GSSI. Numerical Analysis. FS 2025-2026

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Course description

This course is an introduction to some topics of modern numerical analysis. The primary objective of the course is to develop graduate-level understanding of computational mathematics and skills to solve a range real-world mathematical problems on a computer by implementing advanced numerical algorithms using a scientific computing language (such as MATLAB or Python). The main topics covered by the course consist of numerical integration of differential equations, and numerical methods for continuous optimization. Special topics will include delay differential equations and eigenvalue optimization with applications.

Course requirements

Calculus, linear algebra and basic numerical analysis. Previous programming experience in any language may help.

Examination and grading

Written exam and practical (computational) assessment at the end of the course.

Course content

The course will cover the following topics

Numerical integration of differential equations

I. Quadrature

- I.1. Quadrature formulas, order conditions;
- I.2. Error analysis;
- I.3. Superconvergence and orthogonal polynomials;
- I.4. Gauss quadrature.

Initial Value Problems for ODEs

II. One step methods for ODEs

- II.1. Euler method;
- II.2. Runge-Kutta methods;
- II.3. Stiff problems;
- II.4. Collocation methods;
- II.5. Parabolic PDEs.

III. Adaptation to DDEs

Delay differential equations.

- III.1. Delay models;
- III.2. Integrating numerically delay differential equations.

Numerical optimization

IV. Gradient methods

   IV.1. Gradient systems;
   IV.2. Classical gradient methods;
   IV.3. Conjugate direction methods.

V. Constrained optimization

   V.1. Penalty methods;
   V.2. Projection methods.

Matrix nearness problems and eigenvalue optimization

VI. Eigenvalue optimization

   VI.1. Variational properties of eigenvalues;
   VI.2. Structured spectral optimization;
   VI.3. Applications to robust stability;
   VI.4. Applications to data science.

Recommended previous knowledge in numerical analysis

I. Numerical linear algebra

   I.1. Stability and conditioning of linear systems and eigenvalue problems;
   I.2. LU decomposition;
   I.3. Cholesky decomposition;
   I.4. QR decomposition;
   I.5. Iterative methods;
   I.6. Methods for computing eigenvalues;
   I.7. Power methods;
   I.8. QR method.

II. Nonlinear Equations

   II.1. Fixed point iteration;
   II.2. Bisection method;
   II.3. Secant method;
   II.4. Newton's method;

III. Interpolation and approximation

   III.1. Polynomial interpolation (Lagrange form, Newton form);
   III.2. Interpolation error and Chebyschev polynomials;
   III.3. Convergence (Runge's phenomenon);
   III.4. Round-off error influence on interpolation;
   III.5. Interpolation by Spline Functions;
   III.6. Least Squares Approximation

References

[1] R. Fletcher, *Practical methods of optimization.* Second edition. Wiley-Interscience [John Wiley & Sons], New York, 2001. xiv+436 pp.

[2] W. Gautschi, *Numerical analysis. An introduction.* Birkhäuser Boston, Inc., Boston, MA, 1997.

[3] N. Guglielmi and C. Lubich: Matrix nearness problems and eigenvalue optimization, https://arxiv.org/abs/2503.14750

[4] E. Hairer, S. Nørsett and G. Wanner, *Solving ordinary differential equations. I. Nonstiff problems.* Second edition. Springer Series in Computational Mathematics, 8. Springer-Verlag, Berlin, 1993.

[5] J. Nocedal and S.J. Wright, Numerical Optimization. Second edition, Springer, New York, 2006.

[6] J. Stoer and R. Bulirsch, *Introduction to numerical analysis. Translated from the German by R. Bartels, W. Gautschi and C. Witzgall.* Third edition. Texts in Applied Mathematics, 12. Springer-Verlag, New York, 2002.