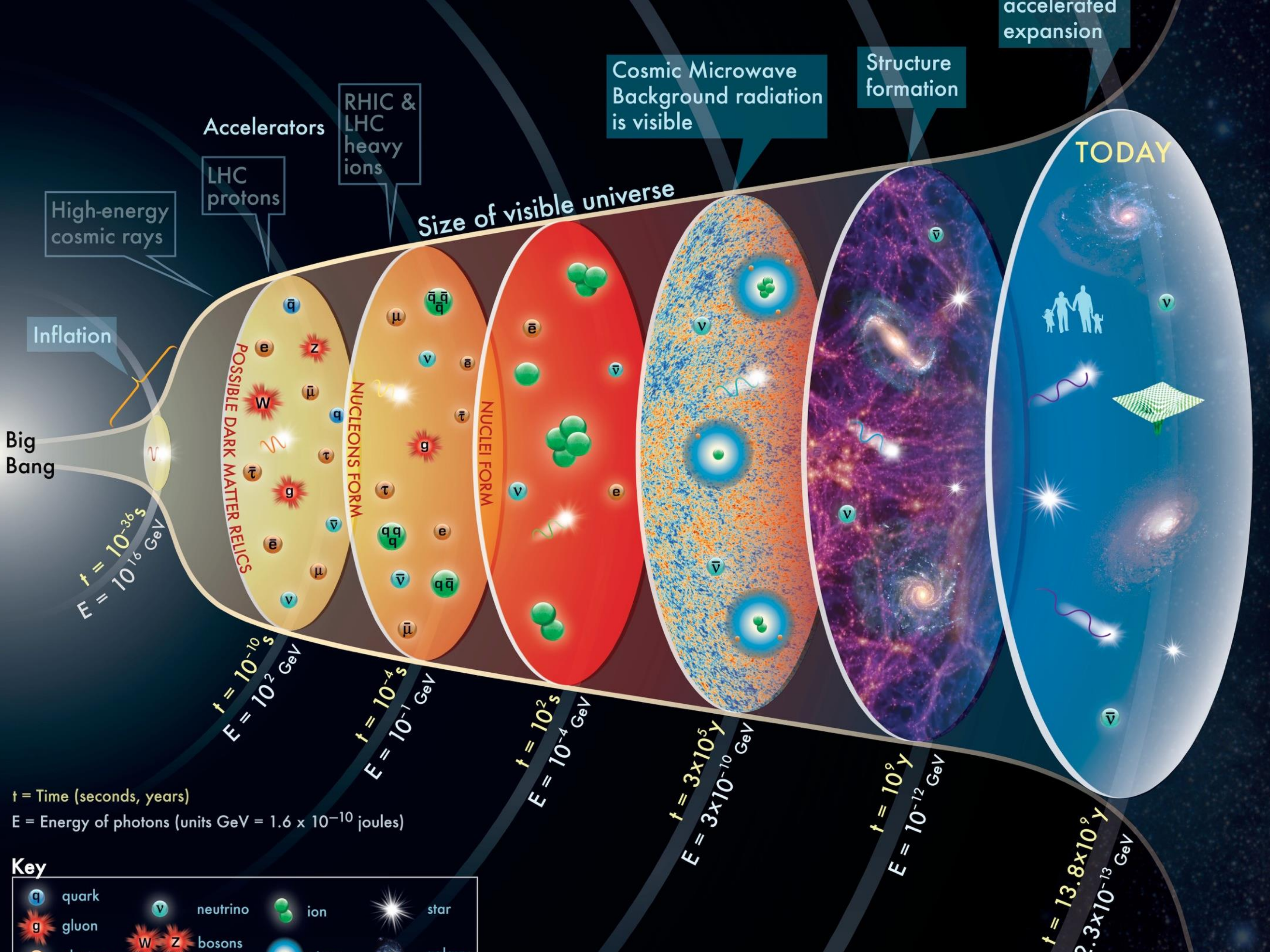
Inflationary Cosmology: Past, Present, and Future

Andrei Linde

In memory of Veniamin Sergeyevich Berezinsky


accelerated
expansion
Cosmic Microwave
Background radiation
is visible
Structure
formation
RHIC &
LHC
heavy
ions
Accelerators
LHC
protons
High-energy
cosmic rays
Inflation
Size of visible universe
TODAY
Big
Bang
POSSIBLE DARK MATTER RELICS
NUCLEONS FORM
NUCLEI FORM
t = 10^-36 s
E = 10^16 GeV
t = 10^-10 s
E = 10^2 GeV
t = 10^-4 s
E = 10^-1 GeV
t = 10^2 s
E = 10^-4 GeV
t = 3x10^5 y
E = 3x10^-10 GeV
t = 10^9 y
E = 10^-12 GeV
t = 13.8x10^9 y
E = 2.3x10^-13 GeV
t = Time (seconds, years)
E = Energy of photons (units GeV = 1.6 x 10^-10 joules)
Key
quark
gluon
neutrino
bosons
ion
star

Why do we need inflation?

Hard Art of the Universe Creation

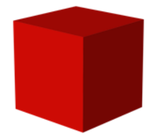
According to the hot Big Bang theory, the total number of particles did not change much during the expansion of the universe, so the universe at the Planck time was supposed to contain at least ~10^{90} particles. At the Planck time $t = O(1)$, there was one particle per Planck length $ct = O(1)$.

Thus, at the Planck time $t = 1$, the whole universe consisted of 10^{90} causally disconnected parts of size $ct = O(1)$. Such parts did not know about each other. If someone wanted to create the universe at the Planck time, he/she could only make **a Small Bang** in a tiny part of the universe of a Planck size $ct = O(1)$, **containing a single particle**.
Everything else was beyond causal control.

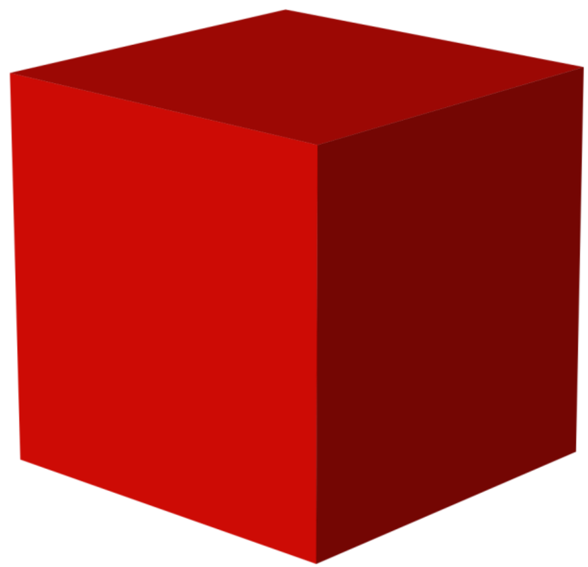
Is it possible to start with less than a milligram of matter (or nothing at all) and produce 10^{90} particles from it?

Basic idea of inflation:

Take a box with large vacuum energy density V



In an expanding universe this box grows in size. It has the same vacuum inside, with the same ENERGY DENSITY. The total energy inside GROWS, it is proportional to the volume.



de Sitter space

de Sitter 1917
Levi-Civita 1917

Then, the vacuum decays and produces an enormous number of protons, proportional to the volume of the box.

Original idea of inflation:

One of the Einstein equations for the flat empty universe with vacuum energy density V_0 (cosmological constant) is

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{V_0^2}{3}$$

It has a solution describing an exponentially growing (inflating) universe:

$$a = a_0\, e^{Ht}$$

The total vacuum energy of such universe grows even faster, as volume

$$E = E_0\, e^{3Ht}$$

If eventually this vacuum state decays, it produces exponentially many elementary particles with exponentially large energy.

Alan Guth 1980

If something looks too good to be true…

If the universe is empty, how can anyone know that it expands?

The universe with a constant positive vacuum energy V_0 is **de Sitter space**, which looks **expanding** in one system of coordinates, **collapsing** in another system of coordinates, and **static** in yet another coordinates.

If there is no preferable coordinate system, there is **no preferable time** when the vacuum state decays. Therefore, vacuum decays chaotically, and the universe becomes grossly inhomogeneous, unsuitable for life. After a year of investigation, Guth and Hawking concluded that this scenario cannot be improved.

Breaking the rules

A solution of this problem was found in 1981 ("new inflation"):

Instead of a vacuum state with a constant vacuum energy V_0, one should consider a slowly moving scalar field with a not very steep potential $V(\phi)$. If the potential is too steep – no inflation. If it is too flat – the universe becomes grossly inhomogeneous due to quantum fluctuations.

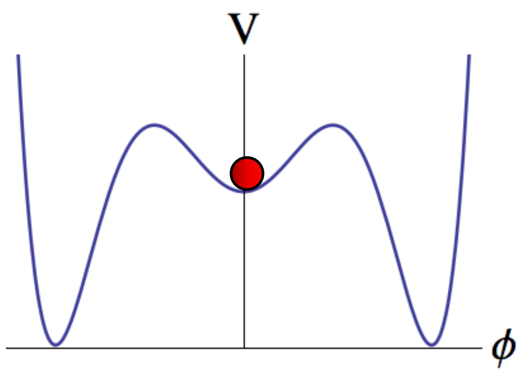
Inhomogeneities are inversely proportional to $dV/d\phi$. That is why one could not improve the old inflation scenario where $dV/d\phi = 0$.

And then, in 1983, it was realized ("chaotic inflation") that it is better **not** to rely on the idea that the universe was born in the hot Big Bang.

