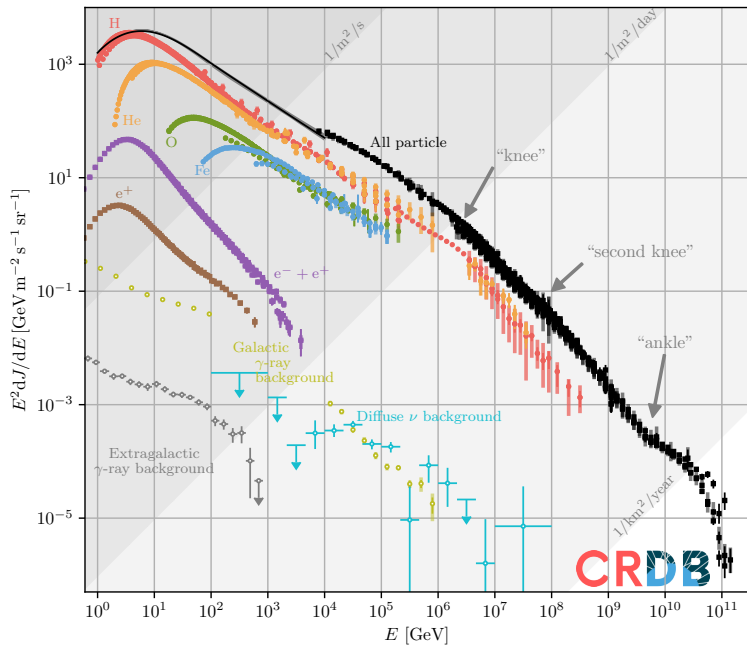


Galactic cosmic rays: an interstellar laboratory

Philipp Mertsch

GSSI colloquium, L'Aquila
11 March 2026

The cosmic ray spectrum

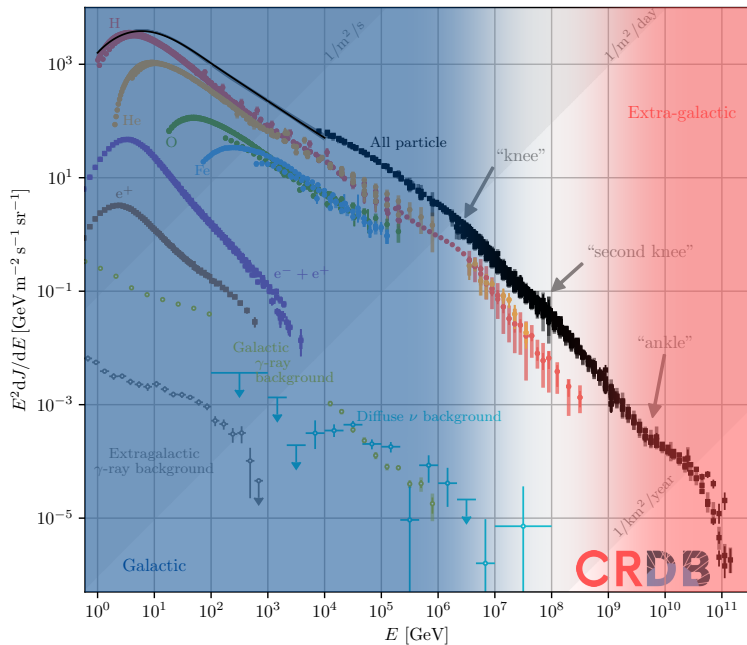


Intensity dJ/dE

$$\frac{dJ}{dE} \equiv \frac{(\# \text{particles})}{\Delta t \Delta A \Delta \Omega \Delta E}$$

- ~ 12 orders of magnitude in energy
- \sim power law $dJ/dE \propto E^{-3}$ with some features

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Why are cosmic rays interesting?

Cosmic ray origin

- What are the sources of cosmic rays?
- Century-old problem!
- Astrophysical interest

Ingredient

Cosmic rays ...

- produce diffuse emission
- ionise and heat
- provide gravitational support
- drive winds
- generate turbulence

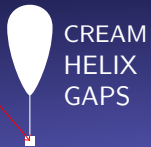
Exotic probe

- Indirect searches of dark matter
- Dark matter cools cosmic rays, cosmic rays scatter dark matter
Bringmann et al. (2019), Ng et al. (2019)
- Primordial anti-matter?

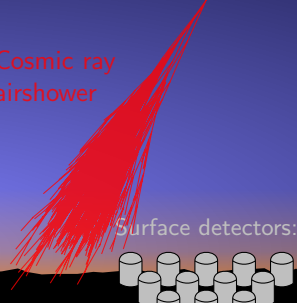
Space experiments: AMS-02, CALET, DAMPE, *Fermi-LAT*



Balloon experiments:



Cosmic ray
airshower



Cherenkov telescopes:
HESS, VERITAS, MAGIC



Fluorescence detectors:

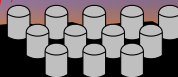
Auger, TA



IceCube

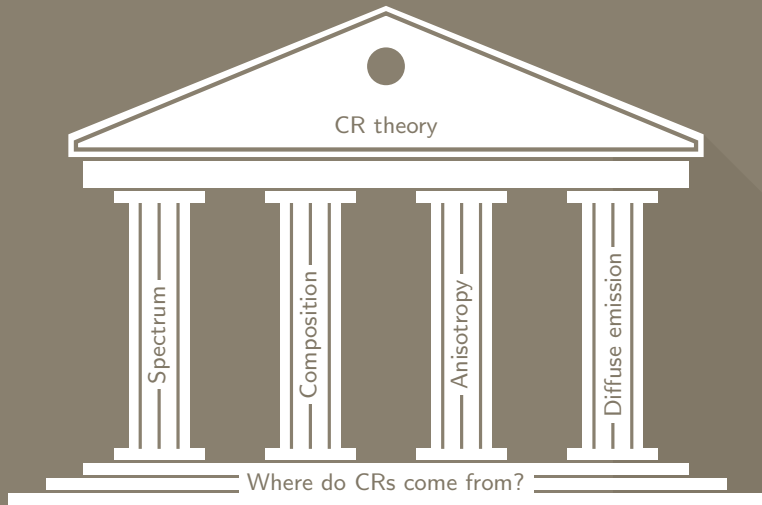


Surface detectors:



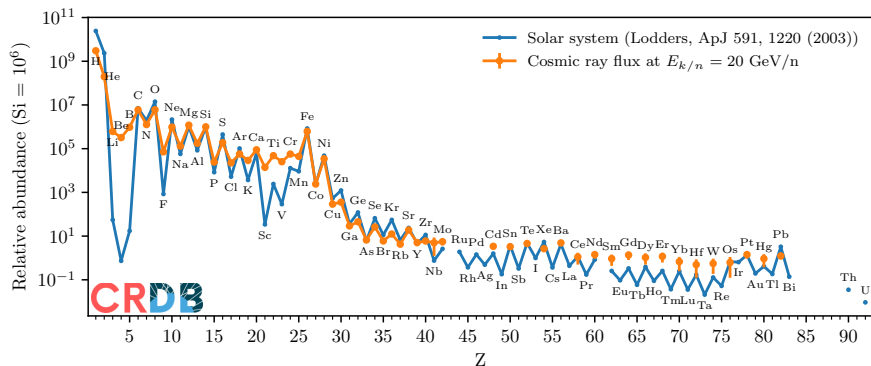
HAWC, IceTop, Auger, TA, LHAASO





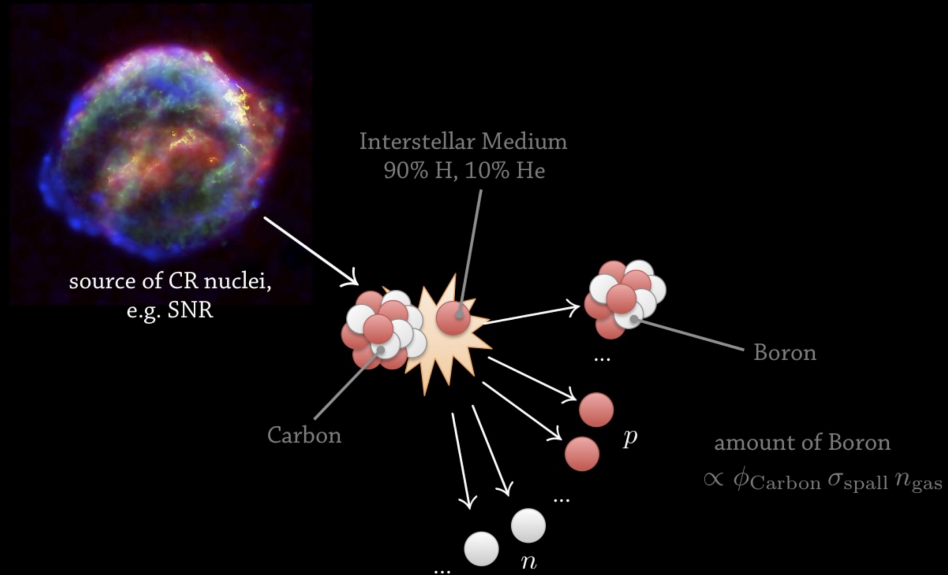
Composition

- Some species have same abundances in CRs and in solar system → **primaries**
- Other species are overabundant with respect to solar abundances:

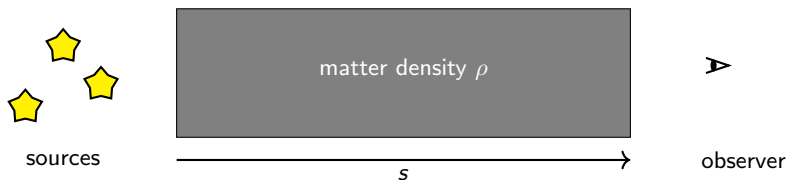


→ Must have been produced during the transport → **secondaries**

Secondaries from spallation

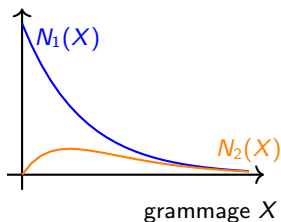


Slab model



- Define CR grammage:
$$X \equiv \int ds' \rho(s') = s\bar{\rho}$$
- Consider number of primary and secondary CRs, N_1 and N_2 :

$$\frac{dN_1}{dX} = -\frac{N_1}{\lambda_1}$$
$$\frac{dN_2}{dX} = -\frac{N_2}{\lambda_2} + \text{BR}_{1 \rightarrow 2} \frac{N_1}{\lambda_1}$$



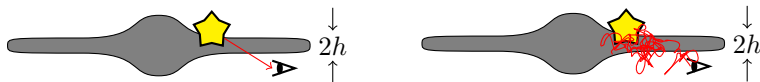
with $1/\lambda_{1,2} = \sigma_{1,2}/m$ the specific cross-section

- Observations: $N_2/N_1 \simeq 0.3 \rightarrow X \simeq 7.2 \text{ g cm}^{-2}$

- Where does the grammage come from?
- If CRs traverse the Galactic disk, every crossing contributes

$$\Delta X \sim hm_N n_{\text{gas}} \simeq (100 \text{ pc})(1.7 \times 10^{-24} \text{ g})(1 \text{ cm}^{-3}) \simeq 5 \times 10^{-4} \text{ g cm}^{-2}$$

- ($1 \text{ pc} \simeq 3.1 \times 10^{18} \text{ cm}$)



CRs must cross the disk many times, e.g. through **diffusion**

- Residence time in disk:

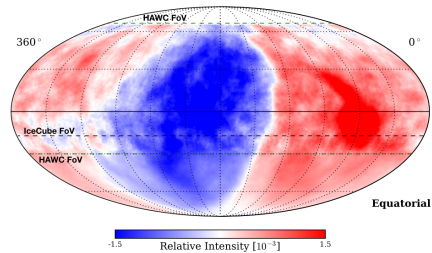
$$t_{\text{esc}} = \frac{s}{v} = \frac{X}{v \bar{\rho}} = \frac{X}{v m_N \bar{n}_{\text{gas}}} \simeq 3 \times 10^6 \text{ yr}$$

for $n_{\text{gas}} = 1 \text{ cm}^{-3}$

Anisotropy (I)

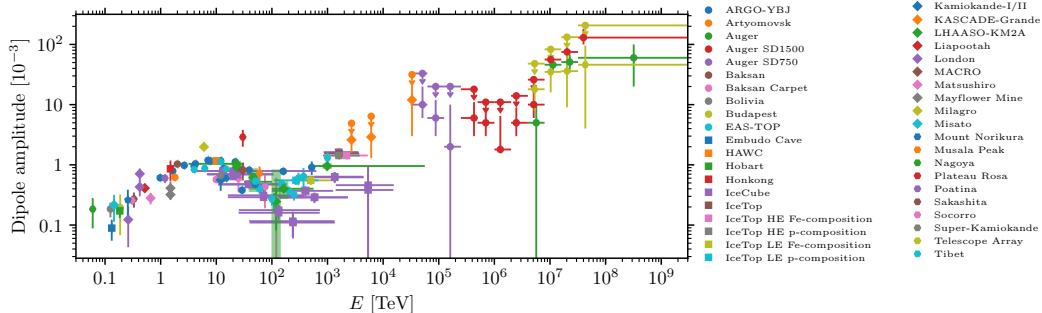
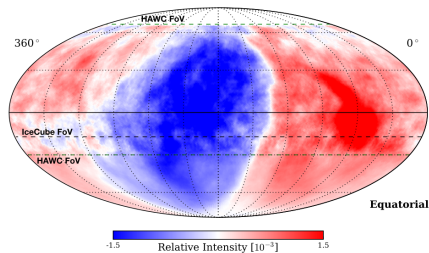
- Angular distribution of CRs is very isotropic

- E.g., the dipole anisotropy $a \equiv \frac{\phi_{\max} - \phi_{\min}}{\phi_{\max} + \phi_{\min}}$



