Illustration by Lia Halloran

RINGDOWN MODELING IN GENERAL RELATIVITY

LAURA PEZZELLA
SUPERVISOR: ANDREA MASELLI

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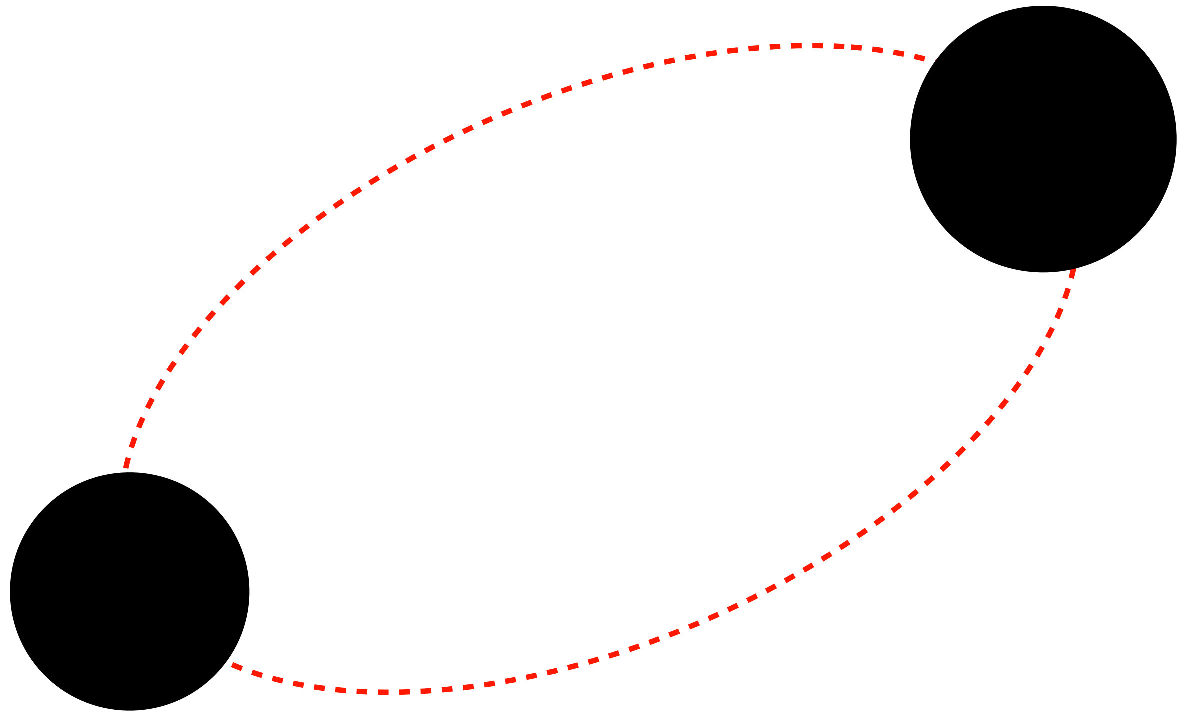
BLACK HOLE BINARIES