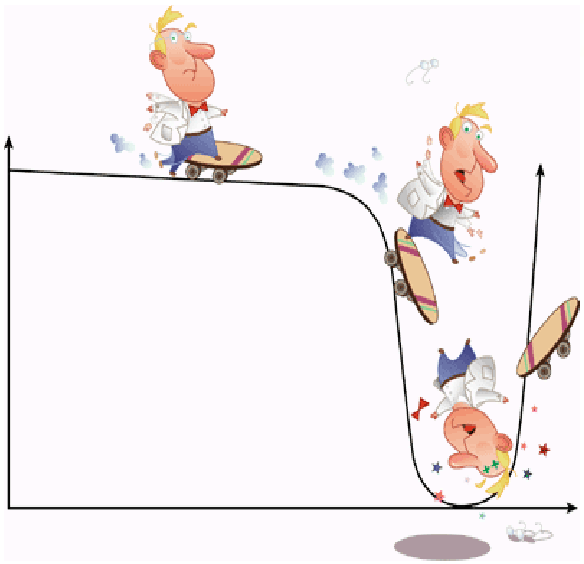
Inflation

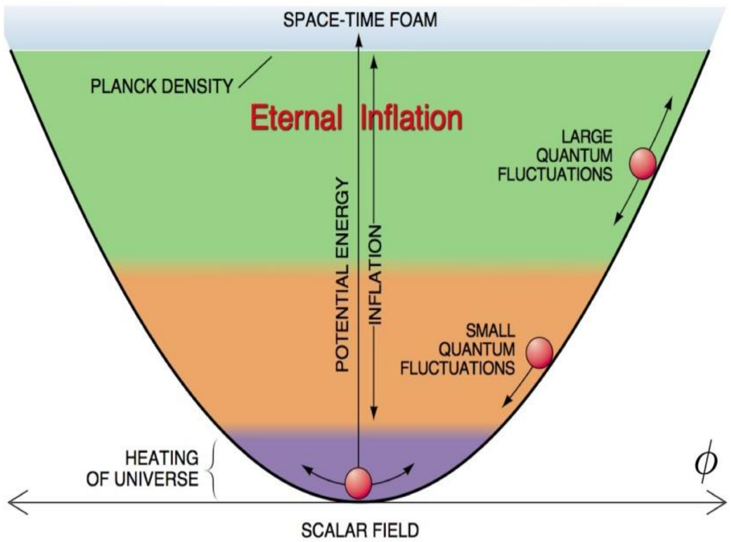
Starobinsky, 1980 – modified gravity, $R + R^2$
a complicated but almost working model

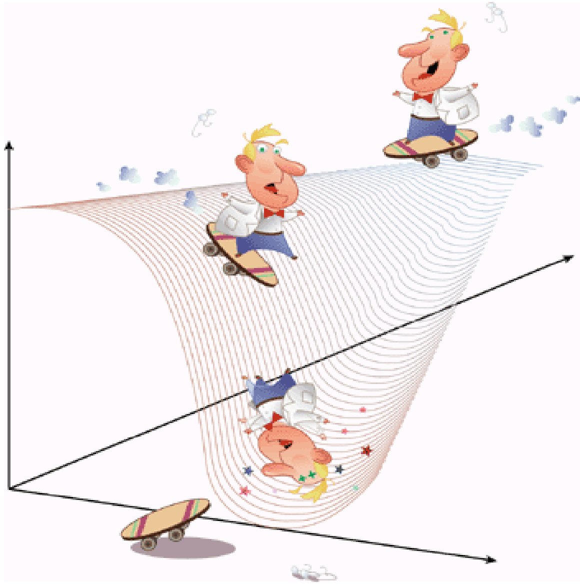
Guth, 1980 - old inflation (inflation in a false vacuum)



$$V(\phi) = \frac{m^2}{2}\phi^2$$







A.L., 1982 - new inflation

1983 - chaotic inflation

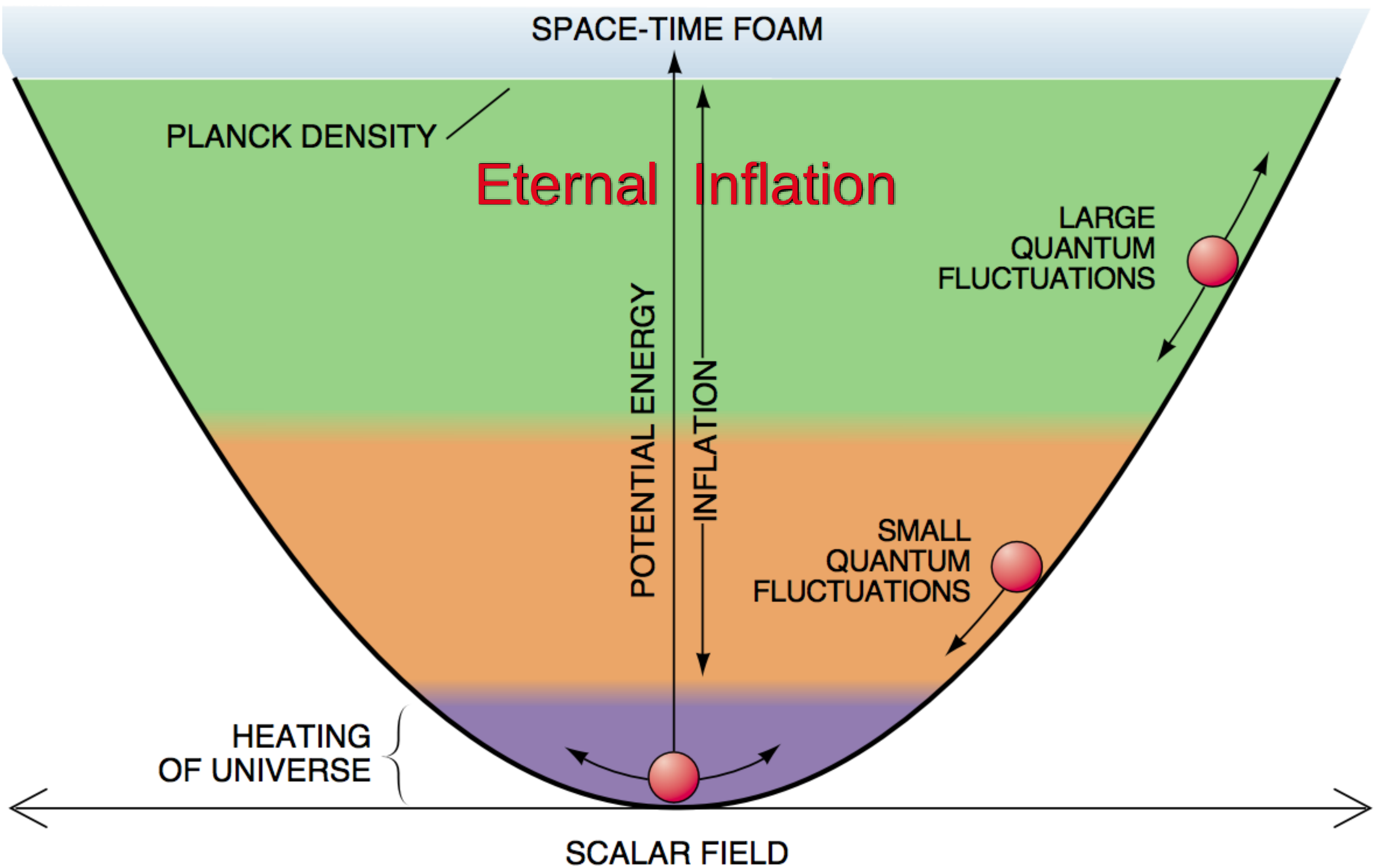
1991 - hybrid inflation

The simplest chaotic inflation model

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}\partial\phi^2 - \frac{1}{2}m^2\phi^2$$

AL 1983, 1986

$$\ddot{\phi} + 3H\dot{\phi} = -m^2\phi$$



Consider a tiny universe of a smallest possible size 10^{-33} cm at the Planck density. If the potential energy of the scalar field in this domain was greater than its kinetic and gradient energy, it starts growing fast.

Within 10^{-42} s the universe becomes homogeneous and completely dominated by the potential energy of the scalar field.

Equation for the scalar field

$$\ddot{\phi} + 3H\dot{\phi} = -m^2\phi$$

Einstein's equation

$$H = \frac{m\phi}{\sqrt{6}}$$

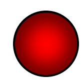
The solution shows that the universe grows approximately exponentially. At the end of inflation, the universe grows up by a factor

$$e^{\phi_0^2/4}$$

Here ϕ_0 is the initial value of the field.

A newborn universe could be as small as 10^{-33} cm and as light as 10^{-5} g

(it could be born from nothing at all…)



$$l \sim 10^{-33}\ \text{cm}$$

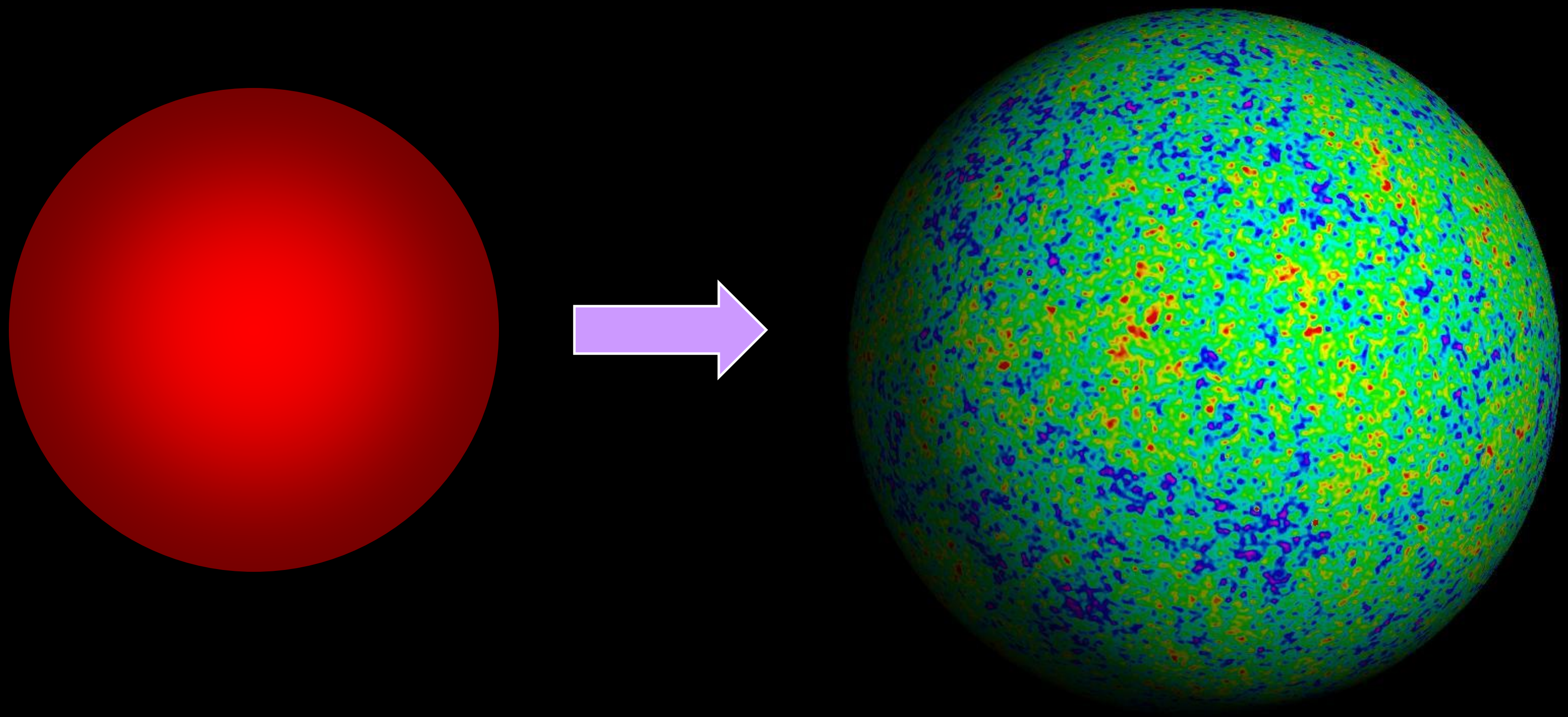
$$m \sim 10^{-5}\ \text{g}$$

Inflationary universe 10^{-35} seconds old

$$10^{10000000000}$$

in ANY units of length

The universe after inflation becomes huge and almost absolutely uniform, but quantum fluctuations make it slightly non-uniform. This leads to formation of galaxies and tiny perturbations of the temperature of the universe



