

Introduction

Lorentz invariance is one of the most fundamental symmetries of modern physics. Several phenomenological approaches to quantum gravity [1], suggest Lorentz invariance violation (LIV) at high energies as a possible extension of the Standard Model that incorporates quantum gravity effects. One of the key consequences of LIV is its impact on fundamental properties of physical processes, such as cross sections, energy thresholds, and mean free paths. Astrophysical and cosmological observations of ultra-high-energy cosmic rays (UHECRs) [2], as well as high-energy gamma rays [3], neutrinos [4] and gravitational waves [5] propagating over cosmological distances, are particularly sensitive to these effects and therefore provide stringent constraints on the magnitude of LIV.

The Modified Dispersion Relation

A central feature of LIV is the modification of the standard energy-momentum dispersion relation. These **Modified Dispersion Relations (MDRs)** [6, 7] can be expressed in terms of the particle energy as:

$$E_i^2 = m_i^2 + p_i^2 + m_{\text{eff},i}^2 \quad \text{where} \quad m_{\text{eff},i}^2 = \sum_{n \geq 0} \eta_{i,n} \frac{E_i^{n+2}}{M_{Pl}^n}, \quad (1)$$

with index i indicating particle type (i.e. electron, photon, etc.) and $M_{Pl} = 1.22 \times 10^{19}$ GeV is the Planck mass¹. $\eta_{i,n}$ are the LIV coefficients, which determine the magnitude of the violation of order n for particles of type i

¹Natural units are used in the notation

The case of photons: superluminal vs subluminal LIV

While the LIV framework defined in Eq. 1 applies to all particle species, we now restrict our discussion to the case of photons. The MDR can then be written as:

$$E_\gamma^2 = p_\gamma^2 + m_{\text{eff},\gamma}^2, \quad (2)$$

Depending on the sign of the LIV coefficients η_n entering inside $m_{\text{eff},\gamma}^2$, photon propagation can be classified into two distinct regimes: **superluminal** ($m_{\text{eff},\gamma}^2 > 0$) and **subluminal** propagation ($m_{\text{eff},\gamma}^2 < 0$). In both cases, the photon group velocity is altered, acquiring an energy dependence and being respectively increased or reduced with respect to the standard speed of light in vacuum. This feature can be probed through searches for time delays between photons of different energies emitted by transient astrophysical sources [8, 9], leading to constraints on the LIV coefficients.

LIV effects in the extragalactic propagation of UHE photons

UHE photons (γ_{UHE}) propagating in the extragalactic space interact with background photons (γ_{bkg}), producing electron-positron pairs, through the **Breit-Wheeler (BW) process** $\gamma_{\text{UHE}} + \gamma_{\text{bkg}} \rightarrow e^+e^-$. Assuming LIV present only in the photon sector and neglecting its effects on the background photons, the **threshold energy** ϵ_{thr} and cross section [10] for this process can be written as:

$$\epsilon_{\text{thr}}^{\text{LIV}} = \frac{4m_e^2 - m_{\text{eff},\gamma}^2}{2E_\gamma(1 - \cos\theta)} \quad (3)$$

$$\sigma_{\text{BW}}^{\text{LIV}} = \frac{\alpha^2 \pi}{E_\gamma \epsilon (1 - \cos\theta)} \left(1 + \left(1 + \frac{2m_{\text{eff},\gamma}^2}{E_\gamma \epsilon (1 - \cos\theta)} \right)^2 \right) \log \left(\frac{E_\gamma \epsilon (1 - \cos\theta) + m_{\text{eff},\gamma}^2}{m_e^2} \right). \quad (4)$$

