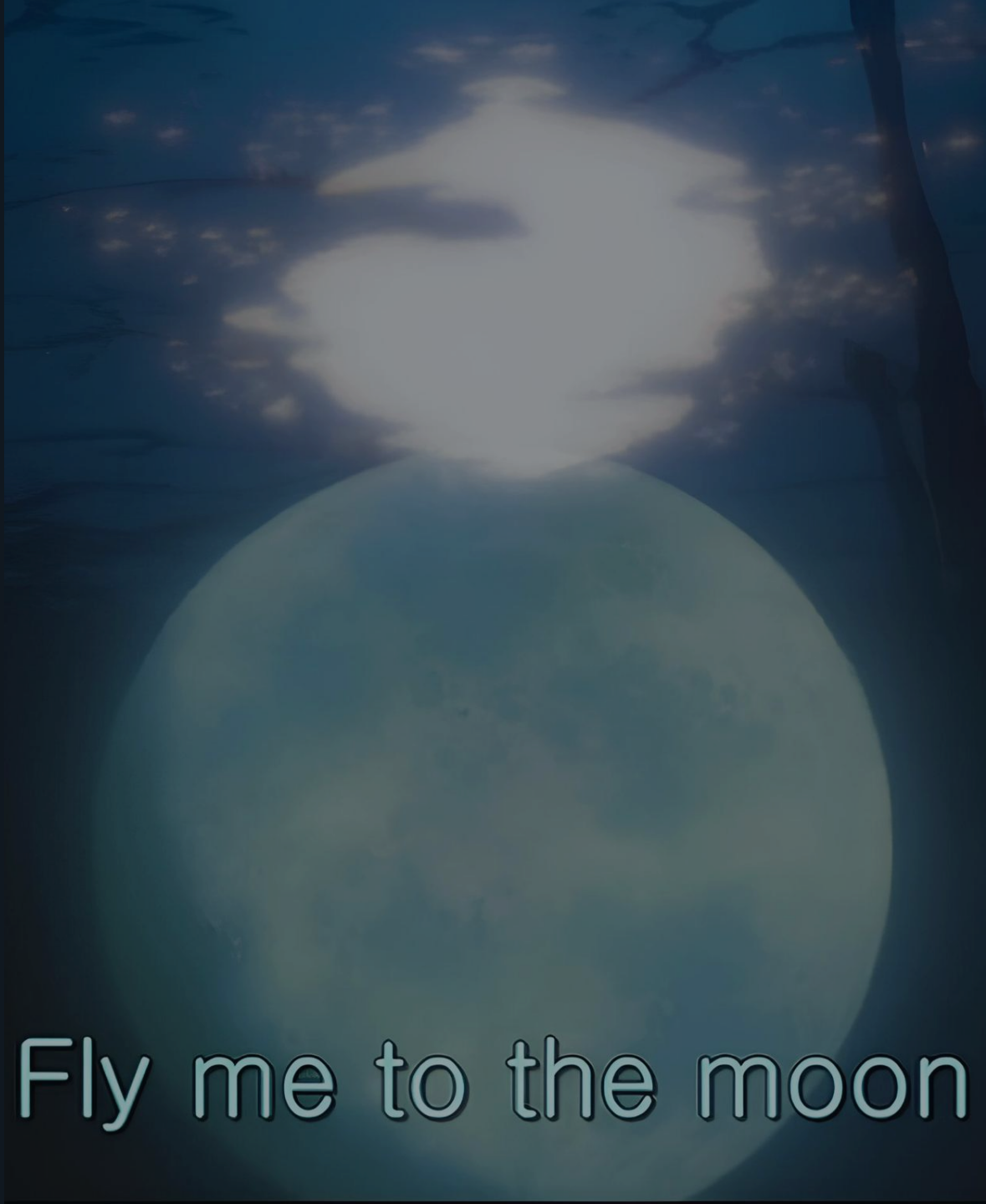
**From using the Moon to detect gravitational waves, to
using gravitational waves to probe the Moon.**

Q&A

Thank you for your attention!
Questions & discussions?