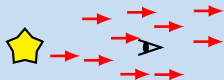
Anisotropy (I)

- Angular distribution of CRs is very isotropic
- E.g., the dipole anisotropy $a \equiv \frac{\phi_{\max} - \phi_{\min}}{\phi_{\max} + \phi_{\min}}$
- Between a few GeV and a PeV: $a = \mathcal{O}(10^{-4} \dots 10^{-3})$



Anisotropy (II)

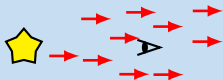
Between a few GeV and a PeV: $a = \mathcal{O}(10^{-4} \dots 10^{-3})$



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- See, e.g., electro-magnetic radiation

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- Need to isotropise CRs
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- Scattering of charged particles with turbulent magnetic field isotropises particle directions
- Particles perform a random walk in space:

$$\langle (\Delta r)^2 \rangle \propto \Delta t$$

- The constant of proportionality is called the **diffusion coefficient** κ

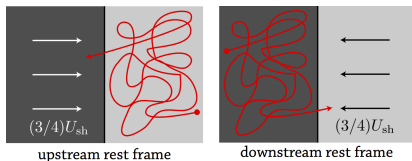
Mathematical description

Ginzburg & Syrovatskii (1964)

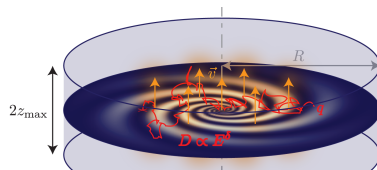
Transport equation

$$\frac{\partial \psi_j}{\partial t} - \nabla \cdot (\kappa \cdot \nabla \psi_j - \mathbf{u} \psi_j) + \frac{\partial}{\partial p} \left(\frac{p}{3} (\nabla \cdot \mathbf{u}) \psi_j \right) = q_j$$

↓
Application to blast wave:



↓
Application to galactic halo:



Shock acceleration

Source spectrum: $q(\mathcal{R}) \propto \mathcal{R}^{-2.4 \dots -1.9}$

Axford, Leer, Skadron (1977); Krymskii (1977); Bell (1978);

Blandford, Ostriker (1978)

Diffusive escape

Observed spectrum: $\psi(\mathcal{R}) \propto \frac{q(\mathcal{R})}{\kappa(\mathcal{R})}$ e.g. $\frac{\mathcal{R}^{-2.2}}{\mathcal{R}^{0.6}} \propto \mathcal{R}^{-2.8}$

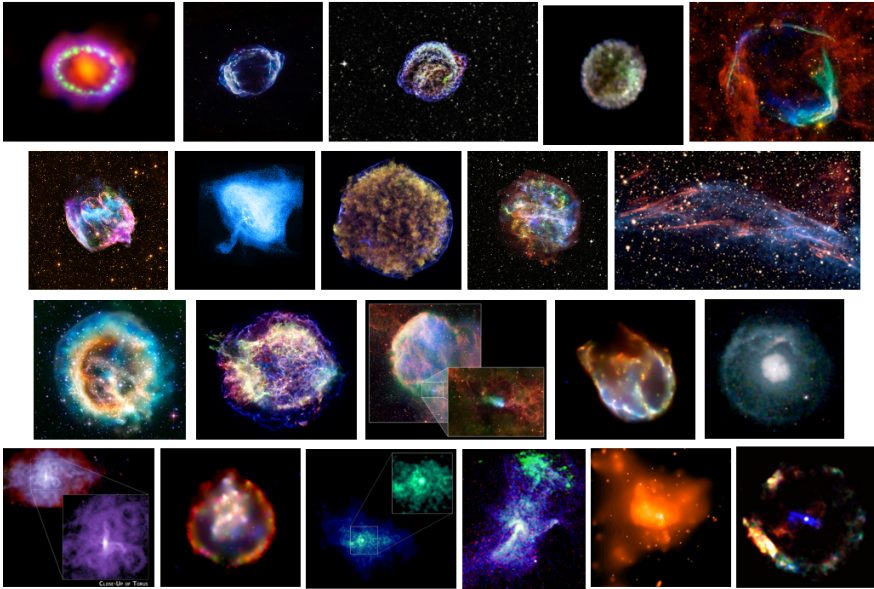
Outline

- ① Introduction
- ② Stochastic modelling of cosmic rays
- ③ Stochastic diffuse emission
- ④ Field line transport and particle transport in synthetic turbulence
- ⑤ Summary & conclusion

Outline

- 1 Introduction
- 2 Stochastic modelling of cosmic rays
- 3 Stochastic diffuse emission
- 4 Field line transport and particle transport in synthetic turbulence
- 5 Summary & conclusion

Supernova remnants have long been considered the sources of cosmic rays

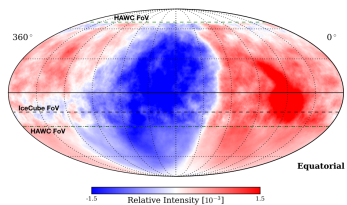


- 1 Observations
- 2 Energetics
- 3 Shock acceleration

Lopez and Fesene (2018)

We do not know individual sources of *local* cosmic rays

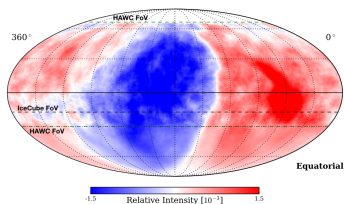
Anisotropies:



Problem: cosmic rays diffuse

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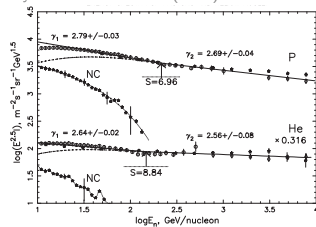
Anisotropies:



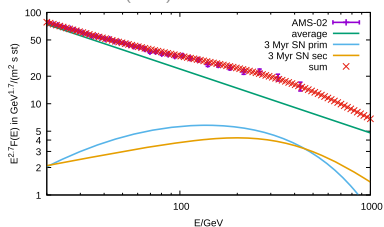
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Spectrum:

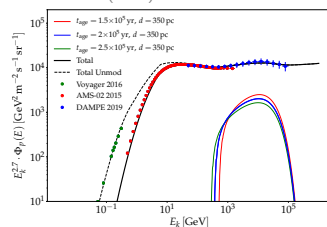
Erlykin and Wolfendale (2012)



Kachelrieß *et al.* (2018)

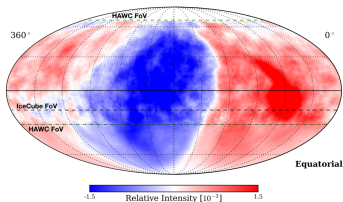


Fornieri *et al.* (2020)



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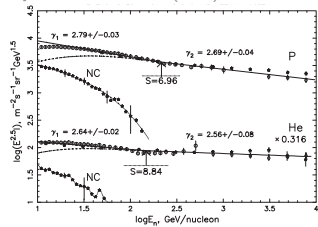
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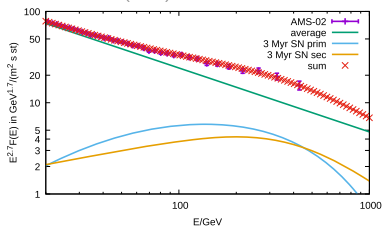
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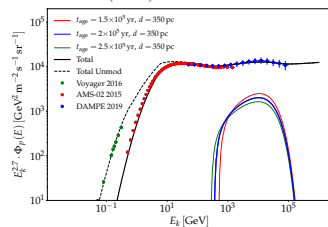
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Fornieri *et al.* (2020)

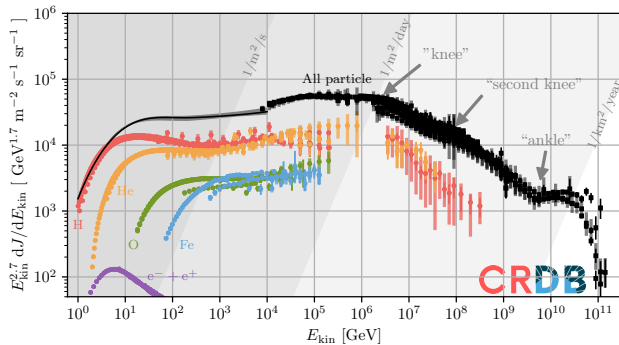


Supernova remnant paradigm

1 000 - 100 000 “active” supernova remnants in the Galaxy

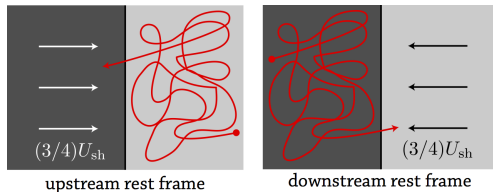
Genolini *et al.* (2017)

Galactic sources should accelerate to E_{knee} , probably via shock acceleration



- $E_{\text{knee}} \simeq 3 \text{ PeV}$:
either maximum energy of source or
change in transport regime
- $E_{\text{max}} \gtrsim 3 \text{ PeV}$ for protons

Axford, Leer, Skadron (1977); Krymskii (1977); Bell (1978); Blandford, Ostriker (1978)



Small energy gain ΔE , little particle loss ΔN per cycle

$$\left. \begin{aligned} \frac{\Delta E}{E} &\propto \frac{U_{\text{sh}}}{c} \\ \frac{\Delta N}{N} &\propto \frac{U_{\text{sh}}}{c} \end{aligned} \right\} \Rightarrow \frac{dN}{dE} \propto E^{-2}$$

We do not understand how supernova remnants accelerate to E_{knee}

What is E_{max} ?

- Equate age with acceleration time: $t_{\text{age}} = t_{\text{acc}} = 8 \frac{\kappa}{U_{\text{sh}}^2}$ ← Diffusion coefficient

- Assume Bohm diffusion: $\kappa = \frac{c \ell_{\text{mfp}}}{3} = \frac{c r_{\text{g}}}{3} = \frac{c E_{\text{max}}}{3 q B}$ ← Gyro radius

- Hillas-like relation: $\Rightarrow E_{\text{max}} \simeq \frac{U_{\text{sh}}^2}{c} q B t_{\text{age}}$ or $\frac{U_{\text{sh}}}{c} q B R$

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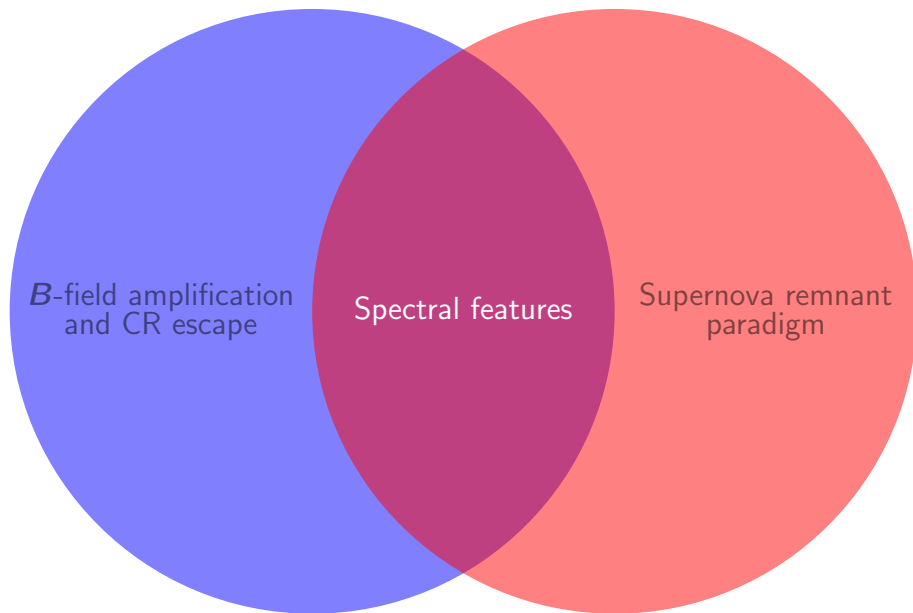
- With typical values: $U_{\text{sh}} = 10^4 \text{ km s}^{-1}$, $B = 1 \mu\text{G}$, $t_{\text{age}} = 10^3 \text{ yr}$

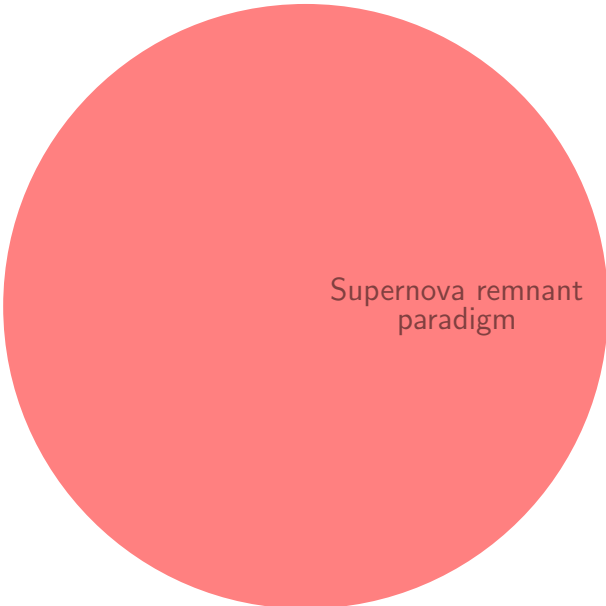
$$\Rightarrow E_{\text{max,b}} \simeq 100 \text{ TeV} \ll E_{\text{knee}}$$

Lagage and Cesarsky (1983)

- 1 Choosing larger t_{age} does not help: $U_{\text{sh}} \propto t_{\text{age}}^{-3/5}$, so E_{max} decreases with time
- 2 Need to amplify B -field to $B \simeq 100 \mu\text{G}$

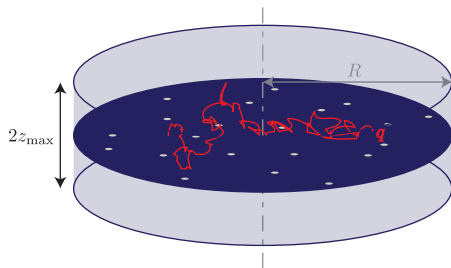
Combining standard ingredients, we predict novel spectral features





Supernova remnant
paradigm

The number of sources contributing to CR flux decreases with energy



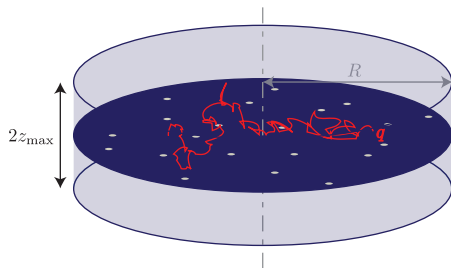
- Residence time: $t_{\text{esc}} = \frac{z_{\max}^2}{2\kappa}$
- Diffusion distance: $R = \sqrt{2\kappa t_{\text{esc}}} = z_{\max}$
- Source density: $\sigma = \frac{\nu t_{\text{esc}}}{\pi R_{\text{disk}}^2}$
- Source number: $N_{\text{src}} = \sigma \pi R^2 = \nu t_{\text{esc}} \frac{z_{\max}^2}{R_{\text{disk}}^2}$

With typical parameters (for details → [Appendix](#)):

$$\mathcal{R} = 10 \text{ GV}, \quad 10 \text{ TV}, \quad 10 \text{ PV}$$

$$N_{\text{src}} \simeq 2 \times 10^5, \quad 2000, \quad 40$$

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$$N_{\text{src}} \simeq 2 \times 10^5, \quad 2000, \quad 40$$

Need to run Monte Carlo simulations
instead of cherry-picking catalogues!

