SCALAR-INDUCED GRAVITATIONAL WAVES

From fundamental physics in early universe to PBH evolution

Anjali Abirami Kugarajh

Supervisors: Prof. Andrea Maselli, Prof. Sabino Matarrese and Dr. Angelo Ricciardone

GRAN SASSO SCIENCE INSTITUTE
SCHOOL OF ADVANCED STUDIES
Scuola Universitaria Superiore

INFN LNGS
Istituto Nazionale di Fisica Nucleare
Laboratori Nazionali del Gran Sasso

10.10.2024

Primordial Universe

Inflationary cosmology predicts the generation of quantum vacuum fluctuations

Cosmological Perturbation Theory

$$\mathbf{A} = \mathbf{A}^{(0)} + \delta^{(r)}\mathbf{A}$$

Scalar perturbation

Vector perturbation

$$ds^2 = a^2(\eta)\left[-\left(1 + 2\phi^{(1)} + \phi^{(2)}\right)d\eta^2 + \left(\partial_i w^{(1)} + \frac{1}{2}\partial_i w^{(2)} + \frac{1}{2}w_i^{(2)}\right)d\eta dx^i \right.$$
$$\left. \left\{\left(1 - 2\psi^{(1)} - \psi^{(2)}\right)\delta_{ij} + D_{ij}\left(\chi^{(1)} + \frac{1}{2}\chi^{(2)}\right) + \frac{1}{2}\left(\partial_i \chi_j^{(2)} + \partial_j \chi_i^{(2)} + \chi_{ij}^{(2)}\right)\right\} dx^i dx^j\right]$$

Tensor perturbation

Primordial Universe

Inflationary cosmology predicts the generation of quantum vacuum fluctuations

Scalar perturbations → Observed in CMB temperature anisotropies and density fluctuations

Tensor perturbations → Observed as gravitational waves

Primordial Gravitational Waves

Tensor perturbations → Observed as gravitational waves

GWs weakly interact with matter

Primordial GW come in a form of stochastic background (SGWB);
superposition of incoming GWs

SCALAR-INDUCED GW

Secondary tensor modes produced due to the coupling of first-order scalar fluctuations