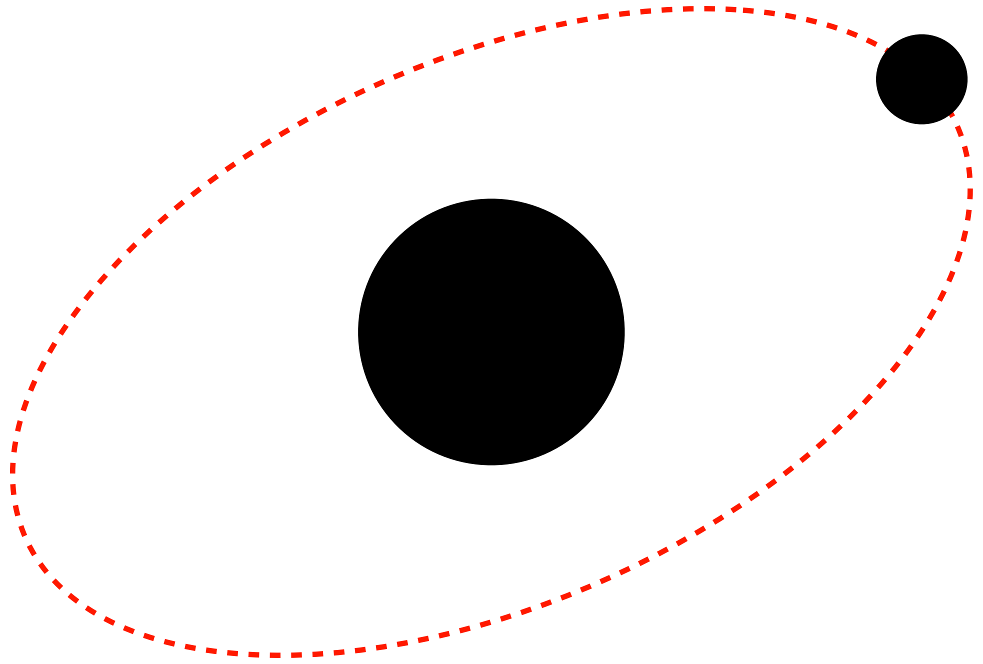
THREE REGIMES



Comparable masses

$$q = m_2/m_1 \sim 1$$

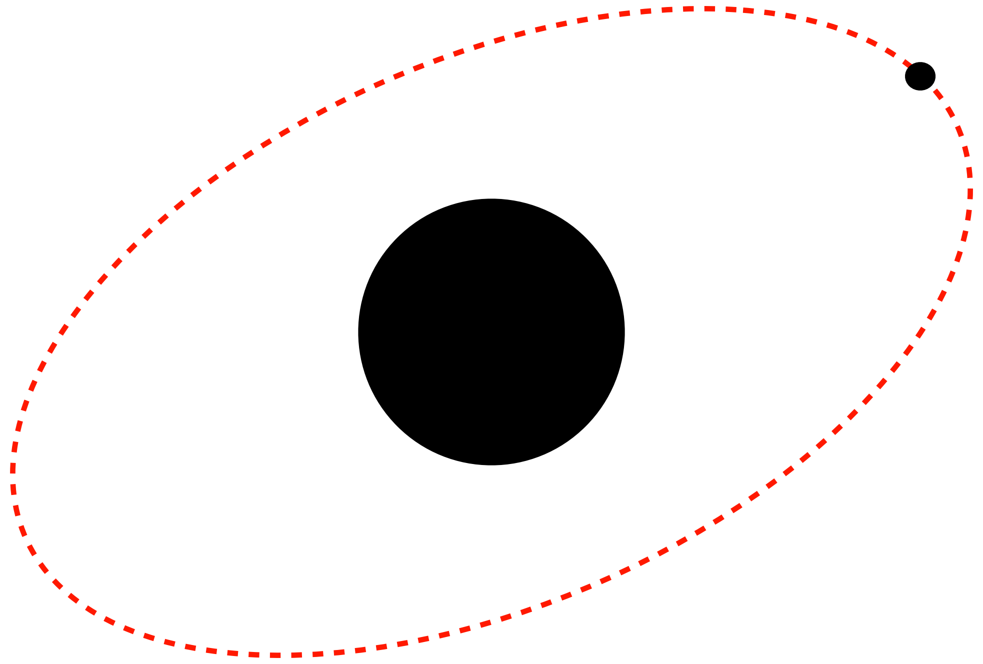
LVK - ET/CE



Intermediate Mass Ratio

$$q = m_2/m_1 \sim 10^{-1} - 10^{-3}$$

ET/CE - LGWA - LISA



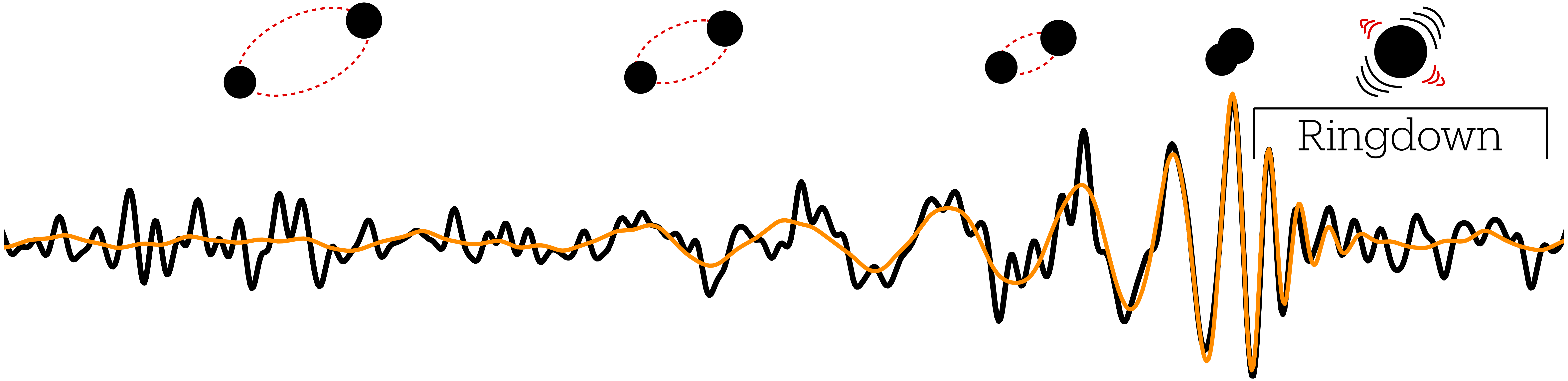
Extreme Mass Ratio

$$q = m_2/m_1 \lesssim 10^{-4}$$

LISA

RINGDOWN

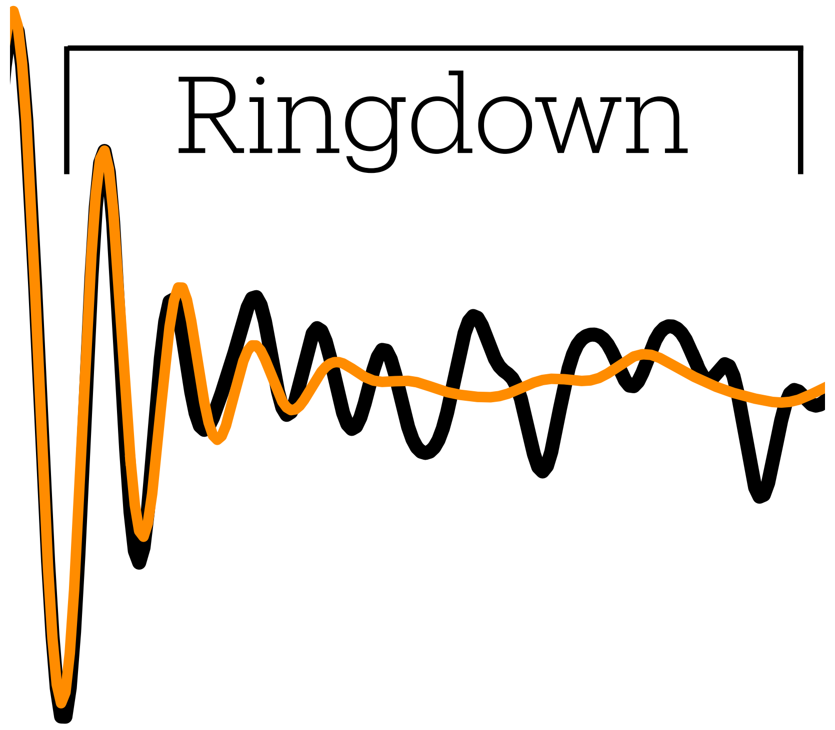
QUASI NORMAL MODES EXCITATION



- The ringdown waveform originates from the **distorted final product** of the merger.
- The **gravitational signal** emitted during the ringdown is well modeled by a **superposition** of damped sinusoids.
- The characteristic complex frequencies are called quasi-normal modes (**QNMs**)

RINGDOWN

QUASI NORMAL MODES EXCITATION



$$h(t) = \sum_{\ell mn} A_{\ell mn} e^{-t/\tau_{\ell mn}} \cos(\omega_{\ell mn} t + \phi_{\ell mn})$$

- QNMs **frequencies** depend entirely on the **final BH's parameters** (**mass M** and **spin J** in vacuum GR)
- The **amplitudes** and **phases** of the signal depend on the **dynamics** of the **specific process** that formed the BH

Illustration by Lia Halloran

PROJECT I :

