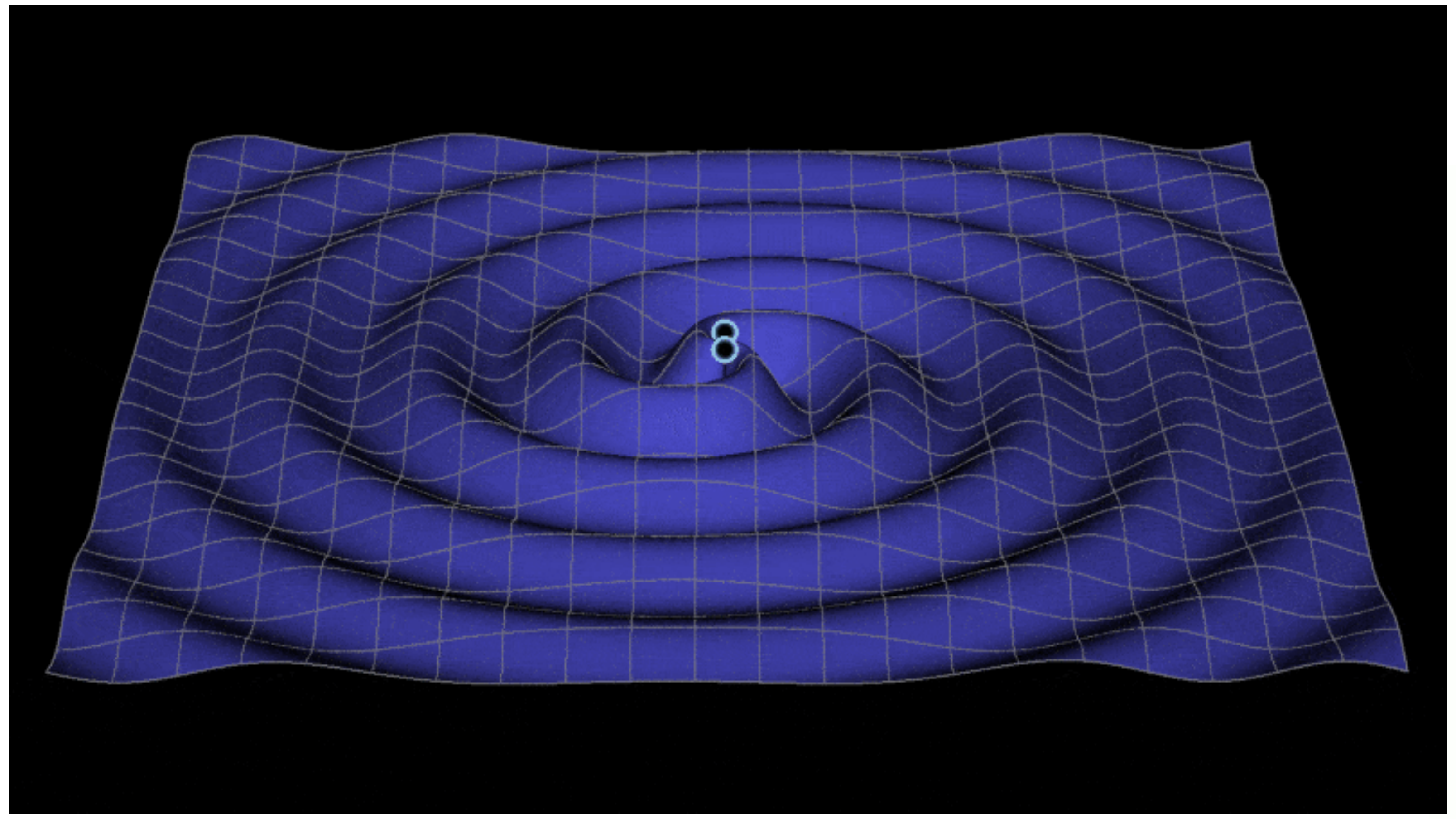
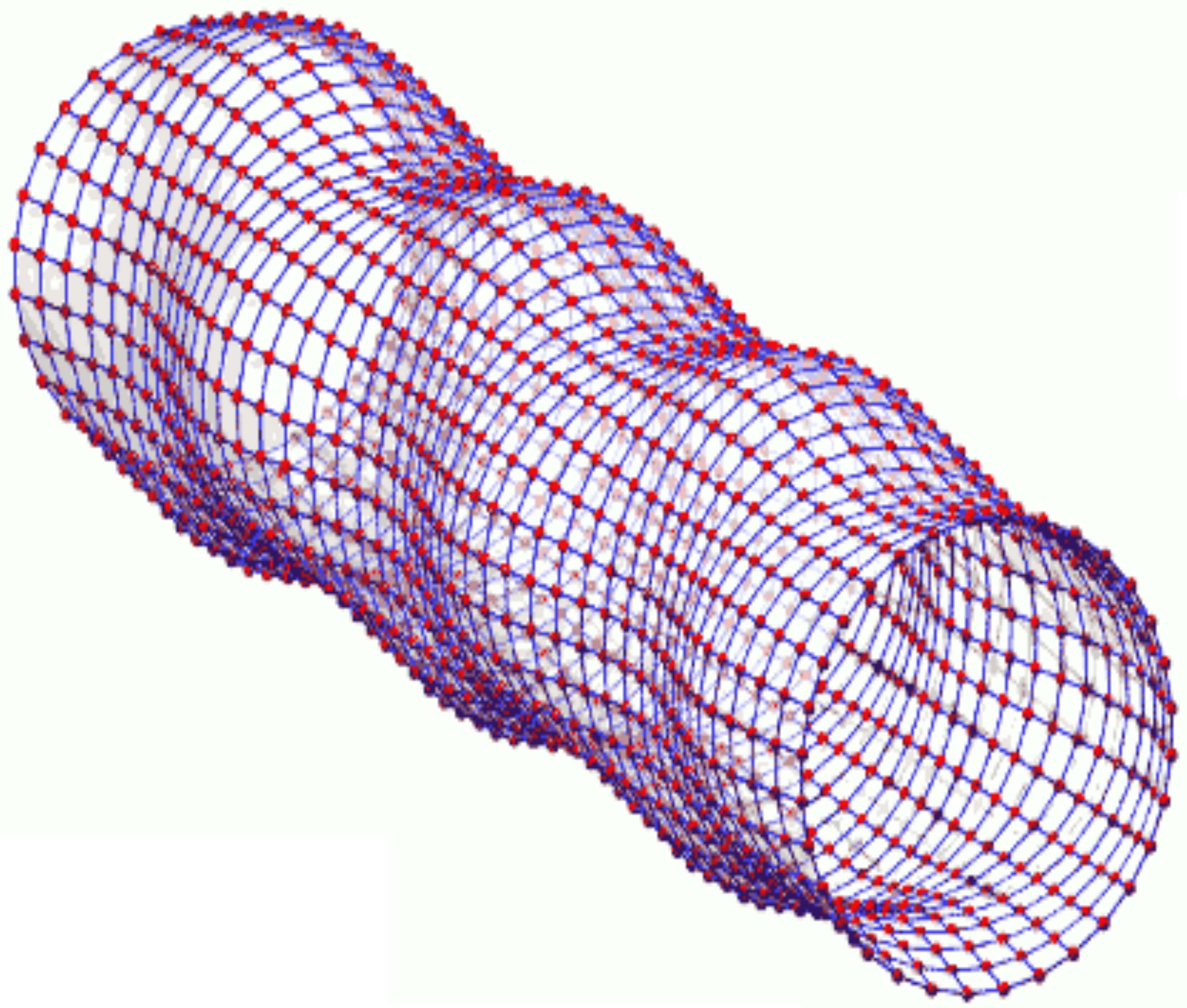
Welcome to the ACME tutorial session!





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Slides based on **Ulyana Dupletsa** and **Jacopo Tissino**'s slide presented at the ET Scienza e Tecnologia meeting in Assisi

https://agenda.infn.it/event/38405/contributions/218803/

Very recommended paper based / using GWFish

Branchesi et al. - *JCAP* 07 (2023) 068

ET collaboration, Abac et al. 2023, 2503.12263

Dupletsa et al. - *Astron.Comput.* 42 (2023) 100671

Banerjee et al. - Astron.Astrophys. 678 (2023) A126

Cozzumbo et al. - *JCAP* 05 (2025) 021

Loffredo et al. Astron.Astrophys. 697 (2025) A36

Ronchini et al. Astron.Astrophys. 665 (2022) A97

Dupletsa et al. *Phys.Rev.D* 111 (2025)

Benetti et al. (2025), 2509.07849

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When we build Einstein Telescope, how many compact binary signals will it be able to detect?
How well will it localize them in the sky?
How well will it measure their parameters?

Branchesi+ (2023)

ET collaboration, Abac+ (2025)

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Matched filtering and GW detection

$$d(t) = h_\theta(t) + n_{\text{Gaussian}}(t) + n_{\text{non-Gaussian}}(t)$$

We want to find $h(t)$ when $|h| \ll |n|$.

The Gaussian component of the noise has a Power Spectral Density (PSD) $S_n(f)$

$$\langle \tilde{n}(f)\tilde{n}^*(f') \rangle = \frac{1}{2} S_n(f)\delta(f - f')$$