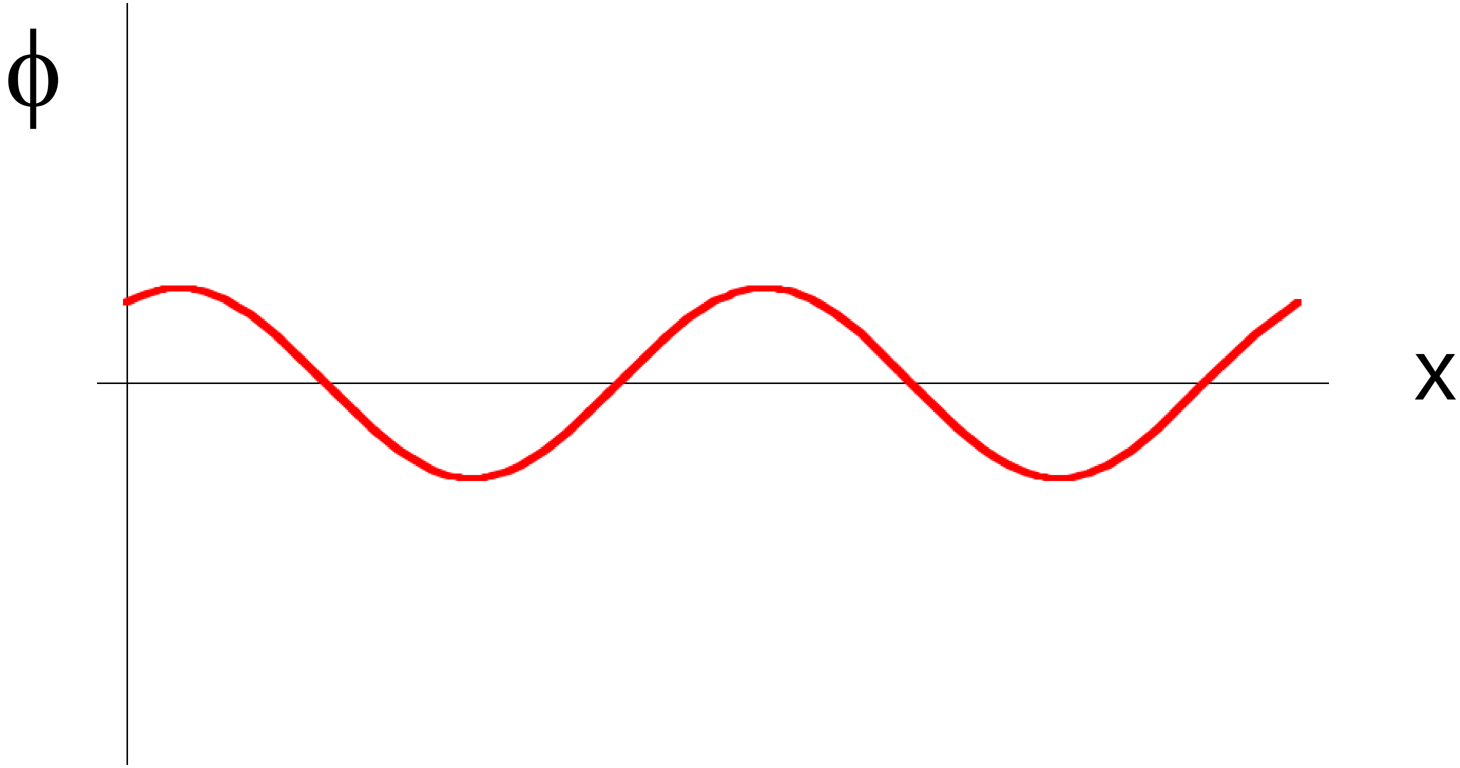
Origin of structure:

In this theory, original inhomogeneities are stretched away, but new ones are produced from **quantum fluctuations** amplified during the exponential growth of the universe.

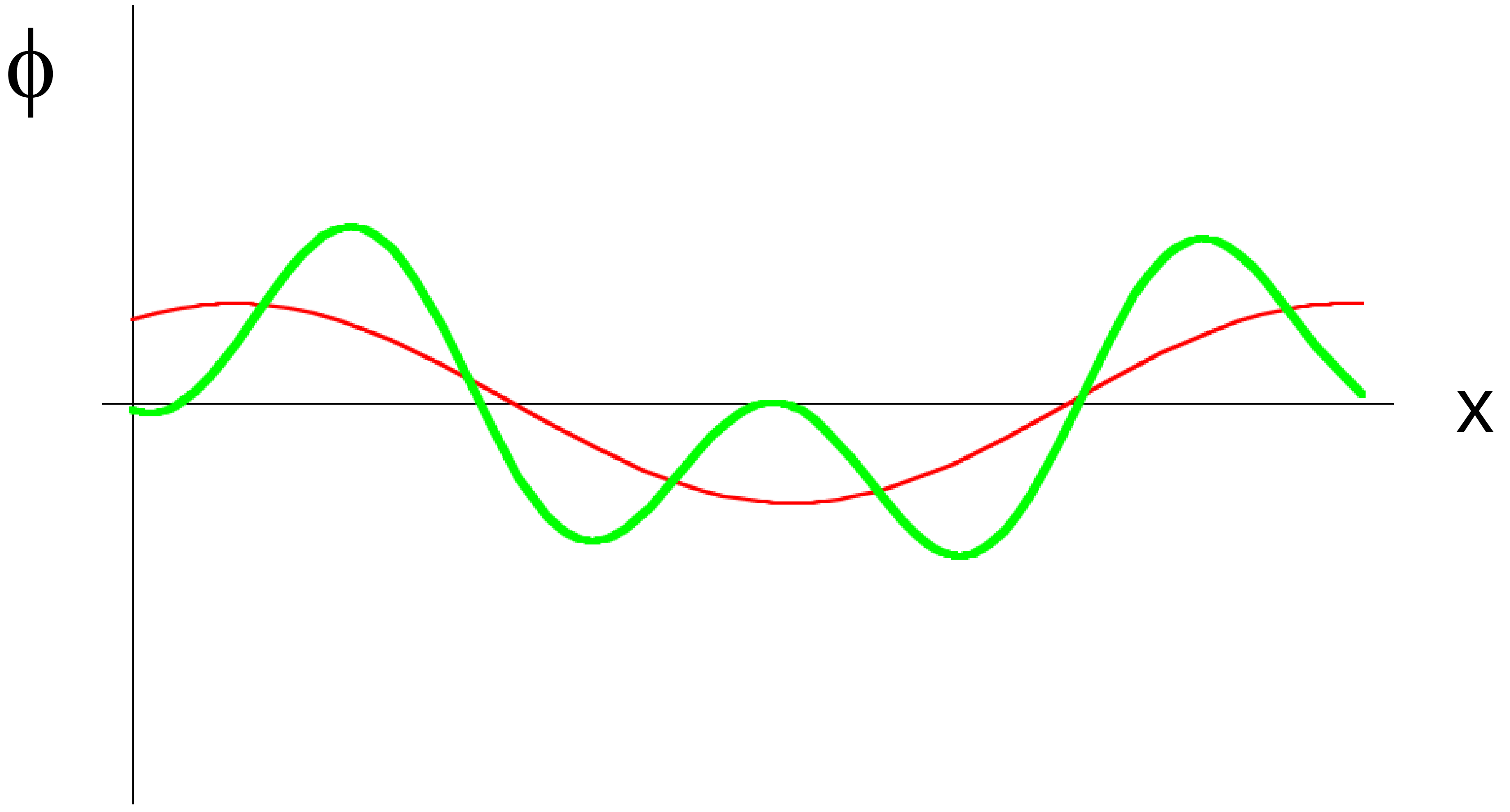
Mukhanov and Chibisov 1981

Galaxies are children of quantum fluctuations produced in the first 10^{-35} seconds after the birth of the universe.

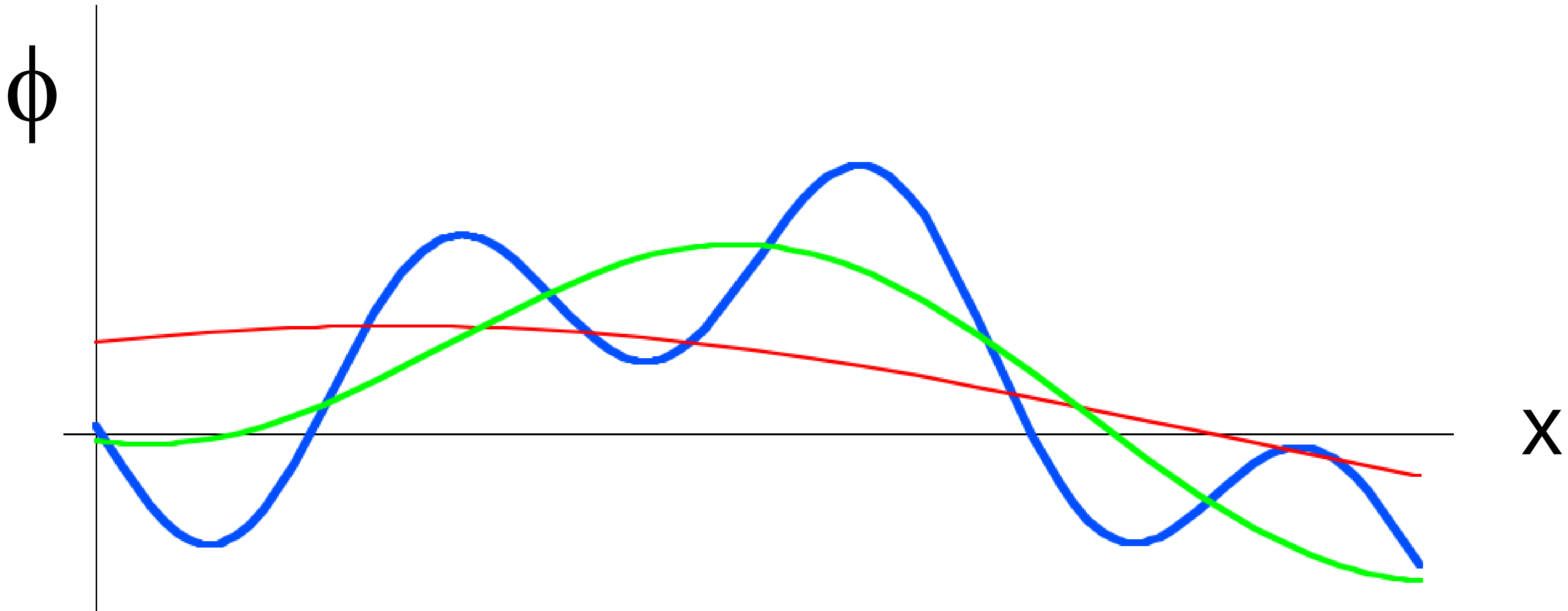
Quantum fluctuations produced during inflation



