

Cosmic Ray Transport & Self-Generated Turbulence

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MHD & Plasma waves

From the **Liouville theorem** for a collisionless plasma, the evolution in the **phase space** of each particle's species is described by the **Vlasov equation**:

$$\frac{\partial f_s}{\partial t} + \vec{v} \cdot \vec{\nabla}_x f_s + \frac{q_s}{m_s} \left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B} \right) \cdot \vec{\nabla}_v f_s = 0$$

Summing the Vlasov equations of **thermal electrons and protons**, the equation for the **mass and momentum conservation**, known as **ideal MHD equations**, are obtained:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0$$

$$\rho \left[\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \vec{\nabla} \vec{v} \right] = -\vec{\nabla} P + \frac{1}{4\pi} (\vec{\nabla} \times \vec{B}) \times \vec{B}$$

From perturbative analysis, one can see that waves with **wave-vector** $\vec{k} \parallel B_0$, known as **Alfvén waves**, travel at the Alfvén speed:

$$\omega^2 = k^2 v_A^2 \quad v_A = \frac{B_0}{\sqrt{4\pi\rho}}$$

Cosmic Rays diffusion

$$m\gamma \frac{d\vec{v}}{dt} = \frac{q}{c} [\vec{v} \times (\vec{B}_0 + \delta\vec{B})]$$

$$\frac{d\mu}{dt} = \frac{q\delta B}{mc\gamma} (1 - \mu^2)^{1/2} \cos[\Omega t \mp kv\mu t \mp \phi]$$

$$\left\langle \frac{\Delta\mu\Delta\mu}{\Delta t} \right\rangle = \left(\frac{q\delta B}{mc\gamma} \right)^2 (1 - \mu^2) \frac{\pi}{\mu v} \delta \left[k \mp \frac{\Omega}{v\mu} \right]$$

The **pitch angle diffusion** on magnetic perturbations has a **resonance condition** that depends on the **test-particle pitch angle** (μ): $k_{\text{wave}} = \frac{1}{R_L\mu}$. \Rightarrow A spectrum of turbulence $\delta B(k)$ is able to diffuse a spectrum of cosmic rays. For a deeper treatment, see **P. Blasi short course, HE-1: Particle acceleration in astrophysical plasma**.

DSA & Supernovae paradigm

$$\frac{\Delta E}{E} = \frac{4}{3}(u_1 - u_2) \quad N(> E_k) = N_0 \left(\frac{E_k}{E_0} \right)^{-\gamma}$$

$$n(E) \sim E^{-\gamma-1}$$

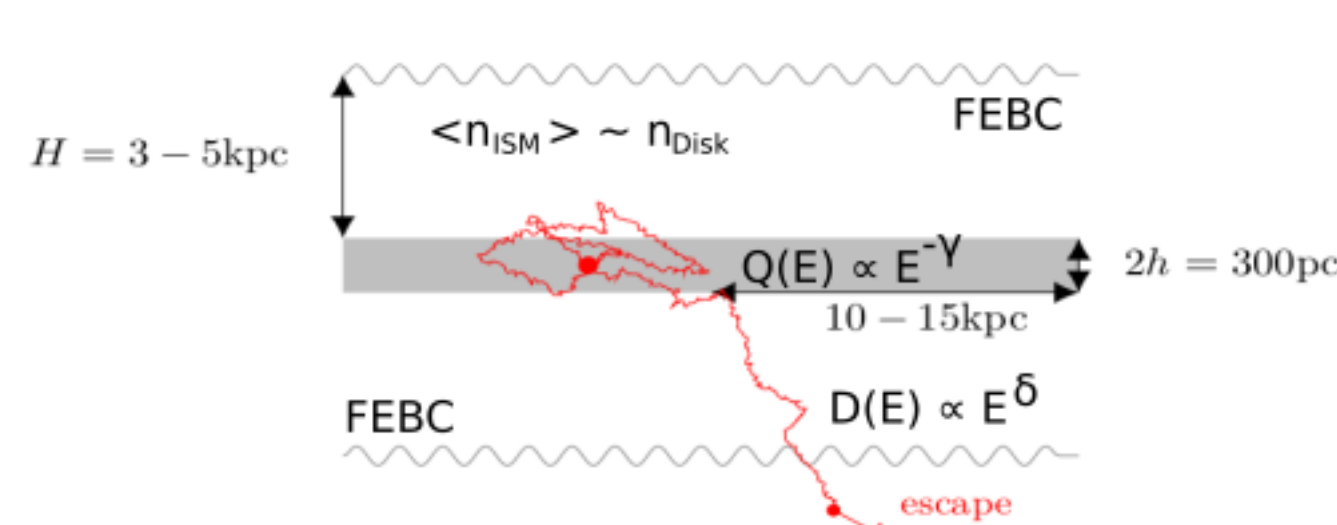
$$P_{\text{cross}} = 1 - 4u_2 \quad \text{strong shock: } \gamma + 1 = 2$$

