

Information Field Theory for Ultra-High Energy Cosmic Ray studies



Igor Vaiman, Carmelo Evoli, Philipp Mertsch

Information theory for fields

Bayes' theorem for **signal** and **data**:

$$\mathcal{P}(s|d) = \frac{\mathcal{P}(d|s)\mathcal{P}(s)}{\int ds\mathcal{P}(d|s)\mathcal{P}(s)}$$

Cast into information-theoretic form:

$$\mathcal{H} \equiv -\ln \mathcal{P}$$

Information Hamiltonian

$$\mathcal{Z}(d) \equiv \int e^{-\mathcal{H}(d,s)} ds$$

Partition sum

$$\mathcal{H}(d,s) = \mathcal{H}(d|s) + \mathcal{H}(s)$$

$$\mathcal{P}(s|d) = \frac{\exp(-\mathcal{H}(d,s))}{\mathcal{Z}}$$

Boltzmann distribution

This gives a useful analogy to apply ideas from statistical mechanics and field theory to inference:

- Free hamiltonian = linear measurement with Gaussian noise; exact solution (Wiener)
- Interaction terms = non-linearity and non-Gaussianity; approximation or numerical treatment required
- Classical limit = maximum a posteriori estimation
- Mean field approximation = best approximation of the signal by Gaussian field (Variational Inference)
- Hamiltonian dynamics → Hamiltonian Monte-Carlo sampling

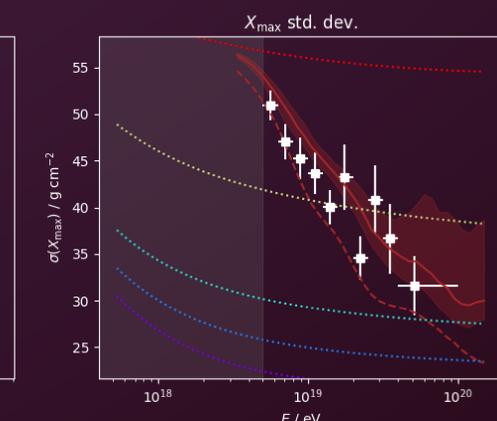
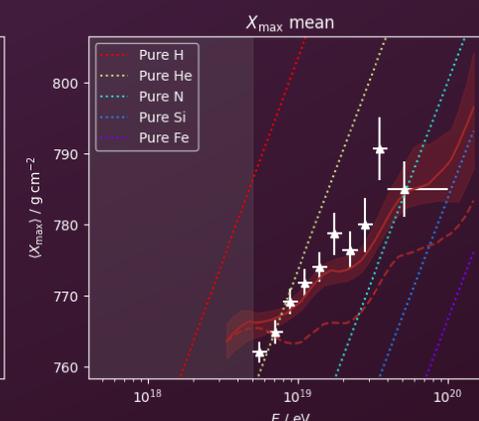
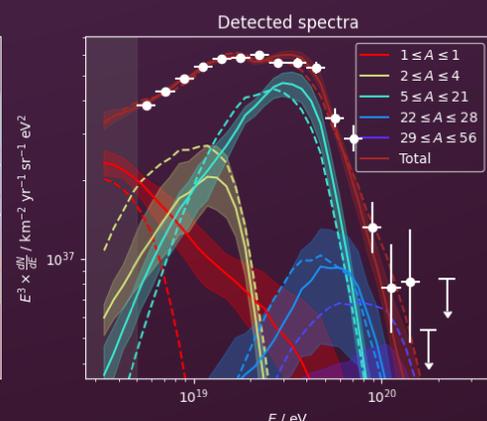
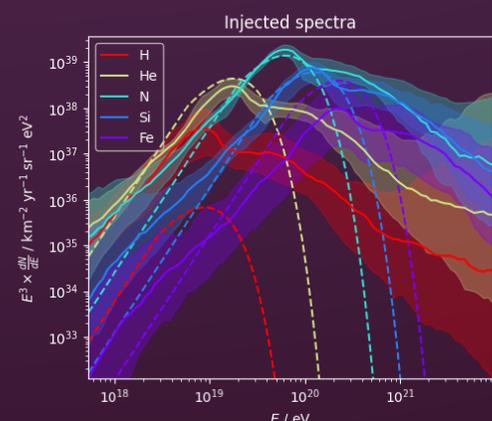
Enßlin, T. A. Information Theory for Fields. Annalen der Physik 531, (2019)

Combined fit of UHECR spectrum and composition with IFT (combined ift)

Pierre Auger Observatory measures **energy spectrum** and **mass composition** (through X_{max}) of cosmic rays up to the highest energies ever observed. These observables carry information about UHECR sources and propagation.