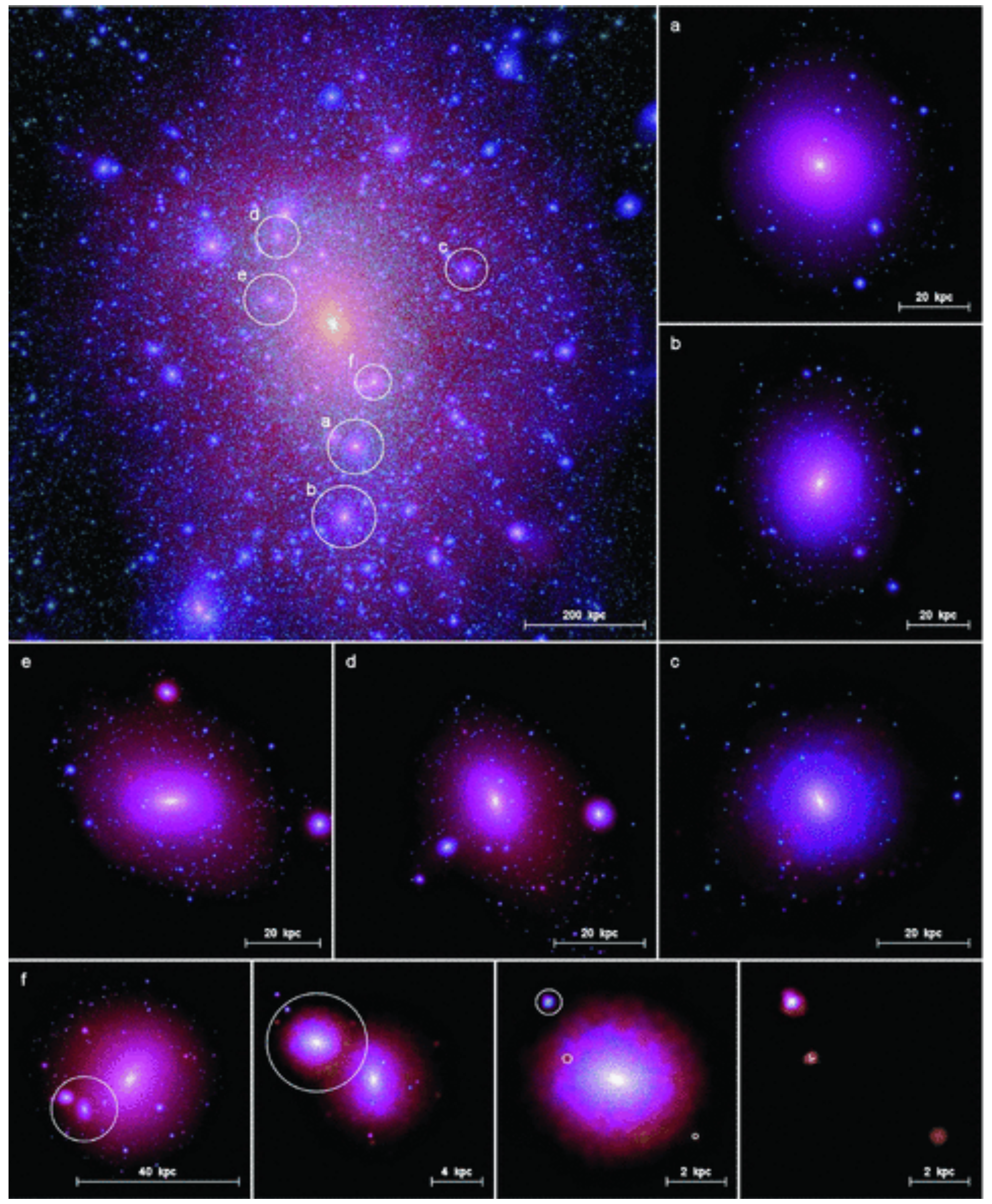
BLACK HOLES IN ENVIRONMENT

BASED ON PRD **111**, 064026 (2025), WITH K. DESTOUNIS, A. MASELLI, V. CARDOSO

Laura Pezzella | October 13rd 2025

DIRTY BLACK HOLES

WHEN VACUUM IS NOT ENOUGH



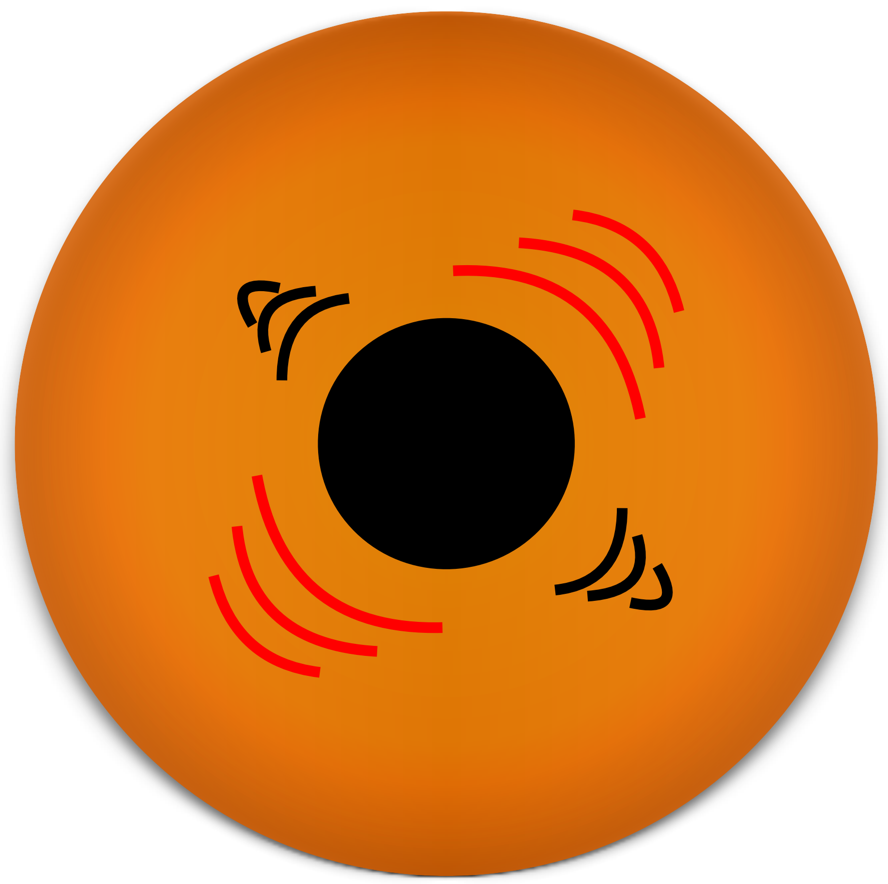
V. Springel+, 2008

- Compact objects evolve **embedded** in a variety of **gas/ matter fields**, which may leave detectable imprints on GW.
- Dark matter may cluster at the **center of galaxies** and close to BHs, affecting the **dynamics of compact binaries** and the **propagation of gravitational waves**.
- Popular density profiles: Hernquist, NFW, Einasto

BLACK HOLES AND ENVIRONMENT

SOME QUESTIONS ARISE

- Can we **infer properties** on the **environment** in which binaries evolve?
- Are there **systematic effects** in **waveform modeling** due to environmental effects?
- How do different **computational methods** behave when **extending** the search for QNMs from **vacuum GR** to the case of **dirty black holes**?



THE PERTURBATION SCHEME

DECOUPLING AXIAL AND POLAR COMPONENTS

The **gravitational perturbations** of a non-spinning BH background induced by a massless probe field

$$g_{\mu\nu} = g^0_{\mu\nu} + h_{\mu\nu} \xrightarrow{\text{can be decoupled into}} h_{\mu\nu} = h^{\text{pol}}_{\mu\nu} + h^{\text{ax}}_{\mu\nu}$$

where the **polar components** share the same parity as the scalar spherical harmonics $(-1)^{\ell}$ and the **axial components** the opposite parity $(-1)^{\ell+1}$

For a **spherically symmetric** background the two families **decouple**.

BLACK HOLE AND DM HALO

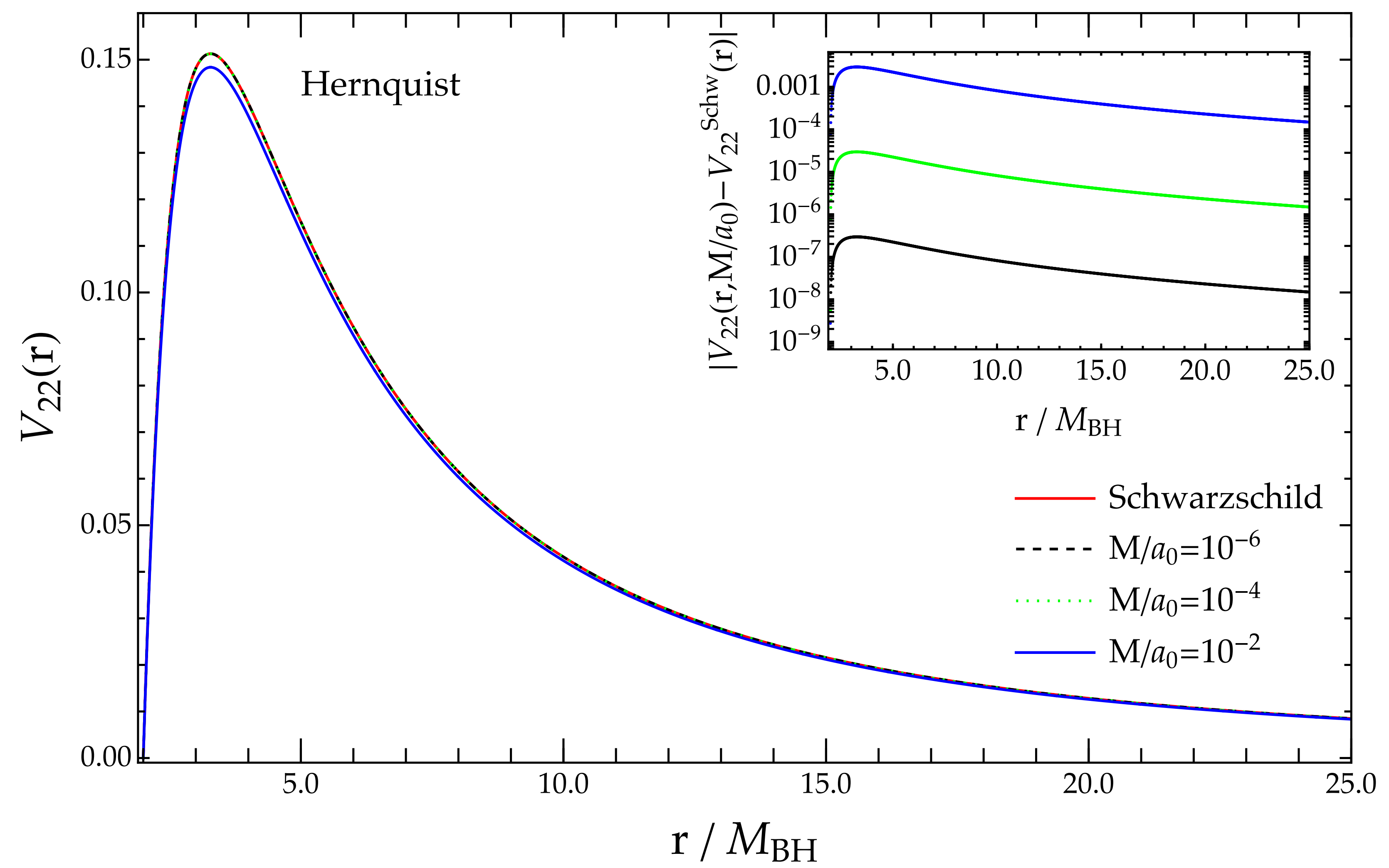
AXIAL MODES

Solving the field equation in **Fourier space**, **the master equation** is obtained

$$\frac{d^2\Psi_{\ell m}}{dr_*^2} + \left[\omega^2 - V_\ell\right]\Psi_{\ell m} = 0 \longrightarrow V_\ell = \frac{f(r)}{r^2}\left[\ell\left(\ell+1\right) - \frac{6m(r)}{r} + m'(r)\right]$$

LP+, PRD 111 (2025)

- The halo affects the structure of the **potential**, as well the **boundary conditions** of the wave propagation at the horizon and at infinity
- Axial modes **do not couple** to matter



CONSTRUCTING THE BACKGROUND

NUMERICAL PROFILES AS BACKGROUND

The BH solutions **embedded within numerical profiles** are constructed as follows:

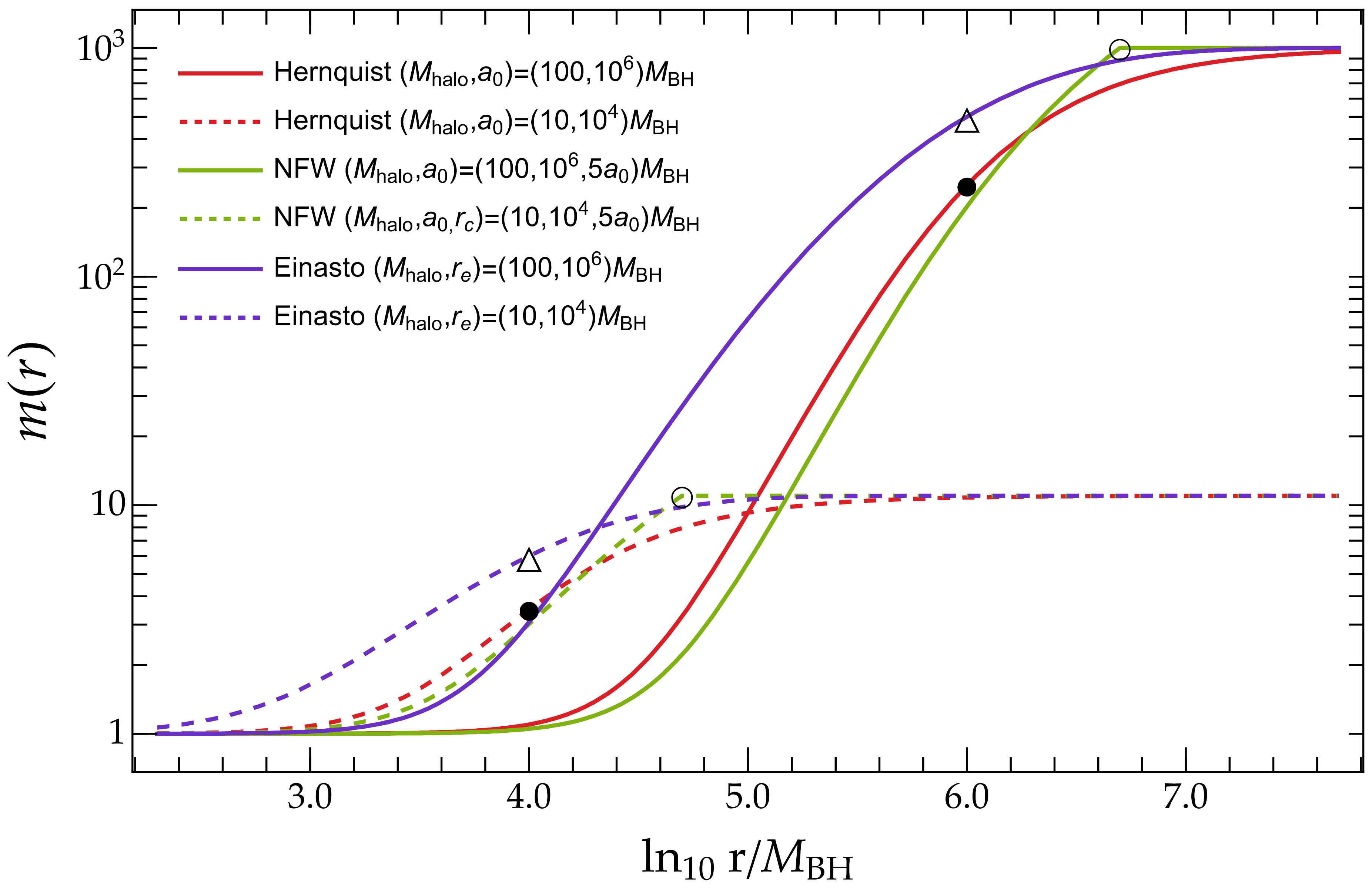
- Take the spherically-symmetric, static spacetime, with $m(r)$ a **generic function**;
- Prescribe the **density profile** $\rho(r)$. The mass profile is recovered by solving the **continuity equation** $m'(r) = 4\pi r^2 \rho(r)$;
- The **metric function** $f(r)$ and the **tangential pressure** $P_t(r)$ are determined from the G_{rr} component from the **Bianchi identities**, respectively, as

$$\frac{f'(r)}{f(r)} = \frac{2m(r)/r}{r - 2m(r)}, \quad 2P_t(r) = \frac{m(r)\rho(r)}{r - 2m(r)}.$$

SPACETIME SOLUTIONS

GENERAL FEATURES

- The solution is **asymptotically flat**
- The event horizon is at $r = 2M_{\rm BH}$
- $M_{\rm BH} \ll M \ll a_0$ ensures there are no curvature singularities outside the horizon
- To mimic galaxy observations, $a_0 \gtrsim 10^4$
- $M_{\rm ADM} = M + M_{\rm BH} = m(\infty)$

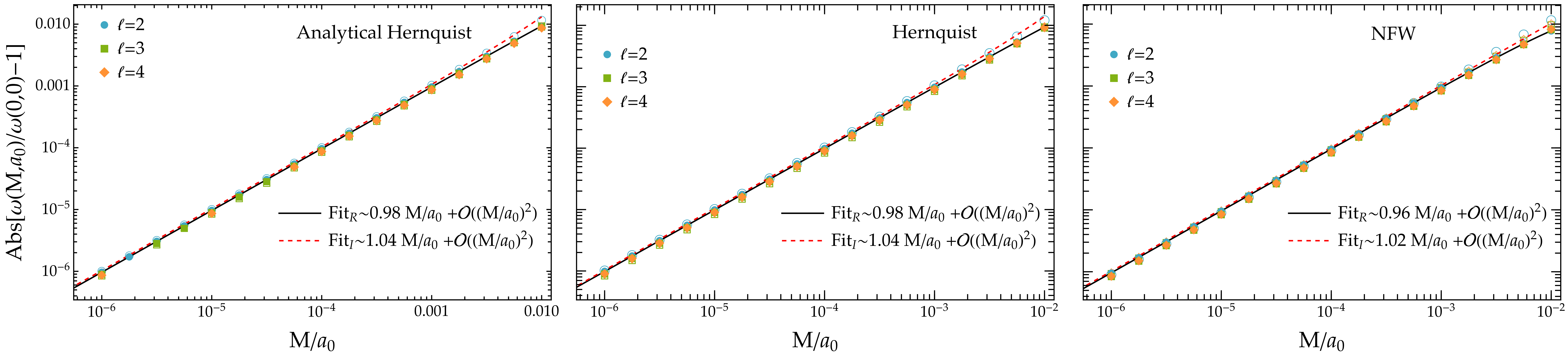


E. Figueiredo+, PRD 107 (2023)

BLACK HOLES AND HALOS

A RESCALING EFFECT

LP+, PRD 111 (2025)



☑ The leading order changes in the QNMs can always be described in terms of a **gravitational redshift** $\omega\left(M, a_0\right) = \omega_{\text{vac}}\left(1 - M/a_0\right)$

QNM IN ENVIRONMENT

RESULTS AND PROSPECTS

First **ab initio calculation of QNMs** of a BH spacetime in a **DM environment**

My contributions:

- Developed a **fully numerical approach** to treat any dark matter distribution $\rho(r)$
- Found a universal relation between the matter environment and the **redshifted** vacuum QNM
- In the axial case, the frequencies are **rescaled** by a factor which depends on the **compactness** of the halo

What can be done:

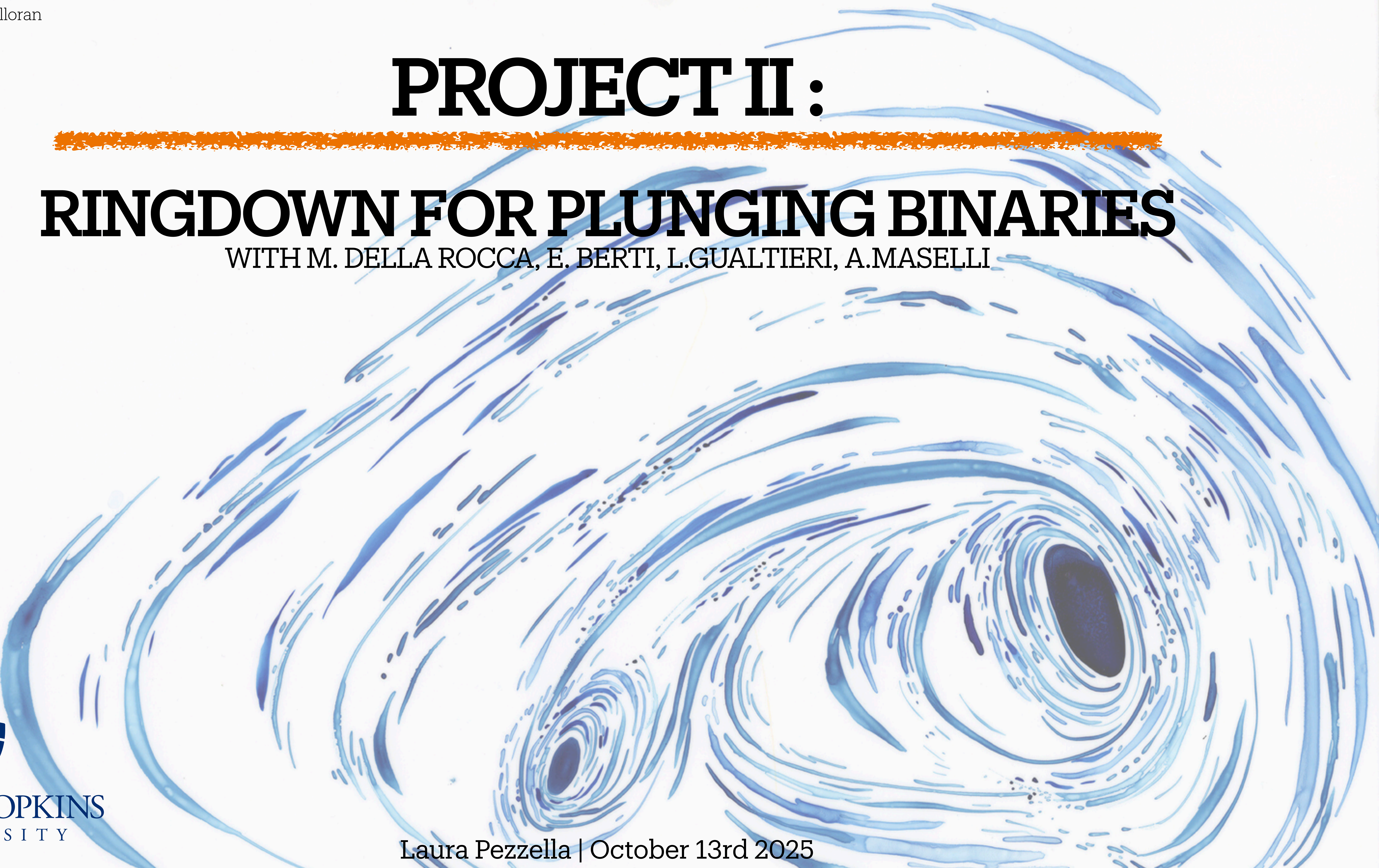
- [ ] Exploring the **polar sector**
- [ ] **Detectability** of halo parameters