Tensor perturbation

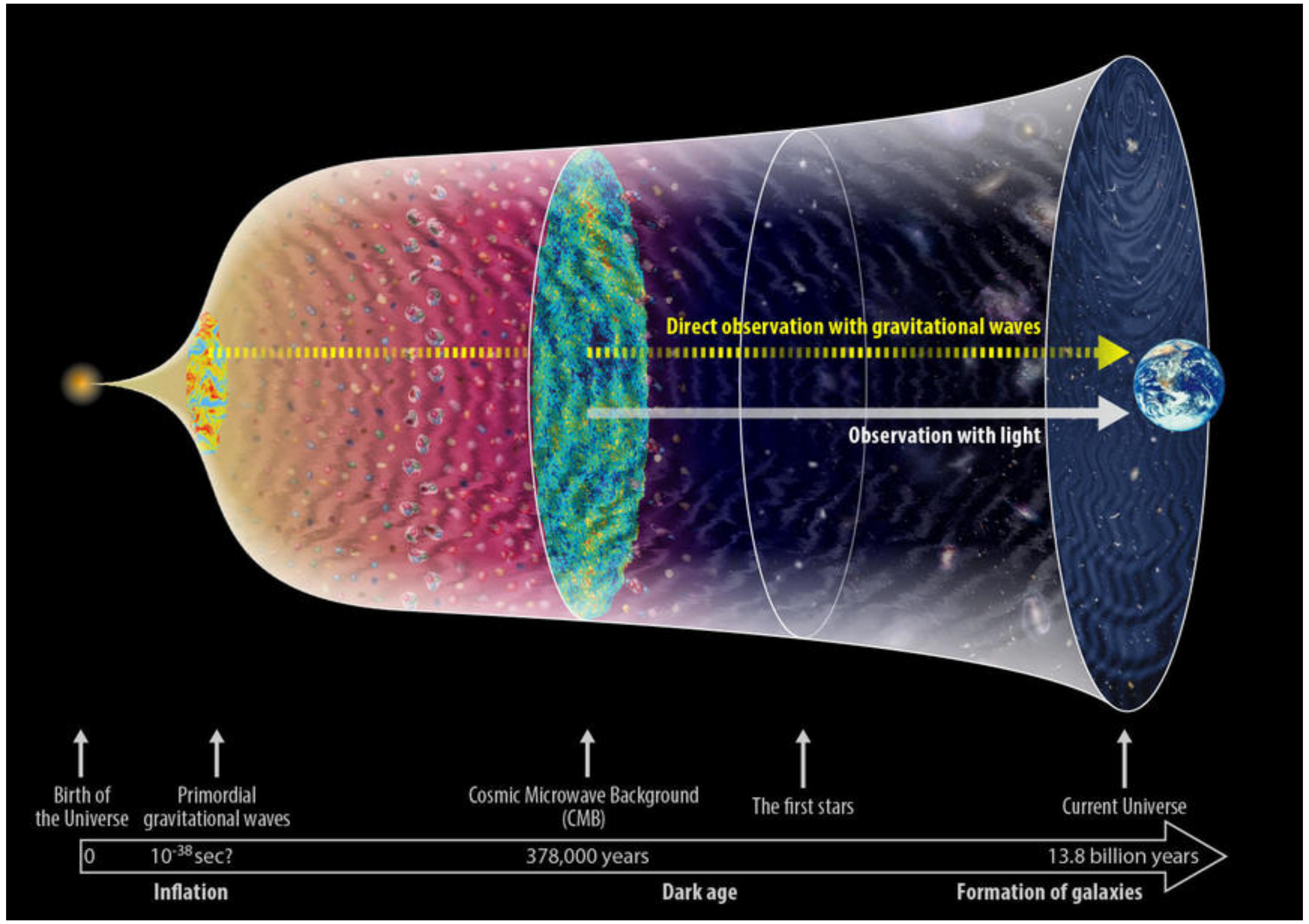
Scalar perturbation

$$\chi_j^{i(2)''} + 2\mathcal{H}\chi_j^{i(2)'} - \nabla^2 \chi_j^{i(2)} = \partial^i \phi^{(1)} \partial_j \phi^{(1)} + \ldots$$

Scalar perturbations	**Tensor perturbations**
First-order scalar spectrum →	Sources the generation of second-order GWs

DETECTION

Characteristic frequencies fall in the band of current and future GW interferometers

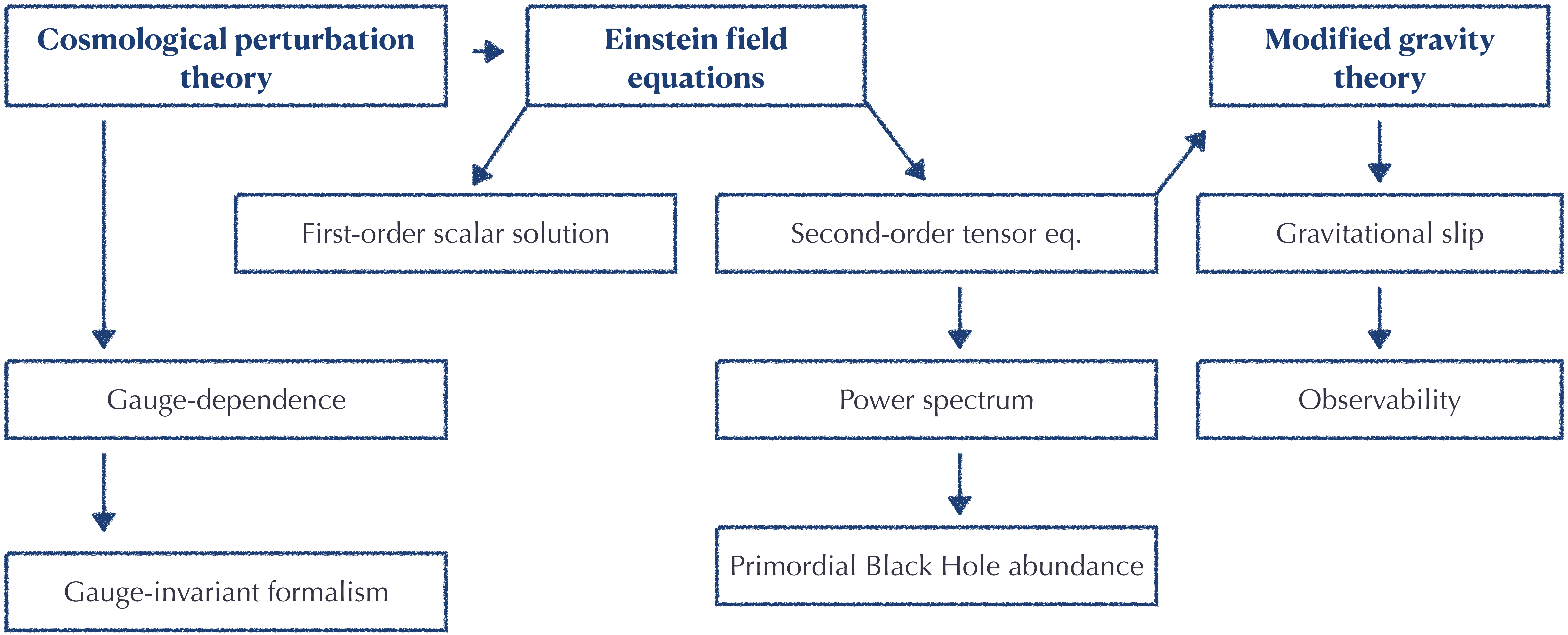


Pulsar Timing Arrays (PTA) observation providing strong evidence for a SGWB & compatibility with “scalar-induced” background