We can apply a linear filter $f(t)$ to try to increase the outcome signal $\hat{\rho}$

$$\hat{\rho} = \int \mathrm{d}t \; d(t+\tau) f(t)$$

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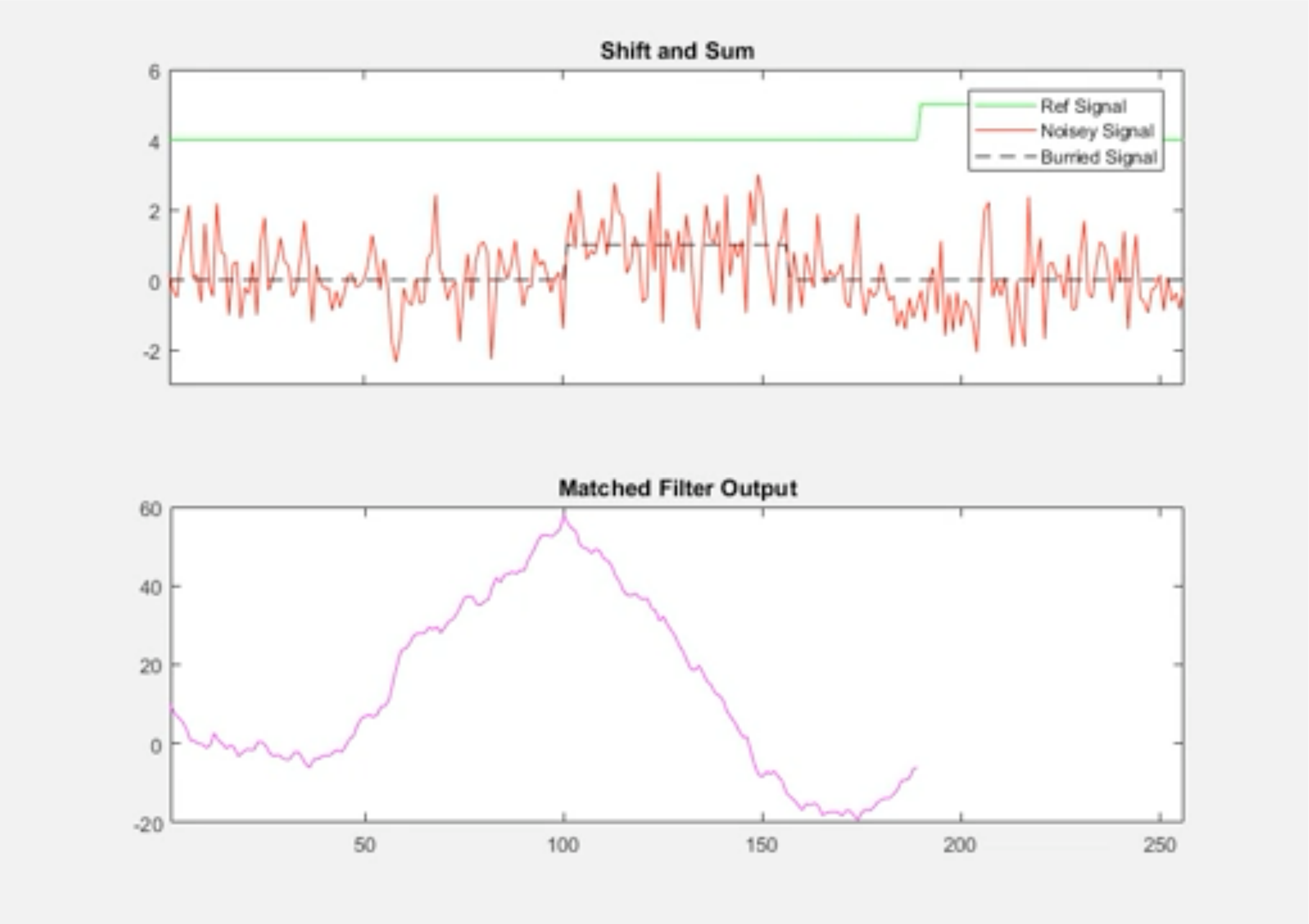
Matched filtering and GW detection



Matched filtering and GW detection

We want to maximize the "**distinguishability**" of the signal: we can quantify it with the signal-to-noise ratio

$$\frac{S}{N} = \frac{\hat{\rho}(\text{a signal is present})}{\text{root-mean-square of } \hat{\rho}}$$

Ignoring the non-Gaussian part of the noise, the **optimal** solution is $\hat{\rho} \propto (d|h)$, where

$$(a|b) = 4\Re \int_0^\infty \frac{a(f) b^*(f)}{S_n(f)} \mathrm{d}f \,,$$

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Matched filtering and GW detection

The optimal SNR is obtained when

$$\rho = \frac{\hat{\rho}}{\hat{\rho}_{\mathrm{rms}}} = \frac{S}{N} = \frac{(d\,|\,h)}{\sqrt{(h\,|\,h)}} \qquad \text{or} \qquad \mathcal{F}\{f(t)\} = \tilde{K}(f) \propto \frac{\tilde{h}(f)}{S_n(f)}$$

$n(t)$ is a zero-mean, stationary, Gaussian random process ($\langle n(t) \rangle = 0$)

$$\rho_{\mathrm{opt}} = \sqrt{(h\,|\,h)} = 2\sqrt{\int_0^\infty \mathrm{d}f\, \frac{|\,h(f)\,|^2}{S_n(f)}}$$

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SNR vs. FAR

What is a "high enough" value for the SNR?

Without time shifts nor non-Gaussianities, the SNR would simply follow a χ^2 distribution with two degrees of freedom: "five σ" significance with a threshold of $\rho = 5.5$.

In real data this has to be estimated through **injections**:

$$\mathrm{FAR} = \mathrm{FAR}_8 \exp\left(-\frac{\rho - 8}{\alpha}\right) .$$

For BNS in O1: $\alpha = 0.13$ and $\mathrm{FAR}_8 = 30000\mathrm{yr}^{-1}$.

How often you would expect a noise fluctuation to produce a trigger **as strong as or stronger than your candidate**, purely by chance.

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GW data analysis

Suppose we measure $d = h_\theta + n$ where our model for $h(t;\theta)$ depends on several parameters (typically between 10 and 15)

We can estimate the parameters θ using Bayes' theorem and **exploring the posterior distribution**

$$p(\theta\,|\,d) = \mathscr{L}(d\,|\,\theta)\pi(\theta) = \mathscr{N} \exp\left\{ (d\,|\,h_\theta) - \frac{1}{2}(h_\theta\,|\,h_\theta) \right\} \pi(\theta)$$

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GW data analysis: MCMC

The posterior is explored **stochastically** (with MCMC, nested sampling...) yielding many samples θ_i distributed according to $p(\theta|d)$, with which can compute summary statistics:

- mean $\langle\theta_i\rangle$,
- variance $\sigma_i^2 = \langle(\theta_i - \langle\theta_i\rangle)^2\rangle$,
- covariance $\mathcal{C}_{ij} = \langle(\theta_i - \langle\theta_i\rangle)(\theta_j - \langle\theta_j\rangle)\rangle$.

At this stage, we are not making any approximation, and the covariance matrix is just a **summary** tool - the full posterior is still available.

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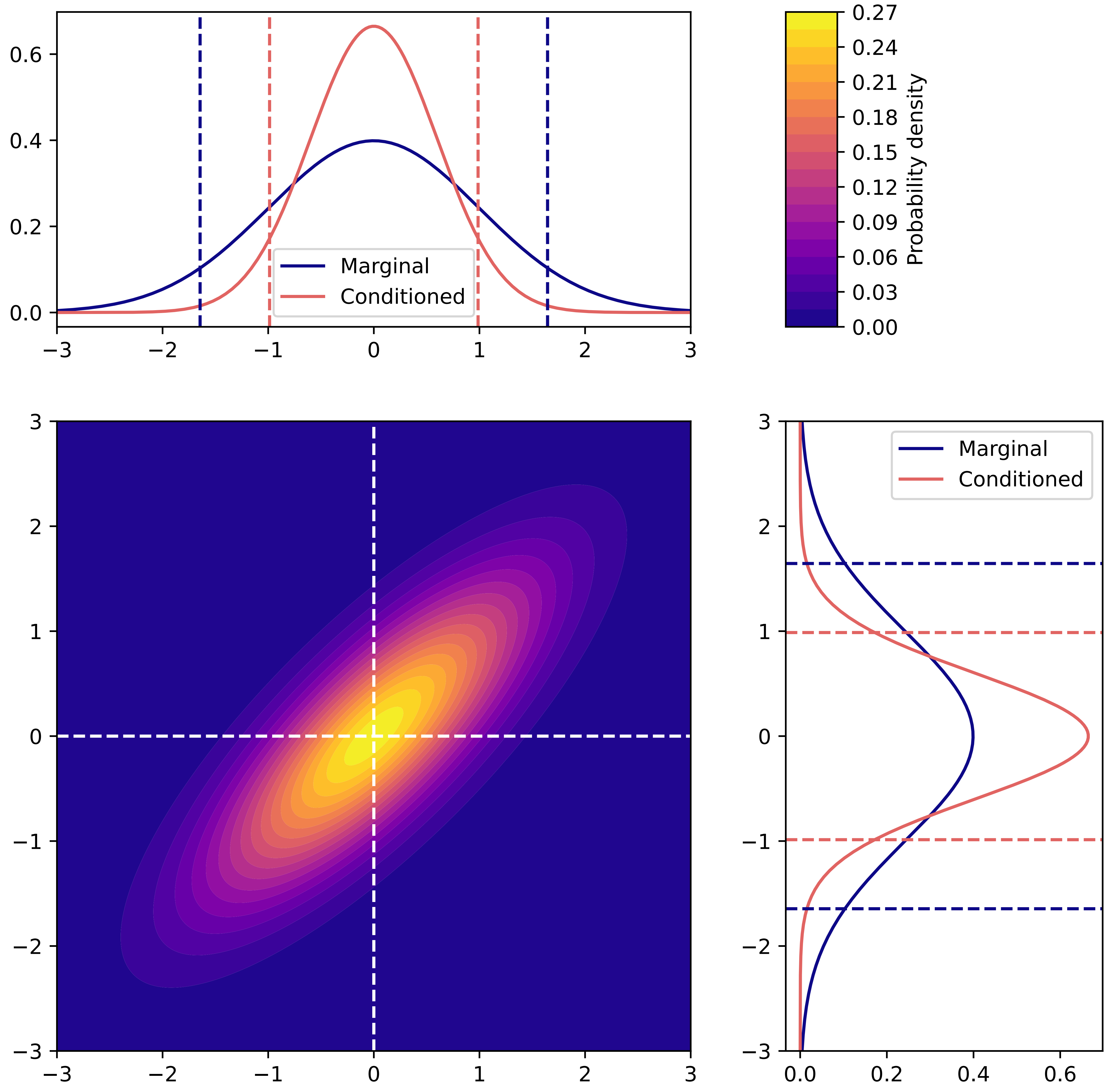
$$\mathcal{C} = \begin{bmatrix} 1 & 0.8 \\ 0.8 & 1 \end{bmatrix}$$