Illustration by Lia Halloran

PROJECT II :

RINGDOWN FOR PLUNGING BINARIES

WITH M. DELLA ROCCA, E. BERTI, L.GUALTIERI, A.MASELLI



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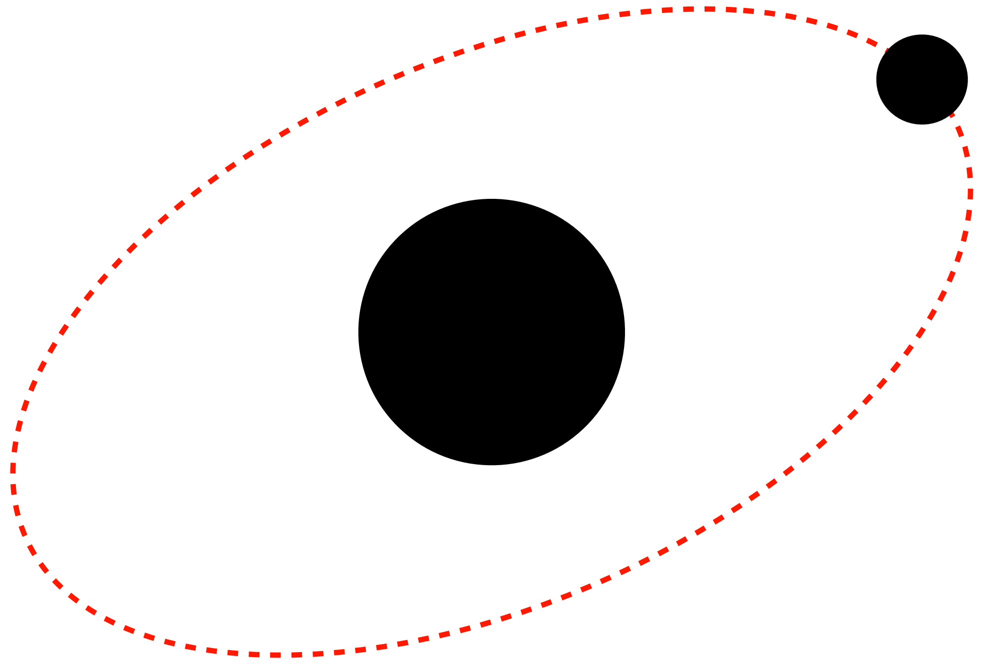
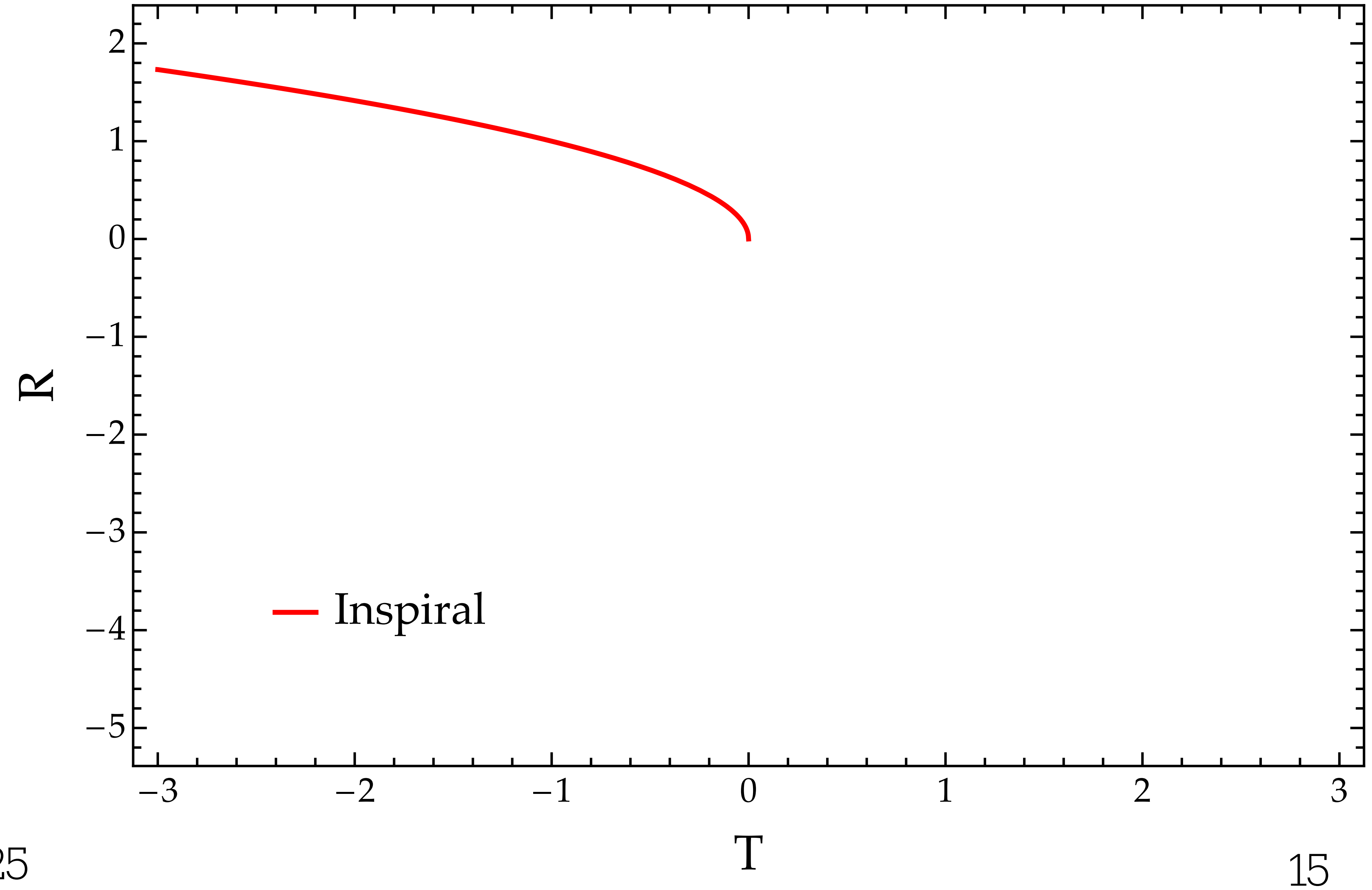
EVOLUTION OF PLUNGING BINARIES

THREE STAGES

- **Inspiral regime**: the orbit of the secondary body evolves adiabatically. Extreme mass ratio inspiral simulations end to the ISCO;