Small quantum fluctuations of all physical fields exist everywhere. They are similar to waves, which appear and then rapidly oscillate, move and disappear. Inflation stretched them, together with stretching the universe. When the wavelength of the fluctuations becomes sufficiently large, they stop moving and oscillating, and do not disappear. They look like frozen waves.



When expansion of the universe continues, new quantum fluctuations become stretched, stop oscillating, and freeze on top of the previously frozen fluctuations.



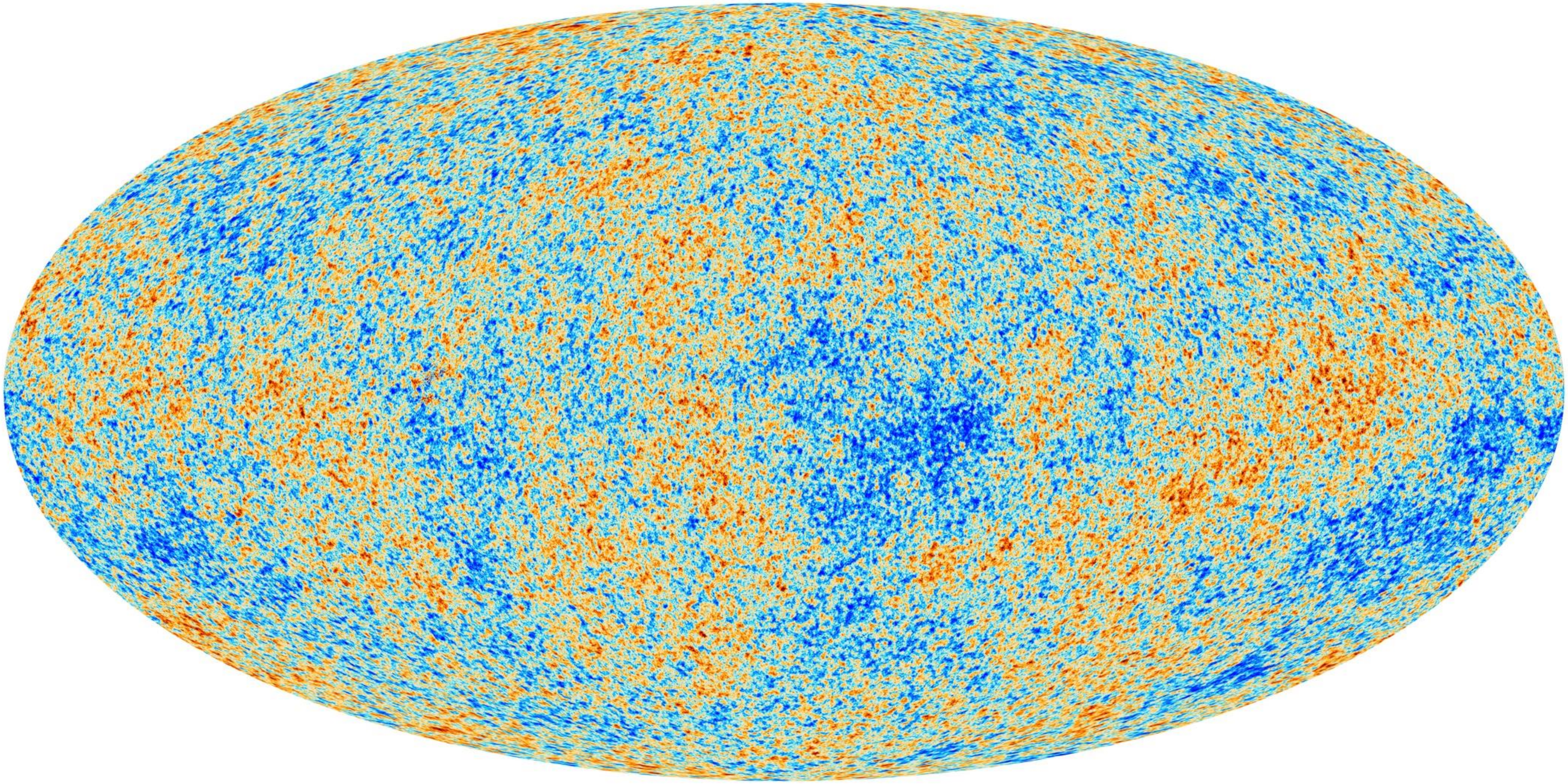
This process continues, and eventually the universe becomes populated by inhomogeneous scalar field. Its energy takes different values in different parts of the universe. These inhomogeneities are responsible for the formation of galaxies.

Sometimes these fluctuations are so large that they can increase the value of the scalar field in some parts of the universe. Then inflation in these parts of the universe occurs again and again. In other words, the process of inflation becomes eternal.

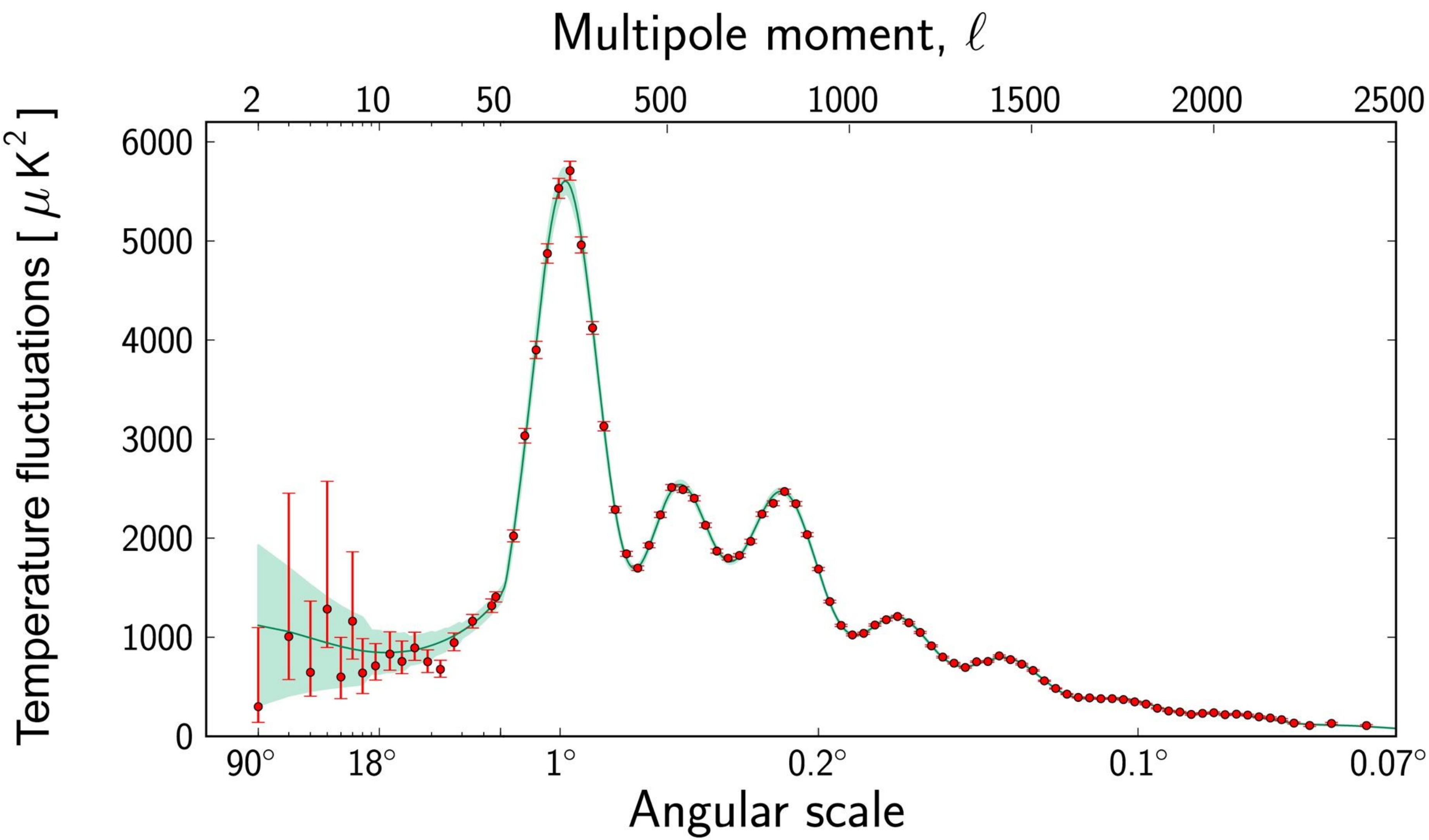
We will illustrate it now by computer simulation of this process.

Planck satellite: Perturbations of temperature

This is an image of **quantum fluctuations produced by inflation 10^{-35} seconds after the Big Bang**. These tiny fluctuations were **stretched by inflation** to incredibly large size, and now we can observe them **using all sky as a giant photographic plate**



Planck satellite: Perturbations of temperature (red dots) and predictions of inflationary theory (green line)



Inflation and Planck 2018

$$\Omega = 1.009 \pm 0.0018$$

Universe is flat with accuracy about 10^{-2}

Planck + SPT + BAO

$$n_s = 0.965 \pm 0.004$$

Spectrum of perturbations is nearly flat

According to Planck 2018, non-inflationary HZ spectrum with $n_s = 1$ is ruled out at a better than 6σ level, just as predicted in 1981 by Mukhanov and Chibisov. (This is an important prediction of inflation, similar to asymptotic freedom in QCD.)

$$f_{\rm NL}^{\rm local} = 0.91 \pm 5$$

Agrees with predictions of the simplest inflationary models with accuracy $O(10^{-4})$.