Gravitational-wave Tomography of the Moon
H. Yan & J. Harms, PRL 137, 021409 (2026)

Han YAN, yanhanphy@pku.edu.cn



Fly me to the moon

Appendix A1 · Sensitivity analysis for lunar seismic GW detection

Scaling intuition

$$\sqrt{S_h} \sim \frac{\sqrt{S_n}}{T_{r/h}}$$

Low-frequency bound (~mHz)

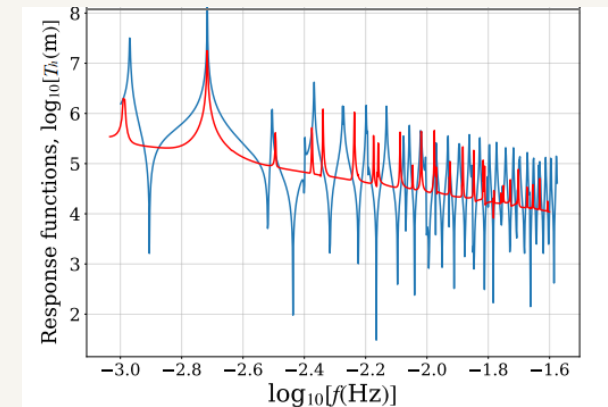
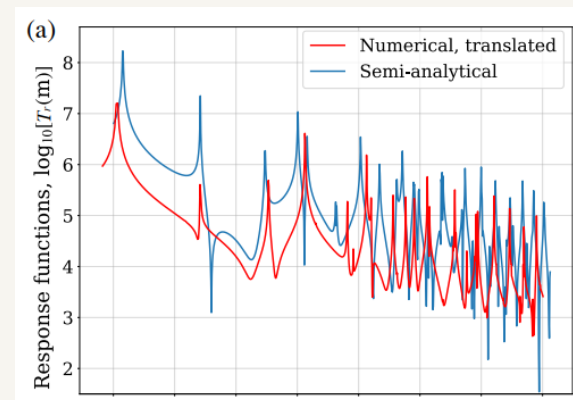
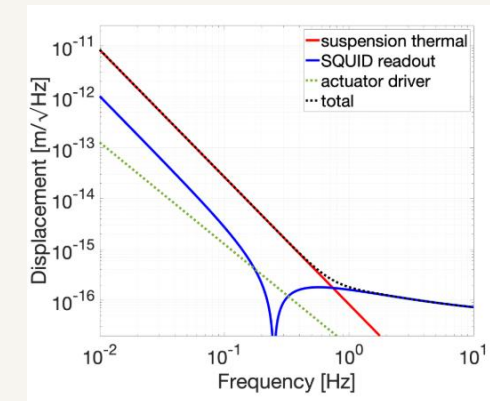
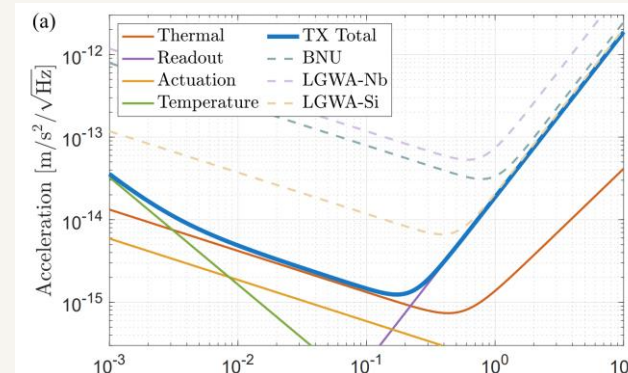
$T_{r/h} \sim$ lunar radius

$\sqrt{S_n} \propto f^{-2}$ (thermal noise)

High-frequency bound (~1 Hz)

$T_{r/h} \propto f^{-2}$

$\sqrt{S_n} \sim \text{const.}$ (readout noise)



Noise spectrum and lunar response per unit strain

Sources: Zhang et al. 2025; Ajith et al. 2025; Xia et al. 2026; original presentation material