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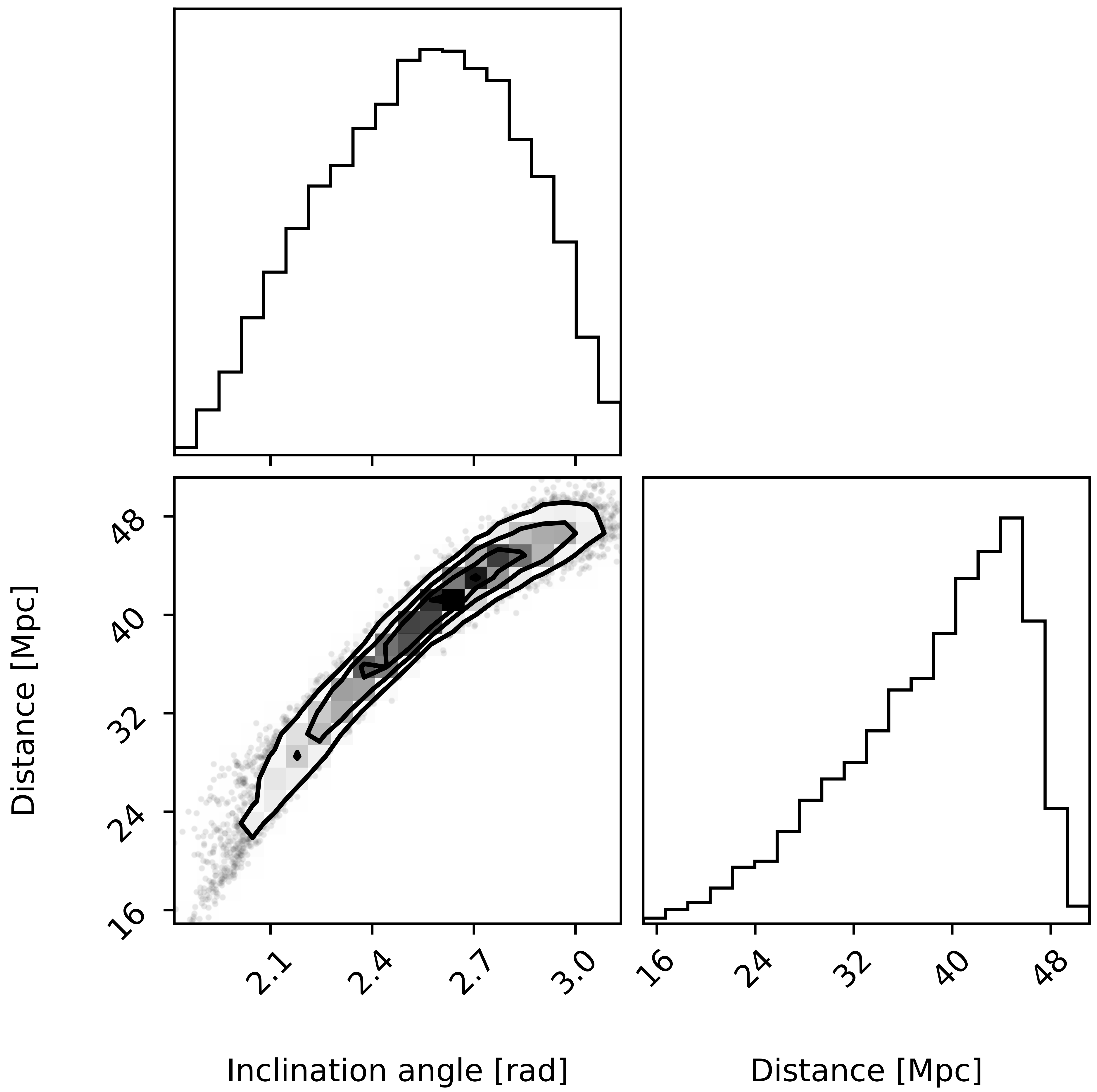
48
40
32
24
16
Distance [Mpc]
2.1
2.4
2.7
3.0
16
24
32
40
48
Inclination angle [rad]
Distance [Mpc]

Parameters dependence of CBC signals

$$h_+(t) = \frac{4}{r}\left(\frac{GM_c}{c^2}\right)^{5/3}\left(\frac{\pi f_{\rm gw}}{c}\right)^{2/3}\frac{1+\cos^2\theta}{2}\cos(2\pi f_{\rm gw} t_{\rm ret} + 2\phi)\,,$$

$$h_\times(t) = \frac{4}{r}\left(\frac{GM_c}{c^2}\right)^{5/3}\left(\frac{\pi f_{\rm gw}}{c}\right)^{2/3}\cos\theta\,\sin(2\pi f_{\rm gw} t_{\rm ret} + 2\phi)\,,$$

$$\downarrow$$

$$h(t) = D^{ij}h_{ij}(t)$$

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Parameters dependence of CBC signals

Component masses m_1 and m_2 with $\sigma_x/x \sim 10\,\%$. They combination, **chirp mass** is constrained at $\sigma_x/x \sim 0.1\,\%$

$$\mathcal{M}_{\mathrm{c}} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{3/5}}$$

Mass ratio $q = m_1/m_2$ at $\sigma_x/x \sim 20\,\%$. The combination $M = m_1 + m_2$ is however better constrained at $\sigma_x/x \sim 3\,\%$

The mass we measure are in **detector frame** thus depending from the one in source frame by a factor $(1 + z)$. There is no way from the gravitational signal itself to disentangle source-frame mass from redshift.

$$M_{\mathrm{obs}} = M_{\mathrm{source}}(1 + z)$$

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Intrinsic Parameters

Higher order source properties

- Spin: the components of the spin of each star aligned to the orbital angular momentum, χ_{1z} and χ_{2z} measured at $\sigma_x/x \sim 3$ and 10 respectively. The effective spin $\chi_{\rm eff} = (m_1\chi_{1z} + m_2\chi_{2z})/(m_1 + m_2)$ is constrained at $\sigma_x/x \sim 1$

- The tidal deformability of the star Λ_1 and Λ_2 constrained at $\sigma_x/x \sim 1.5$ while the effective parameter $\tilde{\Lambda}$ is better constrained $\sigma_x/x \sim 0.6$

$$\Lambda_i = \frac{2}{3}\left(\frac{R_i c^2}{G m_i}\right)^5$$

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Extrinsic Parameters

- The luminosity distance is a self-calibrated measure of the distance to the event, because $h \propto 1/d_L$ and constrained at $\sigma_x/x \sim 20\,\%$

  However, d_L is strongly degenerate with the inclination angle ι, measured as the angle between the orbital angular momentum of the binary and the vector connecting detector and source. Both ι and d_L scales the amplitude

  In the highest-order harmonic expansion ($h(t) = \sum_{\ell\geq 2} \sum_{|m|\leq\ell} h_{\ell m}(t) \; {}^{(-2)}Y_{\ell m}(\iota, \phi)$, with $\ell = 2$, $m = \pm 2$) we have $h_+ \propto \left(1 + \cos^2\iota\right)$ and $h_\times \propto \cos\iota$

- The arrival time $t_{\mathrm{geocentric}}$, the phase ϕ, the polarisation angle ψ and the sky position $(\mathrm{ra}, \mathrm{dec})$

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Extrinsic Parameters

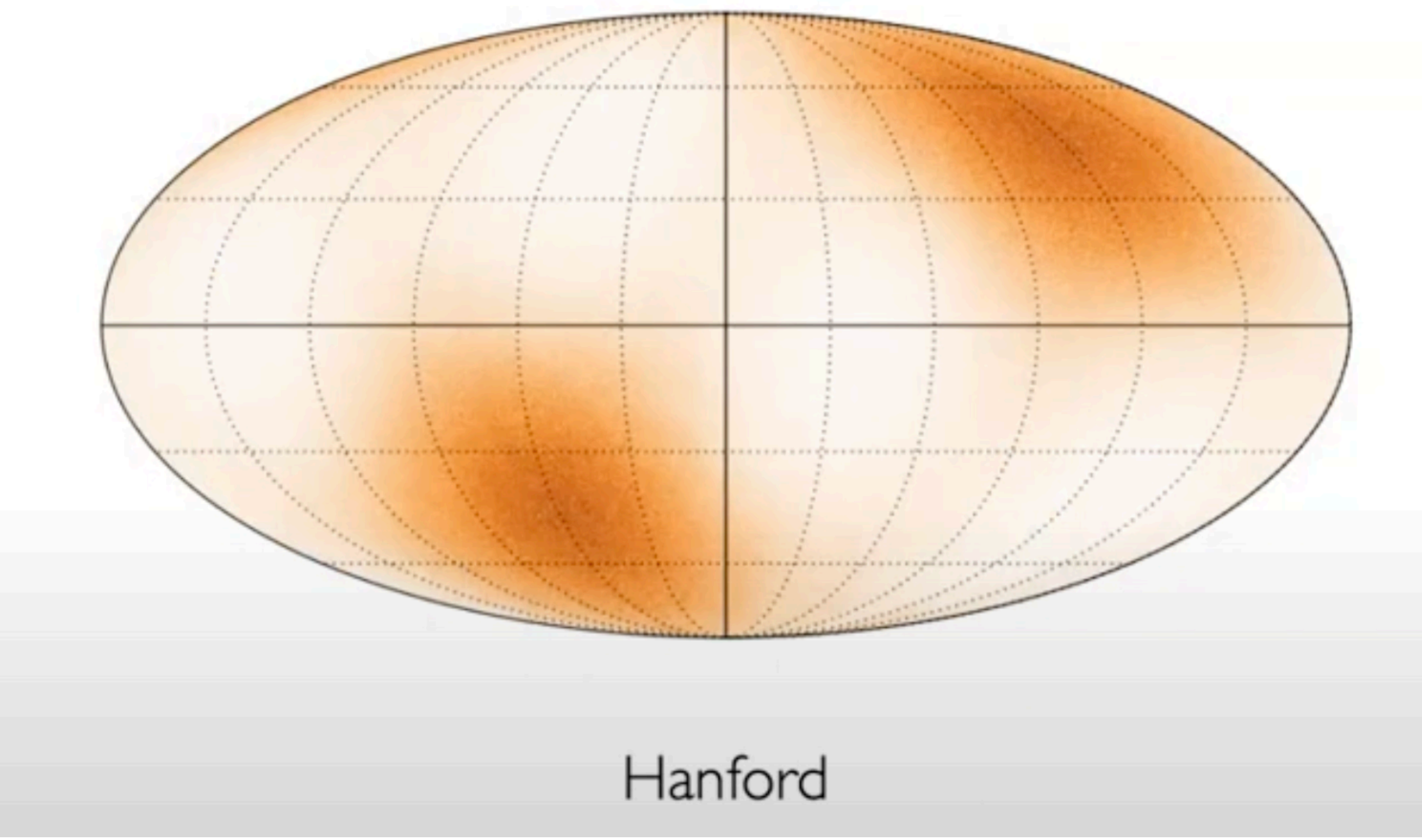
The 1σ **sky area** in steradians can be written in the Gaussian case as:

$$\Delta\Omega_{1\sigma} = 2\pi|\cos(\text{dec})|\sqrt{\sigma^2_{\text{ra}}\sigma^2_{\text{dec}} - \text{cov}^2_{\text{ra, dec}}}$$

and it satisfies $p(\text{source within } \Delta\Omega) = 1 - \exp(-\Delta\Omega/\Delta\Omega_{1\sigma})$.
With this we can compute the 90 sky area in square degrees:

$$\Delta\Omega_{90\%} = -\log(1 - 0.9)\Delta\Omega_{1\sigma}\left(\frac{180 \text{ deg}}{\pi \text{ rad}}\right)^2$$