Source density is oftentimes considered smooth, while it really is discrete

Transport equation

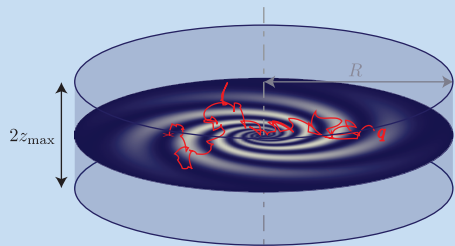
$$\frac{\partial \psi}{\partial t} - \nabla \cdot \kappa \cdot \nabla \psi + \dots = q$$

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Transport equation

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Smooth source density



$$q = q(\mathbf{r}, t, E) = \nu \rho(\mathbf{r}) Q(E)$$

SN rate

smooth function

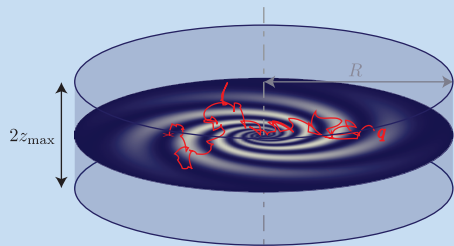
spectrum

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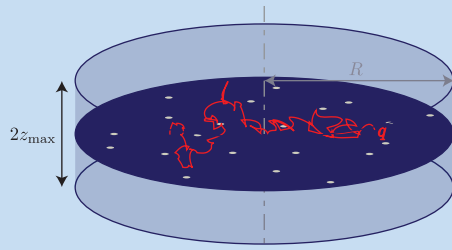
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spectrum

Discrete sources



$$q = q(\mathbf{r}, t, E) = \sum_i \delta^{(3)}(\mathbf{r} - \mathbf{r}_i) \delta(t - t_i) Q(E)$$

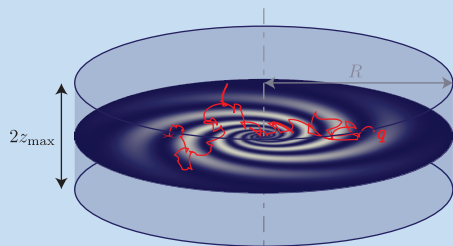
\mathbf{r}_i drawn from $\rho(\mathbf{r})$ and t_i from uniform distribution

Source density is oftentimes considered smooth, while it really is discrete

Transport equation

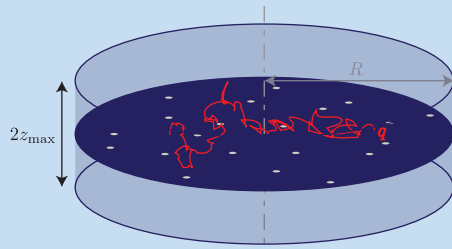
$$\frac{\partial G}{\partial t} - \nabla \cdot \kappa \cdot \nabla G + \dots = \delta^{(3)}(\mathbf{r} - \mathbf{r}_i) \delta(t - t_i) Q(E)$$

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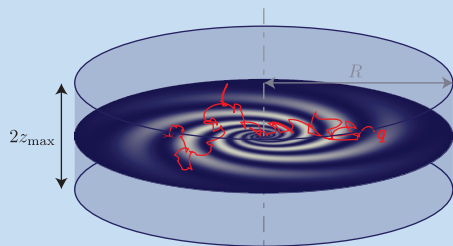
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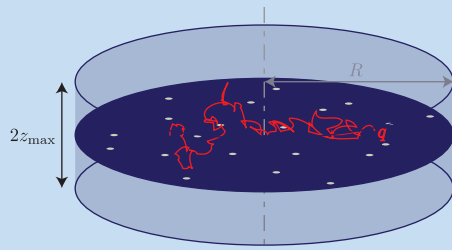
Smooth source density



$$q = q(\mathbf{r}, t, E) = \nu \rho(\mathbf{r}) Q(E)$$

$$\psi(\mathbf{r}, t, E) = \int d^3 r' dt' \nu \rho(\mathbf{r}') G(\mathbf{r} - \mathbf{r}', t - t', E)$$

Discrete sources



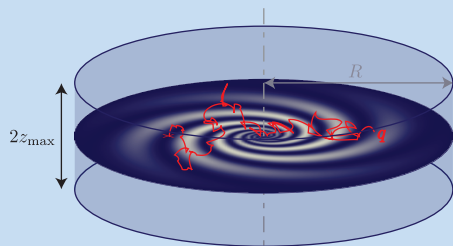
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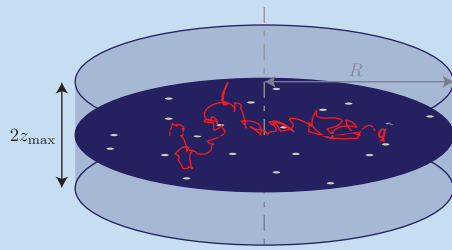
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Stochastic nature of sources implies fluctuations in spectrum

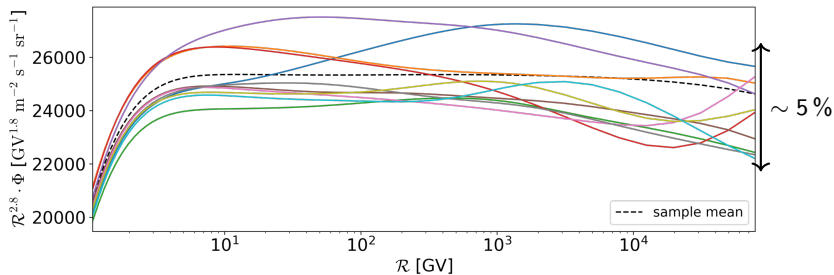
- Solution:
$$\psi(\mathbf{r}, t, p) = \sum_i G(\mathbf{r} - \mathbf{r}_i, t - t_i, E)$$
- \mathbf{r}_i, t_i are random variables $\Rightarrow \psi(\mathbf{r}, t, p)$ is random variable
- Mean:
$$\langle \psi(\mathbf{x}, t, p) \rangle = \int d^3 \mathbf{r}' dt' \nu \rho(\mathbf{r}') G(\mathbf{r} - \mathbf{r}', t - t', E)$$

Can we use $\psi - \langle \psi \rangle$ to find sources?

Stochastic nature of sources implies fluctuations in spectrum

- Solution:
$$\psi(\mathbf{r}, t, p) = \sum_i G(\mathbf{r} - \mathbf{r}_i, t - t_i, E)$$
- \mathbf{r}_i, t_i are random variables $\Rightarrow \psi(\mathbf{r}, t, p)$ is random variable
- Mean:
$$\langle \psi(\mathbf{x}, t, p) \rangle = \int d^3 \mathbf{r}' dt' \nu \rho(\mathbf{r}') G(\mathbf{r} - \mathbf{r}', t - t', E)$$

Can we use $\psi - \langle \psi \rangle$ to find sources?

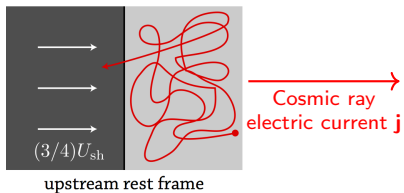




B-field amplification
and CR escape

The Bell instability can amplify B-fields

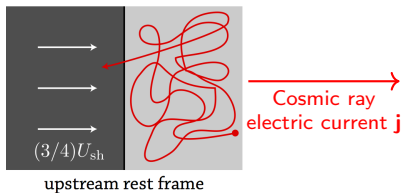
Bell (2004)



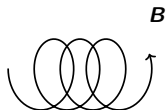
- If B -field too weak, particles escape
- Electric current j
- Waves modes unstable in the presence of current j

The Bell instability can amplify B-fields

Bell (2004)



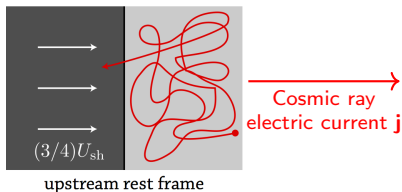
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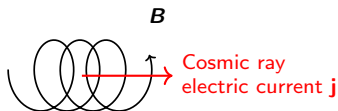
- CRs with gyroradius r_g tied to field lines
- Instability saturates once $\lambda \sim r_g$
- B -field density satisfies $\epsilon_B \sim \frac{U_{sh}}{c} \epsilon_{CR}$

The Bell instability can amplify B-fields

Bell (2004)



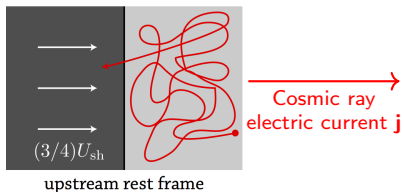
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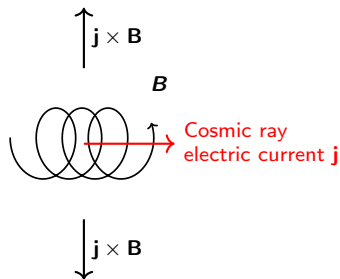
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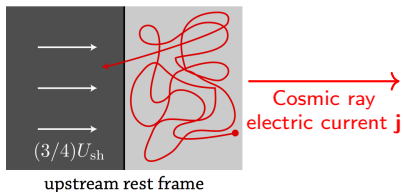
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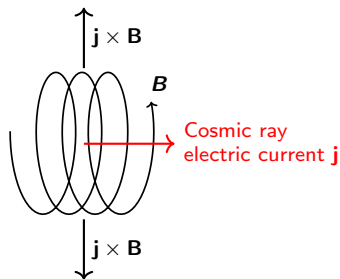
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The Bell instability can amplify B-fields

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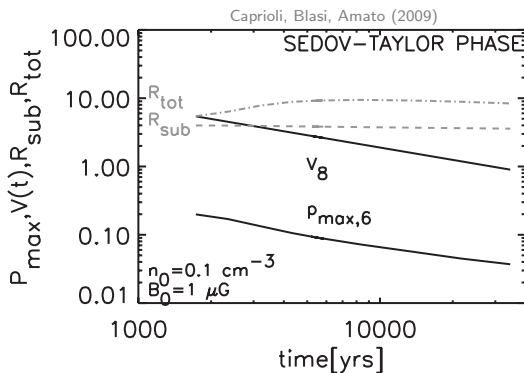
- If B -field too weak, particles escape
- Electric current \mathbf{j}
- Waves modes unstable in the presence of current \mathbf{j}



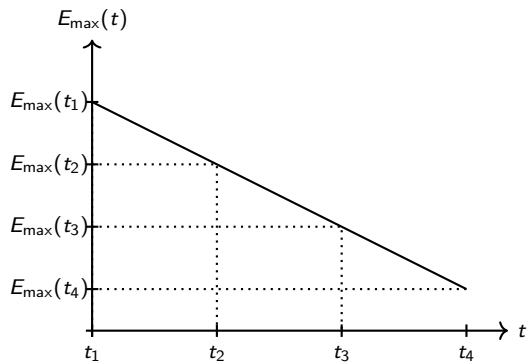
- CRs with gyroradius r_g tied to field lines
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- B -field density satisfies $\epsilon_B \sim \frac{U_{sh}}{c} \epsilon_{CR}$

The highest CR energies can be achieved at start of Sedov-Taylor phase

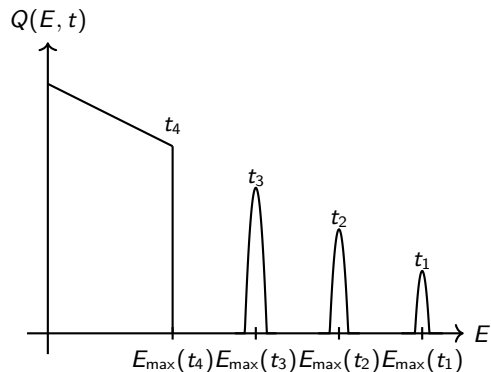
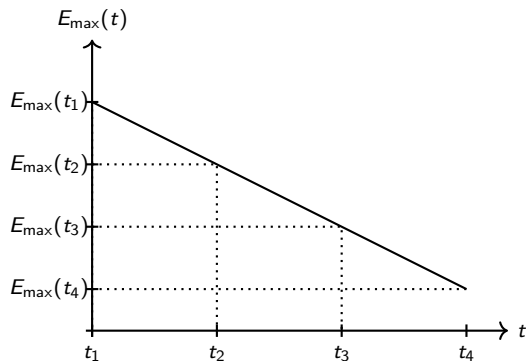
- Shock speed U_{sh} enters into growth rate γ through escape current j
 - Saturation field $B \propto U_{\text{sh}}^{3/2}$
 - $U_{\text{sh}} \propto t_{\text{age}}^{-3/5}$
- $E_{\text{max}} \propto U_{\text{sh}}^2 B t_{\text{age}} \propto t_{\text{age}}^{-11/10}$