Appendix A2 · Response of the spherical Moon

Normal-mode representation of the GW-driven displacement field

$$\begin{aligned}\vec{\xi}_n(\mathbf{x}, \omega_g) &= h_0 \sum_{m=-2}^2 \frac{I_n f^m}{\omega_n^2 - \omega_g^2 + i\omega_n\omega_g/Q_n} \mathbf{s}_{n2m}^S(\mathbf{x}) \\ \vec{\xi}(\mathbf{x}, \omega_g) &= \sum_n \vec{\xi}_n(\mathbf{x}, \omega_g)\end{aligned}$$

$$\begin{aligned}\mathbf{s}_{nlm}^S &= U_{nl}(r)\mathbf{P}_{lm} + V_{nl}(r)\mathbf{B}_{lm} \\ &= U_{nl}(r)\hat{\mathbf{r}}\mathcal{Y}_{lm} + \\ &\quad \frac{V_{nl}(r)}{\sqrt{l(l+1)}}[\hat{\boldsymbol{\theta}}\partial_\theta + \hat{\boldsymbol{\phi}}(\sin\theta)^{-1}\partial_\phi]\mathcal{Y}_{lm}.\end{aligned}$$

$$I_n \equiv \int_0^R \partial_r \mu(r) \left[U_n(r) + \frac{3}{\sqrt{6}} V_n(r) \right] r^2 dr$$

The overlap integral I_n measures how efficiently the GW couples to each spheroidal mode.

Appendix A3 · Fisher formalism and parameter space

$$\mathcal{F}_{ij} = \sum_n \sum_{\alpha=1}^3 \frac{1}{\sigma_{\alpha,n}^2} \frac{\partial O_{\alpha,n}}{\partial p_i} \frac{\partial O_{\alpha,n}}{\partial p_j} + (\mathbf{C}_p^{-1})_{ij}$$

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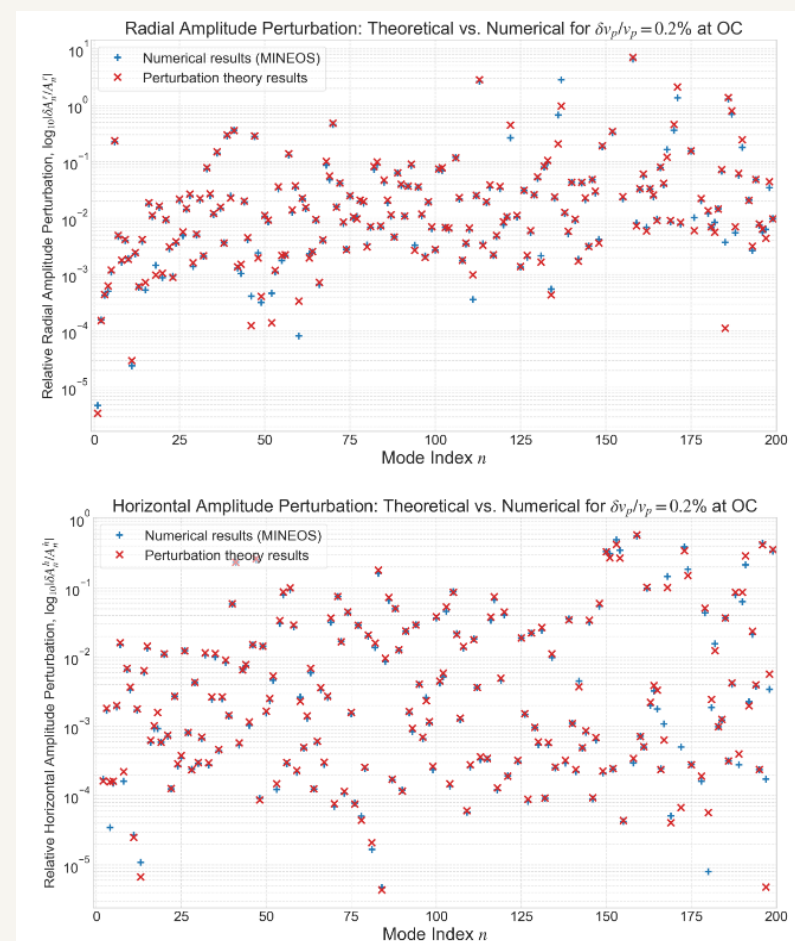
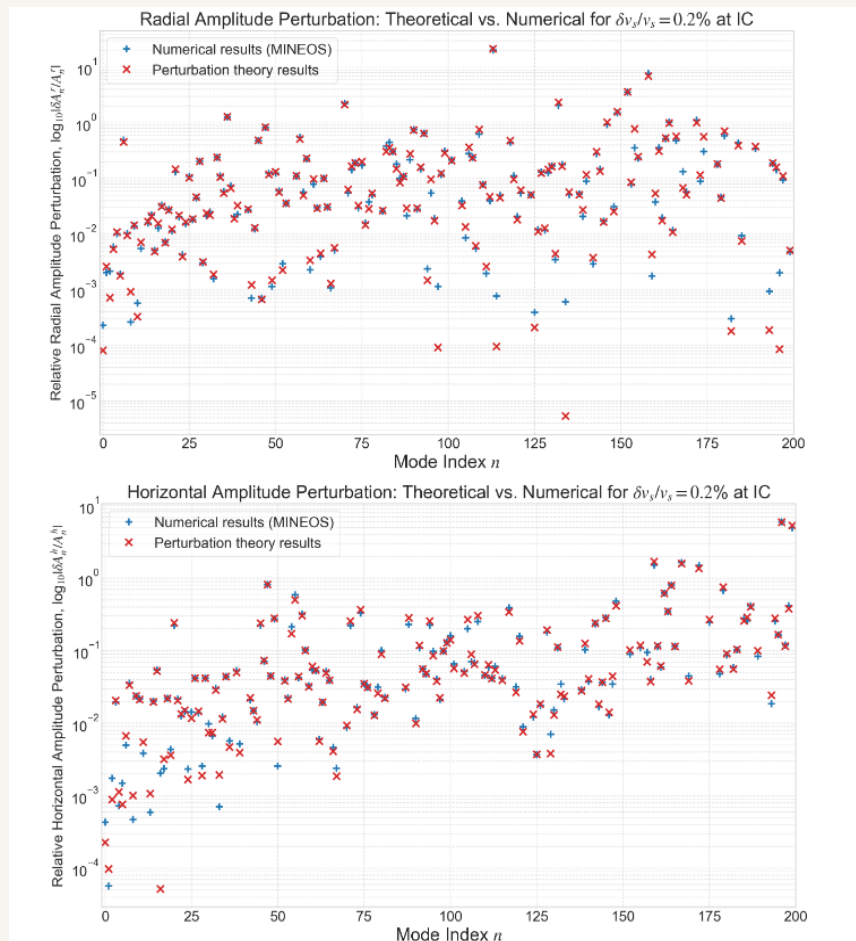
Baseline lunar model

