Mathematical Challenges in Quantum Mechanics
Book of abstract – Contributed Talks

Gran Sasso Science Institute, L'Aquila (IT)
9-14 February 2025

Schedule of the Contributed Talk Sessions

Session A – Auditorium

Monday

Session B –Main Lecture Hall

Monday

Tuesday

Tuesday

Thursday

Thursday

Friday

Friday

Titles & Abstracts (Alphabetical order)

Danko Aldunate – Pontificia universidad catolica de Chile

Nonlinear Dirac Equation: Characterization of a Gap Property for the Linearized 1D Soler Model

We establish for the 1D Soler model with power nonlinearities $f(s) = s|s|^{p-1}$, $p > 0$, that the upper-right block operator L_0 of the linearized operator satisfies: its ground states -2ω and 0 are its only two eigenvalues in the gap of its essential spectrum iif $p \geq 1$. To that effect, starting from this property for the Gross–Neveu model ($p = 1$) established in [1], we make use on the one hand of a minmax principle for operators with a gap to obtain the result for $p > 1$, and we derive on the other hand the singular part of the resolvent of the Gross–Neveu operator for the result when $p < 1$. Our second main result is the simplicity of generalized eigenfunctions at the thresholds of the essential spectrum for Dirac operators with potential. These results apply in particular to lower-left block operator L_μ.

Giulia Elena Aliffi – University of Catania

A generalized form of the Schrödinger equation for the Resonant Tunneling Diode

A Resonant Tunnelling Diode (RTD) is a semiconductor device that thanks to his properties (low power, high-speed, compatibility with MOSFETs...) has gained, over the years, increasing relevance in the electronic field. The aim of my talk is to introduce a fourth order Schrdinger equation from the Kane dispersion relation and calculate new transparent boundary conditions for it. In addition, stationary solutions of a RTD in a ballistic regime are shown and compared with the simulations already existing in literature. Finally, a more generalized expression for the Schrödinger equation and the current are given

Dimitrios Ampelogiannis – King's College London

Rigorous lower bounds on diffusion in chaotic quantum spin chains

Rigorously establishing the laws of hydrodynamics from the microscopic dynamics of quantum many-body systems is a fundamental goal of mathematical physics, with many open problems. One such problem is proving that systems exhibit diffusive transport, by lower-bounding the respective transport coefficients. In this talk we will discuss quantum spin chains, where one can rigorously establish a general lower bound on the Onsager matrix, which implies a bound on the diffusion constant. We will then see how this bound can be applied to non-integrable spin chains, with general nearest-neighbor interactions, to get a rigorous non-zero lower bound on energy diffusion, thus providing an important proof-of-concept that one can indeed rigorously establish diffusion (macroscopic) starting from the Hamiltonian dynamics (miscroscopic).

Giuliano Angelone – Sapienza Università di Roma

Classical echoes of quantum boundary conditions

In this talk I will discuss the classical limit of a non-relativistic particle in a one-dimensional box, considering all the possible quantum boundary conditions that make the kinetic-energy operator self-adjoint. In particular, after determining the Wigner functions associated with the eigenfunctions of the Hamiltonian, I will analyze their behavior in the high-energy regime. I will show that these quantum systems split into two classes: while all the systems belonging to the first class collapse to the same classical system, the systems in the second class display a residual dependence on the quantum boundary conditions even in the classical limit.

Alex Bols – ETH Zurich

Many-body Fu-Kane-Mele index

We define a $\mathbb{Z}_2$-valued index for stably short-range entangled states of two-dimensional fermionic lattice systems with charge conservation and fermionic time reversal symmetry. The index takes its non-trivial value precisely if the 'fluxon', the state obtained by inserting a π-flux through the system, transforms under time reversal as part of a Kramers pair. This index extends the Fu-Kane-Mele index of free fermionic topological insulators to interacting systems.

Morris Brooks – University of Zurich

The effective mass of the Fröhlich polaron at strong coupling

In this talk we will investigate the Fröhlich polaron in the strong coupling limit, which is a model describing the interactions of a charged particle, e.g. an electron, with a polarizable environment. Notably, the model is simple enough to allow for rigorous mathematical proofs, while giving rise to a multitude of interesting and non-trivial phenomena, such as an effectively increased mass of the electron. In particular, we will discuss a recent proof concerning the validity of the celebrated Landau-Pekar formula for the effective mass of the Fröhlich polaron at strong couplings, which has been an outstanding open problem in mathematical physics conjectured by Spohn in 1987.

Domenico Cafiero – Politecnico di Milano

Homogenization regime for zero-range interactions

We consider a non-relativistic quantum particle in $\mathbb{R}^3$ subject to several point-like δ-interactions. We investigate the so-called homogenization limit, making reference to a scaling regime where the intensities of the single interactions and the distances between the points go to zero as the number of centers grows, while the total interaction strength remains finite. Under suitable assumptions on the centers' positions, we show that the quadratic form associated to the N-point interactions operator Γ-converges in the weak

and strong topology of $L^2(\mathbb{R}^3)$ to the quadratic form associated to a Schrödinger operator with a regular potential. As a consequence, we obtain the convergence of the corresponding Hamiltonians in the strong resolvent sense.
Based on a joint work with Michele Correggi and Davide Fermi (Politecnico di Milano).