Time-dependence of B-field amplification determines CR escape



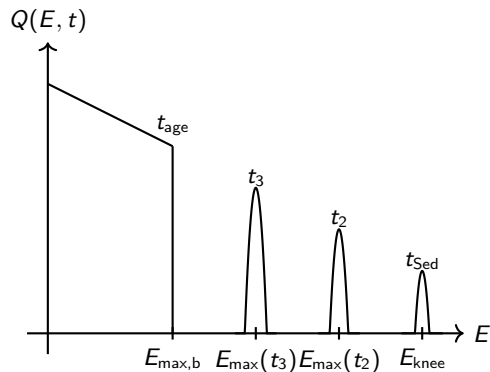
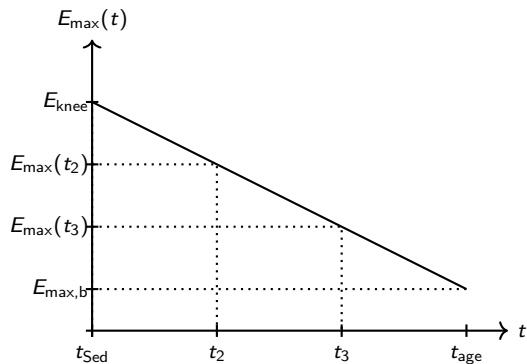
Time-dependence of B-field amplification determines CR escape



also Gabici, Aharonian, Casanova (2009); Caprioli, Blasi, Amato (2010);
Blasi and Amato (2012); Thoudam and Hörandel (2012)

- E_{\max} decreases with time
- At any one time t , particles of energy $E_{\max}(t)$ escape
- Ultimately, all particles with $E < E_{\max,b}$ escape

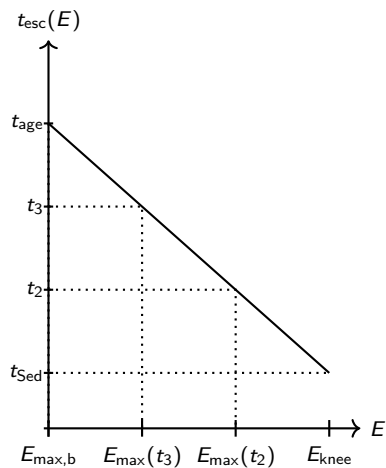
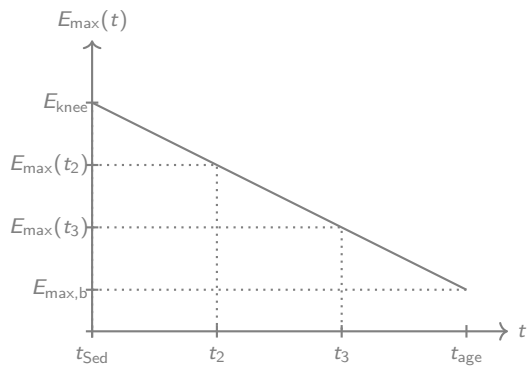
Time-dependence of B-field amplification determines CR escape



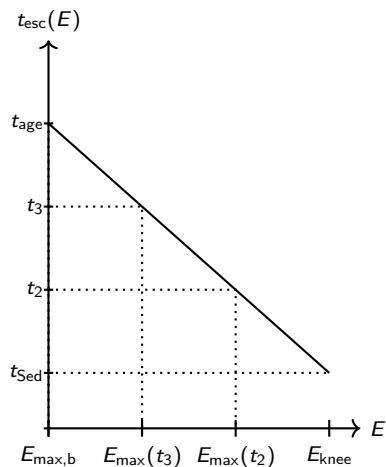
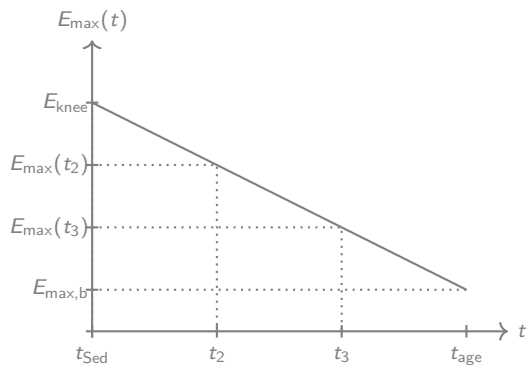
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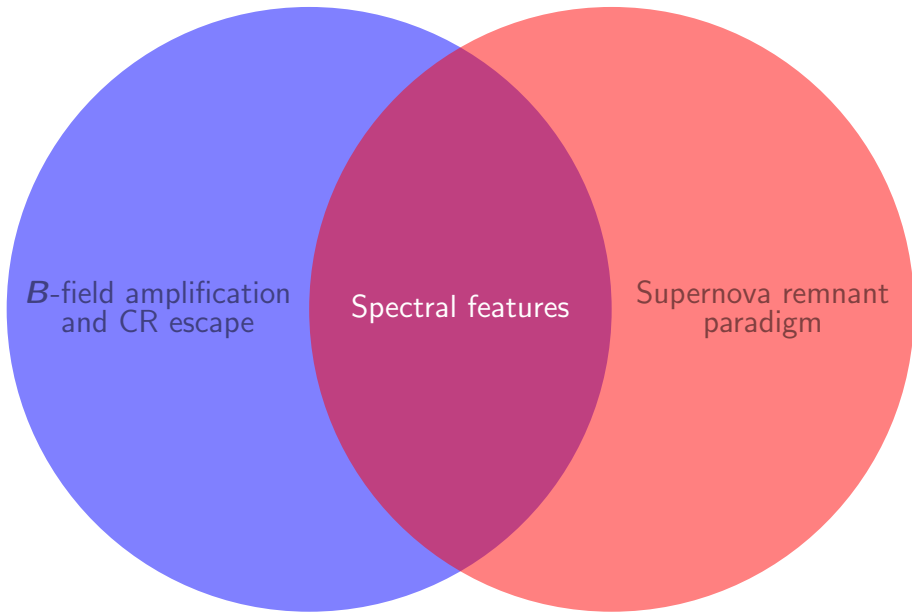
Time-dependence of B-field amplification determines CR escape



Time-dependence of B-field amplification determines CR escape

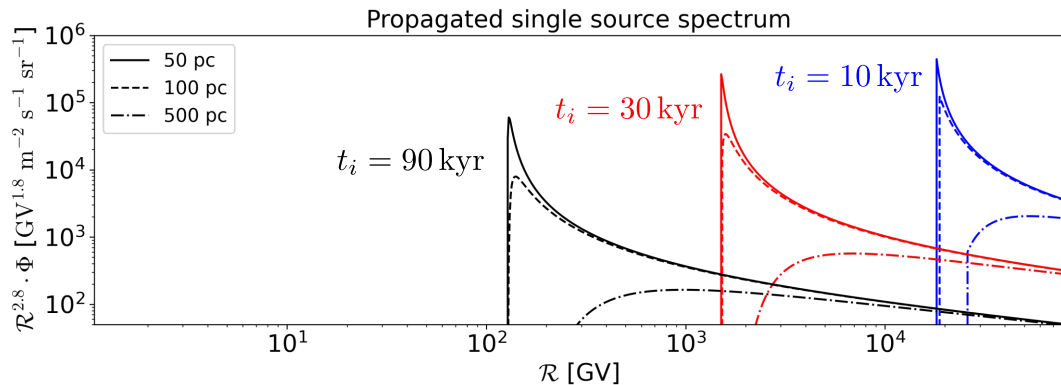


**Cosmic-Ray Energy-Dependent Injection Time
(CREDIT) scenario**



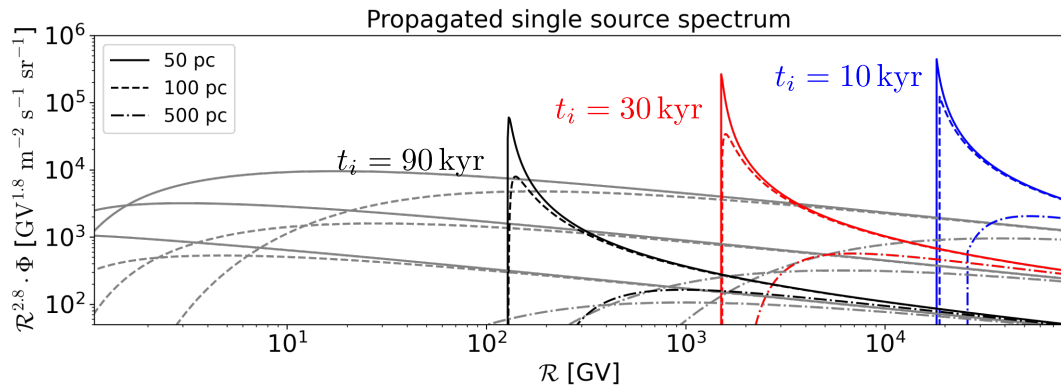
The Green's function has narrow spectral features

$$\frac{\partial G}{\partial t} - \nabla \cdot \kappa \cdot \nabla G = \delta^{(3)}(\mathbf{r} - \mathbf{r}_i) \delta(t - t_i - t_{\text{esc}}(E)) Q(E)$$



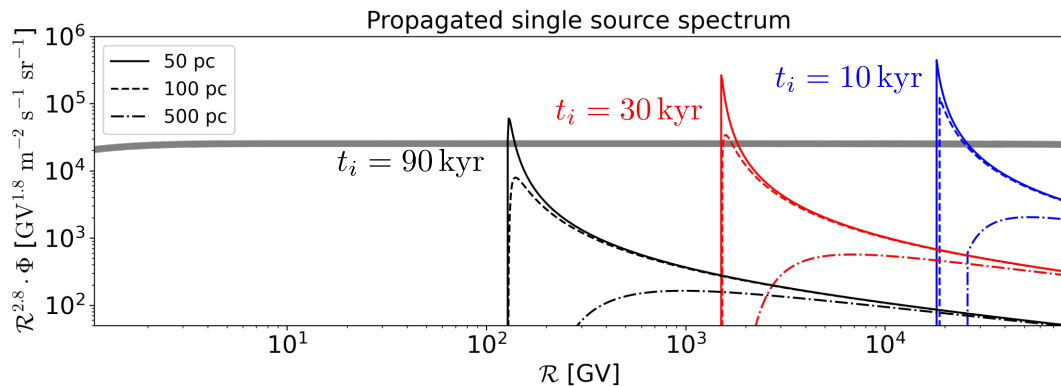
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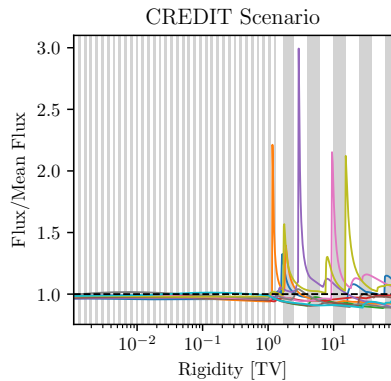
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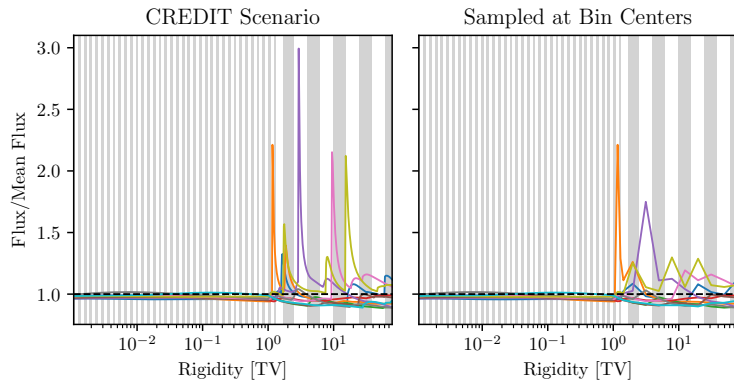
CREDIT scenario predicts dramatic spectral features

Stall, Loo, Mertsch (2025)



CREDIT scenario predicts dramatic spectral features

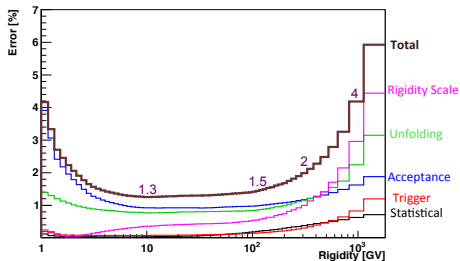
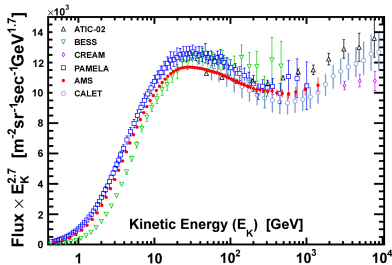
Stall, Loo, Mertsch (2025)



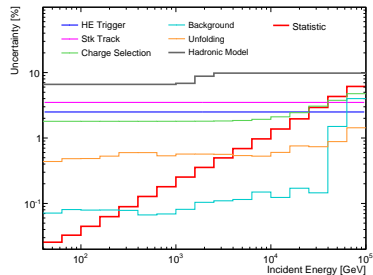
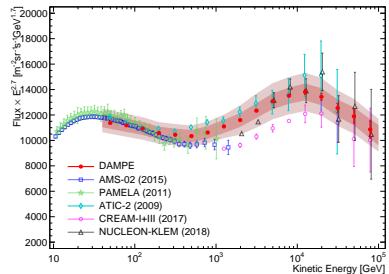
Modern proton data offer unprecedented accuracy