UPSTREAM (fast and cold) DOWNSTREAM (slow and hot)

To explain how **Galactic sources** can reach the **energy of the CR's knee**, something is needed to **amplify the turbulence in the upstream!**

$$\tau_{\text{acc}} = \frac{3}{u_1 - u_2} \left[\frac{D_1}{u_1} + \frac{D_2}{u_2} \right] \sim 1000 \text{ yr} \cdot E_{\text{GeV}}^{1/2}$$

Leaky Box model



$$n(E)_I \propto \begin{cases} Q(E)H^2/D(E) \propto E^{-\gamma-\delta} & \text{if } \tau_{\text{esc}} \ll \tau_{\text{spall}} \\ Q(E)\tau_{\text{spall}} \propto E^{-\gamma} & \text{if } \tau_{\text{esc}} \gg \tau_{\text{spall}} \end{cases}$$

$$n(E)_{II} \propto \begin{cases} n(E)_I H^2/D(E) \propto E^{-\gamma-2\delta} & \text{if } \tau_{\text{esc}} \ll \tau_{\text{spall}} \\ n(E)_I \tau_{\text{spall}} \propto E^{-\gamma} & \text{if } \tau_{\text{esc}} \gg \tau_{\text{spall}} \end{cases}$$

$$n(E)_{II}^{\text{unst.}}/n(E)_{II}^{\text{st.}} \propto \sqrt{H^2/D(E)} \propto E^{-\delta/2} \quad \text{if } \tau_{\text{esc}} \gg \tau_{\text{decay}}$$

Streaming instability

When the **plasma-waves dispersion relation** has an **imaginary component**, the magnetic field fluctuations are **exponentially amplified or damped**.

Including Cosmic Rays in the plasma description (**kinetically**, through their Vlasov equation, or in **MHD**, through their energy density and pressure), leads to the excitation of unstable waves **through their current**.

$$EJ(> E) \gg cU_B \Rightarrow \text{Non-resonant modes}$$

$$EJ(> E) \ll cU_B \Rightarrow \text{Resonant modes}$$

- The CR's current excites unstable magnetic fluctuations.
- The growth of the waves suppresses the CR's diffusion coefficient and develops a pressure gradient that acts on the plasma.
- Both **advection and diffusion** impact the current.