Martino Caliaro – Gran Sasso Science Institute

On the instability of a 1D Gross-Pitaevskii steady flow past a delta potential

In this talk we investigate the dynamics of a one-dimensional Gross-Pitaevskii flow in presence of a static obstacle. The flow of the quantum fluid is obtained by imposing boundary conditions on its density and on its velocity v at spatial infinity. The static obstacle is modeled by a repulsive delta potential. For subcritical values of the potential strength and for subsonic velocities, this system admits two branches of stationary states. By looking at numerical simulations, it is generally argued that the first branch is stable while the second branch is unstable. We employ the method of the Evans function to show the linear instability of the second branch, if the potential strength is small enough. This is a joint work with Paolo Antonelli (GSSI).

Fabrizio Caragiulo – SISSA

Quantum Hall Effect and quasi-periodicity

(Integer) Quantum Hall effect consists in both the exact and robust quantization of transverse conductivity of a 2-dimensional incompressible electron gas and in a relationship between indices defined in the bulk of the 2-dimensional material and excitations at the edge: the so-called bulk-edge correspondence. We prove the full validity of this picture also under weak-quasi periodic perturbations of the model, when only one edge-mode is present, using a combination of rigorous Renormalization Group techniques and lattice Ward identites. Such perturbations are physically relevant in a variety of situations as in Moir-super-lattices.

Giulia Cava – Sapienza Univeristà di Roma

The scaling limit of boundary spin correlations in non-integrable Ising models

We consider non-integrable planar Ising models obtained by perturbing the nearest-neighbor model via a weak, even, finite range potential and we study its critical theory in the half-plane. We prove that the scaling limit of the multipoint boundary spin correlations is the same as for the nearest-neighbor model, up to an analytic multiplicative renormalization constant. The proof is based on an exact representation of the generating function of correlations in terms of a Grassmann integral and on a multiscale analysis thereof; it generalizes the methods developed by G. Antinucci, A. Giuliani and R.L. Greenblatt for the construction of the scaling limit of the bulk energy correlations in cylindrical geometry. This is a joint work with A. Giuliani and R.L. Greenblatt.

Edoardo D'Angelo – University of Milan

Renormalization group flow equations and the Nash-Moser theorem

The Renormalization Group (RG) Equation determines the flow of the effective action under changes in an artificial energy scale, which roughly corresponds to the size of the system under consideration. I report on a rigorous construction of a non-perturbative RG flow equation for the effective action in Lorentzian, possibly curved, spacetimes, generalising the Wetterich equation. I also give the main ideas of a proof of local existence of exact solutions for the RG equation, when a suitable Local Potential Approximation is considered, based on an application of the Nash-Moser theorem.

Elena Danesi – Politecnico di Torino

Stability of thermodynamic equilibria for the Hartree-Fock equation with exchange term

The Hartree-Fock equation admits homogeneous states that model infinitely many particles at equilibrium. The aim of this talk is to present a result on their asymptotic stability in large dimensions.This has been obtained for the equivalent formulation of the equation in the framework of random fields and it includes the exchange term for the first time in the study of these stationary solutions. It is a joint result with C. Collot (CYU), A.S. de Suzzoni (CMLS) and C. Malz (CMLS).

Shahnaz Farhat – Constructor University, Bremen

Expansion of the Many-body Quantum Gibbs State of the Bose-Hubbard Model on a Finite Graph

We consider the many-body quantum Gibbs state for the Bose-Hubbard model on a finite graph at positive temperature. We scale the interaction with the inverse temperature, corresponding to a mean-field limit where the temperature is of the order of the average particle number. For this model it is known that the many-body Gibbs state converges, as temperature goes to infinity, to the Gibbs measure of a discrete nonlinear Schrdinger equation, i.e., a Gibbs measure defined in terms of a one-body theory. In this article we extend these results by proving an expansion to any order of the many-body Gibbs state with inverse temperature as a small parameter. The coefficients in the expansion can be calculated as vacuum expectation values using a recursive formula, and we compute the first two coefficients explicitly.

Stefano Galanda – University of Genoa

Perturbative Equilibrium States for Interacting Fermionic Field Theories

The aim of this talk is to present the perturbative construction, in the framework of perturbative algebraic quantum field theory, of thermal equilibrium states for interacting massive and massless fermionic theories. In particular, the states are proven to exist also in the adiabatic limit. Therefore, part of the talk is devoted to the study of the decay of the correlation functions appearing in the clustering expansion of the interacting state. As an application, we show that the Debye screening of a classical stationary electromagnetic field coupled to a quantized Dirac field at thermal equilibrium for the interacting theory is obtained by explicitly solving the associated semiclassical Maxwell equations.

Ivan Gallo – Politecnico di Torino

The Effect of an Attractive Potential on the existence of NLSE Ground State on Graphs

We investigate the existence of ground states for the Nonlinear Schrdinger equation with an attractive potential on noncompact metric graphs. We present a theorem that extends existing techniques to determine the existence of ground states in our context. Then we prove the existence for both small and large masses using competitor functions. We also extend a nonexistence theorem when the potential is below a certain threshold. The study is inspired by research on quantum waveguides, where the curvature of a thin tube induces an effective, attractive potential.