GW170817



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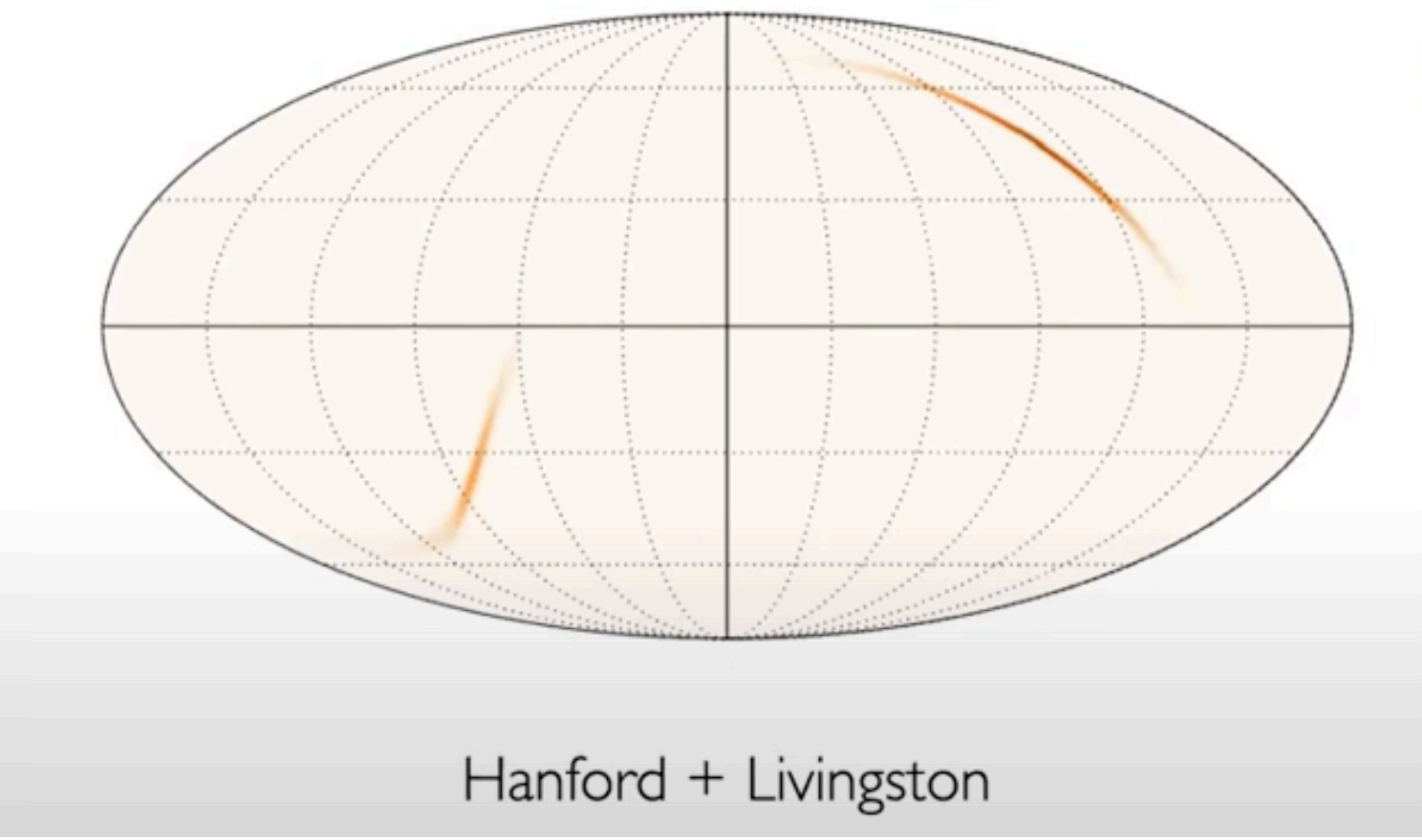

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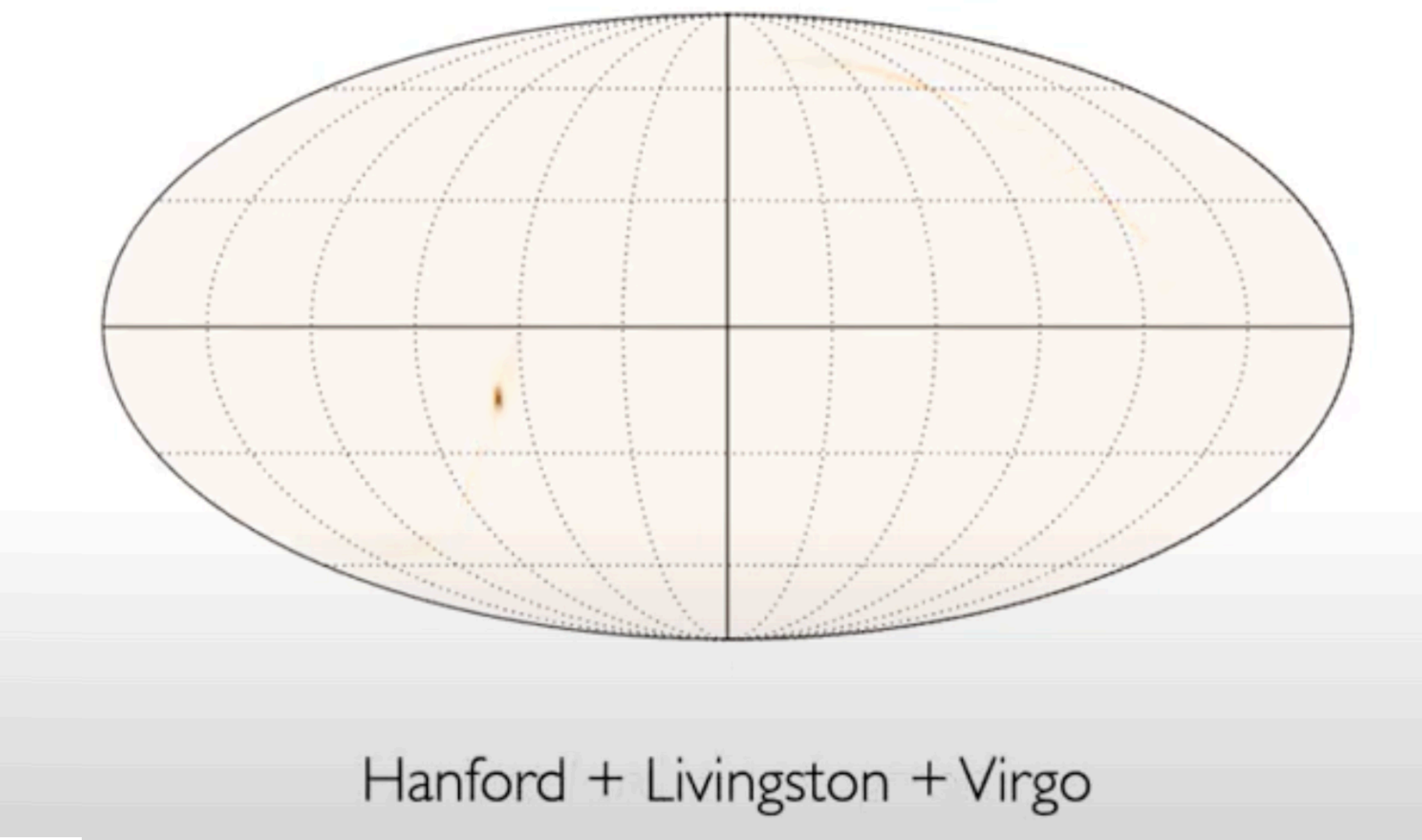

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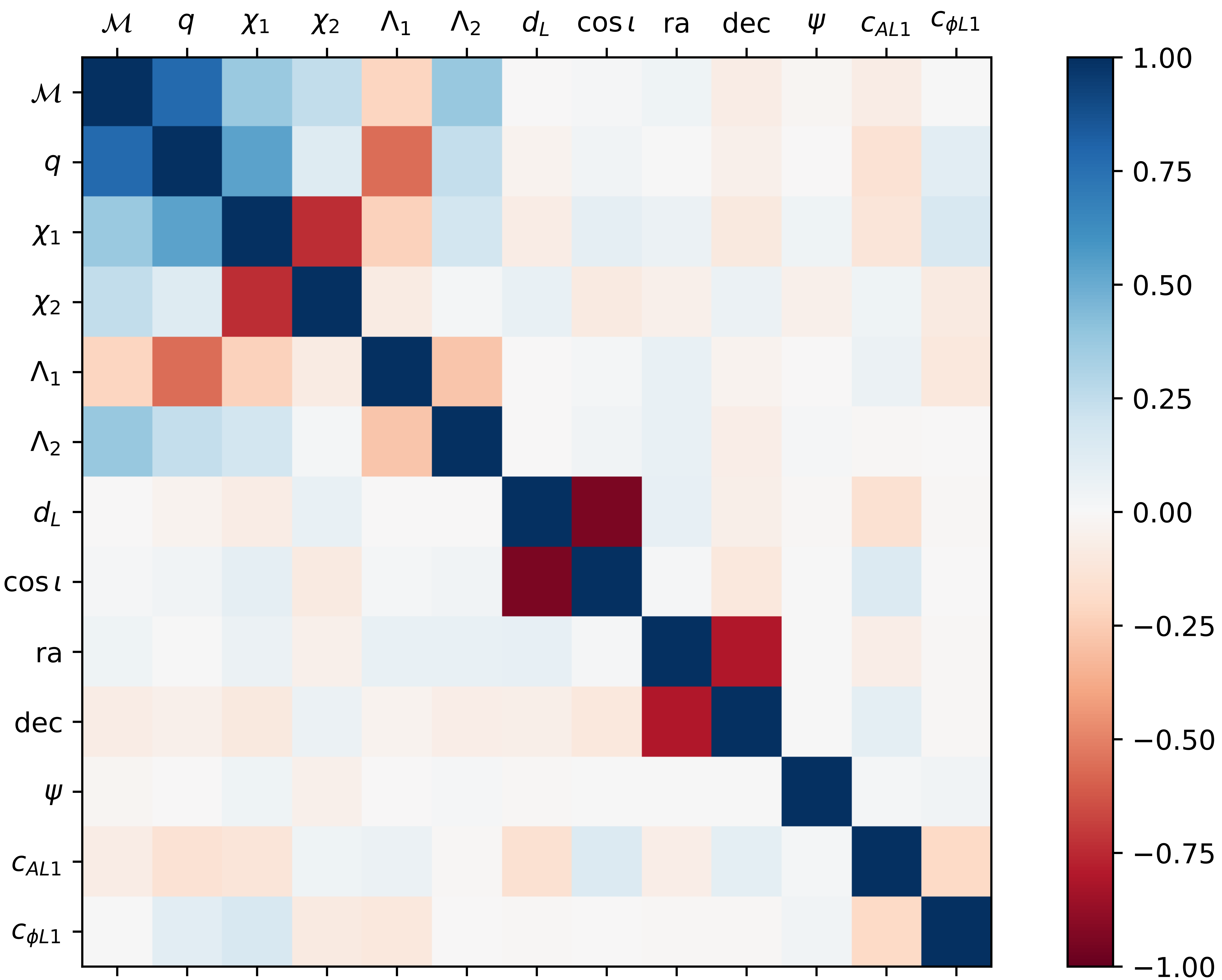
Correlations