V. Choutko (2015), An *et al.* (2019), Aguilar *et al.* (2020),

AMS-02

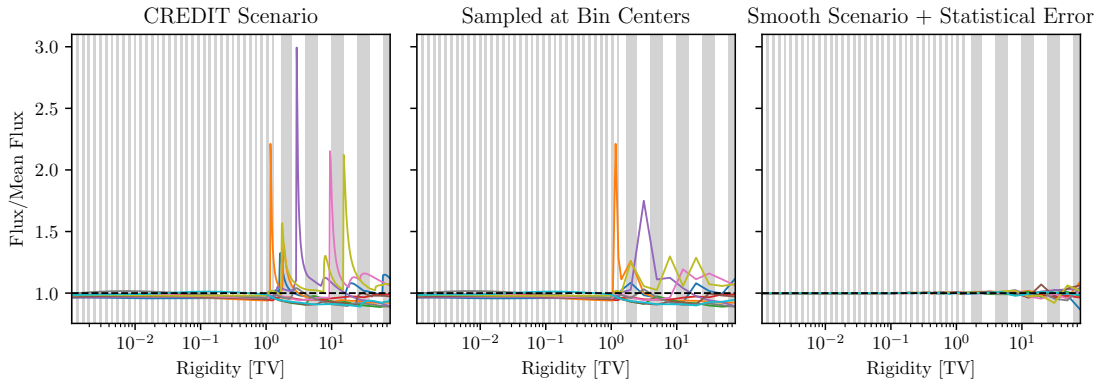


DAMPE



Statistical errors are much smaller than CREDIT features

Stall, Loo, Mertsch (2025)

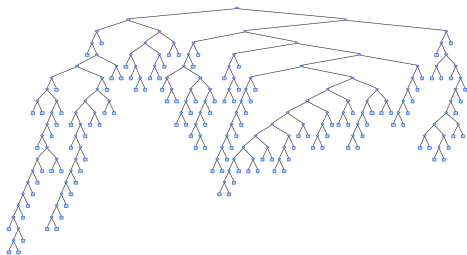


We can confidently discriminate between the different scenarios

Stall, Loo, Mertsch (2025)

Can discriminate features
from statistical fluctuations?

→ Classical machine learning task



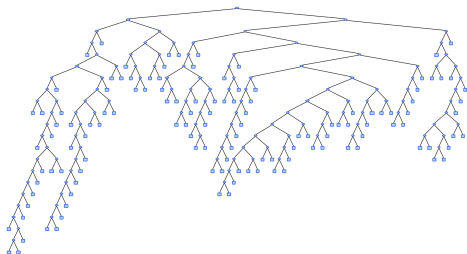
Decision tree

We can confidently discriminate between the different scenarios

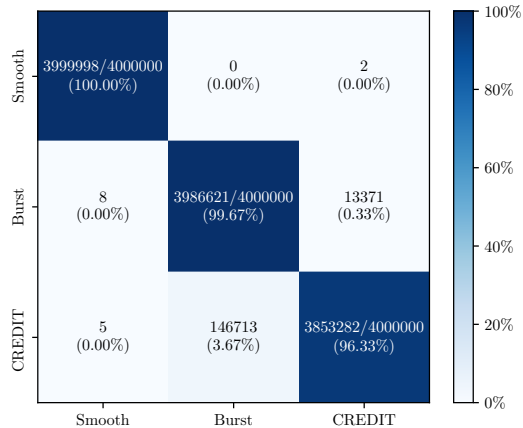
Stall, Loo, Mertsch (2025)

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Decision tree

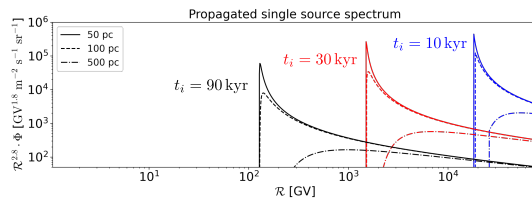


The results are going to be interesting either way

Classifier finds ...

1. CREDIT scenario

→ Investigate sources



The results are going to be interesting either way

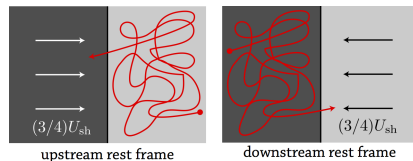
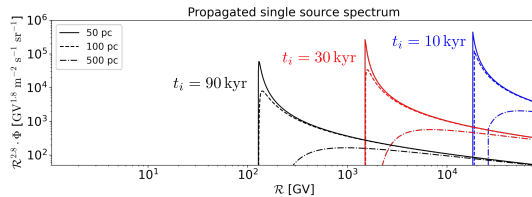
Classifier finds ...

1. CREDIT scenario

→ Investigate sources

2. Burst-like scenario ($E_{\max,b} \rightarrow \infty$)

→ Constraints on acceleration models



The results are going to be interesting either way

Classifier finds ...

1. CREDIT scenario

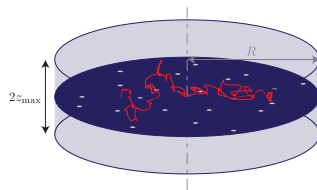
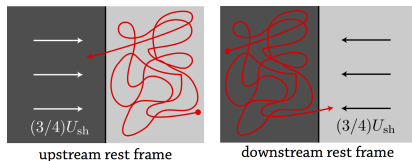
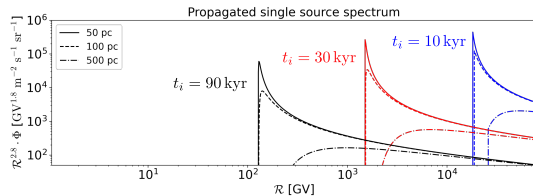
→ Investigate sources

2. Burst-like scenario ($E_{\max,b} \rightarrow \infty$)

→ Constraints on acceleration models

3. Smooth scenario

→ Trouble for supernova remnant paradigm

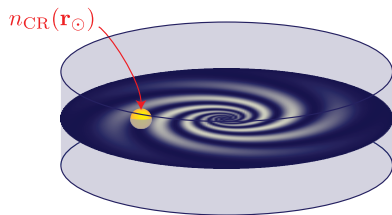


Outline

- ① Introduction
- ② Stochastic modelling of cosmic rays
- ③ Stochastic diffuse emission**
- ④ Field line transport and particle transport in synthetic turbulence
- ⑤ Summary & conclusion

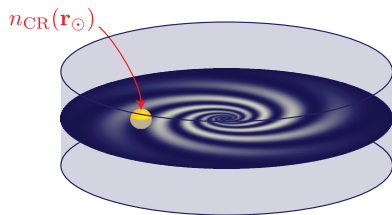
Local fluxes vs diffuse emission

Local fluxes

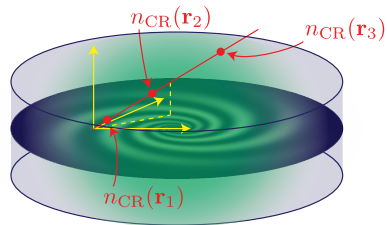


Local fluxes vs diffuse emission

Local fluxes



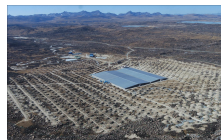
Diffuse emission



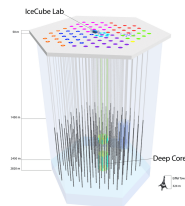
Recently measured at \sim PeV



Tibet AS γ +MD



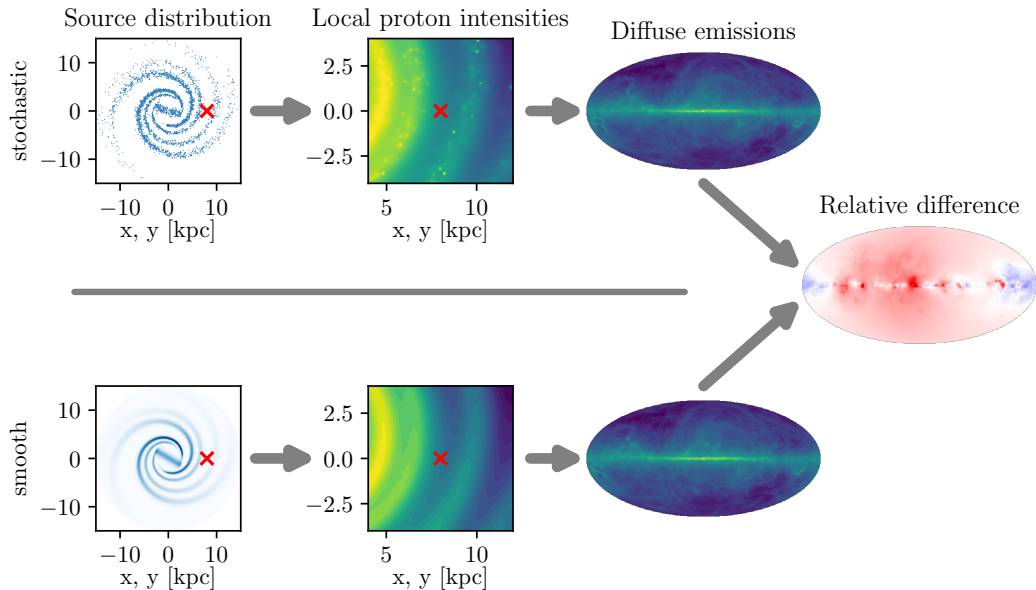
LHAASO



IceCube

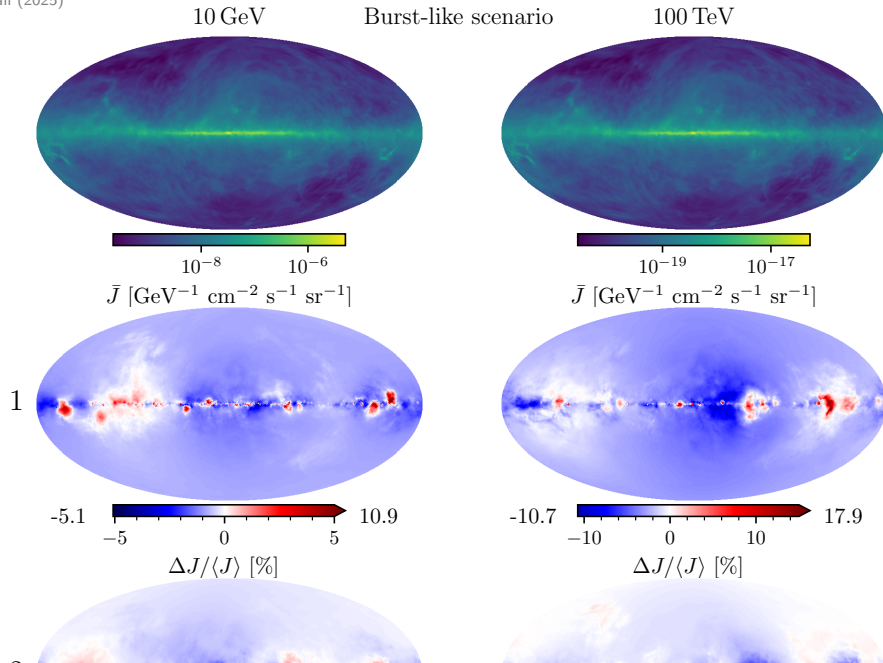
Stochastic diffuse emission

Mertsch & Stall (2025)



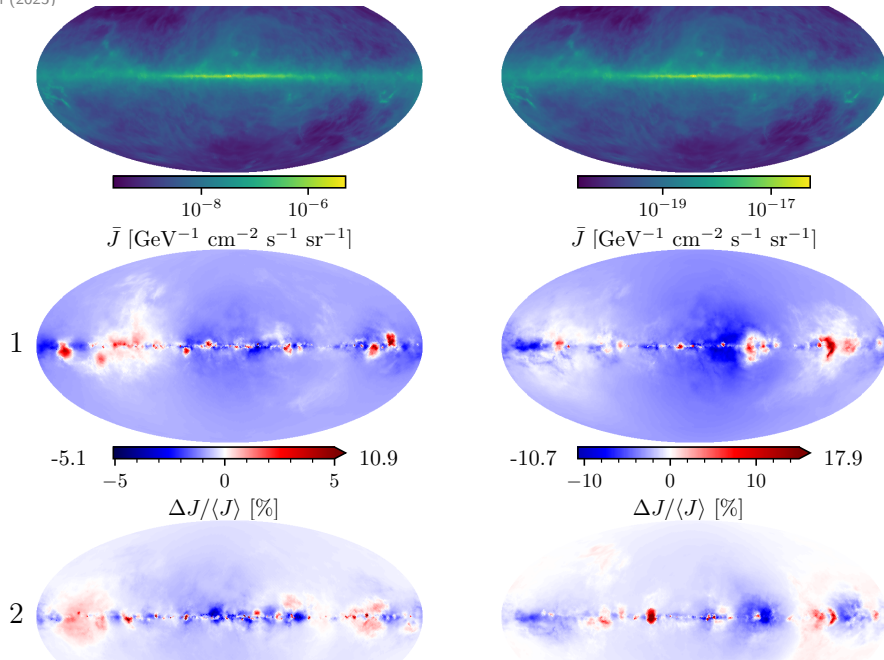
Stochastic diffuse emission

Mertsch & Stall (2025)