An impressive success of inflationary theory

Can we test inflation even better ?

B-modes: a special polarization pattern which can be produced by gravitational waves generated during inflation. A discovery of the gravitational waves of this type could provide a strong additional evidence in favor of inflation.

A.A. Starobinsky, Pis'ma Zh. Eksp. Teor. Fiz. 30 (1979) 719
V.A. Rubakov, M.V. Sazhin, A.V. Veryaskin, Phys.Lett.B 115 (1982)

BICEP/Keck, LiteBIRD and other experiments

A non-discovery of B-modes is fine too: many models predict gravitational waves with a tiny amplitude.

A discovery of inflationary gravitational waves is **NOT** required for proving inflation, but it would be **a great gift indeed**, and not only for inflation, but for investigation of quantum gravity and processes at energies many orders above LHC.

Testing predictions of inflation

1) **The universe is flat, $\Omega = 1$**. (In the mid-90's, the consensus was that $\Omega = 0.3$, until the discovery of dark energy confirming inflation.)

2) The observable part of the universe is **uniform** (homogeneous).

3) It is **isotropic**. In particular, it does not rotate. (Back in the 80's we did not know that it is uniform and isotropic at such an incredible level.)

4) Perturbations produced by inflation are adiabatic

5) Unlike perturbations produced by cosmic strings, inflationary perturbations lead to many **peaks in the spectrum**

6) The large angle TE anti-correlation (WMAP, Planck) is a distinctive signature of **superhorizon fluctuations** (Spergel, Zaldarriaga 1997), ruling out many alternative possibilities

7) Inflationary perturbations should have a **nearly flat (but not exactly flat) spectrum**. A small deviation from flatness is one of the distinguishing features of inflation. It is as significant for inflationary theory as the asymptotic freedom for the theory of strong interactions

8) **Inflation produces scalar perturbations** and **tensor perturbations** with nearly flat spectrum, and it **does not produce vector perturbations**. There are certain relations between the properties of scalar and tensor perturbations

9) In the early 80's it seemed that inflation is ruled out because scalar perturbations are not observed at the expected level 10^{-3} required for galaxy formation. Thanks to dark matter, smaller perturbations are sufficient, and they were **found by COBE**.

10) Scalar perturbations are **Gaussian**. In non-inflationary models, the parameter f_{NL}^{local} describing the level of local non-Gaussianity can be as large as 10^4, but it is predicted to be O(1) in all single-field inflationary models. **Confirmed by Planck.** Prior to the Planck2013 data release, there were rumors that f_{NL}^{local} >> O(1), which would rule out **all** single field inflationary models

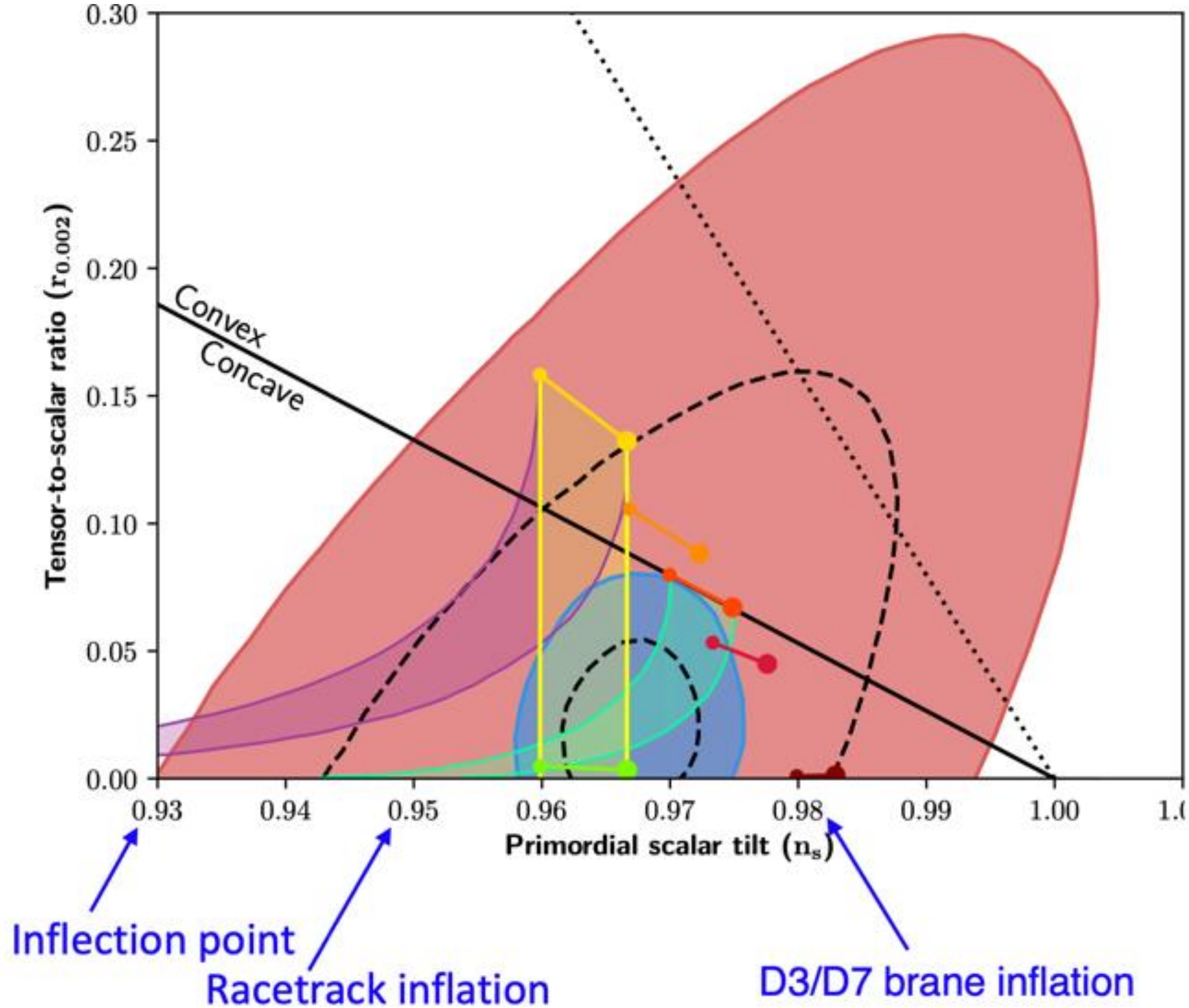
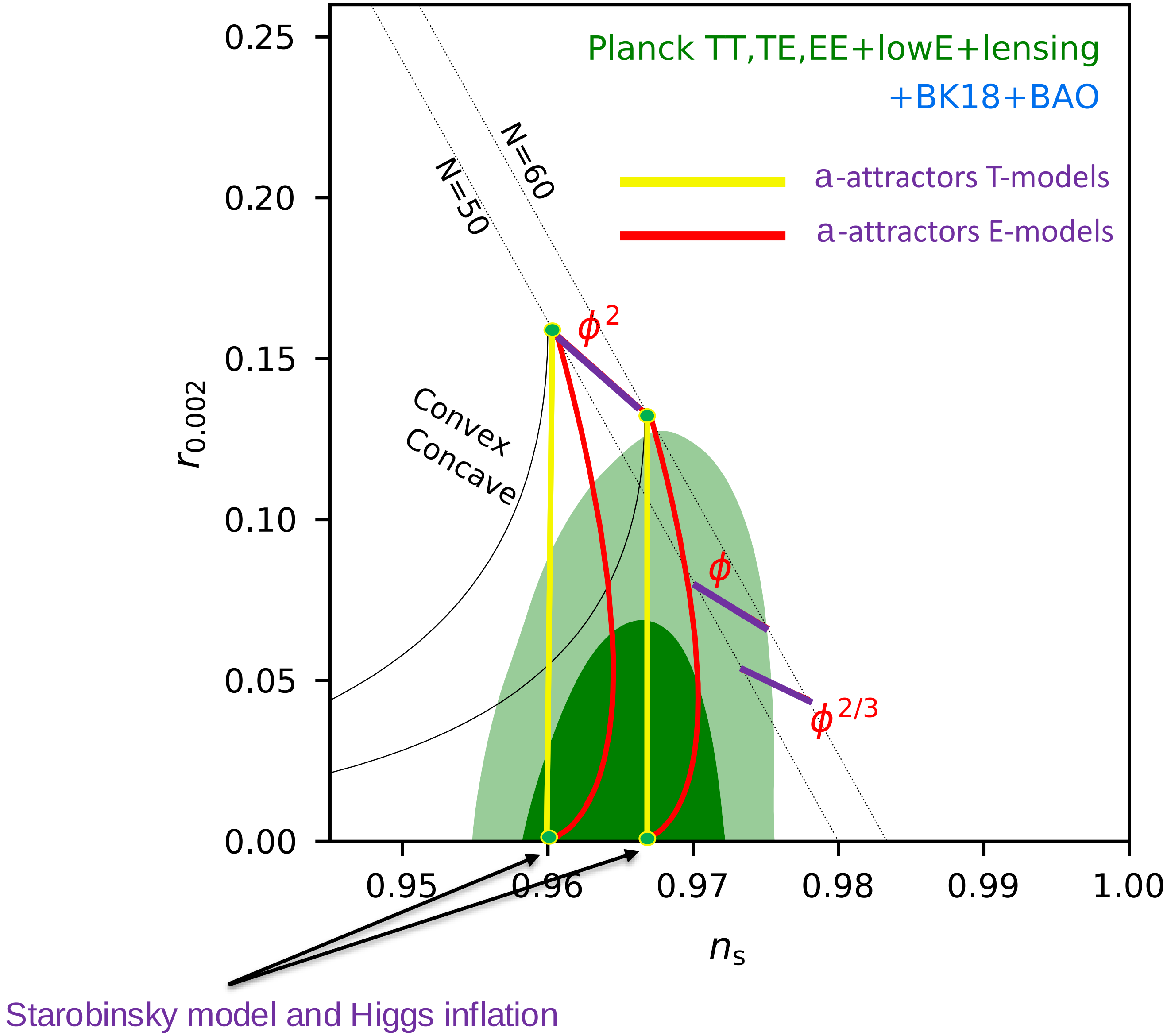


FIG. 5. Many favorite string inflation models from a decade ago, with very low r, are now ruled out by precision data on n_s. 7-year WMAP results [37] are in red, Planck 2018 results [12] are in blue.