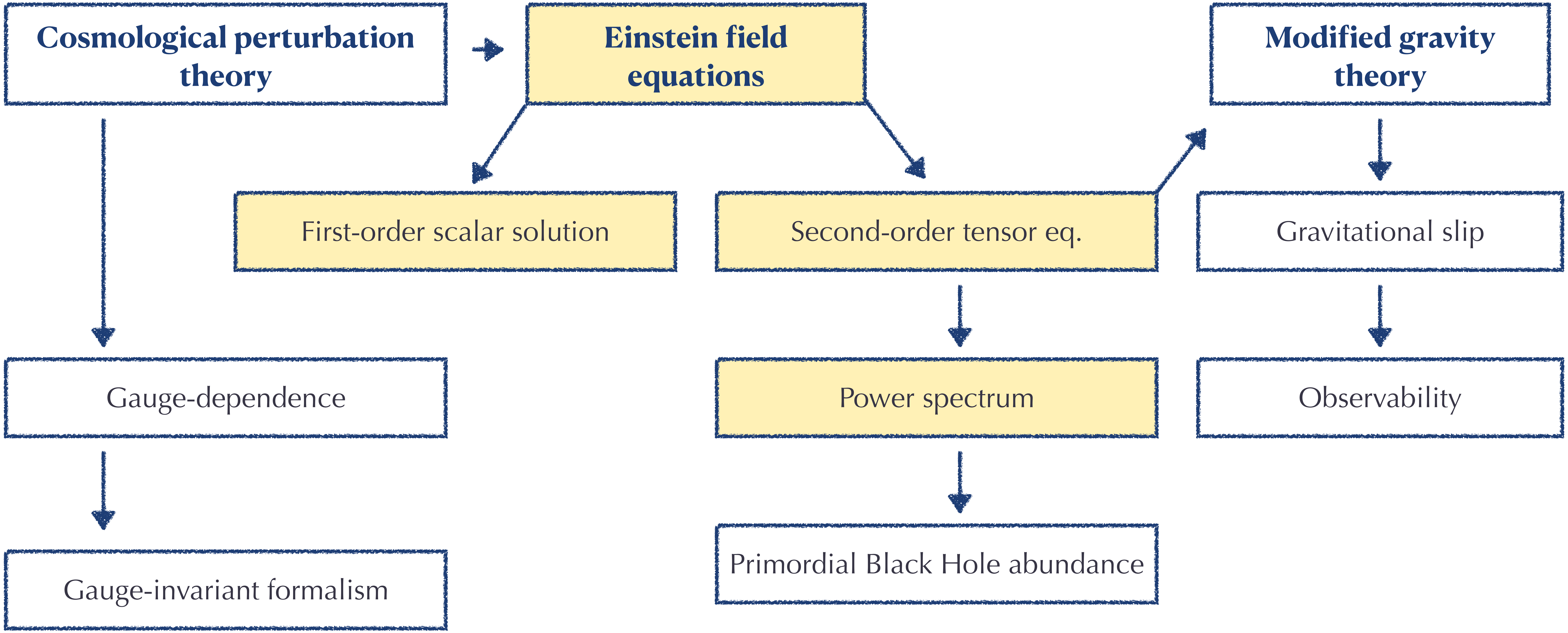
Depends on modeling assumptions highlighting that more data and analysis are needed to discern between cosmological or astrophysical origin

References: arXiv:2307.02399v2

Project structure



Project structure



SIGW E.o.M

$$\chi_j^{i(2)''} + 2\mathcal{H}\chi_j^{i(2)'} - \nabla^2\chi_j^{i(2)} = -4P_{jm}^{li}S_l^m$$

$$\begin{aligned}
S_j^i =\; & \partial^i\phi^{(1)}\partial_j\phi^{(1)} + 2\phi^{(1)}\partial^i\partial_j\phi^{(1)} - 2\psi^{(1)}\partial^i\partial_j\phi^{(1)} - \partial_j\phi^{(1)}\partial^i\psi^{(1)} - \partial^i\phi^{(1)}\partial_j\psi^{(1)} \\
& + 3\partial^i\psi^{(1)}\partial_j\psi^{(1)} + 4\psi^{(1)}\partial^i\partial_j\psi^{(1)} \\
& - \frac{4}{3\mathcal{H}^2(1+w)}\left[\partial^i\left(\psi^{(1)'} + \mathcal{H}\phi^{(1)}\right)\partial_j\left(\psi^{(1)'} + \mathcal{H}\phi^{(1)}\right)\right] \\
& + 4\psi^{(1)}\delta^{ik}\left[\left(\partial_j\partial_k - \frac{1}{3}\nabla^2\delta_{jk}\right)\left(\phi^{(1)} - \psi^{(1)}\right)\right]
\end{aligned}$$