Astrophysical interpretation is done by:

- making simplifying assumptions about the sources' distribution and spectrum
- simulating CR propagation in extragalactic space, either in 1D (only energy losses and spallation) or in 3D (EGMF & GMF effects)
- fitting simulated observables (spectrum, composition, sometimes arrival directions) to the data to constrain the sources in the **combined fit framework**



Pierre Auger Collaboration Combined fit of spectrum and composition data as measured by the Pierre Auger Observatory. J. Cosmol. Astropart. Phys. 2017, 038–038 (2017)
Pierre Auger Collaboration Measurement of the cosmic-ray energy spectrum above 2.5×10^{18} eV using the Pierre Auger Observatory. Phys. Rev. D 102, (2020)

Variational Inference with Gaussian Fields

Maximum Entropy Principle: given only mean and (co)variance, assume Gaussian distribution

Posterior distribution of a high-resolution, multi-dimensional signal is approximated by a Gaussian field – Gaussian Variational Inference (GVI):

- **Prior** can express our expectation of signal's smoothness, similar to regularization; it should be informed by theoretical considerations
- For isotropic and homogenous fields, **spectral representation** of the prior is available & effective
- For high-dimensional problems we must avoid storing full matrices and only work with **implicit operators**

To approximate posterior covariance, we can use Fisher information metric (**Metric GVI**). For significantly non-gaussian posteriors (in case of non-linear response and non-gaussian noise), we can construct a coordinate transformation using geometrical information from the posteior (**geoVI**).

Knollmüller, J. & Enßlin, T. A. Metric Gaussian Variational Inference arXiv:1901.11033 (2019)
Frank, P., Leike, R. & Enßlin, T. A. Geometric Variational Inference. Entropy 23, 853 (2021)

Numerical Information Field Theory

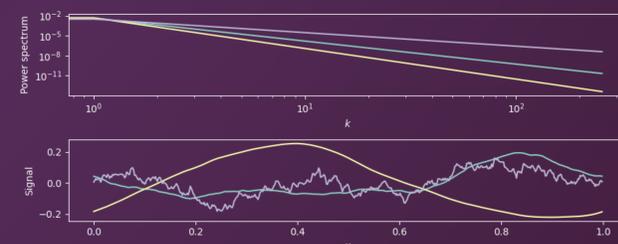
Likelihood implementation must be **fast** and also compute **derivatives**, so:

- Just-In Time compilation from high-level language to CPU/GPU-native code
- Automatic Differentiation (AD), applying chain rule to a program to compute its' gradient without manual implementation

Numerical Information Field Theory (NIFTy) is a Python package achieving both through JAX. It also implements MGVI & geoVI.

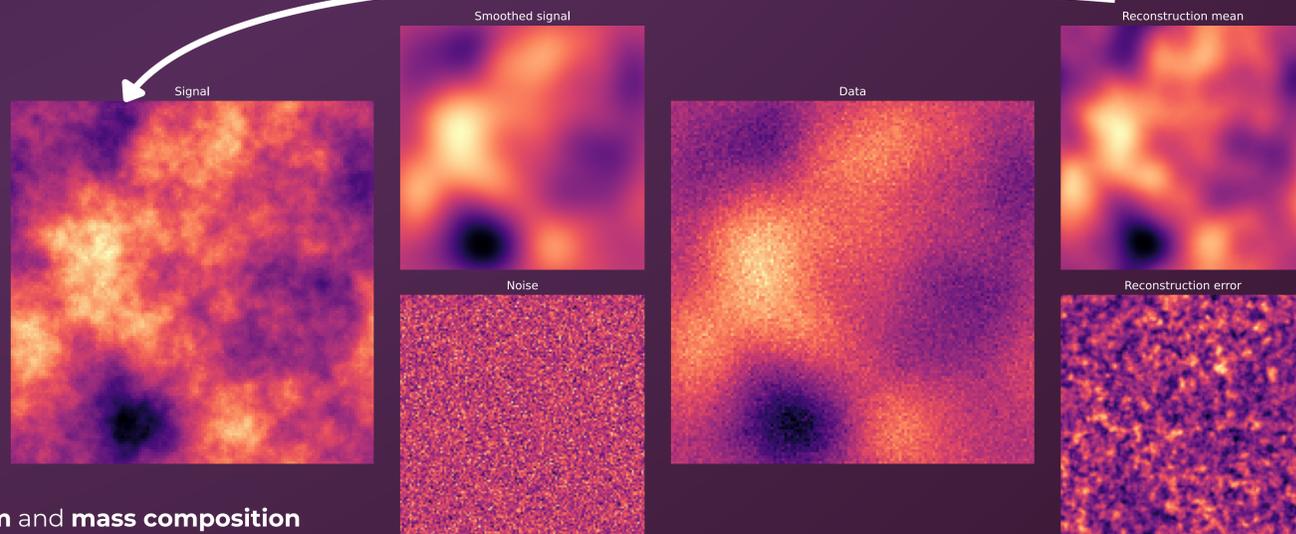


NIFTy provides utilities to specify prior on a random field in frequency domain



Edenhofer, G. et al. Re-Envisioning Numerical Information Field Theory (NIFTy.re): A Library for Gaussian Processes and Variational Inference. JOSS 9, 6593 (2024)

Field reconstruction w/ uncertainty



Our project aims to apply IFT to inference of UHECR source properties:

- **Nonparametric spectrum model** to probe its' features
- Test rigidity-dependence of source spectra to detect possible near-source effects or multiple populations
- In the future, reconstruct source density in 1D and 3D

Reconstruction of source spectra shows promising results, improving the data-model agreement. Further steps include more flexible model, astrophysical interpretation of the result and new observables.