Planck2018 – BICEP/Keck2021 constraints



α-attractors

Kallosh, AL, Roest 2013

To match observations, the simplest chaotic inflation model

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}\partial\phi^2 - \frac{1}{2}m^2\phi^2$$

should be modified:

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}\frac{\partial\phi^2}{(1-\underline{\frac{\phi^2}{6\alpha}})^2} - \frac{1}{2}m^2\phi^2$$

Switch to canonical variables $\phi = \sqrt{6\alpha}\,\tanh\frac{\varphi}{\sqrt{6\alpha}}$

The potential becomes

$$V = 3\alpha\, m^2 \tanh^2\frac{\varphi}{\sqrt{6\alpha}}$$

This model (**α-attractor T-model**) is consistent with observational data for m ~ 10^{-5} and <u>any</u> value of α smaller than O(7).

What is the meaning of α-attractors?

More generally:

$$\frac{L}{\sqrt{-g}} = \frac{R}{2} - \frac{(\partial_\mu \varphi)^2}{2\left(1 - \frac{\varphi^2}{6\alpha}\right)^2} - V(\varphi)$$

In canonical variables

$$\frac{L}{\sqrt{-g}} = \frac{R}{2} - \frac{(\partial_\mu \phi)^2}{2} - V\left(\sqrt{6\alpha}\, \tanh \frac{\phi}{\sqrt{6\alpha}}\right)$$

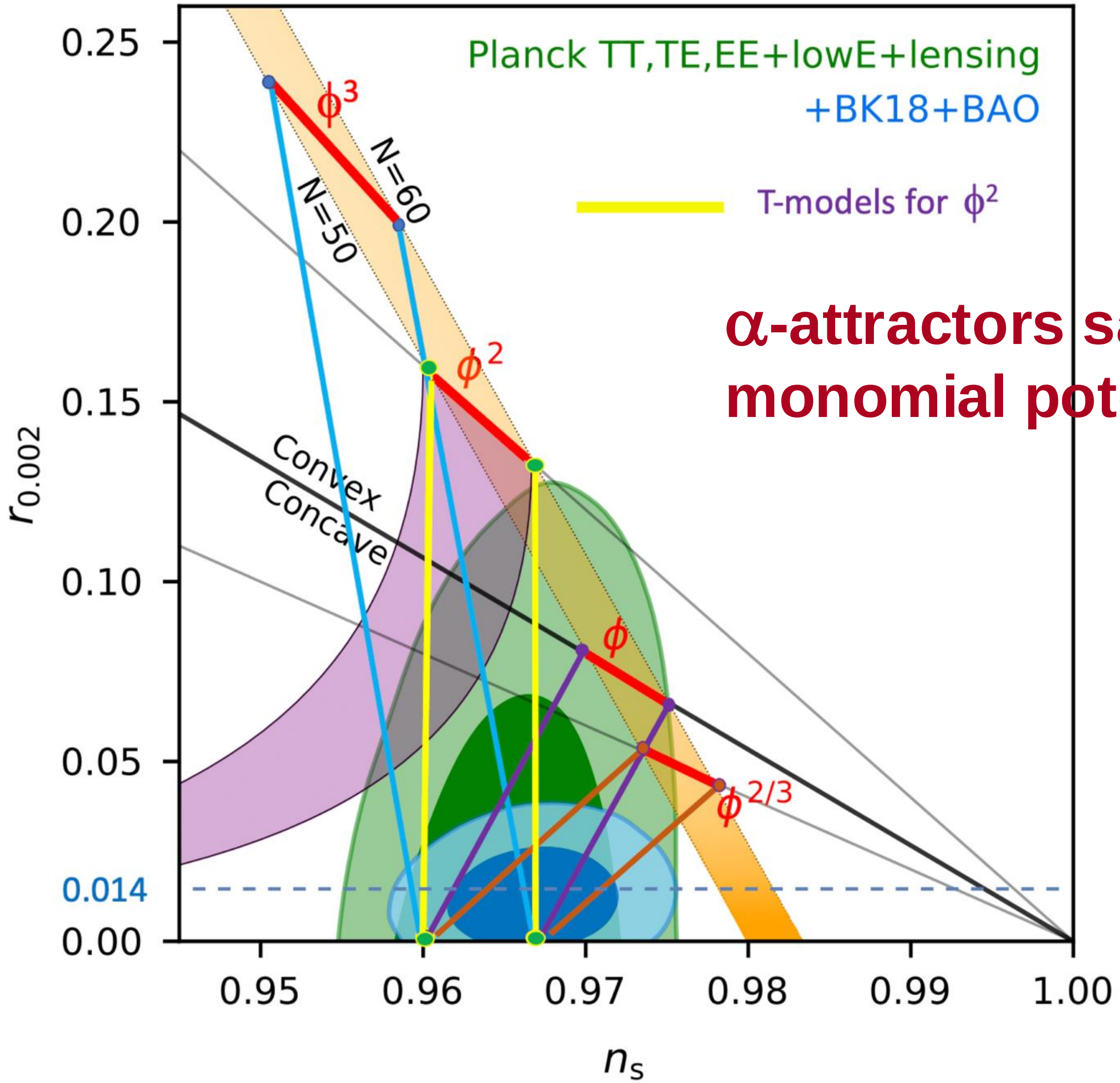
Asymptotically at large values of the inflaton

$$V(\phi) = V_0 - 2\sqrt{6\alpha}\, V_0' \, e^{-\sqrt{\frac{2}{3\alpha}}\phi}$$

Here $V_0' = \partial_\varphi V|_{\varphi=\sqrt{6\alpha}}$ This factor can be absorbed in the redefinition (shift) of the field. **Therefore, at small α, values of n_s and r depend only on V_0 and α**, not on the shape of V(ϕ).

$$n_s = 1 - \frac{2}{N_e}\,, \qquad r = \frac{12\alpha}{N_e^2}$$

Planck2018 – BICEP/Keck2021 constraints



α-attractors saving monomial potentials

E-models of α-attractors

Kallosh, AL, Roest 2014

Start with the model

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{R}{2} - \frac{3\alpha}{4}\frac{(\partial\rho)^2}{\rho^2} - V(\rho)$$

Switch to canonical variables

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{R}{2} - \frac{1}{2}(\partial\varphi)^2 - V(e^{-\sqrt{\frac{2}{3\alpha}}\varphi}).$$

In particular, for $V(\rho) = V_0(1-\rho)^2$ the potential becomes

$$V = V_0\left(1 - e^{-\sqrt{\frac{2}{3\alpha}}\varphi}\right)^2$$

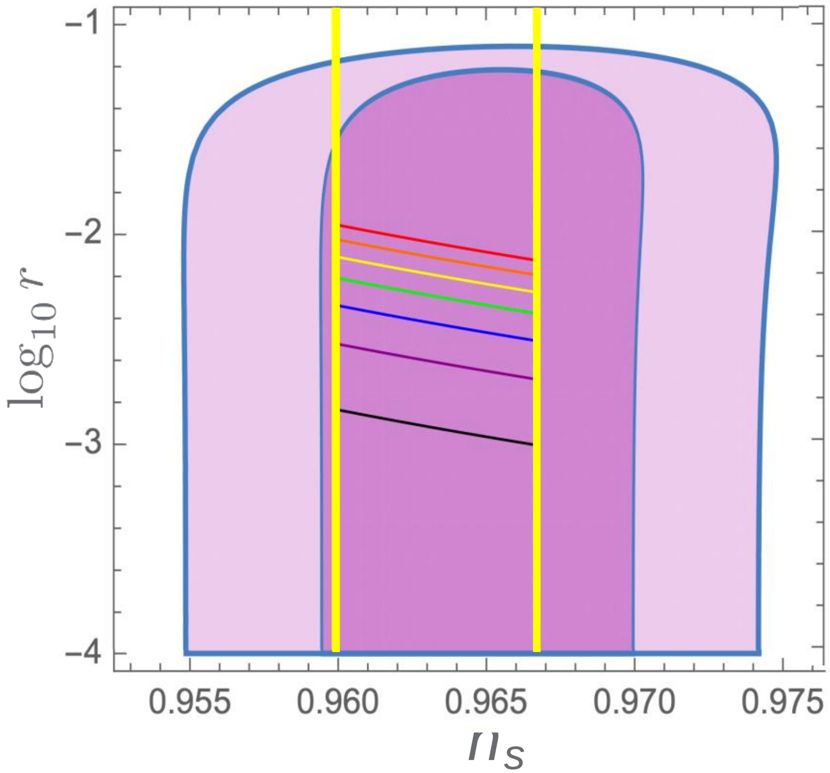
This model (E-model) coincides with the Starobinsky model for α =1. In general case these models predict

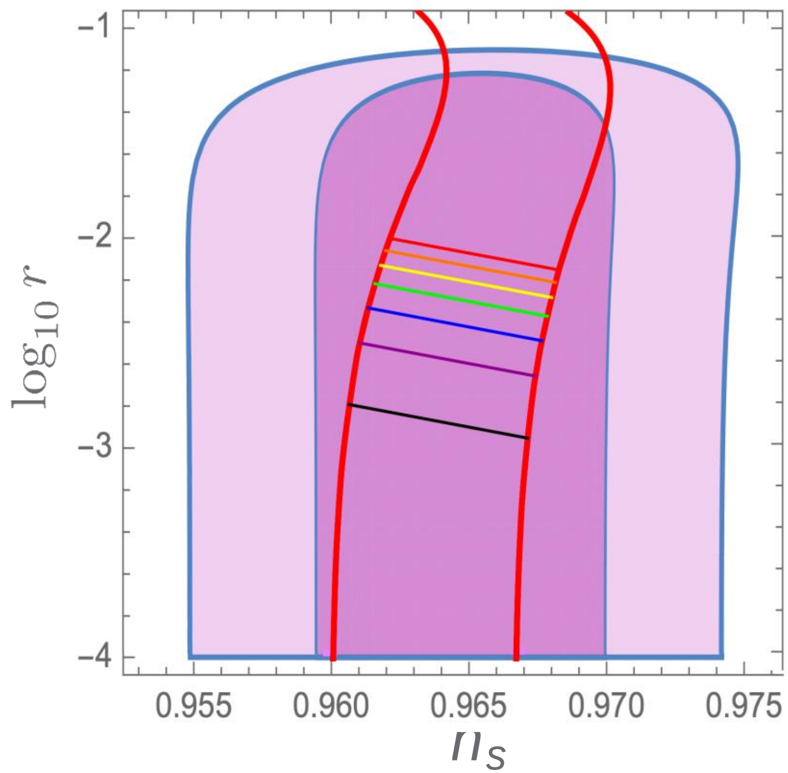
$$n_s = 1 - \frac{2}{N_e}\,, \qquad r = \frac{12\alpha}{N_e^2}$$