→ Anisotropic stress

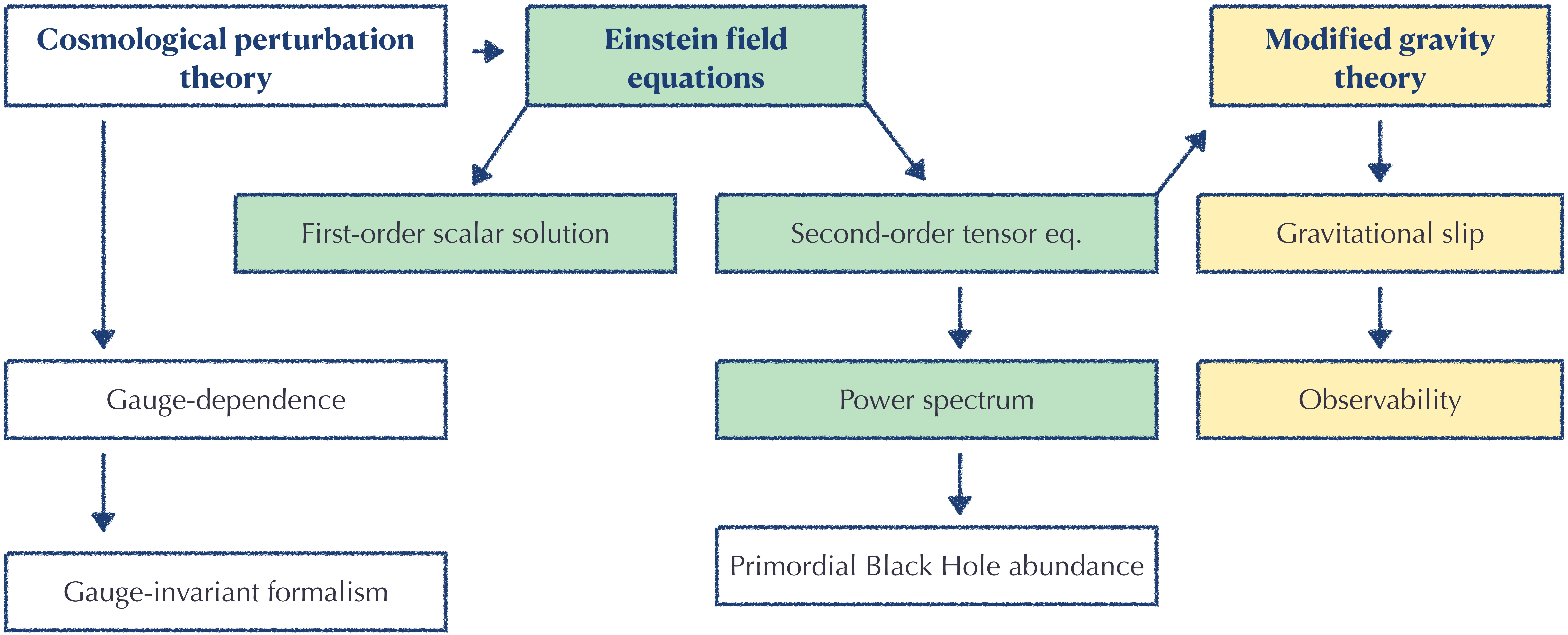
ANISOTROPIC STRESS

Standard GR

- Contribution from free-streaming photon and neutrinos is small and negligible at late times
- At linear-order, scalar potentials are set to be equal to one another, $\phi^{(1)} = \psi^{(1)}$

Mismatch between scalar potentials?

Project structure



Gravitational slip

A non-standard relation between the potentials can arise due to modification in geometric part of Einstein equation

Background quantity

Anisotropic stress source

$$\psi^{(1)} - \phi^{(1)} = \sigma(a)\Pi + \pi_m$$

Combination of linear perturbations

MODIFIED GRAVITY

- $f(R)$ gravity, Lagrangian density f is a function of Ricci scalar
- Model considered: $f(R) = R + \alpha R^2$
- Concentrating on the first-order corrections, i.e. $\mathcal{O}(\alpha)$

$$\psi^{(1)} - \phi^{(1)} = \frac{F^{(1)}}{F^{(0)}}$$

GRAVITATIONAL SLIP in *f*(*R*)

$$\kappa^2 \pi_j^{(1)i} = -\left(1 + 12\alpha a^{-2}\left[\mathcal{H}' + \mathcal{H}^2\right]\right)\left(\partial^i\partial_j - \frac{1}{3}\nabla^2\delta_j^i\right)\left(\phi^{(1)} - \psi^{(1)}\right)$$

$$- 2\alpha a^{-2}\left(\partial^i\partial_j - \frac{1}{3}\nabla^2\delta_j^i\right)\left(-6\psi^{(1)''} - 6\mathcal{H}\phi^{(1)'} - 18\mathcal{H}\psi^{(1)'}\right.$$

$$\left. -12\left[\mathcal{H}' + \mathcal{H}^2\right]\phi^{(1)} - 2\nabla^2\phi^{(1)} + 4\nabla^2\psi^{(1)}\right).$$

<u>NO MODIFICATION</u> to the fluid description - anisotropic stress contribution neglected, i.e. $\pi_j^{i(1)} = 0$

OBSERVABLE

- Fraction of the GW energy density per logarithmic wavelength

$$\Omega_{GW}(\eta, k) = \frac{1}{24}\left(\frac{k}{a(\eta)H(\eta)}\right)^2 \left(\overline{P_\chi(k,\eta)} + \alpha\,\overline{\delta P_\chi(k,\eta)}\right)$$