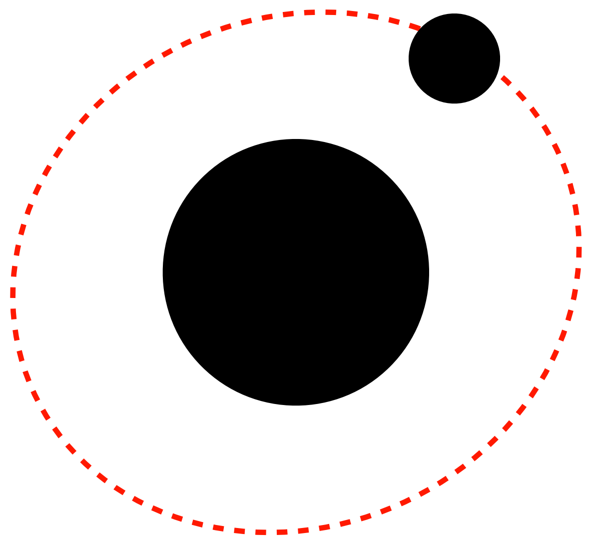


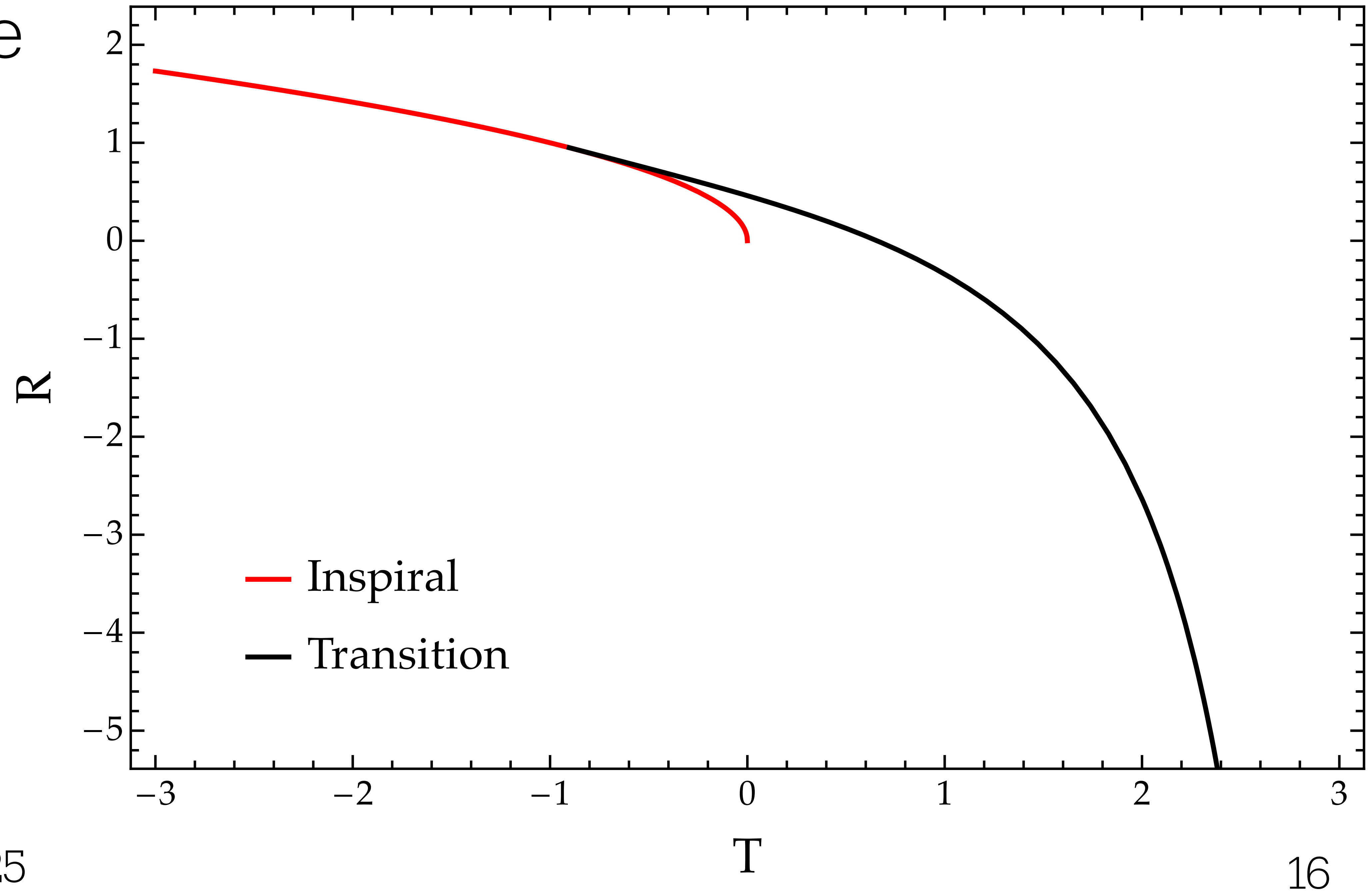


EVOLUTION OF PLUNGING BINARIES

THREE STAGES

- **Inspiral regime**: the orbit of the secondary body evolves adiabatically. Extreme mass ratio inspiral simulations end to the ISCO;
- **Transition regime**: the character of the orbit gradually changes as the secondary approaches the **ISCO** of the primary BH;

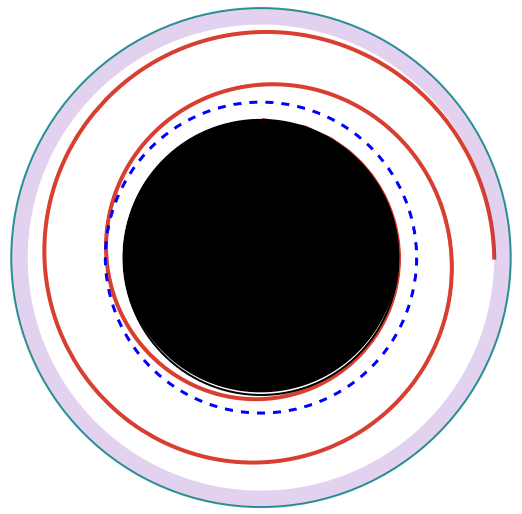


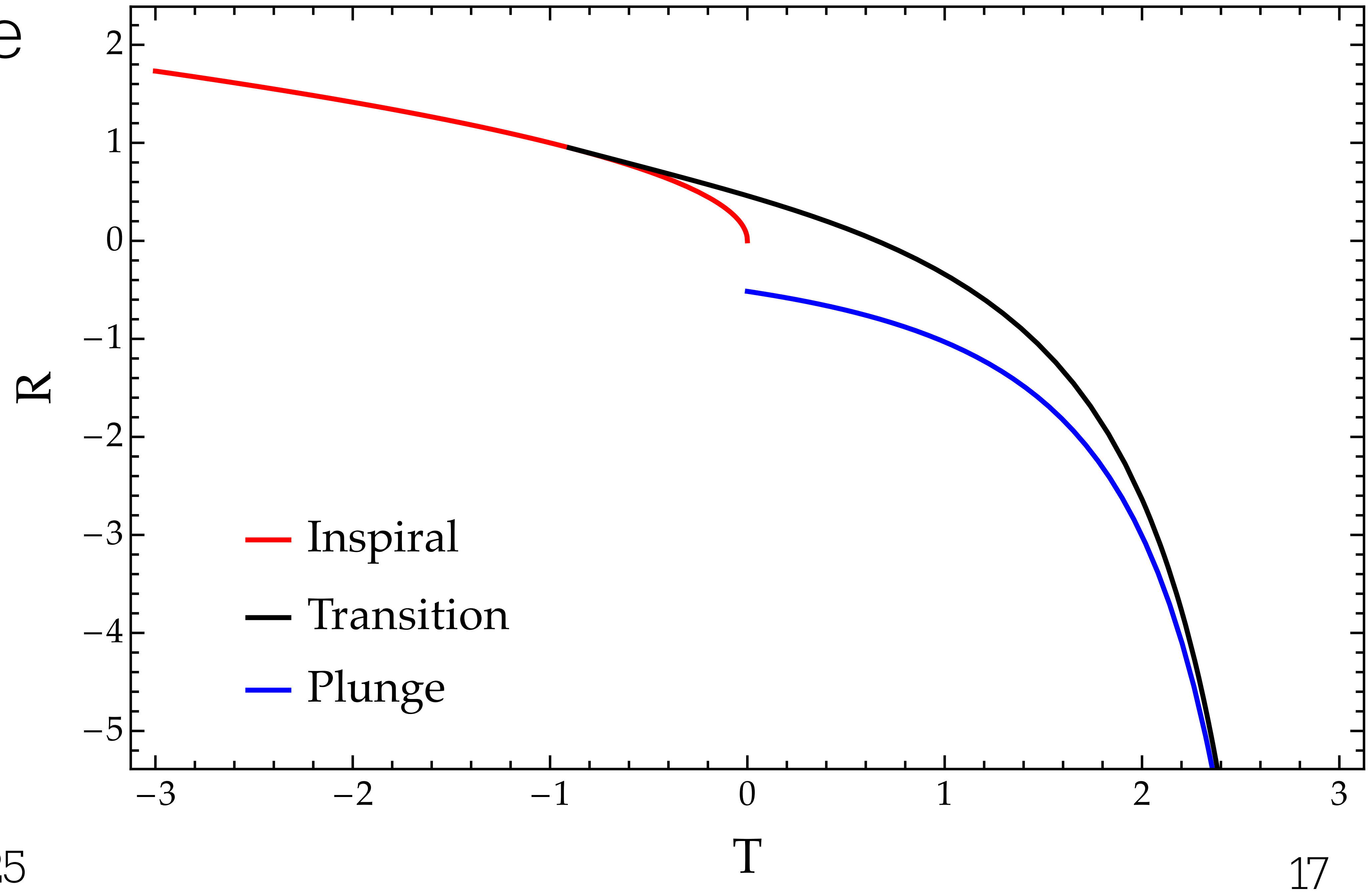


EVOLUTION OF PLUNGING BINARIES

THREE STAGES

- **Inspiral regime**: the orbit of the secondary body evolves adiabatically. Extreme mass ratio inspiral simulations end to the ISCO;
- **Transition regime**: the character of the orbit gradually changes as the secondary approaches the **ISCO** of the primary BH;
- **Plunge**: the body plunges into the horizon along a geodesic.