$$\rho_{ij} = \frac{\mathrm{cov}(\theta_i, \theta_j)}{\sigma_i \sigma_j}$$

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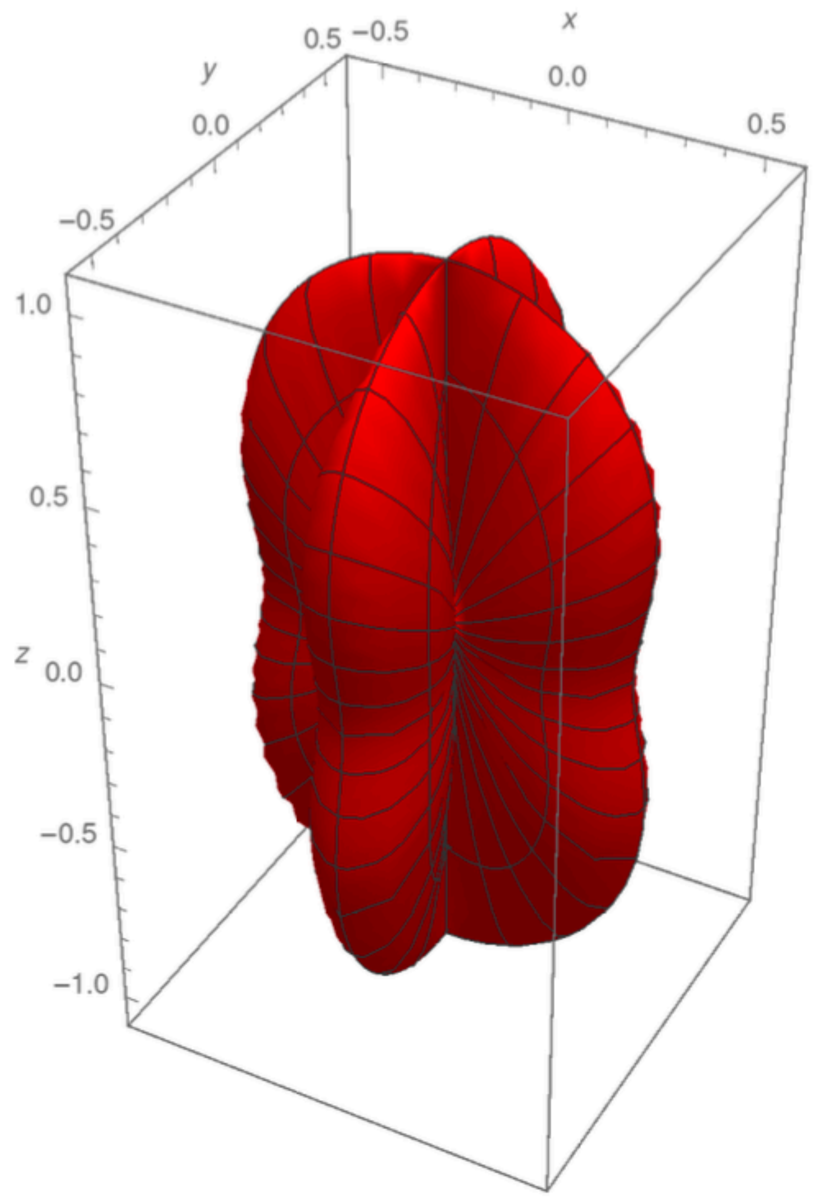
Antenna Pattern

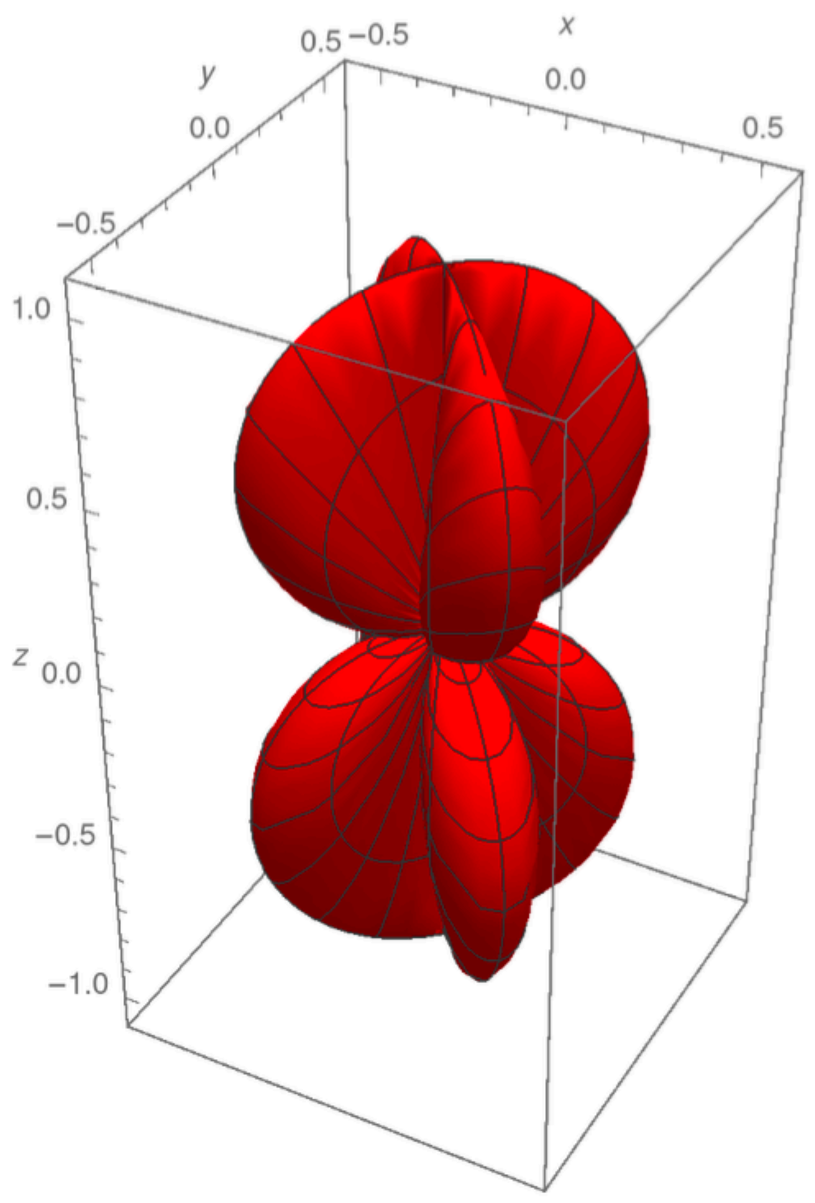
The strain at the detector depends on the antenna pattern:

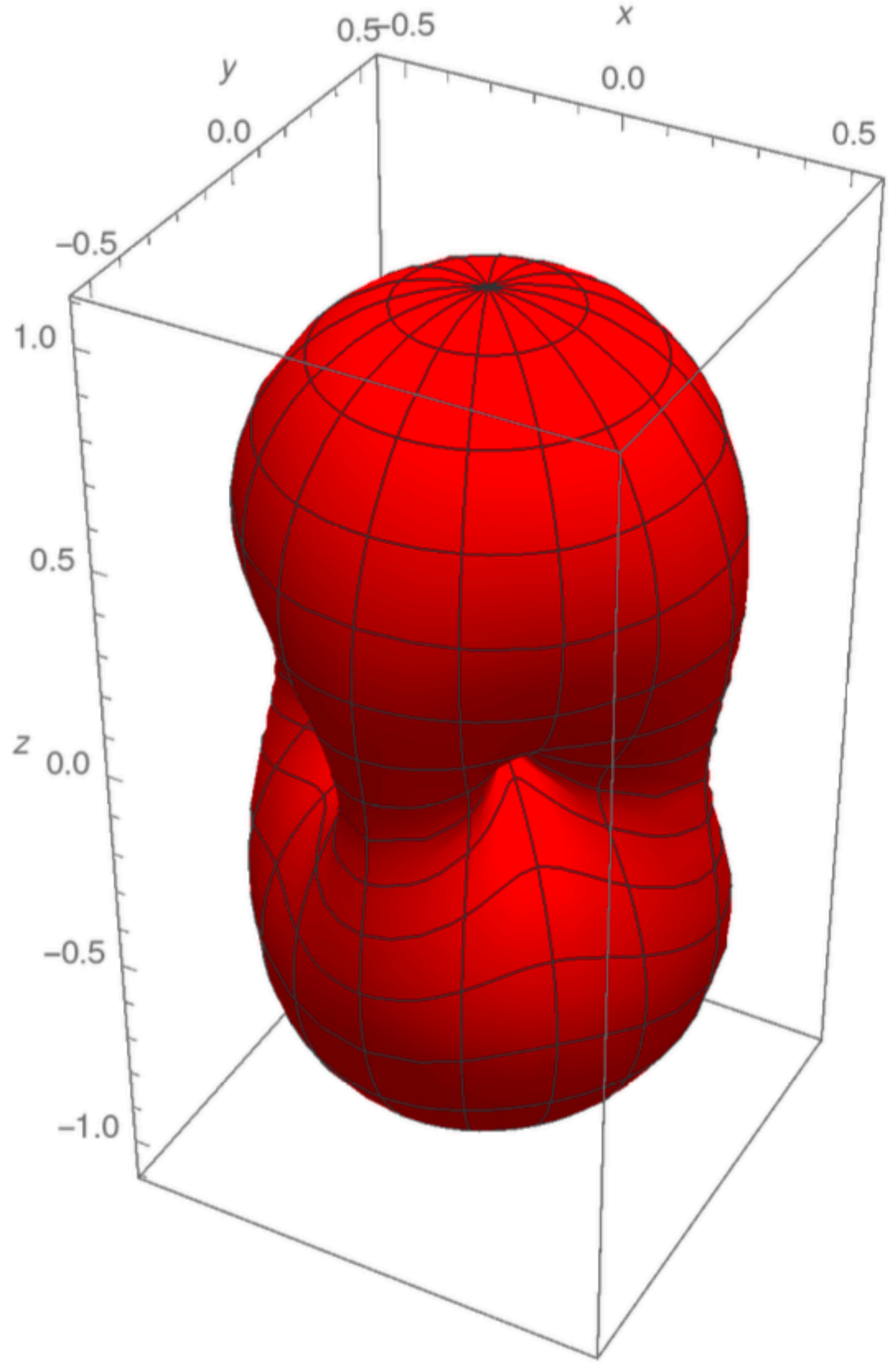
$$h(t) = h_{ij}(t) D_{ij}(t) = h_+(t) F_+(t) + h_\times(t) F_\times(t)\,.$$