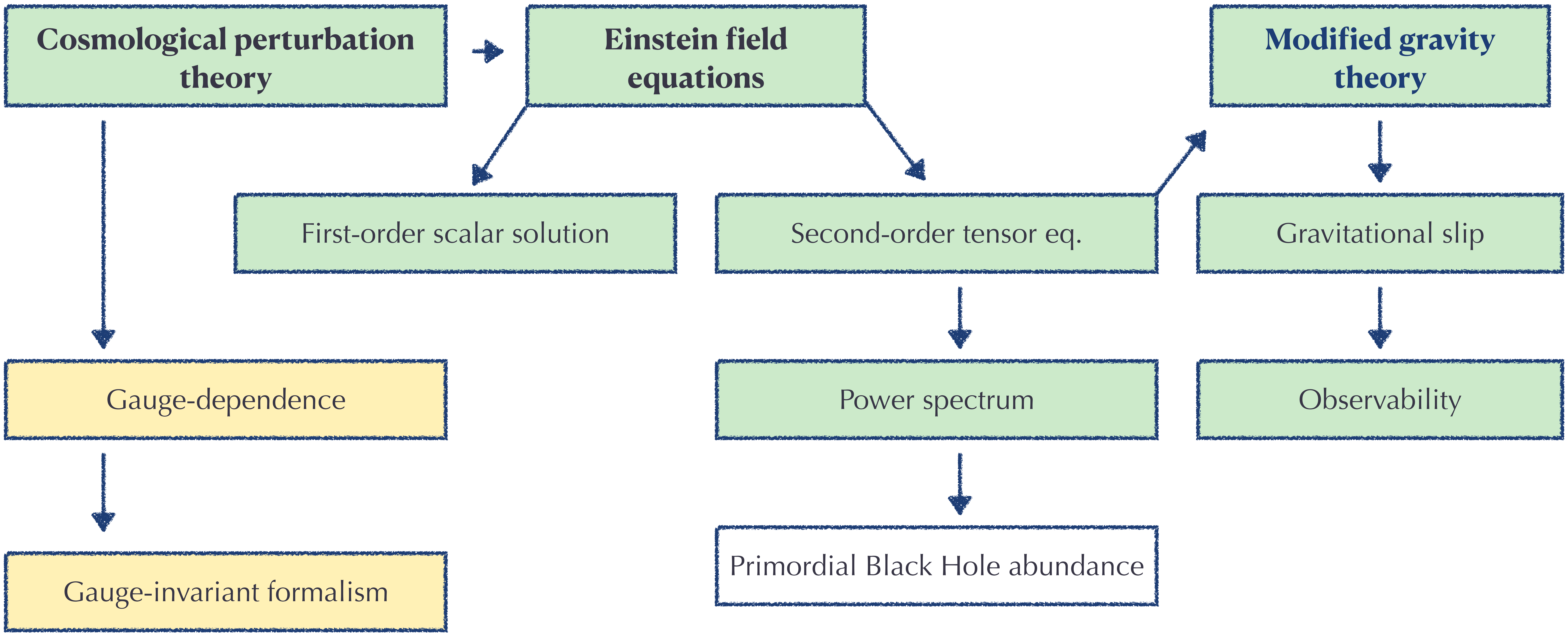
Broad limit on constraint on α

$$\alpha R \ll 1$$

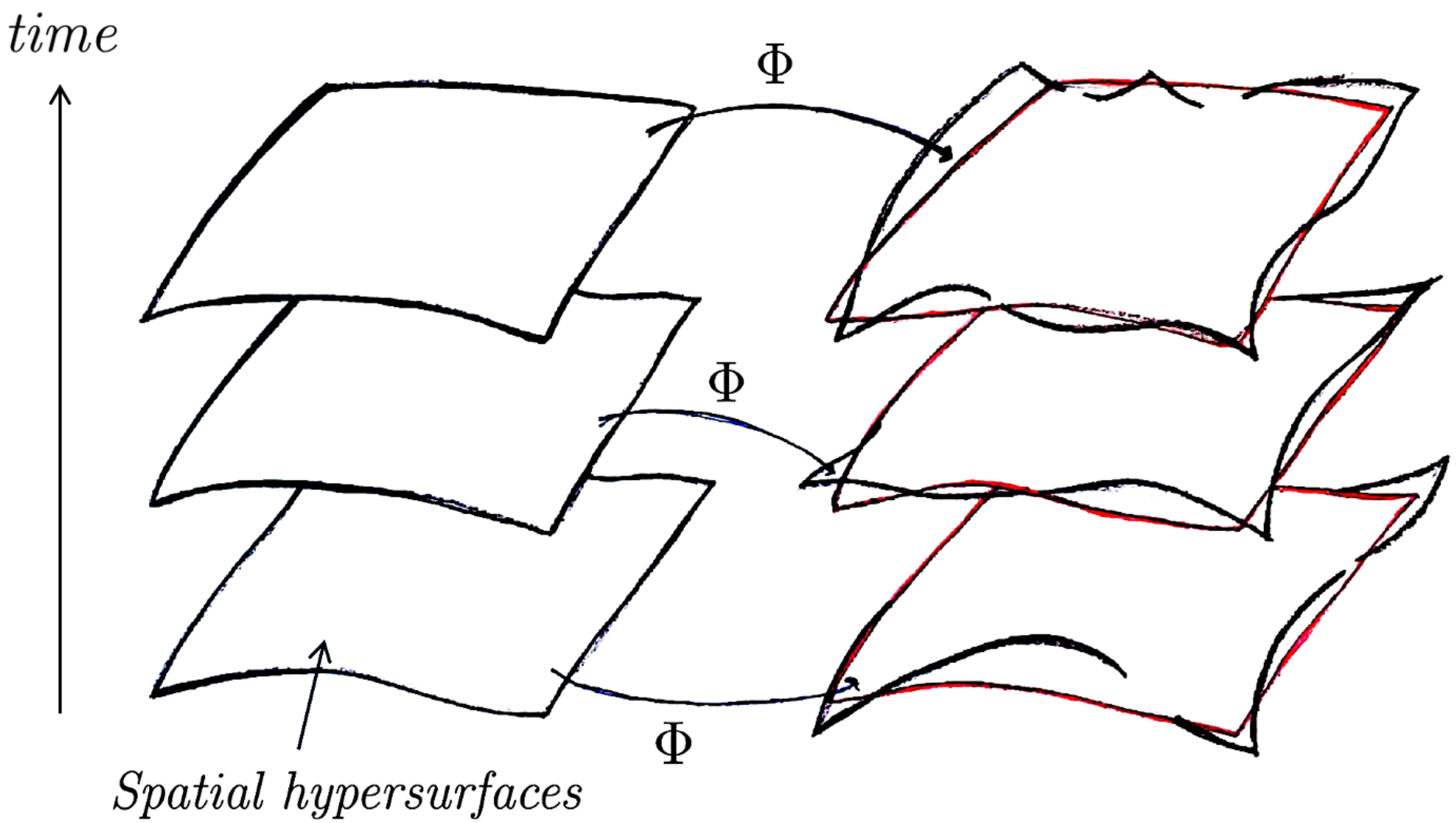
Extra factor in power spectrum

$$\delta P \propto k^4$$

Project structure



Gauge-dependence



Gauge is the mapping between background and perturbed space
Chosen to best suit model and problem

First-order perturbations gauge-dependent,
Source of second-order tensors gauge-dependent

Gauge-invariant formalism

Poisson gauge

$$w^{(r)} = \chi^{(r)} = \chi_i^{(r)} = 0$$

$$\Phi = \phi^{(1)} - \mathcal{H}\sigma - \sigma'$$