Matteo Gallone – SISSA

Quasi-Periodic Ising model and Harris-Luck Criterion

Statistical mechanics models with quasi-periodic modulations describe a variety of physical systems of interest, such as quasi-crystals or experimental setups where quasi-periodicity serves as a practical realization of a disordered background. In such cases, the modulation amplitude is often tunable, and its variation can lead to a transition between a delocalized and a localized phase. The two-dimensional classical Ising model is a paradigmatic example of a critical system, making it particularly interesting to study the effects of a quasi-periodic coupling. According to the Harris criterion, a small-amplitude quasi-periodic modulation is expected to be irrelevant in the renormalization group sense, meaning that the critical exponents remain unchanged. Since the quasi-periodic model lies between the random Ising model (where disorder is always relevant, regardless of strength) and the family of non-integrable "deterministic" Ising models (where a broad class of small perturbations is proven to be irrelevant), understanding its critical behavior is of significant interest. Using rigorous renormalization group techniques and the fermionic representation of the model, we prove that small-amplitude quasi-periodic disorder is irrelevant for a class of intrinsically two-dimensional disorders broader than those studied in previous literature.
This is joint work with Vieri Mastropietro.

Raphaël Gautier – Politecnico di Milano, University of Rennes

Semiclassical limit of entropies and free energies

Entropy and Free energy are central concepts in both statistical physics and information theory with quantum and classical facets. In mathematics these concepts are quite common (dynamical system, probability theory, von Neumann algebras,...). In this work, we study the von Neumann and Wehrl entropies from the point of view of semi-classical analysis. In the first part, we prove the convergence of the von Neumann and Wehrl entropy for quantum Gibbs states (thermal equilibrium) in finite dimensional Hilbert space, after a suitable renormalization. In a second part, we prove the Gamma convergence of the free energies functionals associated to von Neumann and Wehrl entropy, and in particular the convergence of their minimums, and their minimizers are related to the Gibbs states. The main ingredients are the canonical commutation relations and their representation on the Fock space. In particular, we will use coherent states, the Wick and anti-Wick quantization and semiclassical

Leonardo Goller – SISSA

π-Flux Phase Stability in $\mathbb{Z}_2$ Lattice Gauge Theory

The Flux Problem is a famous problem in condensed matter physics, solved by Lieb. It states that the magnetic flux through each plaquette of a square lattice in $2d$ (with either OBC/PBC or PBC/PBC) that minimizes the ground state energy of a system of free electrons at half-filling is π. Such phase, called π-Flux Phase, is known to display emergent Dirac-like low energy excitations.
What happens when we couple the system to a dynamical $\mathbb{Z}_2$ gauge field whose energy is minimized by 0 flux per plaquette?
In this talk, we prove the stability of π-Flux Phase by showing, using Reflection Positivity techniques, that the energy of the fermions at half-filling in a background of N monopoles increases extensively in N. As an application, we compute explicitly the zero temperature diamagnetic susceptibility and the conductivity of the gauge theory, and we show that they coincide with the ones of massless Dirac fermions.

Peter Hearnshaw – Copenhagen University

Eigenvalue Decay Estimates of the Two-Particle Density Matrix for Coulombic Systems

We consider bound states of the quantum system for an atom or molecule. The eigenvalue decay of the corresponding reduced density matrices can tell us about how well N-particle wavefunctions can be approximated using functions of fewer particles, for example with Slater determinants. Certain questions have been answered for the one-particle density matrix, but the eigenvalue decay in the two-particle case brings new challenges. Indeed, the decay rate given by Besov space regularity of the kernel is not optimal. I will talk about our method to improve upon this decay rate, and describe how such questions bring together the theory on singular values of integral operators and regularity of Coulombic wavefunctions. Joint work with A.V. Sobolev.

Lukas Heriban – Czech Technical University in Prague

Dirac operators with non-local singular potentials

In this talk, we will explore new self-adjoint realizations of the Dirac operator associated with non-local δ-shell interactions. This operator is formally expressed as $\mathcal{D}_0 + |F\delta_\Sigma\rangle\langle G\delta_\Sigma|$, where $\mathcal{D}_0$ denotes the free Dirac operator, F and G are matrix valued coefficients, and δ_Σ represents the single layer distribution on a hypersurface Σ. Additionally, we will examine regular approximations for these operators and analyze their non–relativistic limit, i.e., studying the limit as speed of light approaches infinity.

Michal Jex – Czech Technical University in Prague

Classical Density Functional Theory: The Local Density Approximation

We prove that the lowest free energy of a classical interacting system at temperature T with a prescribed density profile $\rho(x)$ can be approximated by the local free energy $\int f_T(\rho(x))\,\mathrm{d}x$, provided that ρ varies slowly over sufficiently large length scales. A quantitative error on the difference is provided in terms of the gradient of the density. Here f_T is the free energy per unit volume of an infinite homogeneous gas of the corresponding uniform density. The proof uses quantitative Ruelle bounds (estimates on the local number of particles in a large system). The talk is based on our two recent papers.

Lukas Junge – Copenhagen University

Expansion of the free energy for dilute bosons in 2 and 3 dimensions

We consider a dilute Bose gas in the thermodynamic limit and prove a bound on the free energy for low temperatures which is in agreement with the conjecture of Lee-Huang-Yang on the excitation spectrum of the system. The technique is allows to handle to difficult but mathematical interesting hard-core potential.

Wilhelm Kroschinsky – University of Bonn

The time-stability of Bose-Einstein Condensates in the Gross-Pitaevskii regime

We revisit the problem of time-stability of Bose-Einstein condensate (BEC) of an initially trapped 3D system of bosons in the Gross-Pitaevskii regime. We prove that the system still exhibits BEC when the trap is switched off, and the effective dynamics is governed by the solution of the time-dependent Gross-Pitaevskii equation. Our main strategy is to control renormalized excitation number operators with respect to the Schrödinger dynamics $t \mapsto e^{-iH_N t}$, rather than controlling the number of excitations with respect to a suitable excitation dynamics $t \mapsto e^{-B_t} e^{-iH_N t}$.