TABLE A1. Radial layers for parameter estimation.

Layer number	r_1 (km)	r_2 (km)
1	0	100
2	100	200
3	200	352
4	353	480
5	481	600
6	600	700
7	700	800
8	800	900
9	900	1000
10	1000	1100
11	1100	1200
12	1200	1300
13	1300	1400
14	1400	1500
15	1500	1600
16	1600	1709
17	1709.1	1725
18	1725.1	1737.1

Radial layers for parameter estimation

Appendix A4 · MINEOS verifications



Examples of amplitude-perturbation comparisons between first-order theory and MINEOS calculations.

Source: Yan & Harms 2026, Supplemental Material; original presentation material

Appendix A5 · Partial derivative example

$$\delta p(r) = \delta p \text{ on } r \in [r_1, r_2]$$

$$\frac{\partial(\delta\omega_n^2)}{\partial(\delta p)} = \sqrt{\frac{2l+1}{4\pi}} \int_{r_1}^{r_2} dr' r'^2 [V_p^{nn}(r') - \omega_k^2 T_p^{nn}(r')]$$

$$\begin{aligned} \frac{\partial(\delta U_n)}{\partial(\delta p)}(r) = & \sqrt{\frac{2l+1}{4\pi}} \sum_{n' \neq n} U_{n'}(r) \\ & \times \int_{r_1}^{r_2} dr' r'^2 \frac{[V_p^{n'n}(r') - \omega_n^2 T_p^{n'n}(r')]}{\omega_n^2 - \omega_{n'}^2} \end{aligned}$$

Example: radially localized density perturbation and its frequency / eigenfunction derivatives.

Source: Supplemental Material; original presentation material

Appendix A6 · What “calibrated GW” means here

In this work, “calibrated” refers to the incident GW source amplitude, not only to instrument calibration.

Required from the GW network

- Time-dependent strain amplitude $h_0(t)$ at the Moon
- Sky position and polarization
- Luminosity distance / source parameters
- Covariance of these quantities

Needed in a realistic inversion

- Propagate h_0 uncertainty into amplitudes
- Include source-population and selection effects
- Model duty cycle and network geometry
- Treat mode identification and response noise

The Fisher result isolates structural information under an ideal calibrated-input assumption.

Appendix A7 · References

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