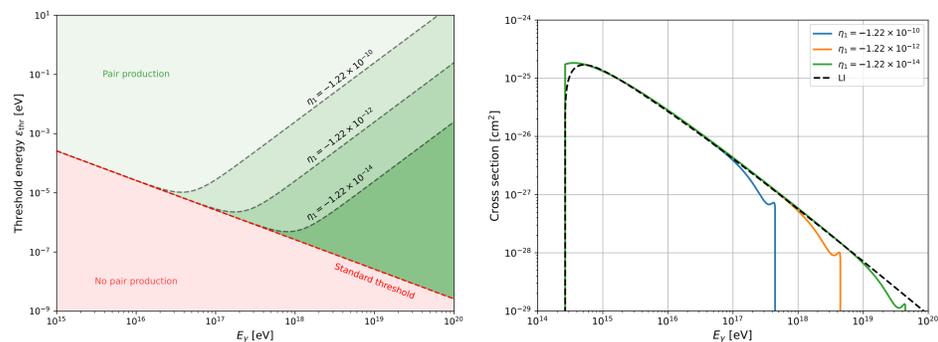


Figure 1. **Left:** energy threshold for BW process $\epsilon_{\text{thr}}^{\text{LIV}}$ as a function of UHE photon energy E_γ for $\theta = \pi$ and different values of η_1 . **Right:** cross section for BW process $\sigma_{\text{BW}}^{\text{LIV}}$ as a function of E_γ for $\theta = \pi$ and different values of η_1 . The black dashed line indicates the standard Lorentz invariant (LI) cross section.

The mean free-path for this process, neglecting cosmological effects, can be computed as:

$$\frac{1}{\lambda_\gamma(E_\gamma)} = \int_{-1}^1 d(\cos\theta) \frac{1 - \cos\theta}{2} \int_{\epsilon_{\text{thr}}^{\text{LIV}}}^\infty d\epsilon n_\gamma(\epsilon) \sigma_{\text{BW}}^{\text{LIV}}(E_\gamma, \epsilon, \theta) \quad (5)$$

where $n_\gamma(\epsilon)$ is the spectral density of CMB photons.

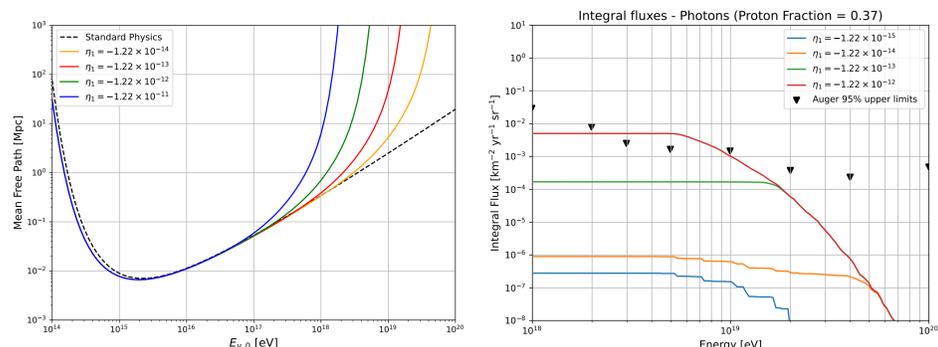


Figure 2. **Left:** Mean free path for UHE photons as function of their energy E_γ and for different values of η_1 . The black dashed line is the mean free path in the standard Lorentz invariant scenario. **Right:** Integral flux of UHE photons as a function of the energy for different values of η_1 .

LIV effects in the development of showers in the Earth atmosphere

When UHE photons reach the Earth atmosphere, they can initiate extensive air showers (EAS). At our relevant energies ($10^{17} - 10^{21}$ eV), the dominant mechanism is the **Bethe-Heitler (BH) process** $\gamma_{\text{UHE}} + N \rightarrow N + e^+ + e^-$. The Lorentz-invariant and LIV cross sections are the following [6, 10]:

$$\sigma_{\text{BH}} = \frac{28Z^2\alpha^3}{9m_e^2} \left(\log \frac{183}{Z^{1/3}} - \frac{1}{42} \right), \quad (6) \quad \sigma_{\text{BH}}^{\text{LIV}} = \frac{8Z^2\alpha^3}{3|m_{\text{eff},\gamma}^2|} \log \frac{1}{\alpha Z^{1/3}} \log \frac{|m_{\text{eff},\gamma}^2|}{m_e^2}, \quad (7)$$