Valentin Kußmaul – University of Stuttgart

Kato's inequality and applications to non-relativistic QED

Kato's distributional inequality relates a magnetic Schrödinger operator, or its semigroup, to the corresponding object without magnetic field. We generalize these results to a setting of Hilbert-space valued L^2-functions and operator valued - that is, quantized - magnetic fields. This allows us to give a short and simple proof of pointwise exponential decay of confined states in non-relativistic quantum electrodynamics. The only previous proof of this result is based on the full machinery of stochastic integration in quantum field theory.
This is joint work with Marcel Griesemer.

Asbjorn Baekgaard Lauritsen – Université Paris Dauphine

Multi-band superconductors have enhanced critical temperatures

Many superconductors are well-described by the Bardeen–Cooper–Schrieffer (BCS) theory of superconductivity. I will introduce this model and describe how the presence of multiple bands (think multiple flavours of charge carries) increases the critical temperature of the superconductor (the temperature below which the material is superconducting). This increase in the critical temperature is either (1) linear or (2) quadratic in the interband coupling if there are (1) multiple equally strongly superconducting bands or (2) a unique strongest superconducting band. Joint work with Joscha Henheik and Edwin Langmann.

Christian Lejsek – Friedrich-Schiller-University Jena

Operator Theoretic Renormalization of the Standard Model of the non-relativistic QED

In the talk I will give the definition of the standard model of non-relativistic QED. I discuss analyticity in the dilation parameter and the coupling constant of the ground state in this model. Furthermore I will introduce the smooth Feshbach map and the renormalization transformation. For the result we introduce a so called generalized Pauli-Fierz transformation, which allows us to apply operator theoretic renormalization as introduced in ”Smooth Feshbach map and operator-theoretic renormalization group methods” by Bach, Chen, Fröhlich and Sigal. This is joint work with D. Hasler.

Sascha Lill – Università degli Studi di Milano

Momentum Distributions of Fermi Gases

We present recent findings on the momentum distribution of a fermionic gas, both in the mean-field and in the dilute regime. More precisely, we consider trial states obtained through bosonization that have been proven to be energetically close to the ground state. For these states, we prove formulas about the momentum distribution, which gives full information about the 2-point function. Our formulas agree with predictions from the physics literature and exhibit a discontinuity, which is characteristic for Fermi liquid behavior. The talk is based on joint work with N. Benedikter, E. L. Giacomelli, A. B. Lauritsen and D. Naidu.

Abdallah Maichine – Mohammed 5 University in Rabat

Thermodynamic limit for electronic systems with 2d symmetry

This talk will be devoted to characterizing 2d and 3d self-adjoint operators that commute with magnetic translations. In particular, we show how to compute their trace per unit surface. Therefore, we apply the thermodynamic limit process to derive a simpler expression for the Hartree Fock energy per unit surface for fermionic systems with 2d symmetries in the presence of magnetic fields.

Stefano Marcantoni – Université Côte d'Azur

Open quantum systems in the ultrastrong coupling limit

In this talk, I will consider a finite-level quantum system linearly coupled to a bosonic reservoir, that is the prototypical example of an open quantum system. I will discuss recent results on the reduced dynamics of the finite system when the coupling constant tends to infinity, i.e. in the ultrastrong coupling limit. In particular, I will show that the dynamics corresponds to a nonselective projective measurement followed by a unitary evolution with an effective (Zeno) Hamiltonian. The connection with the quantum Zeno effect will be discussed. The rigorous proof of the limit is quite simple and can be generalized to the case of a small system interacting with two reservoirs when one of the couplings is finite and the other one tends to infinity. In this second scenario the reduced dynamics is richer and possibly non-Markovian. Joint work with Marco Merkli, arXiv:2411.06817.

Maria Matushko – Steklov Mathematical Institute of RAS

Q-deformed long-range spin chains of Haldane-Shastry type

The Haldane-Shastry spin chain describes N particles on equal spaced sites on a ring with an internal spin degree of freedom. Many special properties of the Haldane-Shastry long-range spin chain naturally arise from a connection with the spin Calogero-Sutherland model. This connection is given by so-called Polychronakos freezing trick — we put the particles of the dynamical model in their equilibrium positions in the corresponding classical mechanics and remove all terms with differential operators from the spin Calogero Hamiltonians. Also this connection helps to describe the spectrum of the Haldane-Shastry spin chain. We discuss a construction of a q-deformed Haldane-Shastry spin chain. We present a commuting set of matrix-valued difference operators in terms of R-matrices (solutions of the Yang-Baxter equation), which satisfy some R-matrix identities. Next, we show how to obtain long-range spin chains using the Polychronakos freezing trick from these models. We discuss the elliptic, trigonometric and supersymmetric spin chains that can be constructed by this method, its properties and integrability.

Nathan Metraud – Universidad del Pais Vasco

Flow Equation for Quadratic Hamiltonians

Quadratic Hamiltonians are very important in many-body quantum fields theory. They appear for instance as effective models for superconductivity and superfluidity in the fermionic and bosonic case respectively. Their studies on more general setting, which go back to the sixties, are relatively incomplete for the fermionic case. In this work, we present a method to N-diagonalize quadratic fermionic Hamiltonians under much weaker assumptions than before using a flow equation. We show that we can implement Bogoliubov transformations through this novel elliptic operator-valued non-linear differential equations.