$$\Psi = \psi^{(1)} + \mathcal{H}\sigma$$

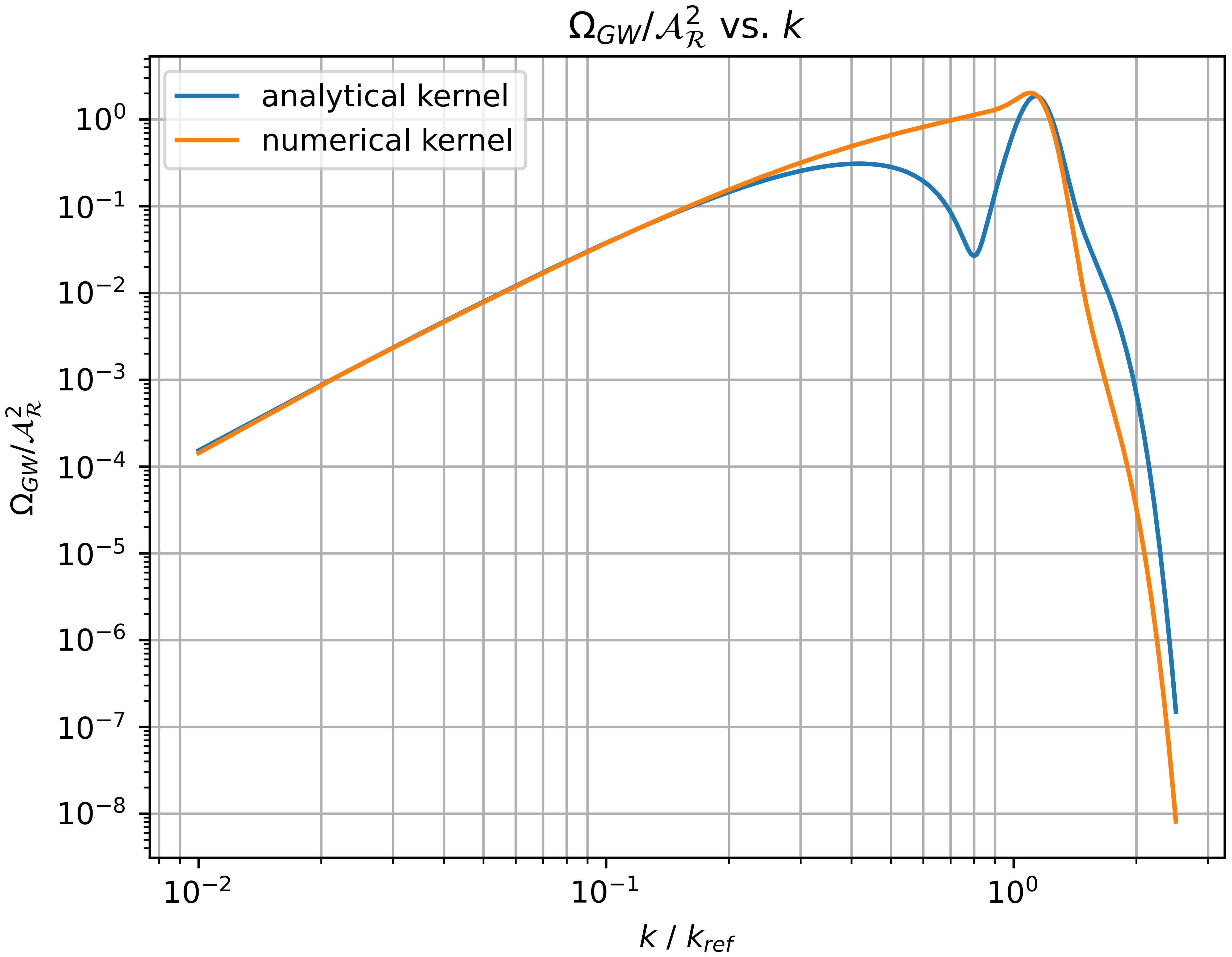
Spatially flat gauge

$$\psi^{(r)} = \chi^{(r)} = 0$$

$$\tilde{\phi}^{(1)} = \phi^{(1)} + \psi^{(1)} - \left(-\frac{\psi^{(1)}}{\mathcal{H}}\right)'$$

$$\tilde{w}^{(1)} = w^{(1)} - \frac{\psi^{(1)}}{\mathcal{H}} - \chi^{(1)\prime}$$

Spectral Energy Density



Ideal solution would be to find a form of the energy density of GWs, which will remain gauge-invariant under second-order gauge transformation

arXiv:1912.00885
arXiv:1911.09689
arXiv:2012.14016

What else?