$|F_+|$



$|F_\times|$





$$\sqrt{(F_+)^2 + (F_\times)^2}$$

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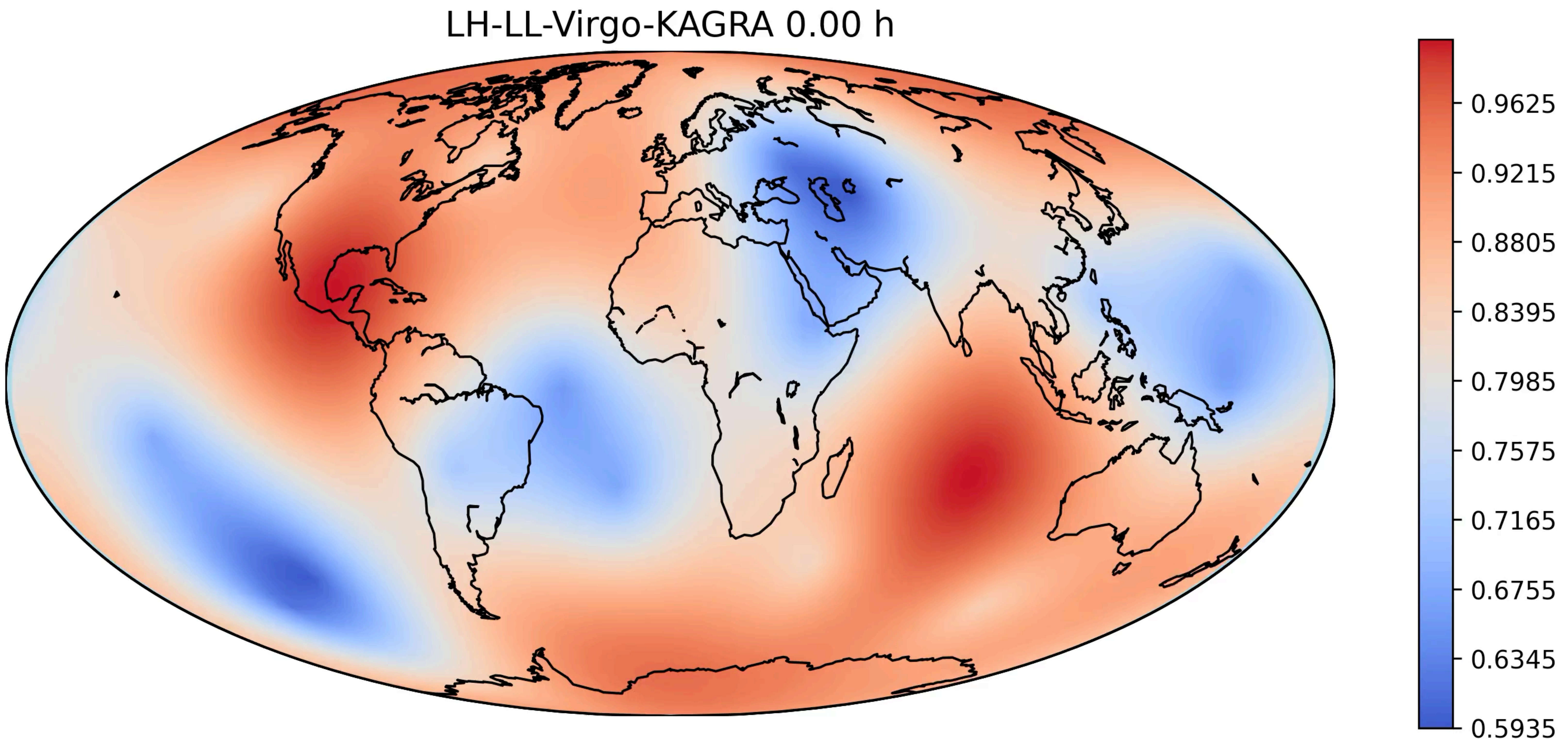
Virgo: antenna Pattern



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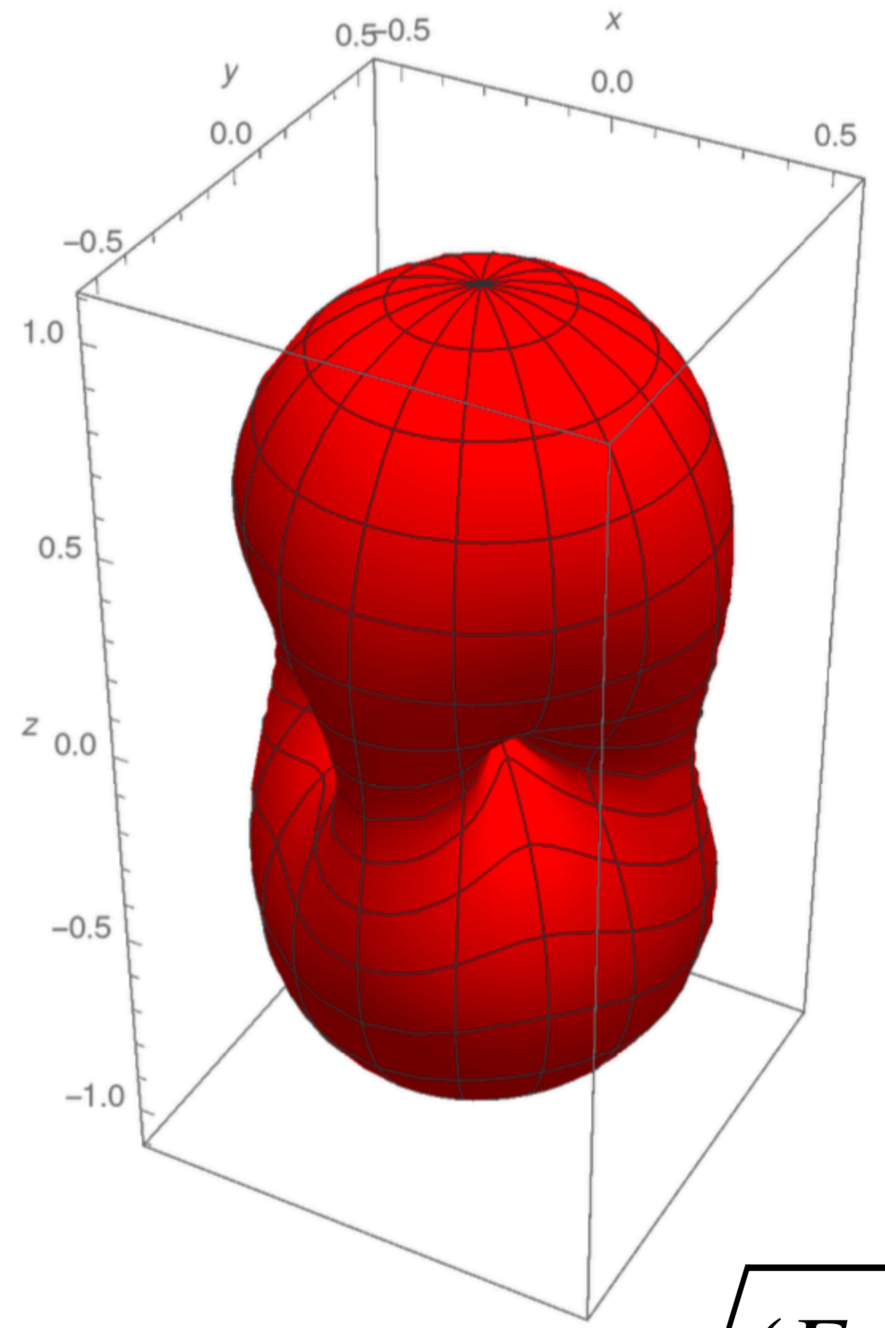
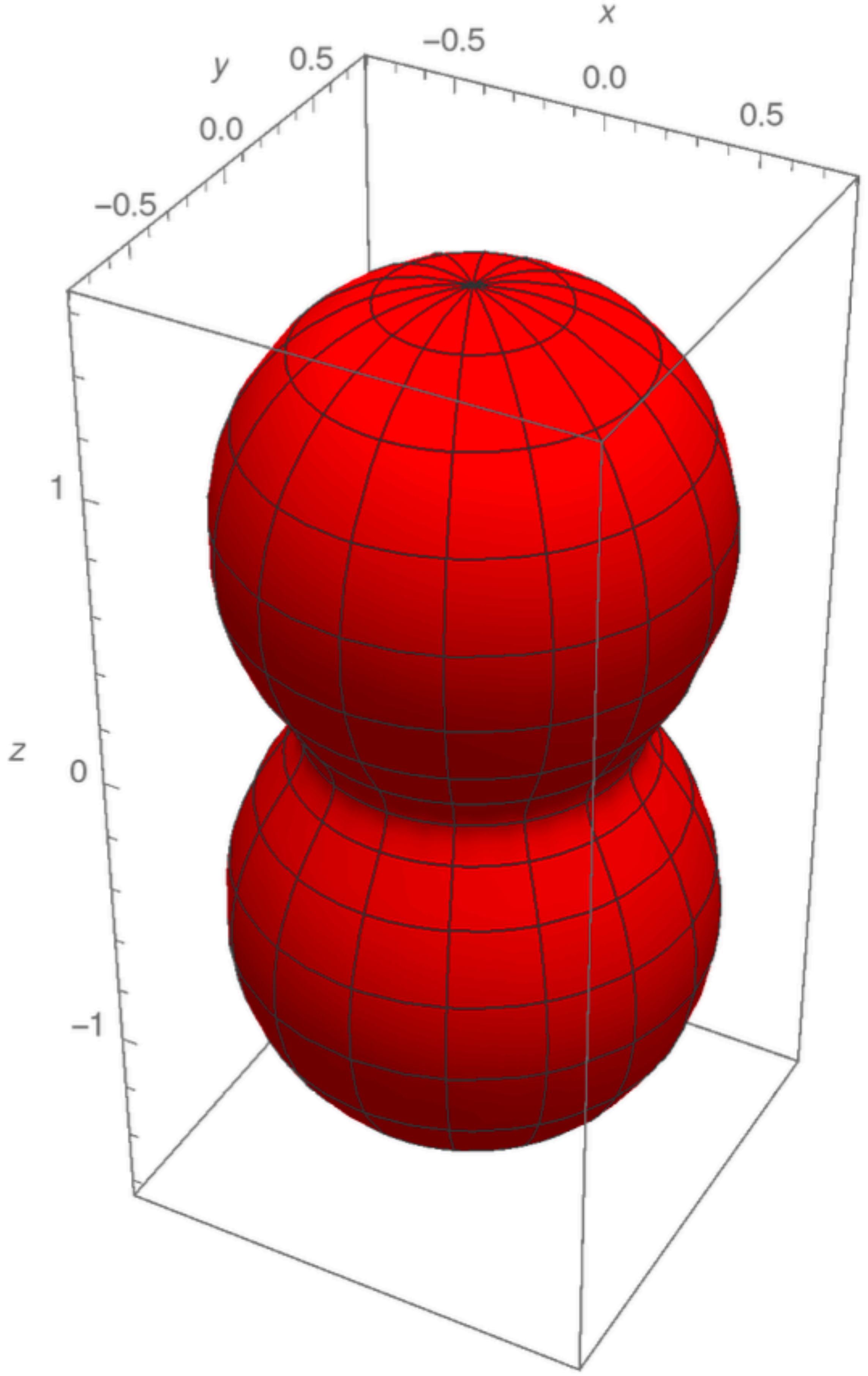