Norbert Mokrzański – University of Warsaw

The Bogoliubov-Bose-Hubbard model: existence of superfluidMott insulator phase transition

We consider a variational approach to the Bose-Hubbard model based on the Bogoliubov theory. We introduce the grand canonical and canonical free energy functional for which we prove the existence of minimizers. By analyzing their structure we show the existence of a superfluid-Mott insulator phase transition at positive temperatures. In particular, we show that this model does not exhibit a quantum phase transition. Joint work with Marcin Napiórkowski.

Umberto Morellini – Università di Pisa

The Pauli-Villars regularised free energy of Dirac's vacuum in purely magnetic fields

The Dirac vacuum is a non-linear polarisable medium rather than an empty space. This non-linear behaviour starts to be significant for extremely large electromagnetic fields such as the magnetic field on the surface of certain neutron stars. Even though the null temperature case was deeply studied in the past decades, the problem at non-zero temperature needs to be better understood. In this work, we present the first rigorous derivation of the one-loop effective magnetic Lagrangian at positive temperature, a non-linear functional describing the free energy of the Dirac vacuum in a classical magnetic field. After introducing our model, we properly define the free energy functional using the Pauli-Villars regularisation technique in order to remove the worst ultraviolet divergences, which represent a well known issue of the theory. The study of the properties of this functional is addressed before focusing on the limit of slowly varying classical magnetic fields. In this regime, we prove the convergence of this functional to the Euler-Heisenberg formula with thermal corrections, recovering the effective Lagrangian first derived by Dittrich in 1979.

Léo Morin – Copenhagen University

A first generic purely magnetic tunneling formula between 2D magnetic wells

Tunneling is a well-known phenomenon in quantum physics: a charged particle in symmetric double-well electric potential is localized in both wells simultaneously. A way to quantify this phenomenon is by estimating the spectral gap between the first two eigenvalues of the corresponding Schrödinger operator. In the semiclassical limit, this gap is exponentially small. Such sharp estimates were obtained by Helffer-Sjstrand in the 80's. When the localization is induced by a magnetic field instead, without electric potential, a similar effect was conjectured by Helffer-Morame in 1996. However, the magnetic case turned out to be more complicated to understand. After recent partial results, we finally proved such a tunneling formula between purely magnetic wells, hence answering a 30 years old problem. Interestingly, the size of the interaction is smaller than the one conjectured in 1996. As we will see during the talk, we had to revisit the Helffer-Sjstrand theory, in light of recent advances in the study of magnetic wells. In particular, our proof relies on the understanding of the dynamical properties of magnetic Laplacians. In the semiclassical limit, we manage to split the effects of the cyclotron and center guide motions.

It turns out that tunneling occurs along the center guide variables only, the cyclotron motion acting on top of it. A key aspect of our analysis is getting exponential decay estimates of eigenfunctions in phase space, where the position and momenta variables are mixed.

Diwakar Naidu – Università degli Studi di Milano

Momentum distribution of a high density fermi gas in the random phase approximation

I will talk about the momentum distribution of an interacting Fermi gas on a 3D torus in the mean field regime. The key tool for deriving the distribution is a rigorous bosonization method. I will start with the construction of a natural trial state and then show the implementation of the bosonization procedure. Finally, I will sketch how we obtain the momentum distribution in the mean-field approximation. The expression for the momentum distribution contains the contribution collective excitations above the Fermi-surface going beyond the precision of Hartree-Fock theory. This result is an extension of the previous result for the momentum distribution by Benedikter-Lill.

Jakob Oldenburg – University of Zurich

Quantum Fluctuations of Many-Body Dynamics around the Gross-Pitaevskii Equation

We consider the evolution of a gas of N bosons in the three-dimensional Gross-Pitaevskii regime (in which particles are initially trapped in a volume of order one and interact through a repulsive potential with scattering length of the order $1/N$). We construct a quasi-free approximation of the many-body dynamics, whose distance to the solution of the Schrdinger equation converges to zero, as N goes to infinity, in the L^2-norm. To achieve this goal, we let the Bose-Einstein condensate evolve according to a time-dependent Gross-Pitaevskii equation. After factoring out the microscopic correlation structure, the evolution of the orthogonal excitations of the condensate is governed instead by a Bogoliubov dynamics, with a time-dependent generator quadratic in creation and annihilation operators. As an application, we show a central limit theorem for fluctuations of bounded observables around their expectation with respect to the Gross-Pitaevskii dynamics. This is joint work with Cristina Caraci and Benjamin Schlein.

Marco Olivieri – Copenhagen University

Lee-Huang-Yang type expansion for the energy of a mixture of Bose gases

We consider a system of two-component bosons interacting through positive inter-species and intra-species pairwise potentials. We prove that, in the thermodynamic limit, the first two terms of the energy density expansion in the dilute regime are universal, i.e., they depend only on the scattering lengths of the potentials. The obtained expansion is analogous to the so-called Lee-Huang-Yang (LHY) expansion for a single species. From the lower bound, we derive a candidate for the correct constant of the second-order term, which matches the LHY constant for a single species as one of the densities approaches zero.

Abderrahman Oularabi – Faculty of Sciences And Techniques Al Hoceima

Charging battery in collisional identical sub-baths

A general framework for the charging process based on a collisional approach is proposed, in which a stream of ancillas an sequentially collides with the system, which directly interacts with the thermal environment. This approach allows for the generation of correlations with independent ancillas, meaning no energy is exchanged between them. We show that in this scenario, the extracted maximal work is a monotonic function of temperature T.