PRIMORDIAL BLACK HOLES

Which density fluctuations are the source of induced GWs?

1. Large fluctuations re-entering horizon can collapse and form PBHs
2. After formation, gas of PBHs can be treated as fluid with density fluctuations, GW produced via gravitational potential of gas of PBHs

Scalar-induced gravitational waves can provide constraints on abundance of PBHs

References: arXiv:1612.06264v2 & arXiv:2012.08151v2

Conclusion

- Detection of scalar-induced gravitational waves has opened up a new door for cosmology
- Can be used to probe various early universe and modified gravity theories
- Gravitational slip signature in the source term seen in the observable spectrum.
- Future studies on connection to Primordial Black Holes

Back up slides

Decomposing scalar potentials

- Defining scalar perturbation as:

$$\phi^{(1)} = \phi^{(1)}_{gr} + \alpha\delta\phi^{(1)}$$

$$\psi^{(1)} = \psi^{(1)}_{gr} + \alpha\delta\psi^{(1)}$$

$$\downarrow$$

$$\phi^{(1)}_{gr}$$

GRAVITATIONAL SLIP in $f(R)$

$$\delta\psi^{(1)} = \delta\phi^{(1)} + 2a^{-2}\left(-6\phi_{gr}^{(1)''} - 24\mathcal{H}\phi_{gr}^{(1)'} - 12\left[\mathcal{H}' + \mathcal{H}^2\right]\phi_{gr}^{(1)} + 2\nabla^2\phi_{gr}^{(1)}\right).$$

- Non-standard relation at first-order $\mathcal{O}(\alpha)$ correction, $\delta\phi^{(1)} \neq \delta\psi^{(1)}$

SIGW E.o.M in $f(R)$ gravity

$$\left(1+12a^{-2}\alpha\left[\mathcal{H}'+\mathcal{H}^2\right]\right)\left(\chi_j^{i(2)''}+2\mathcal{H}\chi_j^{i(2)'}+k^2\chi_j^{i(2)}\right) + 12a^{-2}\alpha\left[\mathcal{H}''+2\mathcal{H}\mathcal{H}'\right]\chi_j^{i(2)'} = S_{gr}+\alpha\delta S,$$

Standard GR

First-order modification

Source in $f(R)$ gravity

$$\delta S(\mathbf{k},\eta) = \mathbf{e}^l_m(\mathbf{k})\delta S^m_l(\mathbf{k})$$
$$= 4\int \frac{d^3\tilde{\mathbf{k}}}{(2\pi)^{3/2}}\mathbf{e}^l_m(\mathbf{k})\tilde{k}_l\tilde{k}^m \Big[6\delta\phi^{(1)}_{\tilde{k}}\phi^{(1)}_{gr,k-\tilde{k}} + 2\eta\delta\phi^{(1)}_{\tilde{k}}\phi^{(1)\prime}_{gr,k-\tilde{k}}$$
$$+2\eta\delta\phi^{(1)\prime}_{\tilde{k}}\phi^{(1)}_{gr,k-\tilde{k}} + 2\eta^2\delta\phi^{(1)\prime}_{\tilde{k}}\phi^{(1)\prime}_{gr,k-\tilde{k}}$$
$$-\left(\frac{\eta^*}{a^*}\right)^2\Big\{\left(96\eta^{-2}+4\tilde{k}^2\right)\phi^{(1)\prime}_{gr,\tilde{k}}\phi^{(1)\prime}_{gr,k-\tilde{k}} + 60\eta^{-2}\phi^{(1)}_{gr,\tilde{k}}\phi^{(1)\prime\prime}_{gr,k-\tilde{k}} + 12\eta^{-1}\phi^{(1)}_{gr,\tilde{k}}\phi^{(1)\prime\prime\prime}_{gr,k-\tilde{k}}$$
$$+36\eta^{-1}\phi^{(1)\prime}_{gr,\tilde{k}}\phi^{(1)\prime\prime}_{gr,k-\tilde{k}} + 12\phi^{(1)\prime}_{gr,\tilde{k}}\phi^{(1)\prime\prime\prime}_{gr,k-\tilde{k}} + 4\tilde{k}^2\eta^{-2}\phi^{(1)}_{gr,\tilde{k}}\phi^{(1)}_{gr,k-\tilde{k}}\Big\}\Big].$$

Power spectrum

- Power spectrum of second-order tensor mode

$$P_\chi(k,\eta) + \delta P_\chi(k,\eta) = 8\int_0^\infty dv \int_{|1-v|}^{1+v} du \left(\frac{4v^2 - \left(1 - u^2 + v^2\right)^2}{4uv}\right)^2 \times \left(I^2(\eta,k,u,v) + 2I(\eta,k,u,v)\delta I(\eta,k,u,v)\right) P_\zeta(ku)P_\zeta(kv)$$