$\sqrt{(F_+)^2 + (F_\times)^2}$

$\times\, 3$ co-located ETΔ



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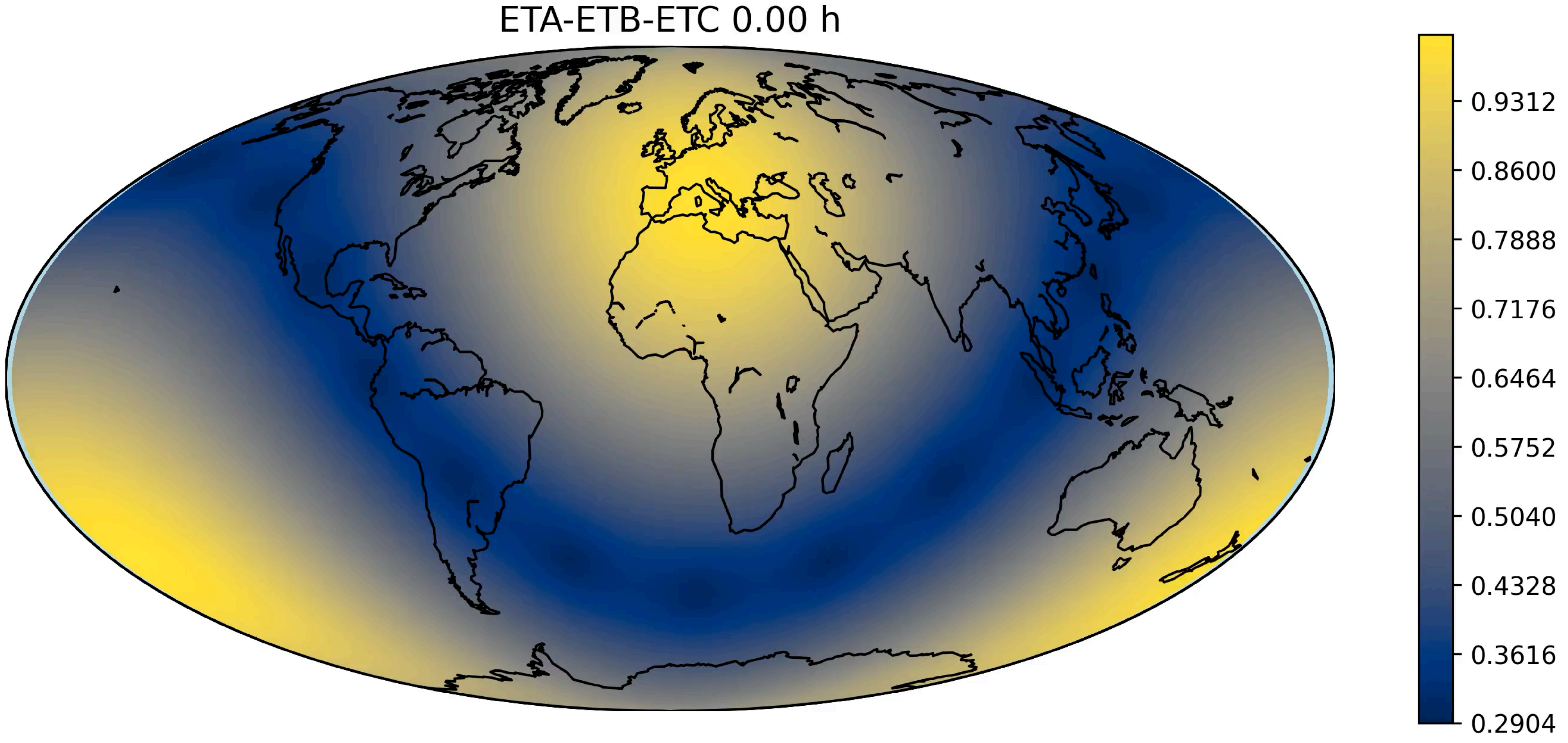
ET: antenna Pattern



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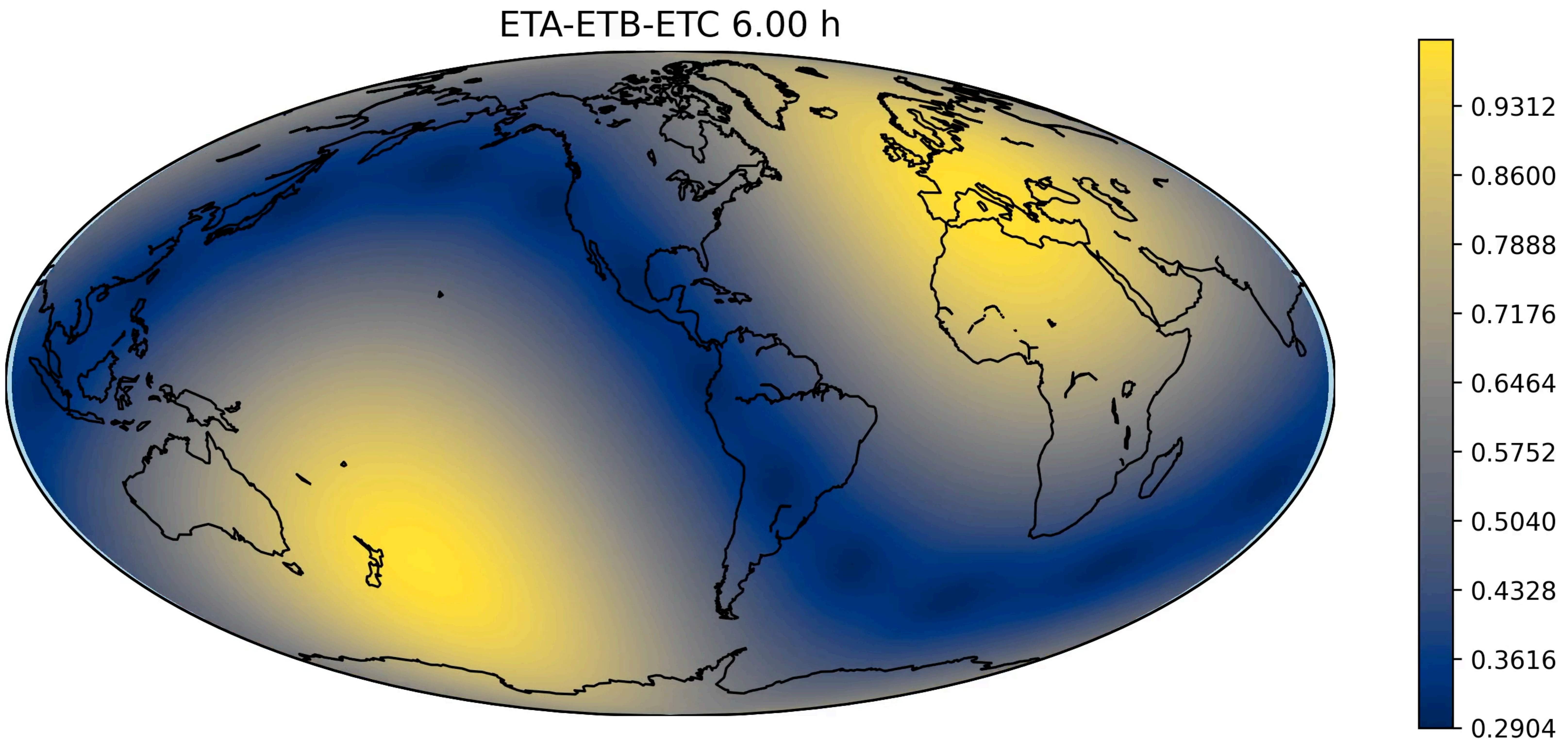
ET: antenna Pattern



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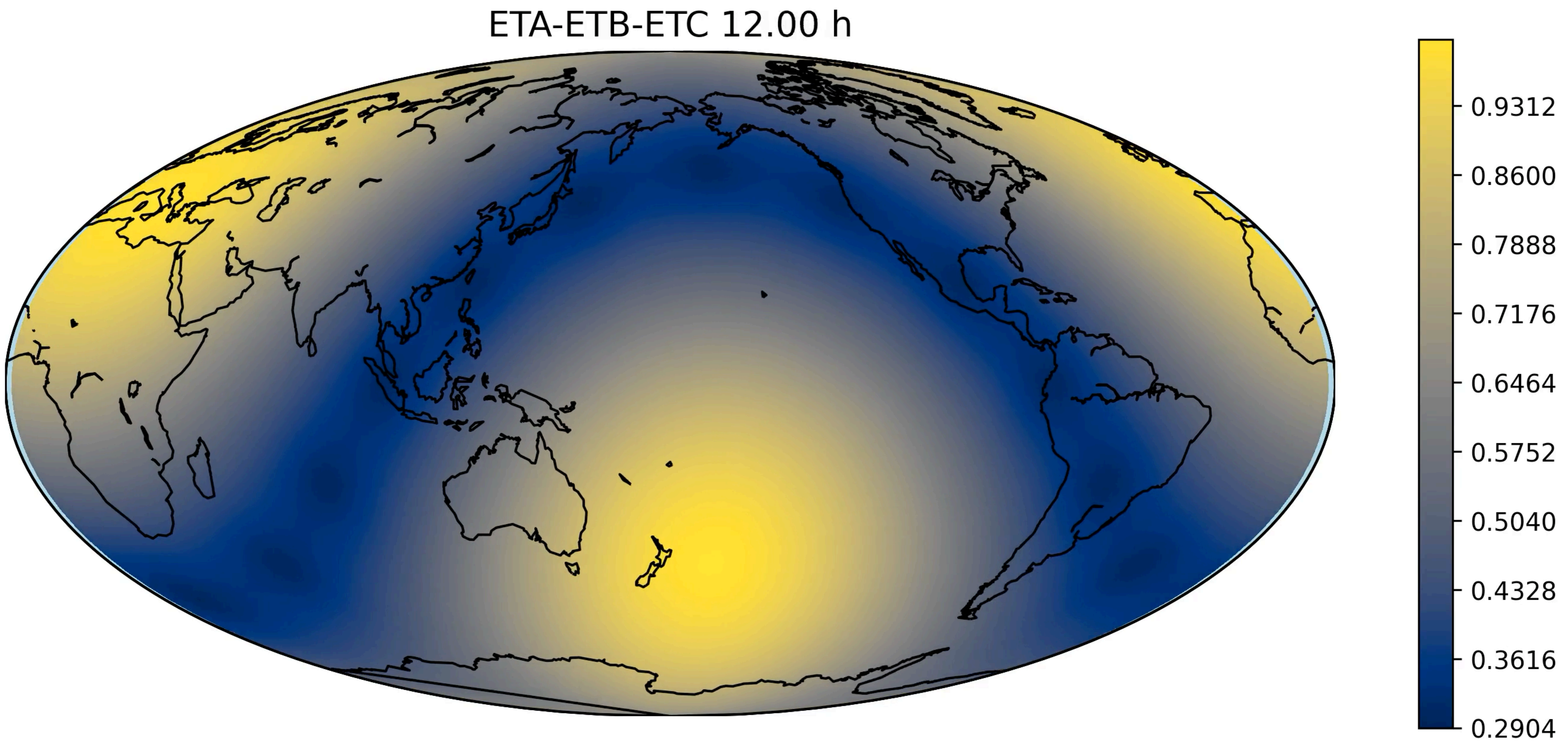
ET: antenna Pattern



