

Celestial Mechanics Advancements: Exploring Orbital Trajectories and Stability through Variational Methods

In recent years, significant advancements in celestial mechanics have emerged through a variational method applied to the n -body problem, leading to the discovery of novel orbital trajectories. This approach involves examining critical points of the Lagrangian action associated with the n -body problem, wherein specific periodic solutions of the dynamical system can be numerically determined by utilising evolutionary algorithms once the problem has been discretised.

Initially, emphasis is placed on scrutinising critical points, particularly minima, of the action functional pertaining to the n -body problem. These critical points, under certain assumptions, represent feasible and physically meaningful solutions of the dynamical system, manifesting as periodic orbits that satisfy the associated differential equations. By employing a blend of stochastic and deterministic algorithms, the parameter space of viable solutions can be explored to ascertain the expression of these orbits.

Subsequently, attention shifts towards assessing the functional stability of the problem. Orbit trajectories identified in the previous phase are mapped as critical points, allowing for an examination of the stability or instability within their respective neighbourhoods. This involves treating the problem as a dynamical system, analysing the gradient of the action functional \mathcal{A} , denoted as $\eta' = -\nabla\mathcal{A}(\eta)$. This approach not only characterises the minima discovered earlier but also delineates the basin of attraction for each minimum, derived from the algorithm's analysis of the initial input data or starting point.

Lastly, attention is directed towards analysing the boundaries of these basins of attraction. It has been established that when two boundaries converge and then separate, the point of separation asymptotically converges to a critical point that is not a minimum. Leveraging conventional algorithms such as the Newton method enables the numerical approximation of these new critical points, which differ from previously identified minima.

Despite its theoretical nature, this methodology holds practical applications in Astrodynamics, particularly in mission design and the deployment of satellite constellations into orbit.

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