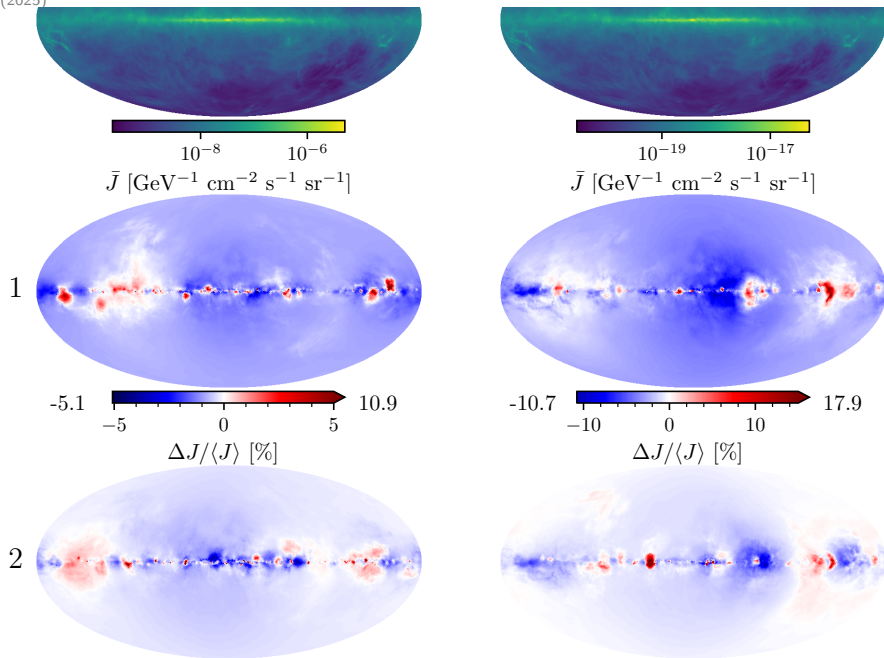
Stochastic diffuse emission

Mertsch & Stall (2025)



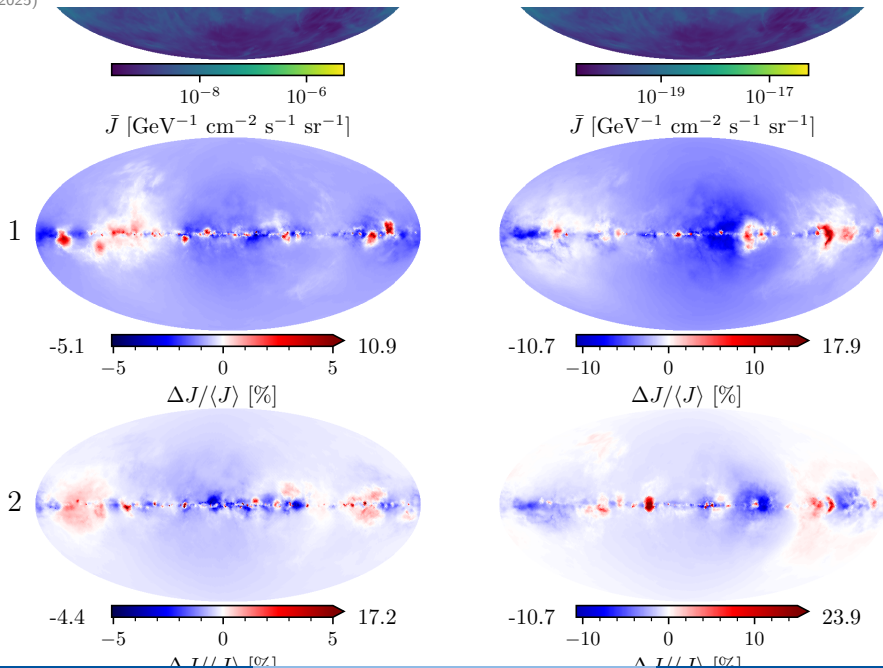
Stochastic diffuse emission

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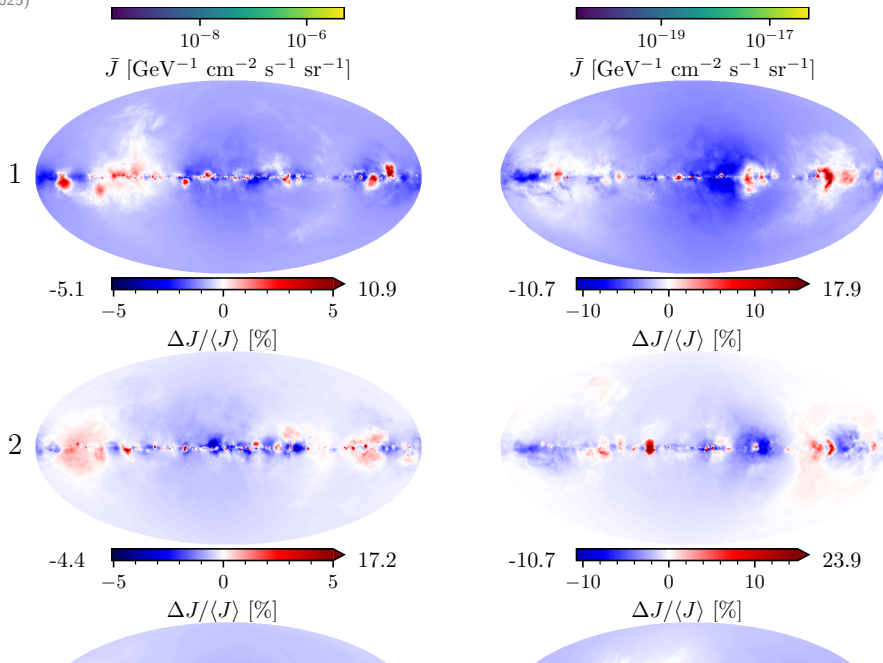
Stochastic diffuse emission

Mertsch & Stall (2025)



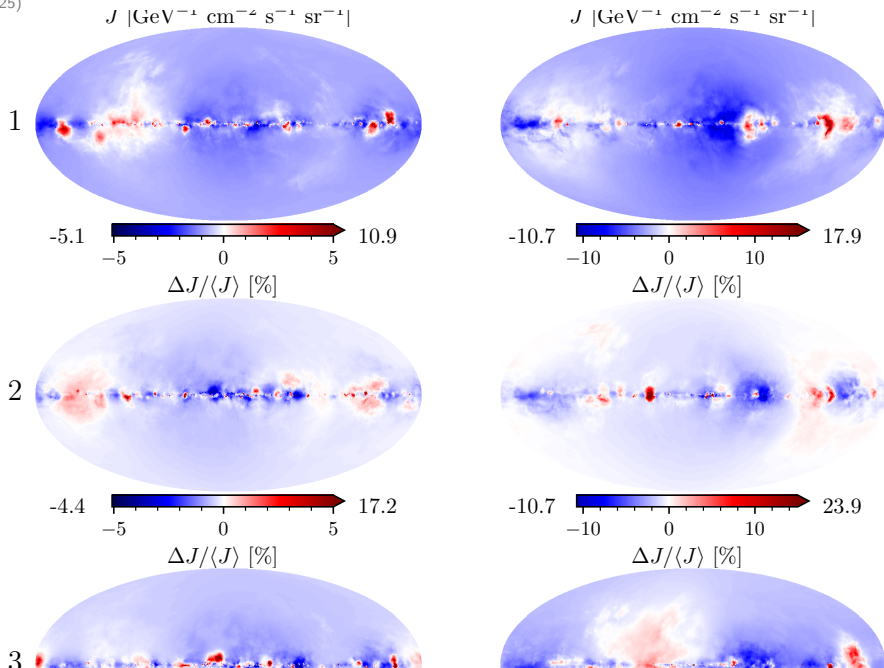
Stochastic diffuse emission

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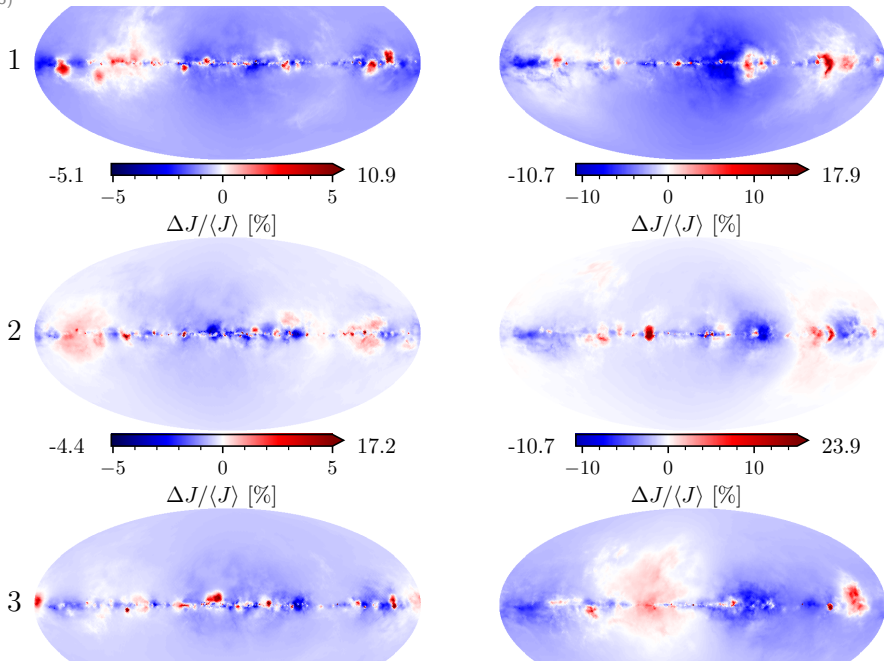
Stochastic diffuse emission

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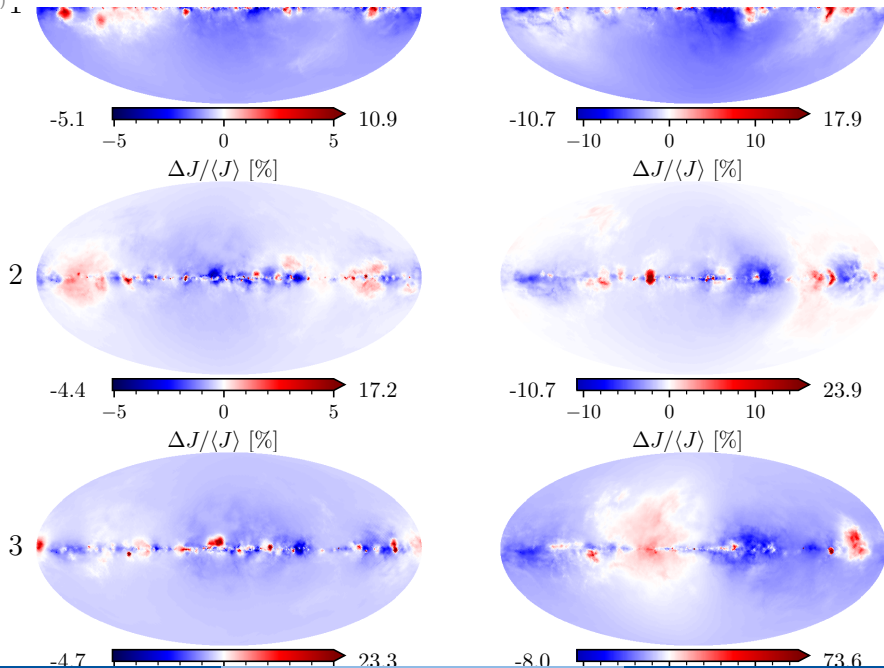
Stochastic diffuse emission

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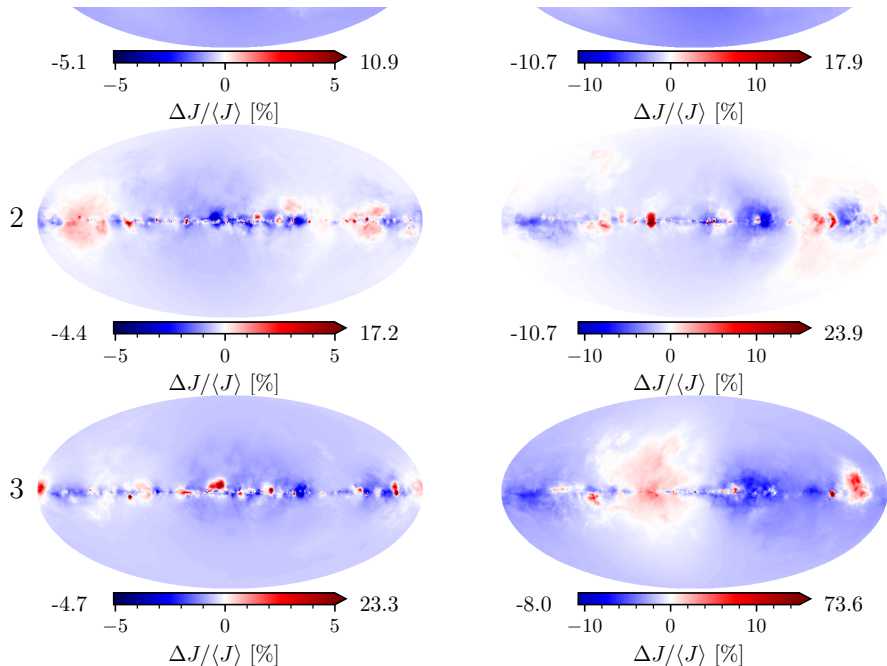
Stochastic diffuse emission

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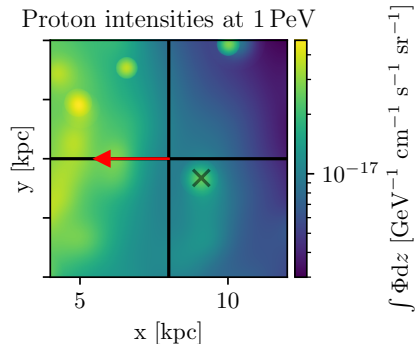
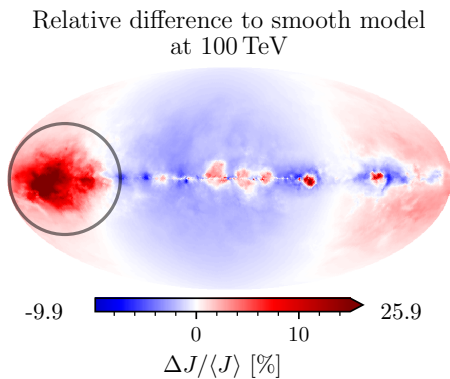
Stochastic diffuse emission