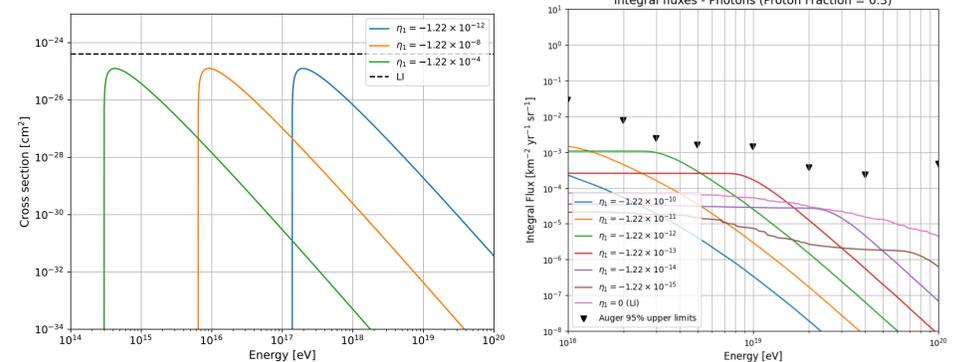


Figure 3. **Left:** Cross section for pair production in the atmosphere as a function of the photon energy, for different η_1 values. Black dashed line indicates the Lorentz invariant cross section. **Right:** Integral flux of UHE photons accounting for atmospheric suppression as a function of energy. Different colored lines for different values of η_1 .

LIV effects in the hadronic sector

LIV effects can also be included in the hadronic sector, in particular for pions. Following Eq. 1, the MDR and the π lifetime can be written as:

$$E^2 = p^2 + m_\pi^2 + m_{\text{eff},\pi}^2 \equiv p^2 + m_{\text{LIV}}^2 \rightarrow \tau = \frac{E}{m_{\text{LIV}}} \tau_0 \quad (8)$$

Since pions dominate secondary particle production in EAS, LIV corrections can alter shower evolution, in particular the muon content of showers [11].

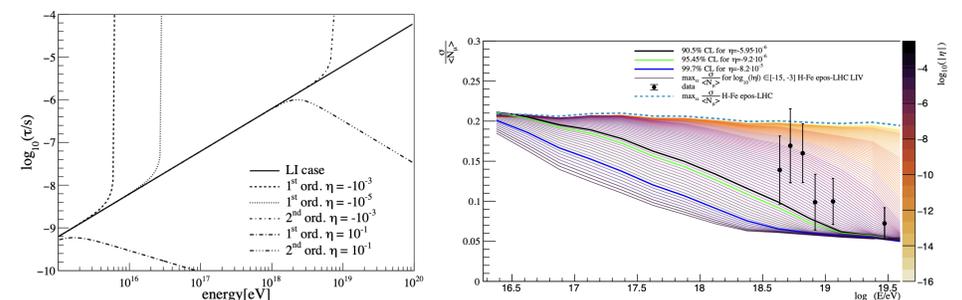


Figure 4. **Left:** Neutral pion lifetime as a function of energy for different LIV scenarios and parameters. Solid line represents Lorentz invariant case. **Right:** Maximum, with respect to relative abundance of Fe nuclei, of relative fluctuations of the number of muons in presence of LIV. Colored lines correspond to different values of η_1 , black points with error bars represent the measured relative fluctuations and the dashed curve represent the standard Lorentz invariant scenario. Figures taken from [11].

Implementing LIV in CORSIKA

Recently, within the COST Action BridgeQG, a dedicated effort is currently underway to implement LIV effects in Monte Carlo air shower simulations **CORSIKA**, enabling the inclusion of LIV directly in the simulations and its study within realistic shower modeling. [12]



Current LIV parameters limits



References

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