- Primordial curvature power spectrum

$$P_\zeta(k) = \frac{\mathcal{A}_\zeta}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\ln^2(k/k_*)}{2\sigma^2}\right)$$

OBSERVABLE

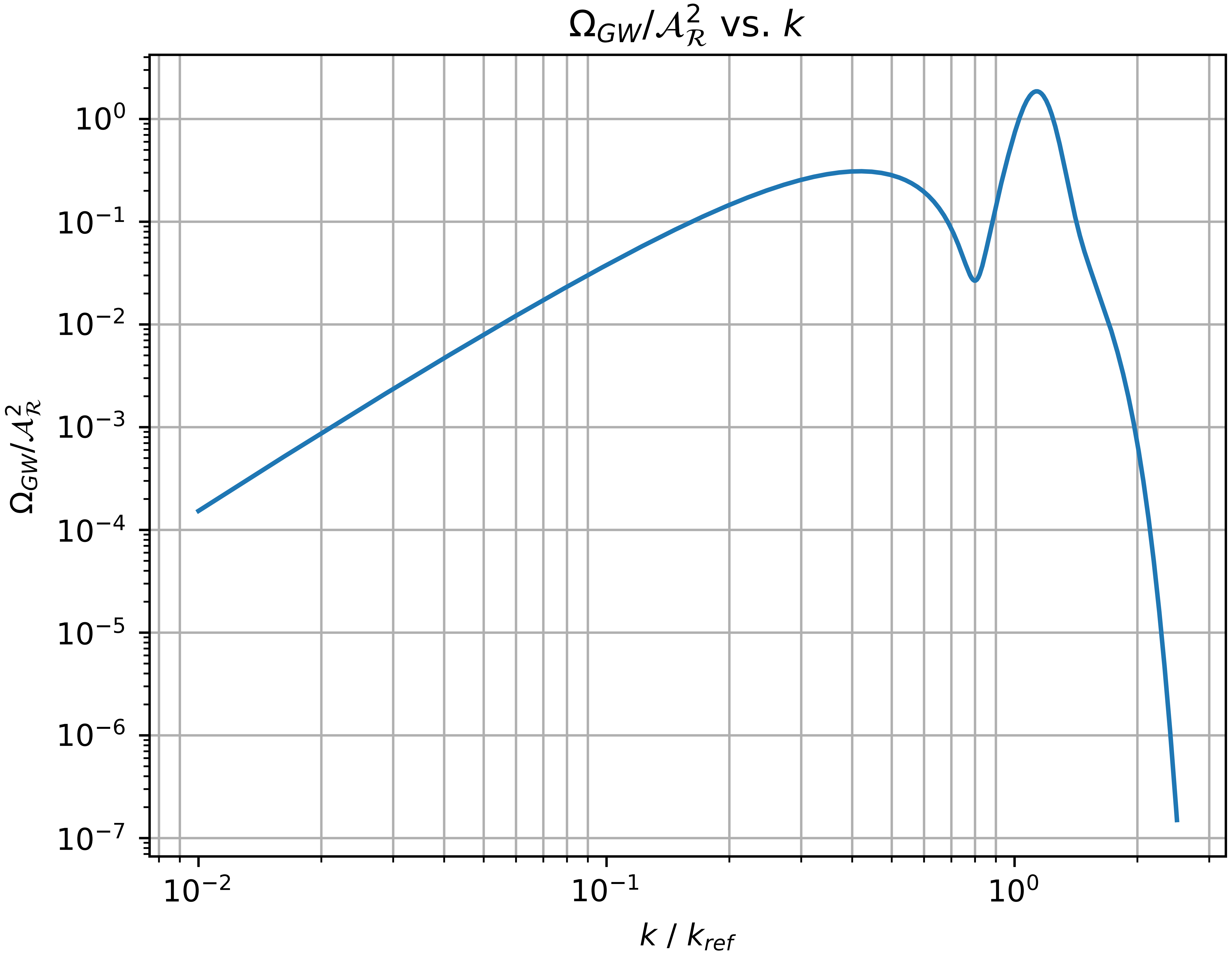
- Fraction of the GW energy density per logarithmic wavelength

$$\Omega_{GW}(\eta, k) = \frac{1}{24}\left(\frac{k}{a(\eta)H(\eta)}\right)^2 \left(\overline{P_\chi(k,\eta)} + \alpha\,\overline{\delta P_\chi(k,\eta)}\right)$$

- Observable spectrum today

$$\Omega_{GW,0}(\eta_0, k)h^2 = \Omega_{r,0}(k)h^2 \left(\frac{g_*(\eta_0)}{g_*(\eta_k)}\right)^{1/3} \Omega_{GW,e}(k)$$

OBSERVABLE


$\Omega_{GW}/\mathcal{A}_{\mathcal{R}}^2$ vs. k
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
$\Omega_{GW}/\mathcal{A}_{\mathcal{R}}^2$
10^{-2}
10^{-1}
10^0
k / k_{ref}

Gauge-transformation

$$\tilde{\phi}^{(1)} = \phi^{(1)} - \mathcal{H}\xi^0 - (\xi^0)'$$

$$\tilde{\psi}^{(1)} = \psi^{(1)} + \mathcal{H}\xi^0$$

$$\tilde{w}^{(1)} = w^{(1)} + \xi^0 - \xi'$$

$$\tilde{\chi}^{(1)} = \chi^{(1)} - \xi$$

Spectral Energy Density