Lubashan Pathirana – Copenhagen University

Random Environment Techniques in Quantum Dynamics: Asymptotics, Random MPS and Quantum Trajectories

We study the asymptotic behavior of time-inhomogeneous quantum dynamical maps, a.k.a. time-inhomogeneous Markovian dynamics in a random environment. We only assume P-divisibility of the dynamics and we do not assume that the dynamic propagators are completely positive, nor we assume that they are trace-preserving. We extend the existing results in the discrete time to the continuous time parameter setting by obtaining almost sure asymptotic limits. Under certain irreducibility conditions we obtain a families of full-rank random states and a family of rank-one super-operators, so that, given an initial time, the dynamical maps to the future is approximated by these two families exponentially-well. Furthermore, under certain mixing conditions we show that the said families of full-rank states and rank-one super-operators approximate the forward-time dynamics of the quantum system, in mean, supper-polynomially. If the dynamical propagators are quantum channel-valued then we see that given an initial time the propagators to the future become exponentially close to a replacement channel. In discrete time parameter we obtain a law of large numbers and a central limit type theorem involving the top Lyapunov exponent of the cocycle. Furthermore we consider a family of random (matrix product states) MPS that are obtained by a finite collection of a random family of Kraus operators. These MPS are distributionally translation invariant (a generalization of the typical translation invariant MPS) and we consider two point correlation inequalities in thermodynamic limits. For quantum trajectories, this setting gives a Markov chain with random transition kernels (a.k.a. Markov chain in a random environment). The environment configuration can be thought of as an external disorder configuration and we provide (computable) conditions so that the quenched probability of a trajectory of any initial state asymptotically approaches the set of pure states (almost surely) for all disorder realizations.

Gabriele Peluso – Sapienza Università di Roma

Topological classification of chiral and particle-hole symmetric insulators in low dimension.

Motivated by the study of periodic Hamiltonians enjoying chiral or particle-hole symmetry, like the SSH model or the Kitaev chain, we present a topological study of families of symmetric functions from d-dimensional tori to the space of rank-fixed projectors on a Hilbert space. Our approach is based on the direct calculation of fundamental groups and homotopy groups on spaces of unitary matrices, building on the existing classification of vector bundles in reduced dimensions. The main results include a classification up to unitary equivalences or homotopies for $d = 0, 1$ or 2 for chiral or particle-hole symmetric projectors within separable Hilbert spaces of minimal or infinite dimension.

Denis Périce – Constructor University Bremen

Mean-Field Dynamics of the Bose-Hubbard Model in High Dimension

The Bose-Hubbard model effectively describes bosons on a lattice with on-site interactions and nearest-neighbor hopping, serving as a foundational framework for understanding strong particle interactions and the superfluid to Mott-insulator transition. We aim to rigorously establish the validity of mean-field approximations for the dynamics of quantum systems in high dimensions, using the Bose-Hubbard model on a square lattice as a case study. Our result provides a trace norm estimate between the first reduced density matrix of the Schrödinger dynamics and the mean-field dynamics in the limit of large dimensions. In this context, the mean-field approximation applies to the hopping amplitude rather than the interaction between particles, resulting in a rich and non-trivial mean-field equation. Unlike the conventional mean-field descriptions derived from large particle number limits that average out interactions, this mean-field equation fully incorporates the non-trivial interactions, capturing the Mott insulator/superfluid phase transition. Our work offers a rigorous justification for a specific case of the highly successful dynamical mean-field theory (DMFT) for bosons, which, somewhat surprisingly, provides many qualitatively accurate results even in three dimensions.

Lorenzo Pettinari – University of Trento

On Classical Aspects of Bose-Einstein Condensation

Berezin and Weyl quantization are renown procedures for mapping classical, commutative Poisson algebras of observables to their non-commutative, quantum counterparts. The latter is famous for its use on Weyl algebras, while the former is more appropriate for continuous functions decaying at infinity. In this work, we define a variant of the Berezin quantization map, which acts on the classical Weyl algebra $\mathcal{W}(E, 0)$ and constitutes a positive *strict deformation quantization*. We use this map as a mathematical tool to compare classical and quantum thermal equilibrium states for a boson gas by computing the classical limit of the latter. For this scope, we first define a purely algebraic notion of KMS states for the classical Weyl algebra and verify that in the finite volume setting there is only one possible KMS state, which can be interpreted as the Fourier transform of a Gibbs measure on some Hilbert space. Subsequently, we perform a thermodynamic limit and show that the limit points of the finite volume classical KMS state manifest condensation in the zero mode, similarly to what happens in the standard formulation of Bose-Einstein condensation. Lastly, we prove that there exist sequences of quantum KMS states for the infinite volume Bose gas, that converge weak-* to classical KMS states. Moreover, as the different thermal phases are preserved by this limit, it is demonstrated that a quantum condensate is mapped to a classical one. (based on arXiv.2411.02626).

Maik Reddiger – Anhalt University of Applied Sciences

On the mathematical theory of the Madelung equations

The Madelung equations are a non-linear system of PDEs, which may be considered as a quasi-classical reformulation of the Schrödinger equation without spin. So far, discussions of the equations have been largely confined to the literature on the foundations of quantum mechanics, while the mathematics community has been focusing on closely related systems of PDEs arising in the context of quantum fluid dynamics. More cooperation between the two communities may shine light on the much debated importance of the Madelung equations to the foundations of quantum theory and potentially lead to more general physical descriptions. Motivated by this observation, in our recent review (Reddiger & Poirier, J. Phys. A: Math. Theor. 56, 193001 (2023)) we have pointed out and clarified the underlying open problem: finding a weak formulation of the Madelung equations, which is equivalent to the respective Schrödinger equation. In particular, a suitable "quasi-irrotationality condition" is to be included in the former system. Indeed, in the mathematics literature notable progress on the problem had already been made by Gasser, Markowich, Antonelli, Marcati and others. In this talk I present some of the progress on this problem, in which physical considerations interplay intimately with PDE theory.