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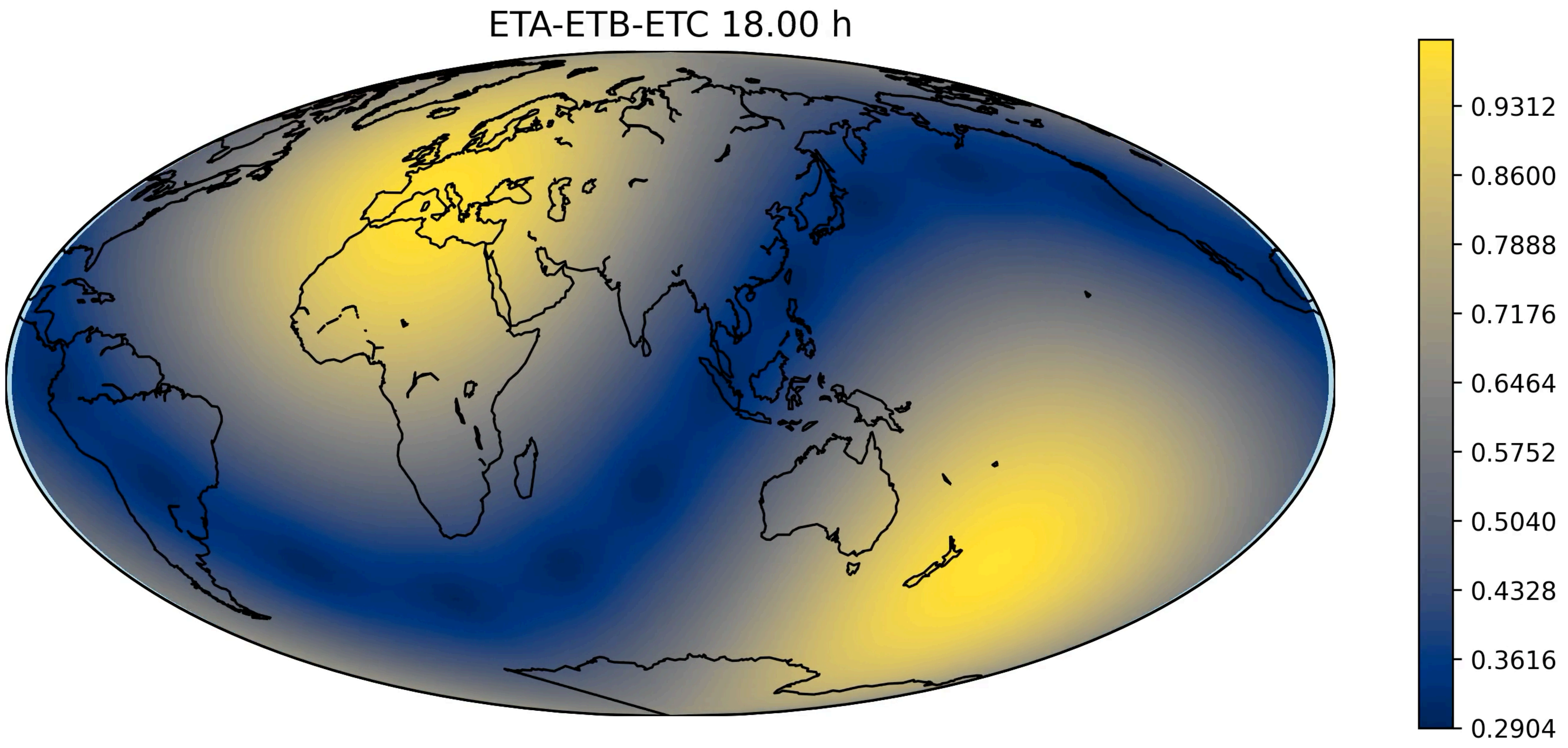
ET: antenna Pattern



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Fisher Matrix

GWFISH

In the Fisher matrix approximation we are approximating the likelihood as

$$\mathscr{L}(d\,|\,\theta) \approx \mathscr{N} \exp\left\{-\frac{1}{2}\Delta\theta_i F_{ij} \Delta\theta^j\right\}$$

$$\Delta\theta_i = \theta_i - \langle\theta_i\rangle$$

$$F_{ij}^{-1} = C_{ij}$$

This is a good approximation in the high-SNR limit, since prior matter less

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Fisher Matrix

The Fisher matrix F_{ij} can computed as the scalar product of the derivative of the waveform and thus computed analytically or numerically

$$F_{ij} = \langle \partial_i \partial_j \mathscr{L} \rangle |_{\theta=\langle\theta\rangle} = (\partial_i h \,|\, \partial_j h) = 4\Re \int_0^{\infty} \mathrm{d}f \, \frac{1}{S_n(f)} \frac{\partial h}{\partial \theta_i} \frac{\partial h^*}{\partial \theta_j}$$

In the case of multiple detectors, we can compute the *network* Fisher matrix by adding the single-detector ones:

$$F_{ij} = \sum_{k=1}^{N} F_{ij}^{k}$$

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The benefits of Fisher Matrix

- The peed of the computation is ~seconds for a single CBC, while a full PE with MCMC estimation of the posterior distribution can take ~10 hours for BBH and even days for BNS
- This is why you can perform big number estimations and perform forecast **population studies**
- It is possible to follow binaries that stay in the detector band for hours, including the motion of the pattern function with the Earth motion, to better localise the source
- Simulate ground based, lunar and space based detectors
- Reliable instrument

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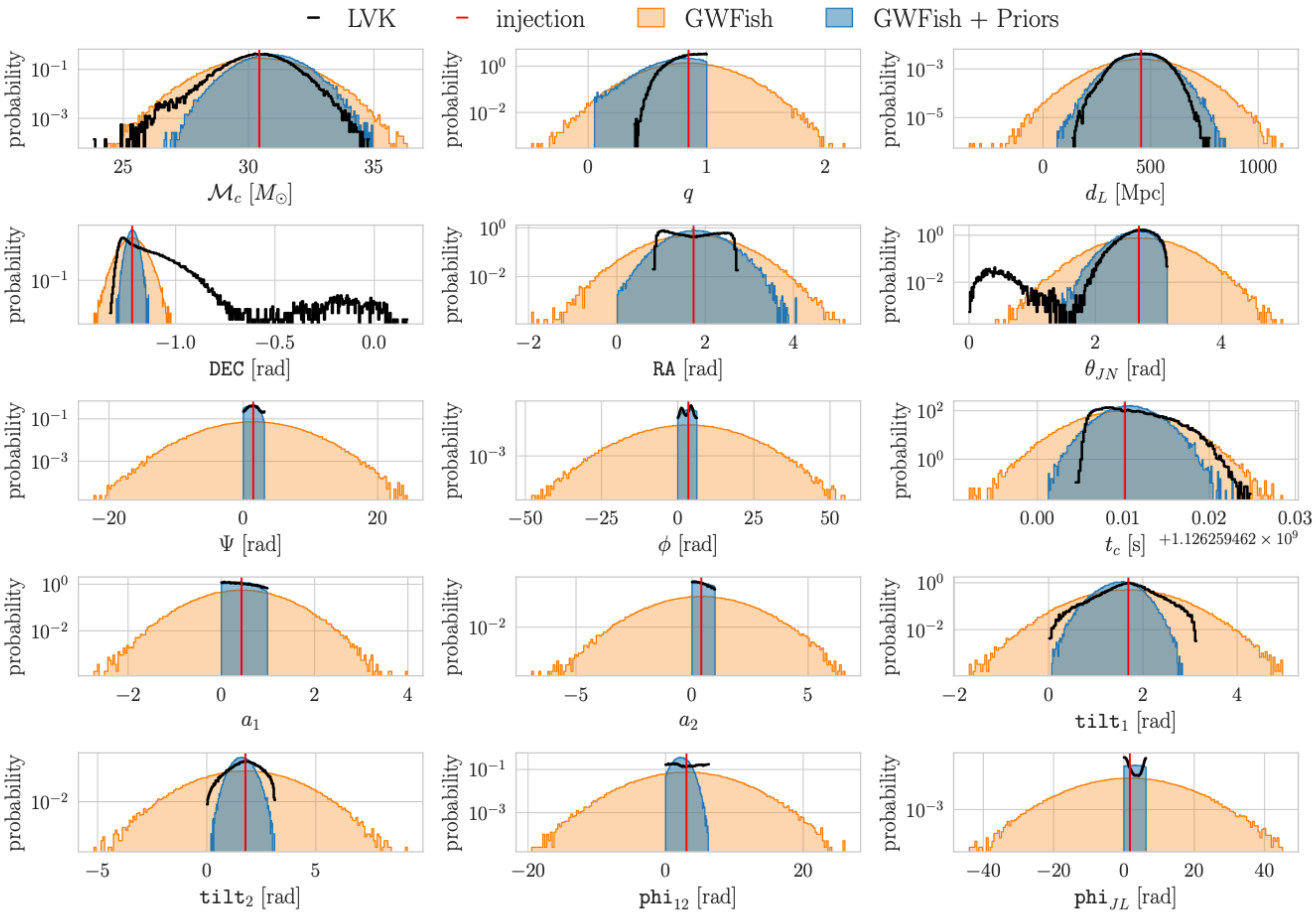
The benefits of Fisher Matrix



Dupletsa+ (2024)

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