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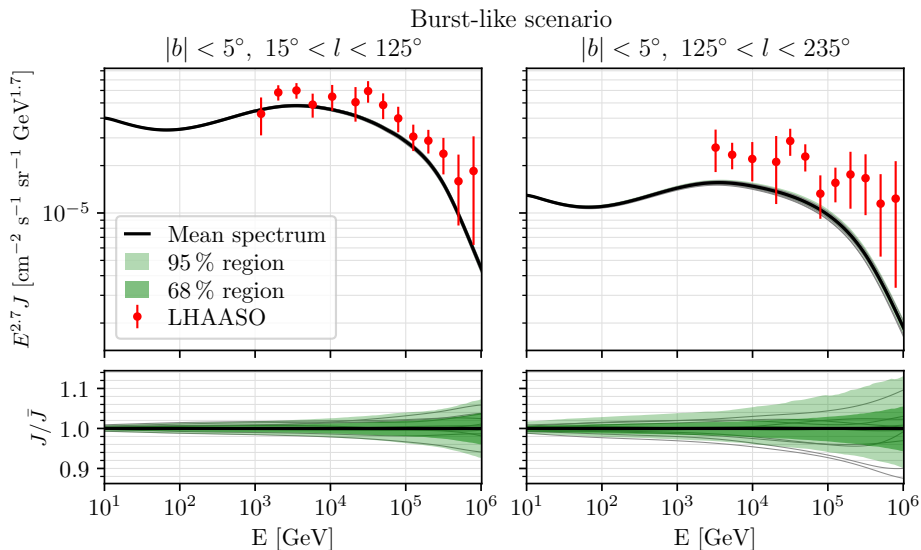
Sources manifest as deviations from the smooth model

Mertsch & Stall (2025)



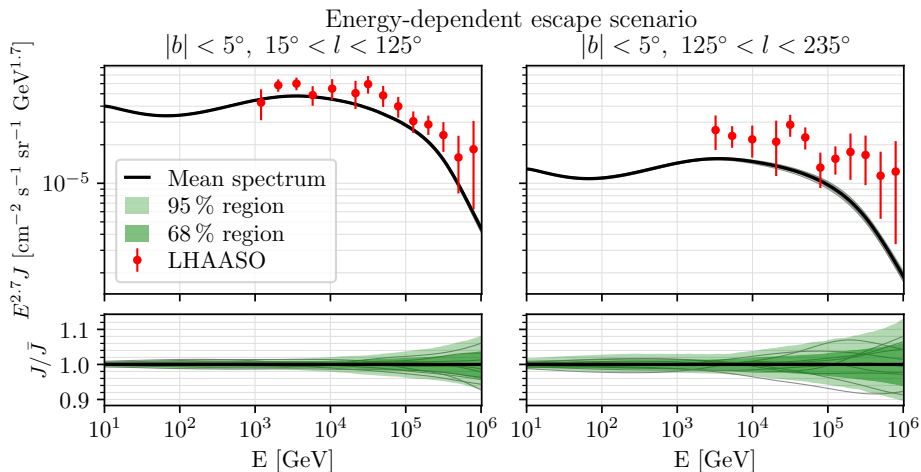
Deviations from the ensemble average can be sizeable

Mertsch & Stall (2025)



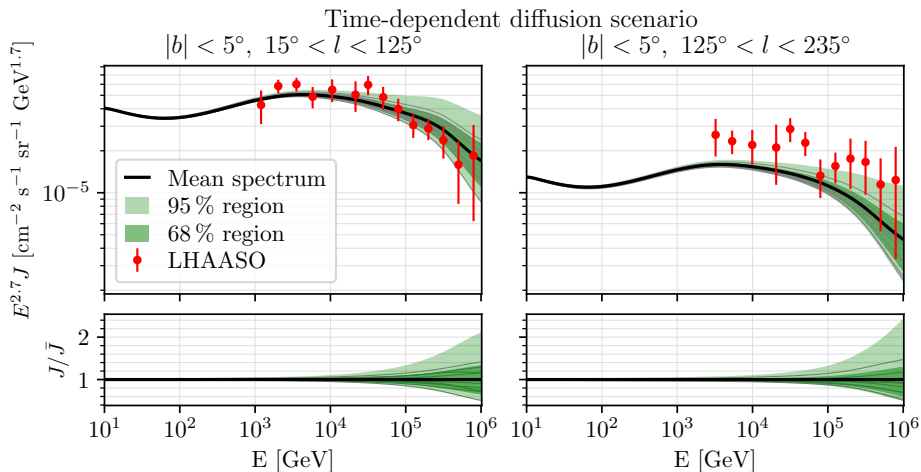
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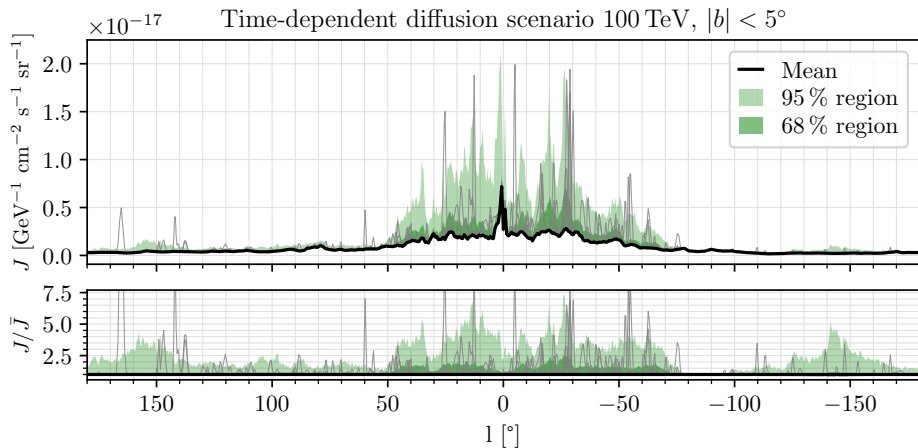
Deviations from the ensemble average can be sizeable

Mertsch & Stall (2025)



Along the galactic plane, intensity can be enhanced by factors of a few

Mertsch & Stall (2025)



Outline

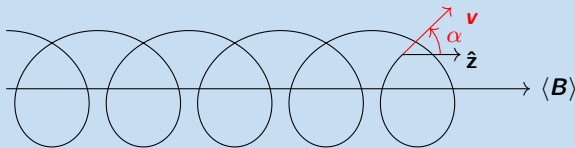
- ① Introduction
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Parallel transport

- Magnetic field: $\langle \mathbf{B} \rangle + \delta \mathbf{B}$ \rightarrow phase-space density: $\langle f \rangle + \delta f$

$$\Rightarrow \frac{\partial \langle f \rangle}{\partial t} + \mathbf{v} \cdot \nabla_r \langle f \rangle = \int_0^t dt \left\langle (\mathbf{v} \times \delta \mathbf{B}) \cdot \nabla_p \left[(\mathbf{v} \times \delta \mathbf{B}) \cdot \nabla_p \langle f \rangle \right]_{\mathbf{r}(t')} \right\rangle$$

Unperturbed trajectory $\mathbf{r}(t)$ characterised by pitch-angle cosine $\mu \equiv \cos \alpha$



$$\Rightarrow \text{Pitch-angle scattering } \frac{\partial \langle f \rangle}{\partial t} + v \mu \frac{\partial \langle f \rangle}{\partial z} = \frac{\partial}{\partial \mu} D_{\mu\mu} \frac{\partial \langle f \rangle}{\partial \mu} \quad \text{with} \quad D_{\mu\mu} \sim \left(\frac{\delta B^2}{B_0^2} \right)^{-1} \left(\frac{r_g}{L_c} \right)^{q-2} \Omega_g$$

$$\Rightarrow \text{For isotropic phase-space density } \bar{f}: \frac{\partial \bar{f}}{\partial t} - \frac{\partial}{\partial z} \kappa_{\parallel} \frac{\partial \bar{f}}{\partial z} = 0 \quad \text{with} \quad \kappa_{\parallel} = \int_{-1}^1 d\mu \frac{(1 - \mu^2)^2}{D_{\mu\mu}}$$

Perpendicular transport = particle transport along field line + transport of field line

--- field line
— particle trajectory



(a)



(b)

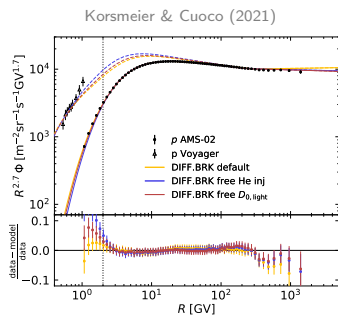


(c)

- (a) Straight field line and gyration
- (b) Wandering field line and gyration
- (c) Wandering field line and diffusion

Particle “jumps” field lines upon scattering

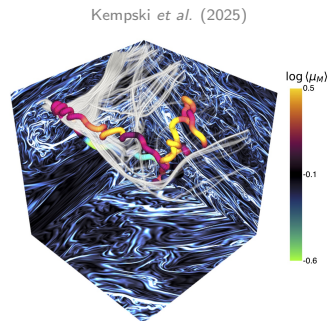
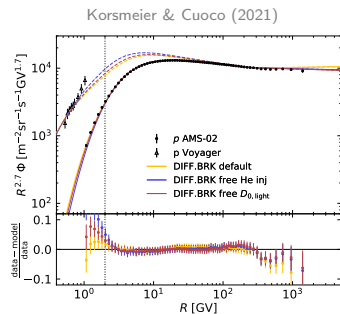
How to model particle diffusion



Phenomenological models

- Parametric form for $\kappa(\mathcal{R})$
- Fit to CR data 😊
- Connection to microphysics? ☹️

How to model particle diffusion



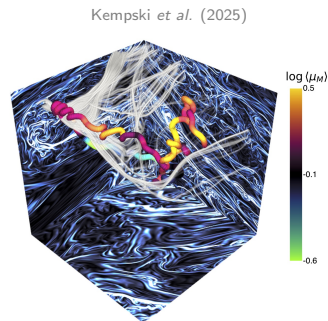
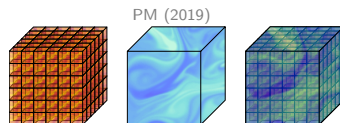
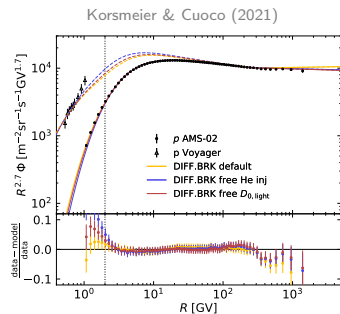
Phenomenological models

- Parametric form for $\kappa(\mathcal{R})$
- Fit to CR data 😊
- Connection to microphysics? ☹️

MHD models

- $\kappa(\mathcal{R})$ computed from microphysics
- First principles 😊
- Do not fit data ☹️

How to model particle diffusion



Phenomenological models

- Parametric form for $\kappa(\mathcal{R})$
- Fit to CR data 😊
- Connection to microphysics? 😞

Synthetic turbulence models

- Parametric form for $\kappa(\mathcal{R})$
- Dynamical range 😊
- Realistic? 😞

MHD models

- $\kappa(\mathcal{R})$ computed from microphysics
- First principles 😊
- Do not fit data 😞

Test particle simulations

Kuhlen, Mertsch, Phan (2025); also Mertsch (2019)



Results depend on:

- 1 Set up realisation of δB on computer
- 2 Propagate a large number of particles for long times
- 3 Rinse and repeat
- 4 Running diffusion coefficients:

$$d_{\parallel}(t) \equiv \frac{1}{2} \frac{d}{dt} \langle (\Delta z)^2 \rangle$$

$$d_{\perp}(t) \equiv \frac{1}{2} \frac{d}{dt} \langle (\Delta r_{\perp})^2 \rangle$$

- Reduced time: Ωt
- Reduced rigidity: $\frac{r_g}{L_c}$
- Turbulence level: $\eta = \frac{\delta B^2}{B_0^2 + \delta B^2}$

MHD equations:

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{V}) = 0$$

$$\rho (\partial_t + \mathbf{V} \cdot \nabla) \mathbf{V} = -\nabla P + \frac{1}{4\pi} (\nabla \times \mathbf{B}) \times \mathbf{B}$$

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{V} \times \mathbf{B})$$

$$\frac{d}{dt} \left(\frac{P}{\rho^\gamma} \right) = 0$$

Alfvén modes

- Incompressible: $\delta\rho, \delta P = 0$
- Restoring force: magnetic tension
- $\delta \mathbf{B} \perp \mathbf{B}_0, \mathbf{k}$

Magnetosonic modes

- Compressible: $\delta\rho, \delta P \neq 0$
- Restoring force: gas + magnetic pressure
- $\delta \mathbf{B} \perp \mathbf{k}$, but with components $\parallel \mathbf{B}_0$

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Isotropic modes

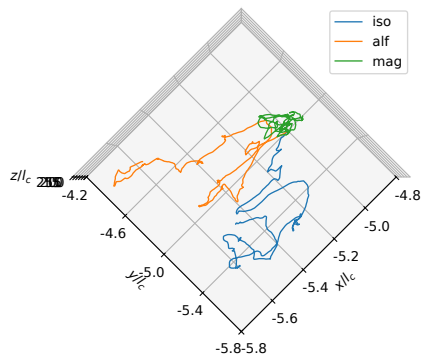
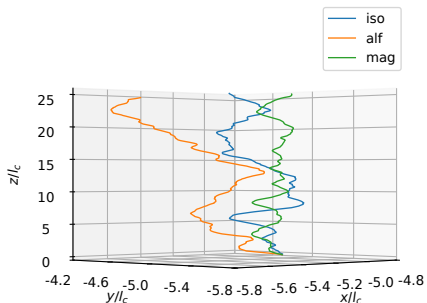
- Equal mixture of Alfvén and magnetosonic
- Same power in all directions

Field lines

Bouchet *et al.* (in prep.)