Andrew Rout – University of Rennes

Gibbs measures and KMS states for the focusing nonlinear Schrödinger equation
Gibbs measures are an important object in the study of low regularity well posedness of nonlinear Hamiltonian PDEs. On the other hand, the KMS condition is a method of singling out equilibrium states in a dynamical system. In this talk, I discuss some recent results on the relationship between KMS states and Gibbs measures for the focusing nonlinear Schrödinger equation. Based on work with Zied Ammari and Vedran Sohinger.

Diane Saint Aubin – University of Zurich

Third order corrections to the ground state energy of a Bose gas in the Gross-Pitaevskii regime

In recent decades, substantial progress has been made in understanding Bose gases. In this talk, we consider a system of N bosons in the Gross-Pitaevskii regime, moving on the unit torus and interacting through a repulsive potential with scattering length $1/N$. We present the derivation of the ground state energy of such a system. While the leading order (of order N) and second order (of order 1) of the ground state energy have been established in recent years, this approach resolves the next order term of order $\log N/N$. The correction to the energy is consistent with predictions of the ground state energy per particle in the thermodynamic limit. This talk is based on joint work with Cristina Caraci, Alessandro Olgiati and Benjamin Schlein.

Grega Saksida – University of Warwick

Lace expansion in statistical mechanics

Lace expansion is a powerful technique first developed by David Brydges and Thomas Spencer in 1985 to study self-avoiding random walks. In 2007, Akira Sakai found a way to apply it to the classical Ising model. The technique has been used on other models as well since then. I will give a short demonstration of the technique on the self-avoiding walk, and show how we hope to use the technique next.

Andreas Schaefer – Université Grenoble Alpes

Dynamical Localization of Quantum Walks on the hexagonal lattice in the regime of strong disorder

We study Quantum Walks (QW) on the hexagonal lattice, where the QW is given by the composition a of shift and coin operator, together with a random phase. We will prove dynamical localization under the condition that the coin matrix used to define the coin operator is close enough to the permutation matrices , which correspond to the permutations (1 2 3) or (1 3 2) and induce full localization. Under dynamical localization we understand the property that the probability to move from a lattice site z to another site y decreases on average exponentially in the distance $|z - y|$, independently of how many steps the QW may take. The proof relies on showing exponential decay of the fractional moments of the resolvent and uses a finite volume method.

Olga Shchegortsova – Lomonosov Moscow State University

Geometric Asymptotics for the Schrödinger Equation with a Delta Potential Localized on the Surface of Codimension 1

We study semiclassical asymptotics for the Schrödinger equation with a delta potential that is the sum of a smooth function $V(x)$ and a delta function localized on a surface of codimension 1:

$$ih\frac{\partial\psi}{\partial t} = -\frac{h^2}{2}\Delta\psi + V(x)\psi + \frac{q(y)}{h}\delta_M\psi, \quad M = \{x \in \mathbb{R}^n : x = r(y)\}, \quad \dim M = n - 1.$$

Initial data can be represented in the form $\psi(x, 0) = e^{i\frac{i}{h}S_0(x)}\varphi_0(x)$. If the initial phase $S_0(x)$ is a real function, we have a rapidly oscillating wave packet. If $S_0(x)$ is a complex function with $\mathrm{Im}S_0(x) \geq 0$, the initial condition takes the form of a Gaussian wave beam, or so-called squeezed state.

The Schrödinger operator with a delta potential is defined using the theory of extensions as a self-adjoint extension of the operator with smooth potential $\hat{H}_0 = -\frac{h^2}{2}\Delta + V(x)$, bounded by the function that equals zero on the domain of the delta function. Its domain consists of functions that satisfy boundary conditions on the carrier of the singularity:

$$\begin{cases} \psi(r(y) - 0, t) = \psi(r(y) + 0, t), \\ h\left(\frac{\partial\psi}{\partial m}(r(y) - 0, t) - \frac{\partial\psi}{\partial m}(r(y) + 0, t)\right) = q(y)\psi(r(y), t). \end{cases}$$

We use a modification of the methods for constructing semiclassical asymptotics with real and complex phases, developed by V.P. Maslov and known as Maslov's semiclassical theory and complex germ theory. This approach is based on the idea that semiclassical asymptotics can be associated with special geometric objects corresponding to Hamiltonian and variational systemscomplex vector bundles over isotropic manifolds. The effect of a delta potential is illustrated by the fact that an incident wave, after interacting with a delta potential, splits into two parts: reflected and transmitted waves. In this case, geometric objects corresponding to the problem need to be modified at the points of support of the delta function. We construct modifications of isotropic manifolds and complex vector bundles corresponding to asymptotic solutions and write the asymptotic solution.