Benchmarks for T-models and E-models

T-models $V_T = V_0 \tanh^2 \frac{\varphi}{\sqrt{6\alpha}}$

E-models $V_E = V_0 \left(1 - e^{\sqrt{\frac{2}{3\alpha}}\varphi}\right)^2$





String theory interpretation of **7 discrete targets for α-attractors**

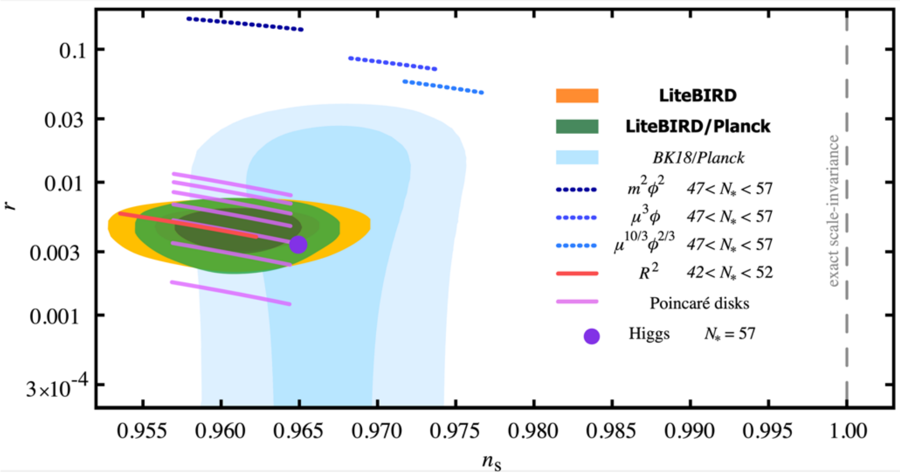
$$3\alpha = 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7$$

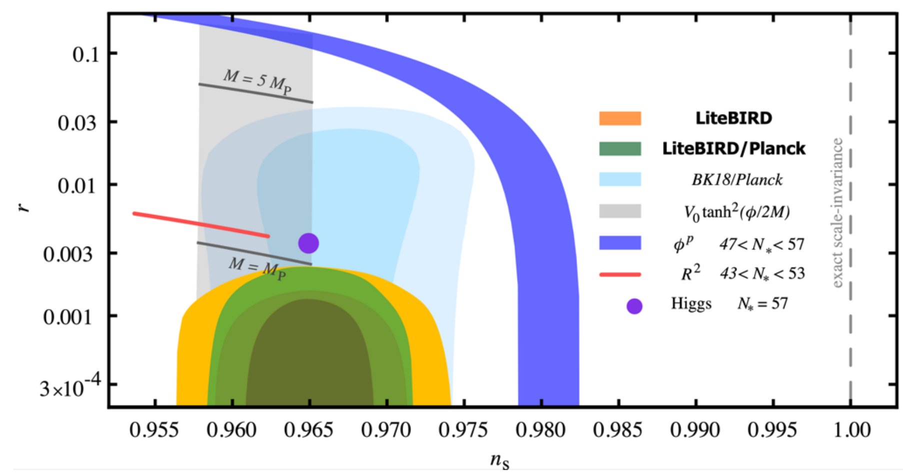
Ferrara, Kallosh 1610.04163. Kallosh, A.L., Wrase, Yamada 1704.04829

LiteBIRD

[Probing Cosmic Inflation with the LiteBIRD Cosmic Microwave Background Polarization Survey]

2202.02773





Are there any good models for the right side of the blue area favored by BICEP/Keck ??

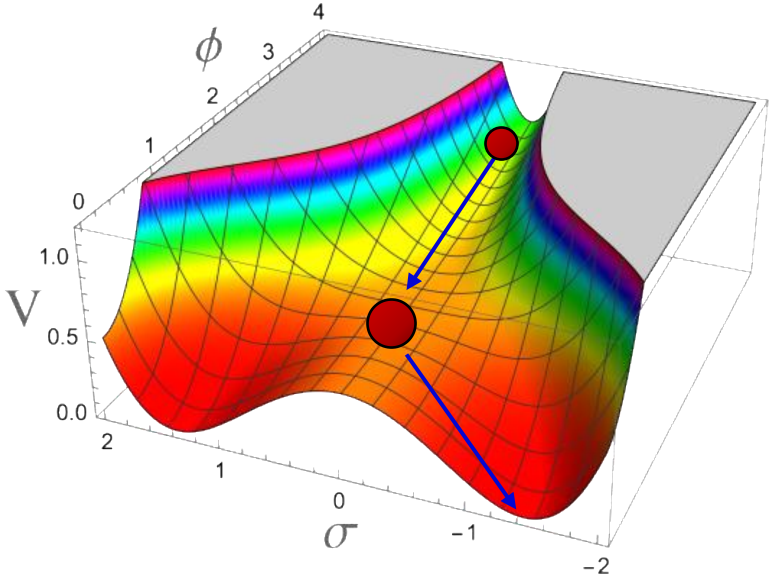
What if H_0 problem changes everything?

There are many attempts to resolve the disagreement between the Planck results and supernova data by considering exotic models of dark energy, etc. Some of these attempts require large values of n_s, all the way to $n_s = 1$. While this is an exotic possibility, it is better to be prepared and consider maximally flexible formulations of inflationary models.

A possible comprehensive solution: Hybrid α-attractors

There is a special class of inflationary models where $n_s = 1$ is an attractor point: Hybrid Inflation

AL 1991, 1994



$$V(\sigma,\phi) = \frac{1}{4\lambda}(M^2 - \lambda\sigma^2)^2 + \frac{m^2}{2}\phi^2 + \frac{g^2}{2}\phi^2\sigma^2$$

For σ = o, it is just the quadratic potential uplifted by $M^4/4\lambda$

$$n_s = 1 - 3\left(\frac{V'}{V}\right)^2 + 2\frac{V''}{V}$$

By increasing the uplifting term $V_{\rm uplift} = M^4/4\lambda$ one can increase V without changing any derivatives of V. Therefore, in the large uplift limit, for generic V we have an **attractor prediction n_s = 1**.

The same conclusion is valid for hybrid α-attractors: In two-field inflationary models one can increase n_s all the way to n_s = 1.

Conclusions:

1. Many predictions of inflationary theory have been tested and confirmed by observations during the last 40 years.

2. Some inflationary models, such as the Starobinsky model, the Higgs inflation, and a broad class of α-attractors can describe all available CMB-inflation-related data by a single parameter.

3. Predictions of α-attractors are stable with respect to significant modifications of the inflaton potential. These models can describe any small value of r, all the way down to $r = 0$.

4. BICEP/Keck results are moving close to the range necessary for testing tensor modes in these models. LiteBIRD would move us much further.

5. α-attractor versions of hybrid inflation can consistently describe a much greater range of n_s, including $n_s = 1$, which is an attractor value in the large uplift regime.

6. Hybrid α-attractors can describe copious production of PBH, while remaining consistent with the Planck/BICEP/Keck data.

Uniformity of our **universe** is explained by **inflation**: Exponential stretching of the universe makes **our part** of the universe almost exactly uniform.

However, the same theory predicts that on a much greater scale, the universe can be (and probably should be) 100% non-uniform.

Inflationary **universe** becomes a **multiverse**

Here comes the multiverse

Pessimist:

If each part of the multiverse is huge, we will never see other parts, so it is **impossible to prove** that we live in the multiverse.

Optimist:

If each part of the multiverse is huge, we will never see other parts, so it is **impossible to disprove** that we live in the multiverse.

I'd rather be an optimist and a fool than a pessimist and right. Albert Einstein

This scenario is **more general** (otherwise one would need to explain why all colors but one are forbidden). Therefore, the theory of the **multi**verse, rather than the theory of the **uni**verse, is the basic theory.

Moreover, even if one begins with a single-colored universe, quantum fluctuations make it multi-colored.

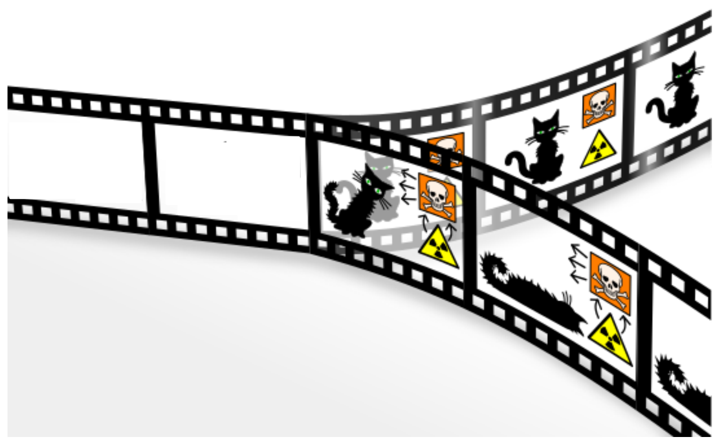
Independently of the discovery or non-discovery of inflationary **tensor** modes (gravitational waves), **scalar** modes produced by **quantum effects** in the early universe are already discovered. Numerous attempts to propose a **non-quantum** mechanism of formation of the large-scale structure of the universe during the last 40 years failed.

Thus, it is time to take very seriously the assumption that **the large-scale structure of the universe was formed due to quantum fluctuations**.

This is the Cosmological Schrodinger Cat story.

A warm-up: Remember the Schrodinger cat

The Universe is similar to it, but more weird, since initially there was no cat, so the movie that we see now shows something that did not exist before inflation



The difference between this picture and the old one is that in quantum field theory (unlike quantum mechanics) the number of particles is not conserved.

In inflationary cosmology, this effect is pushed to its limits. We may start with the universe weighting less than a milligram, without any particles at all, and we may end up with a humongous multiverse consisting of exponentially large parts where all possible outcomes consistent with the underlying physical theory are realized.