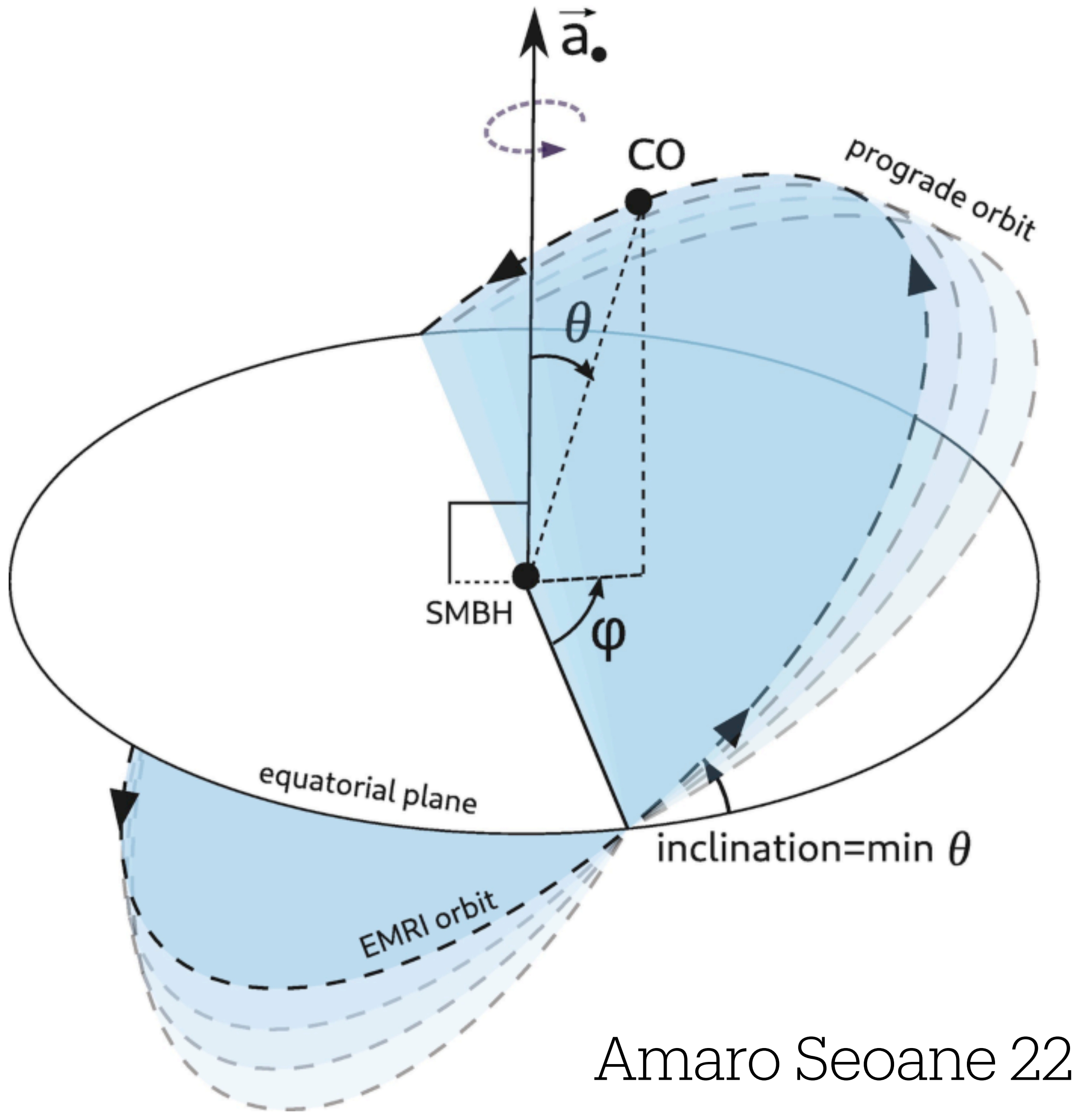




FRAMEWORK

HOW CAN A BINARY BE PARAMETRIZED

- **Kerr** BHs
- Mass ratio $q = m_2/m_1$
- Dimensionless spin parameter $-1 \leq j \leq 1$
- ? **Equatorial** orbits ($\theta = \pi/2$)
- ? **Circular orbit** near the ISCO
- ? Edge-on binary ($\iota = \pi/2$)

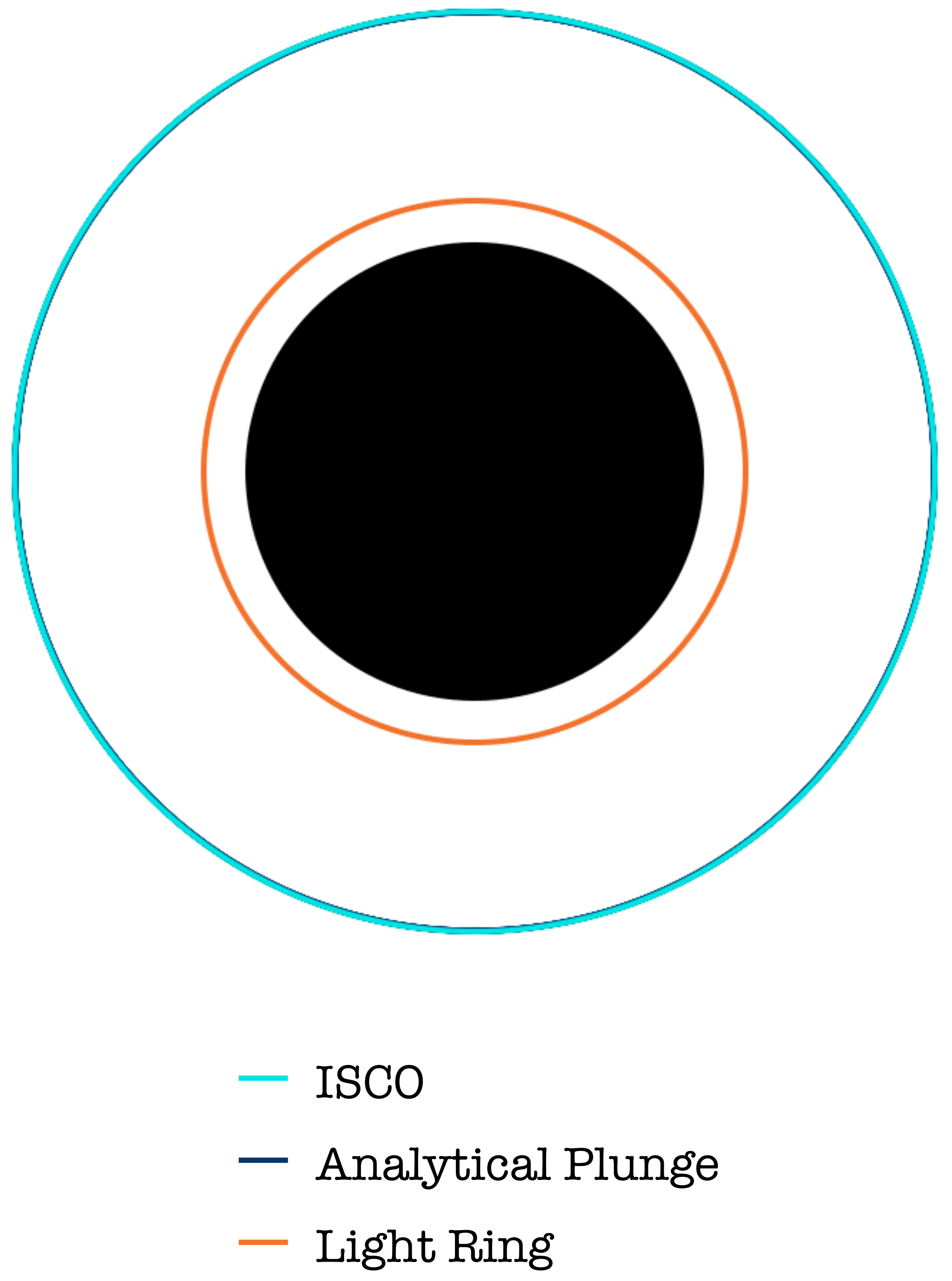


Amaro Seoane 22

PLUNGING GEODESICS

DYSON-VAN DE MEERT

- We studied QNM excitation by a particle plunging from the innermost stable circular orbit (ISCO)
- Dyson and van de Meent have recently found an analytical expression for geodesics plunging in terms of elliptic functions.
- In this model, the plunge is analytically extended up to the ISCO: the transition is not modeled.
- The body is in free fall, with energy and angular momentum fixed at the ISCO values



