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Low-rank splitting integrators for stiff differential equations

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The efficient numerical integration of large-scale matrix differential equations is an important and extensively studied problem in numerical analysis. Large-scale matrix differential equations typically arise from the space discretization of semi-linear partial differential equations. Examples include evolution equations in two space dimensions and differential Lyapunov and Riccati equations. The problems are usually stiff and standard numerical methods require a large amount of computing time and memory.

In the first part of the talk we propose a numerical low-rank approximation that is based on splitting methods. In particular, we split the stiff linear part of the vector field from the non-stiff non-linear one. The linear part is integrated exactly by employing the action of the matrix exponential function. This action is approximated numerically by an interpolation at Leja points. The non-stiff non-linear part, however, is integrated by a projector-splitting method. This provides a dynamic low-rank approximation that does not suffer from possible small singular values. We carry out a rigorous stiff error analysis of the proposed method and illustrate its benefits by numerical examples.

In the second part of the talk, we propose a dynamic low-rank integrator for the Vlasov-Maxwell system. This system of partial differential equations describes a collisionless plasma interacting with an electro-magnetic field. The key idea is to constrain the dynamics of the system to a low-rank manifold by means of a projection on the tangent space. The dynamics of the system is then represented by the low-rank factors, which are determined by solving lower-dimensional partial differential equations. The divergence of the electric field resulting from Maxwell's equations, however, is not consistent with the charge density computed from the Vlasov equation. To remedy this problem we propose a correction based on Lagrange multipliers which enforces Gauss' law up to machine precision.

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