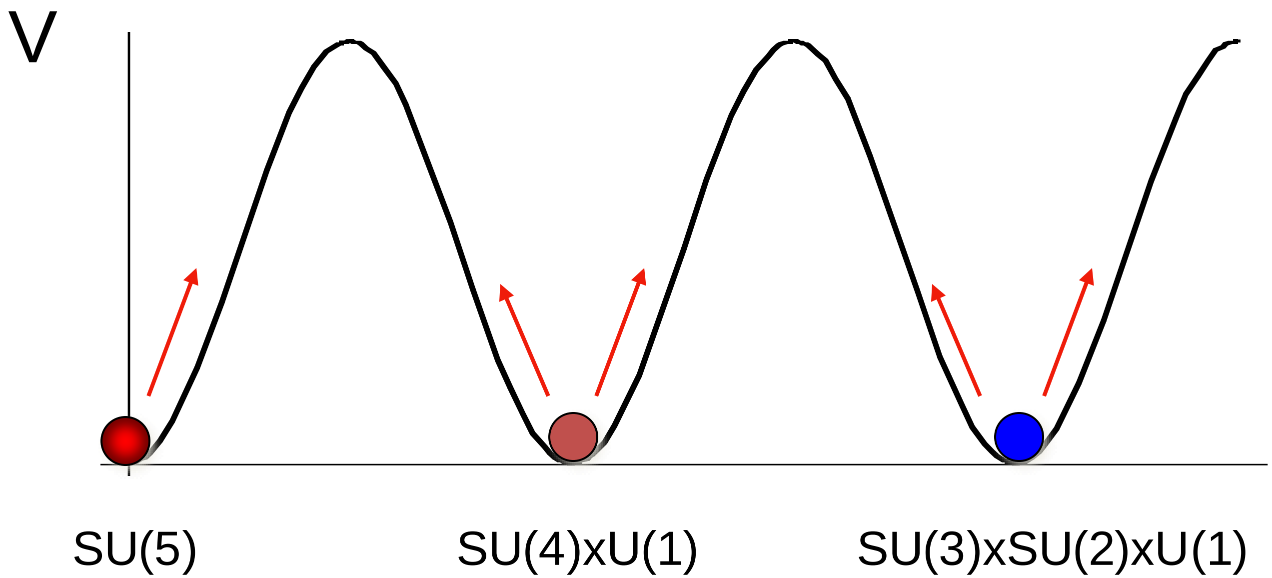
Note that not only in inflation, but in ALL cosmological theories discussed at present (including ekpyrotic, cyclic, conformal, string gas cosmology) the large scale structure of the universe was formed by quantum fluctuations.

Thus positions of the galaxies in the universe and their properties are determined by quantum fluctuations. These positions are completely different in different parts of the universe. Same is true for the values of light scalar fields such as axions. These values determine local values of dark matter and dark energy density, which take different values in different parts of the world.

Welcome to the multiverse

Example: SUSY landscape

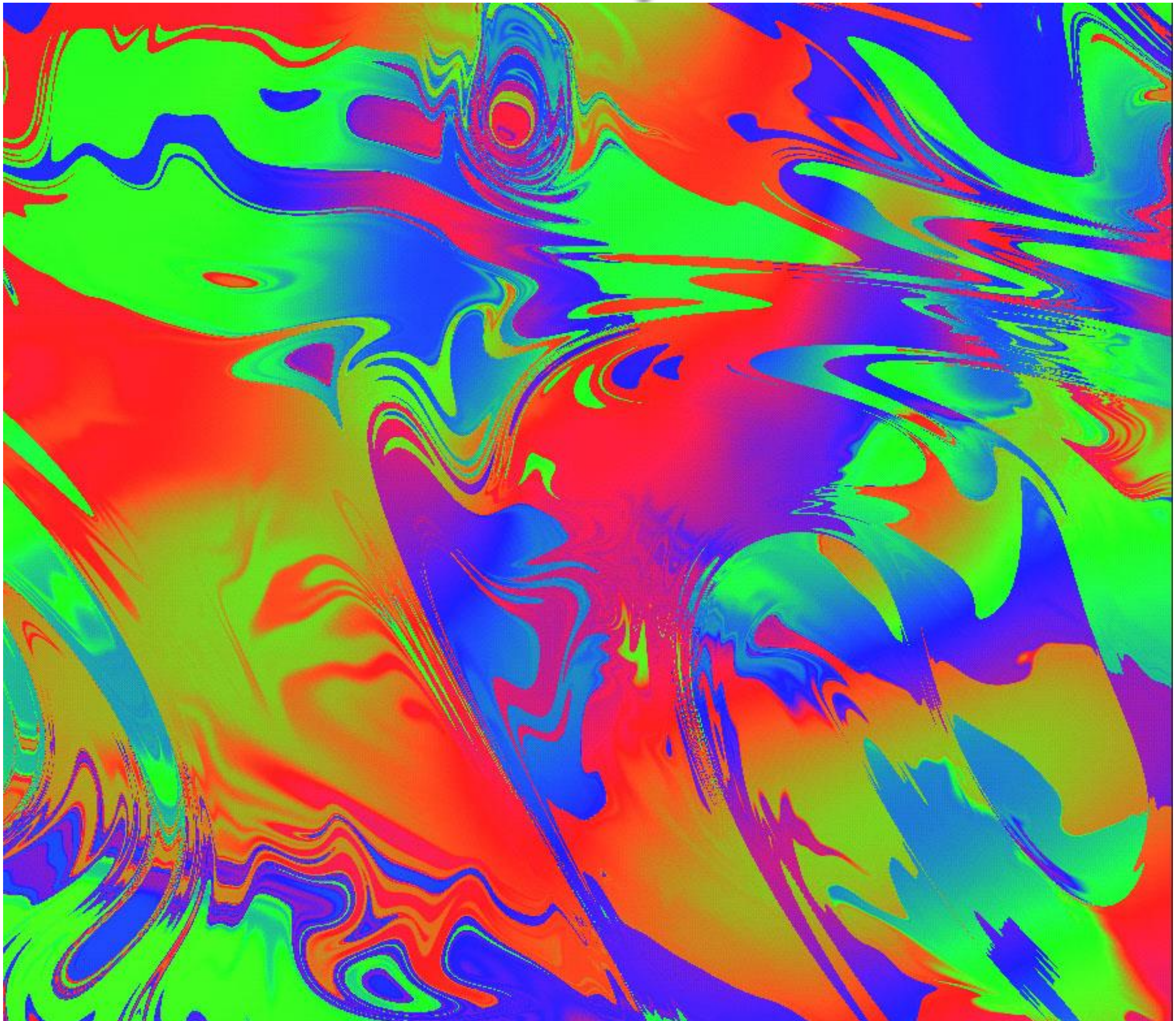
Supersymmetric SU(5)

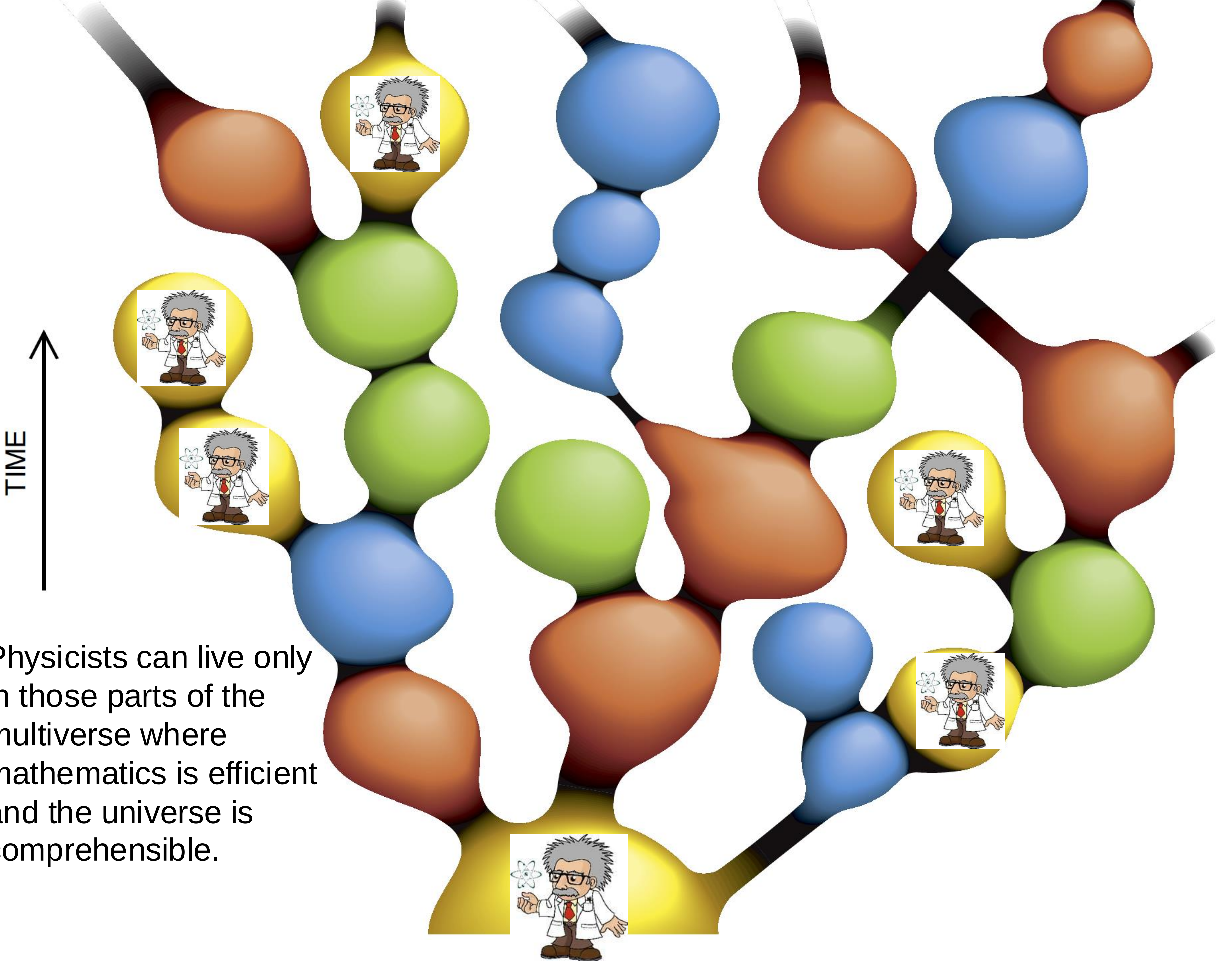


Weinberg 1982: Supersymmetry forbids tunneling from SU(5) to SU(3)xSU(2)XU(1). This implied that we cannot break SU(5) symmetry.

A.L. 1983: Inflation solves this problem. Inflationary fluctuations bring us to each of the three minima. Inflation makes each of the parts of the universe exponentially large. We can live only in the SU(3)xSU(2)xU(1) minimum.

Kandinsky Universe





Physicists can live only in those parts of the multiverse where mathematics is efficient and the universe is comprehensible.