GRAVITATIONAL WAVE EMISSION

FROM THE PLUNGE ORBIT

- Numerically computed adopting the **Sasaki-Nakamura** (SN) formalism
- Second order differential equation

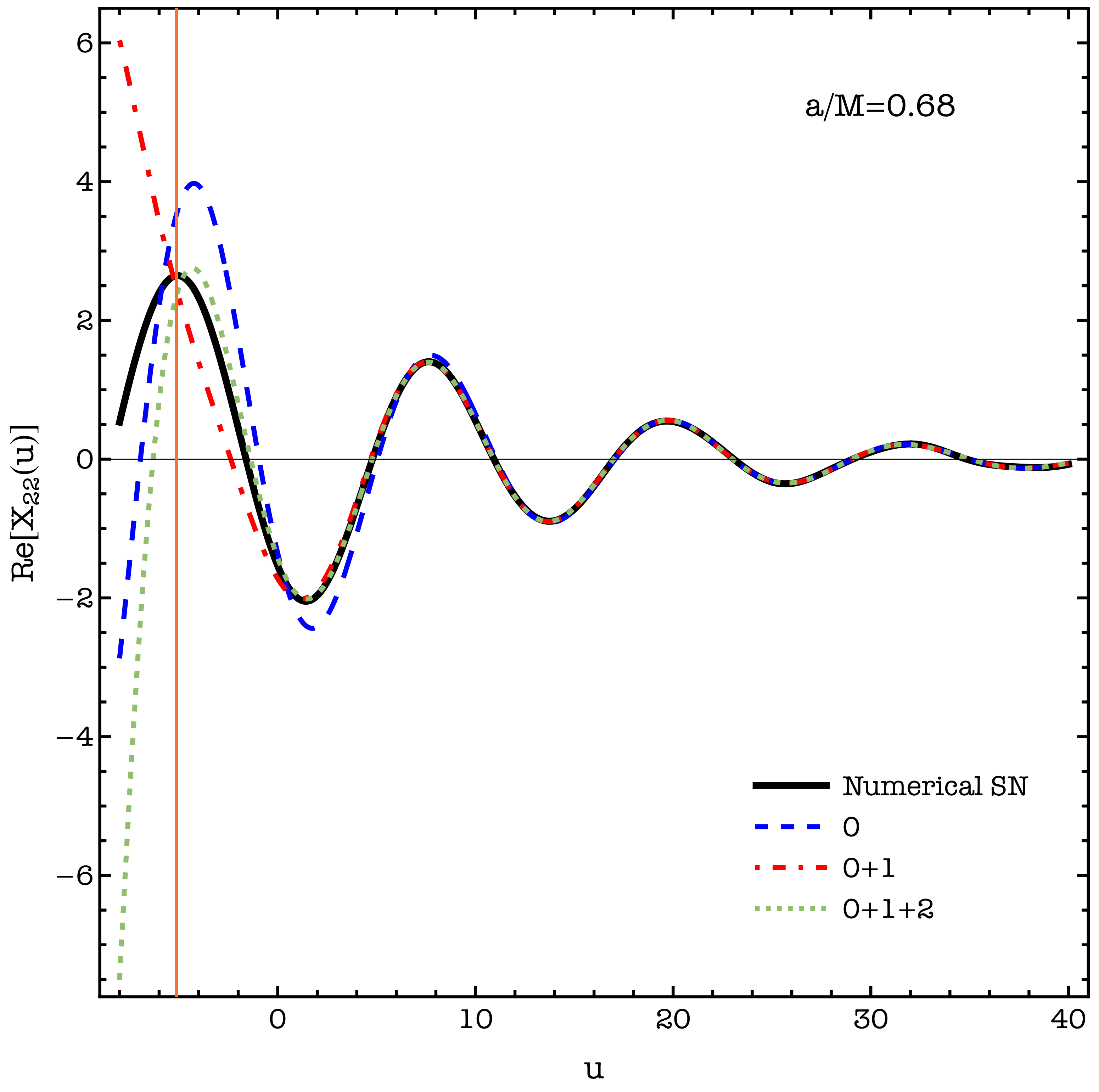
$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r_*^2} - F_{\ell m}(r)\frac{\mathrm{d}}{\mathrm{d}r_*} - U_{\ell m}(r)\right] X_{\ell m\omega}(r_*) = \mathcal{S}_{\ell m\omega}(r)$$

- The **source term** contains all the details of the **source's trajectory**
- The master equation can be solved through the **Green function method**

GRAVITATIONAL WAVE EMISSION

COMPARISON WITH RINGDOWN AMPLITUDES IN TIME DOMAIN

- Waveform can be rewritten as $X_{\ell m\omega} \approx -2\pi \sum_{\ell m} C_{\ell m} e^{-i\omega_{\ell m} u}$
- We quantify the excitation of a quasi-normal mode via the coefficients C_q, which are commonly called **excitation coefficients**.
- Full response function for $a = 0.68M$ and $\ell = m = 2$



RINGDOWN FOR PLUNGING BINARIES

RESULTS AND PROSPECTS

Construction of ringdown waveforms for a **plunging event**

My contributions:

- Developed a code to **treat the plunge** of the secondary in **frequency domain**
- The **complete waveform** can be reconstructed as the **superposition of excitation coefficient**, provided that more modes are included
- New catalogue of **excitation coefficients** for plunging events

What can be done:

- [ ] Characterize the plunge as a function of **eccentricity** and **inclination**
- [ ] Check the **validity** of the method for **comparable mass binaries**

Illustration by Lia Halloran

THANKS FOR THE ATTENTION!

LAURA PEZZELLA
LAURA.PEZZELLA@GSSI.IT
October 13rd 2025

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PUBLICATIONS

- L. Pezzella, K. Destounis, A. Maselli, V. Cardoso, *Quasinormal modes of black holes embedded in halos of matter*, doi.org/10.1103/PhysRevD.111.064026
- M. Della Rocca, L.Pezzella, E. Berti, L.Gualtieri, A.Maselli, Quasinormal ringing of Kerr black holes III. Excitation factors by an equatorially plunging particle, fall 2025

TALKS AND CONFERENCES

- *"Quasi Normal Modes of black holes surrounded by dark matter halos"*, Linking Advances in our Understanding of Theoretical Astrophysics and Relativity to Observations (LAUTARO) Meeting, Università degli Studi di Milano-Bicocca (UNIMIB), Milan (Italy), April 18th 2024
- *"Quasi Normal Modes of black holes surrounded by dark matter halos"*, Women in Theoretical Physics, Galileo Galilei Institute (GGI), Arcetri, Florence (Italy), October 8th 2024
- *"Post-ISCO ringdown: state-of-the-art and future prospects "*, 1st GSSI-JHU MAECI meeting on Gravitational Wave Astrophysics, L'Aquila (Italy), December 17th 2024
- *"Quasinormal modes of black holes embedded in halos of matter"*, Invited Speaker at University of Rome La Sapienza, Rome (Italy), January 22nd 2025
- *"Post-ISCO ringdown"*, Theoretical Horizons in Unraveling Relativity, Astrophysics, and Mergers (THURAM) Meeting, Gran Sasso Science Institute (GSSI), L'Aquila (Italy), May 8th 2025
- *"Environmental imprints on black hole ringdowns"*, 5th Meeting on GW Science in Scandinavia, Copenhagen (Denmark), May 16th 2025
- *"Environmental imprints on black hole ringdowns: quasinormal modes in matter halos"*, XV ET Symposium, Bologna (Italy), May 27th 2025
- *"Environmental imprints on black hole ringdowns: quasinormal modes in matter halos"*, 24th International Conference on General Relativity and Gravitation & 16th Edoardo Amaldi Conference on Gravitational Waves, Glasgow (UK), July 14th 2025