Harman Preet Singh – SISSA

Large scale edge response for $2d$ topological insulators

For a class of non-interacting quantum Hall lattice systems on a cylinder, we consider the edge density and current dynamical responses to a time-adiabatic density-type perturbation, in the infinite volume and zero temperature limit. We prove the validity of the linear response for such quantities, thus justifying Kubos formula. Further, we explicitly evaluate them solely in terms of spectral data at the Fermi points, and of the profile of the perturbation. The proof of validity of linear response on the physically relevant time-scale relies on a precise analysis of the imaginary-time Duhamel series, and in particular on a cancellation for the scaling limit of the higher order correlation functions describing the nonlinear response, also related to bosonization. The precise form of the linear response coefficient is then fixed by lattice conservation laws. Joint work with Marcello Porta, based on part of arXiv:2411.04023.

David Spitzkopf – Charles University

Magnetic transport due to a translationally invariant potential obstacle

We consider a two-dimensional system in which a charged particle is exposed to a homogeneous magnetic field perpendicular to the plane and a potential that is translationally invariant in one dimension. We derive several conditions on such a perturbation under which the Landau levels change into an absolutely continuous spectrum.

Siegfried Spruck – University of Tübingen/Bourgogne

Derivation of the Effective Dynamics for the Bose Polaron

We consider a dense quantum gas consisting of N Bosons evolving in $\mathbb{R}^3$ in the presence of an impurity particle in the mean-field scaling with initially high density ρ and large volume Λ of the gas. In the initial state of the system almost all Bosons are in the Bose-Einstein condensate, with a few excitations. For this system we derive from the microscopic dynamics in the limit of large densities and volumes the effective description by a quantum field theory modelled by the Bogoliubov-Frhlich Hamiltonian and thus prove the existence of a quasi-particle, the Bose polaron.

Jack Thomas – Université Paris-Saclay

Screening in the reduced Hartree-Fock model

Placing a point charge Q at the origin in a vacuum produces a long-range Coulomb potential $\frac{Q}{\epsilon_0 r}$, with r being the distance to the charge and ϵ_0 the dielectric permittivity of the vacuum. However, in a material, the material reorganises itself; electrons move towards positive charges (or away from negative charges) and the total potential (including the Coulomb interactions, and the response from the electrons) is screened. Metals and insulators display fundamentally different screening behaviours. In metals at finite temperature, electrons are mobile and are able to move long distances to fully screen the defect; a simple empirical model being given by the Yukawa potential $\frac{Q}{r}e^{-kr}$. On the other hand, electrons in insulators are tightly bound to the nuclei and are thus unable to move too far from their periodic arrangement. A simple model for the total potential is given by $\frac{Q}{\epsilon r}$ for some $\epsilon > \epsilon_0$; one observes partial screening. This heuristic description has been rigorously proved for the reduced Hartree-Fock model at finite temperature [3], and for insulators at zero temperature [1,2]. In metals at zero temperature, the sharp cut-off in the Fermi-Dirac distribution at the Fermi surface (the surface in reciprocal space separating occupied and unoccupied electron states) causes singularities in Fourier space. As a result, one expects to observe Friedel oscillations; the total potential decays with an algebraic rate with oscillatory tails. The rate of decay and frequency of oscillation depends on the Fermi surface, and thus on the metal, in a non-trivial way. This dependence is only known for the simplest of cases when the external potential is zero, and thus the Fermi surface is spherical (i.e. the Free electron gas). For real metals, the Fermi surface can significantly deviate from this spherical picture. We will discuss ongoing work in explaining this complicated screening behaviour. Joint work with Eric Cances and Antoine Levitt.

[1] E. Cances, A. Deleurence, and M. Lewin. A new approach to the modeling of local defects in crystals: The reduced HartreeFock case, Communications in Mathematical Physics 281.1 (2008), pp. 129177.
[2] E. Cances and M. Lewin. The dielectric permittivity of crystals in the reduced HartreeFock approximation, Archive for Rational Mechanics and Analysis 197.1 (2009), pp. 139177.
[3] A. Levitt, Screening in the finite-temperature reduced HartreeFock model. Archive for Rational Mechanics and Analysis 238.2 (2020), pp. 901927.

François Visconti – LMU Munich

Derivation of Hartree theory for two-dimensional trapped Bose gases in almost Gross-Pitaevskii regime

We study the ground state energy of trapped two-dimensional Bose gases with mean-field type interactions. We prove the stability of second kind of the many-body system and the convergence of the ground state energy per particle to that of a non-linear Schrödinger (NLS) energy functional. Notably, we can take the scaling of the interaction to be arbitrarily close to the Gross-Pitaevskii scaling. As a consequence of the stability of second kind we also obtain Bose-Einstein condensation for the many-body ground states and dynamics for a much improved range of the diluteness parameter. Based on joint work with Lukas Junge.

Marius Wesle – University of Tübingen

Macroscopic Hall-Current Response in Fermionic Infinite-Volume Systems

Given a 2-dimensional system of interacting fermions, the Hall-conductivity is defined as the linear response coefficient that is associated to the current induced in one direction when applying a homogeneous electric field in the perpendicular direction. In this talk I will explain how in infinitely-extended periodic systems of interacting lattice fermions with a spectral gap, one can rigorously realise the linear response definition of the Hall-conductivity described above. By using the NEASS (Non-Equilibrium Almost Stationary State) approach to linear response theory we can rigorously control the induced Hall-current, despite the fact that even a very small homogeneous electric field closes the spectral gap. Our proof recovers a many-body version of the double-commutator formula for the Hall-conductivity and shows, that the current response is purely linear with no polynomial corrections. It also allows for a simple argument that shows that the Hall-conductivity is constant within symmetry protected topological phases. This talk is based on recent joint work with Giovanna Marcelli, Tadahiro Miyao, Domenico Monaco and Stefan Teufel (arXiv:2411.06967).