Integrate field line equation:

$$\frac{d\mathbf{r}(s)}{ds} = \frac{\mathbf{B}_0 + \delta\mathbf{B}(\mathbf{r}(s))}{|\mathbf{B}_0 + \delta\mathbf{B}(\mathbf{r}(s))|}$$



Much less variation in perpendicular direction for magnetosonic than for Alfvénic

Field line diffusion

- Running field-line diffusion coefficient

$$\mathcal{D}_\perp(s) = \frac{1}{4} \frac{d}{ds} \left\langle (x(s) - x_0)^2 + (y(s) - y_0)^2 \right\rangle$$

- Asymptotic field-line diffusion coefficient

$$\mathcal{K}_\perp(s) = \lim_{s \rightarrow \infty} \mathcal{D}_\perp(s)$$

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Alternative parametrisation

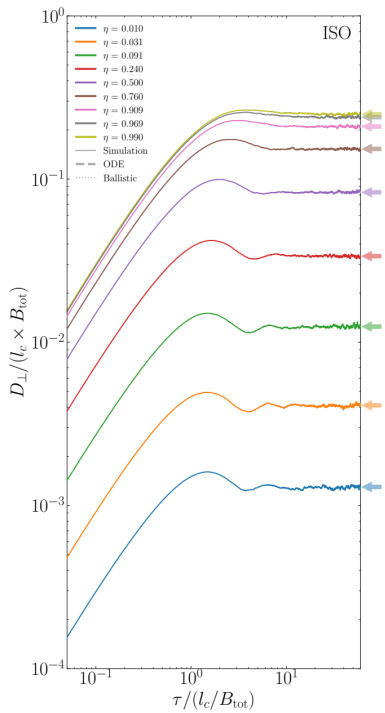
Sonsretter *et al.* (2019)

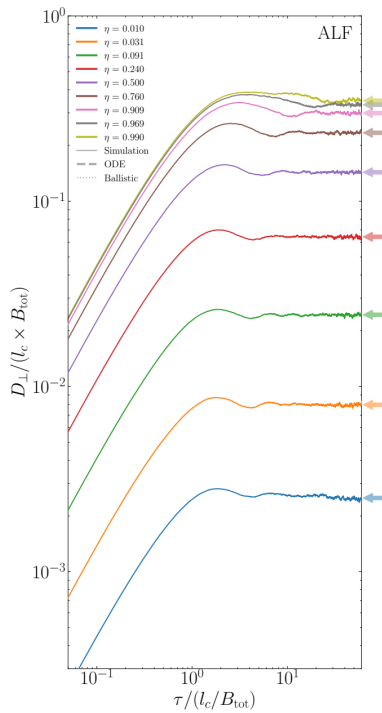
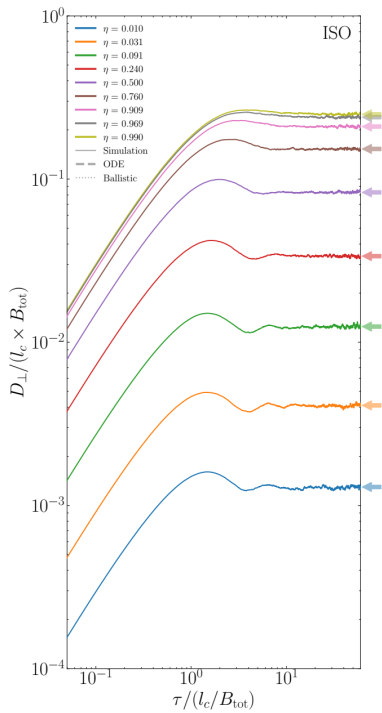
- Reparametrise $s \rightarrow \tau$: $d\tau \equiv \frac{ds}{|\mathbf{B}|} \rightarrow \frac{d\mathbf{r}(s)}{d\tau} = \mathbf{B}_0 + \delta\mathbf{B}(\mathbf{r}(s))$
- Running field-line diffusion coefficient

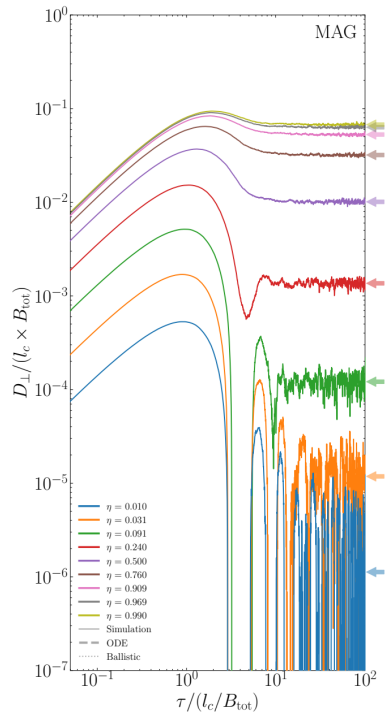
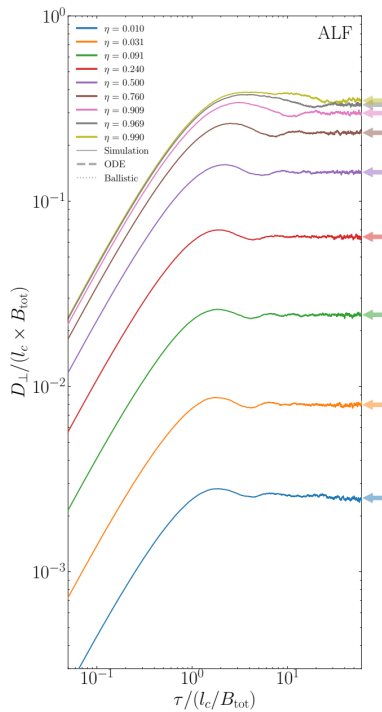
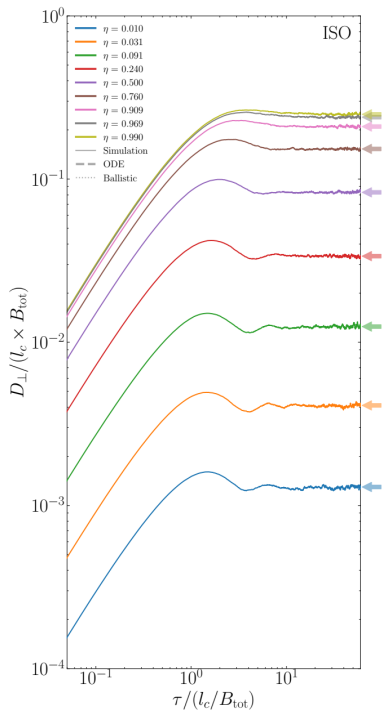
$$D_\perp(\tau) = \frac{1}{4} \frac{d}{d\tau} \left\langle (x(\tau) - x_0)^2 + (y(\tau) - y_0)^2 \right\rangle$$

- Asymptotic field-line diffusion coefficient

$$K_\perp = \lim_{\tau \rightarrow \infty} D_\perp(s)$$



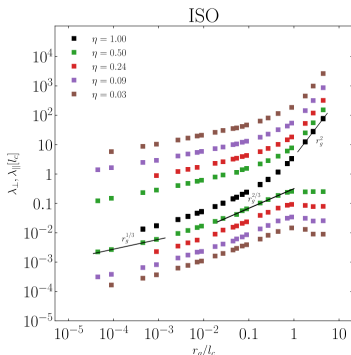




Perpendicular particle transport

Bouchet *et al.* (in prep.)

Turbulence level:
 $\eta = \delta B^2 / (B_0^2 + \delta B^2)$



- $10^{-3} \ll r_g/l_c \ll 1$:
extended transition

- $r_g \ll 10^{-3}$:
 $\lambda_{\perp} \propto (r_g/l_c)^{1/3}$

Dundovic *et al.* (2020),

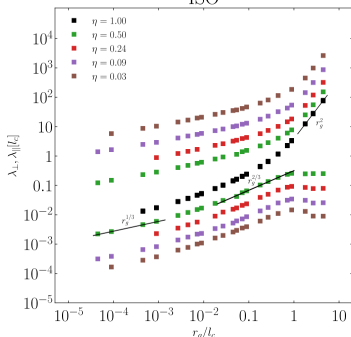
Kuhlen, Phan, Mertsch (2025)

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ISO



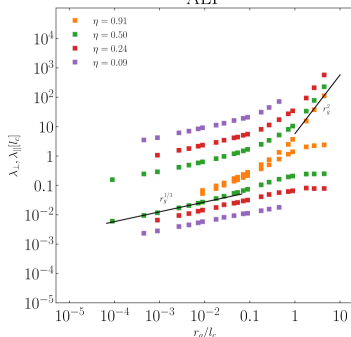
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Dundovic *et al.* (2020),

Kuhlen, Phan, Mertsch (2025)

ALF



- $10^{-3} \ll r_g/l_c \ll 1$:
no extended transition

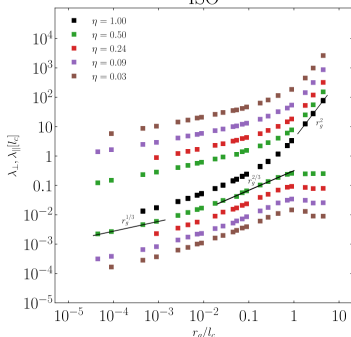
Perpendicular particle transport

Bouchet *et al.* (*in prep.*)

Turbulence level:

$$\eta = \delta B^2 / (B_\infty^2 + \delta B^2)$$

ISO



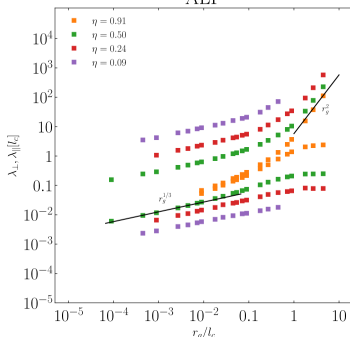
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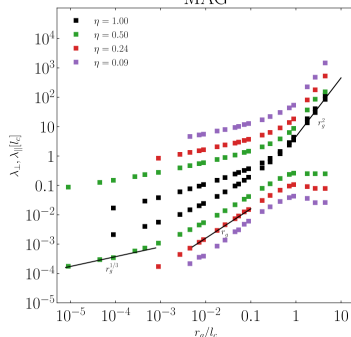
Kuhlen, Phan, Mertsch (2025)

ALF



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no extended transition

MAG

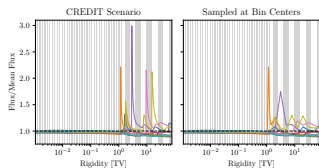


- $10^{-3} \ll r_g/l_c \ll 1$:
extended transition $\lambda_\perp \propto r_g/l_c$

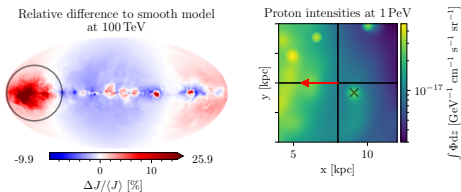
Outline

- ① Introduction
- ② Stochastic modelling of cosmic rays
- ③ Stochastic diffuse emission
- ④ Field line transport and particle transport in synthetic turbulence
- ⑤ Summary & conclusion

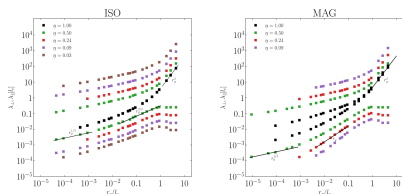
Summary



Shock acceleration and supernova paradigm \rightarrow stark spectral features



Fluctuations in local CR density \rightarrow effect on diffuse γ -ray maps



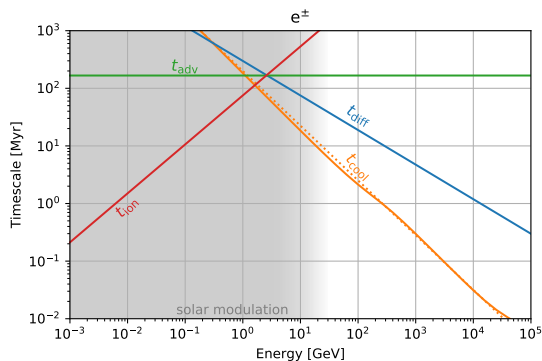
Non-standard transport of magnetic field lines and particles in polarised synthetic turbulence

Outline

⑥ Backup

Time scales

Time scales:



- $t_{\text{diff}} = \frac{z_{\text{max}}^2}{2\kappa}$ with $z_{\text{max}} = 5$ kpc, $\kappa(10 \text{ GV}) = 5 \times 10^{28} \text{ cm}^2 \text{ s}^{-1}$
- t_{cool} : KN cross-section with $\rho = \{0.26, 0.6, 0.6, 0.1\} \text{ eV cm}^{-3}$ for CMB, IR, opt, UV; $3 \mu\text{G}$ B-field
- t_{ion} : $n_{\text{H}} = 0.5 \text{ cm}^{-3}$ (WIM) and $n_{\text{H}} = 0.5 \text{ cm}^{-3}$ (WNM) and 100 pc wide gas disk

In a diffusion model with $E^{-\Gamma}$ sources in disk:

- $\phi(E) \propto E^{-\Gamma-\delta}$ if diffusion dominated
- $\phi(E) \propto E^{-\Gamma-(\delta+1)/2}$ if cooling dominated