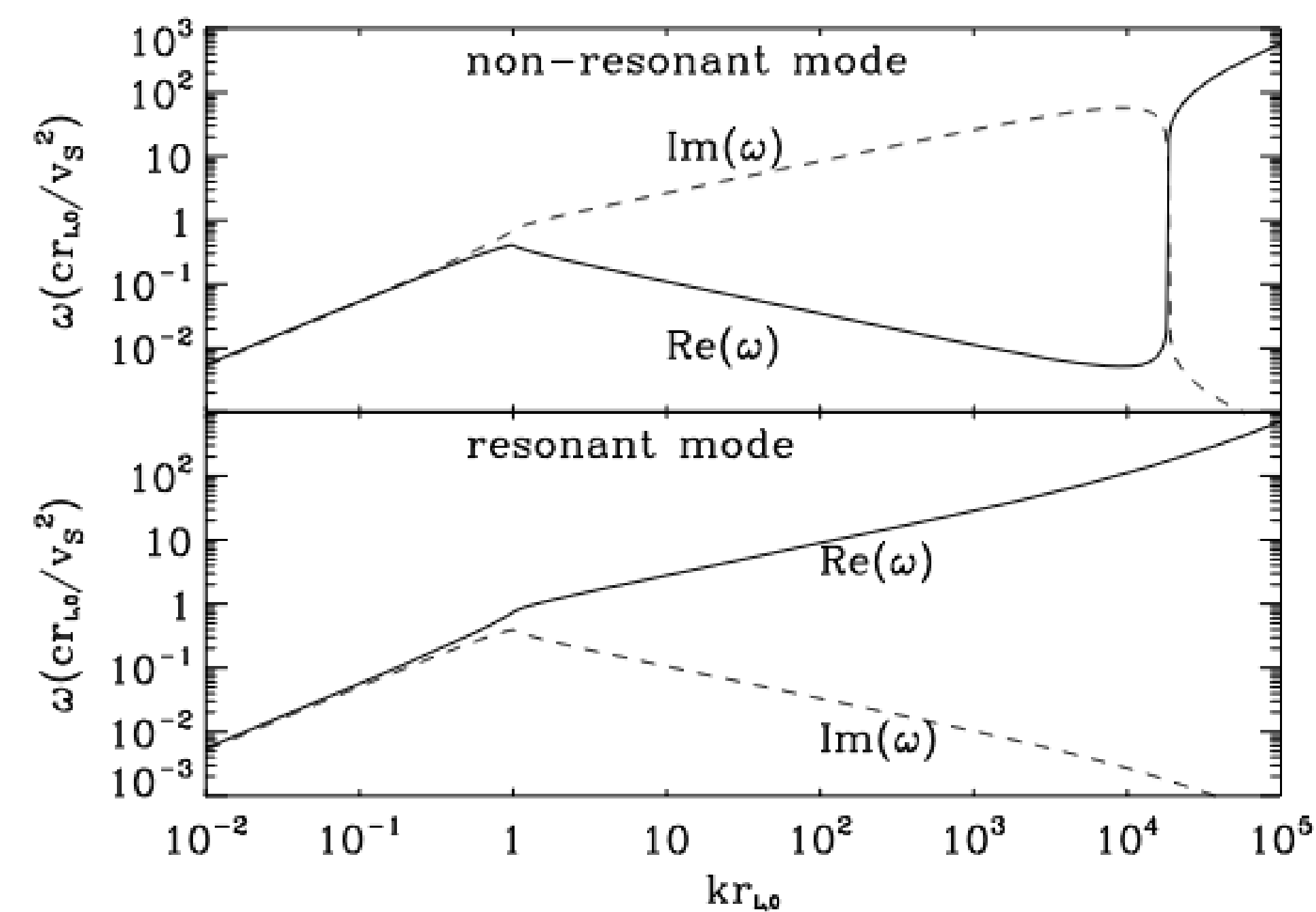


Figure: Amato, E. and Blasi, P. "A kinetic approach to cosmic-ray-induced streaming instability at supernova shocks" (2009), MNRAS. Bell, A. R. "Turbulent amplification of magnetic field and diffusive shock acceleration of cosmic rays" (2004), MNRAS

In general, these are **non-linear effects**, studied **phenomenologically, semi-analytically** and through **numerical simulations**:

- PIC**: Both plasma and CRs are treated as particles (**fully kinetic**).
- Hybrid**: plasma electrons are treated as a fluid (MHD), CRs and ions as particles (**kinetic**).

Why do we need these? What can we measure?

- Secondary products** (γ , ν).
- Primary / Secondary nuclei** (H/He, B/C ...).
- Diffusion** acts in **rigidity** (prop. + acc.), while **spallation** acts in **Energy/nucleon** (prop.).

Non-linear DSA in Supernovae

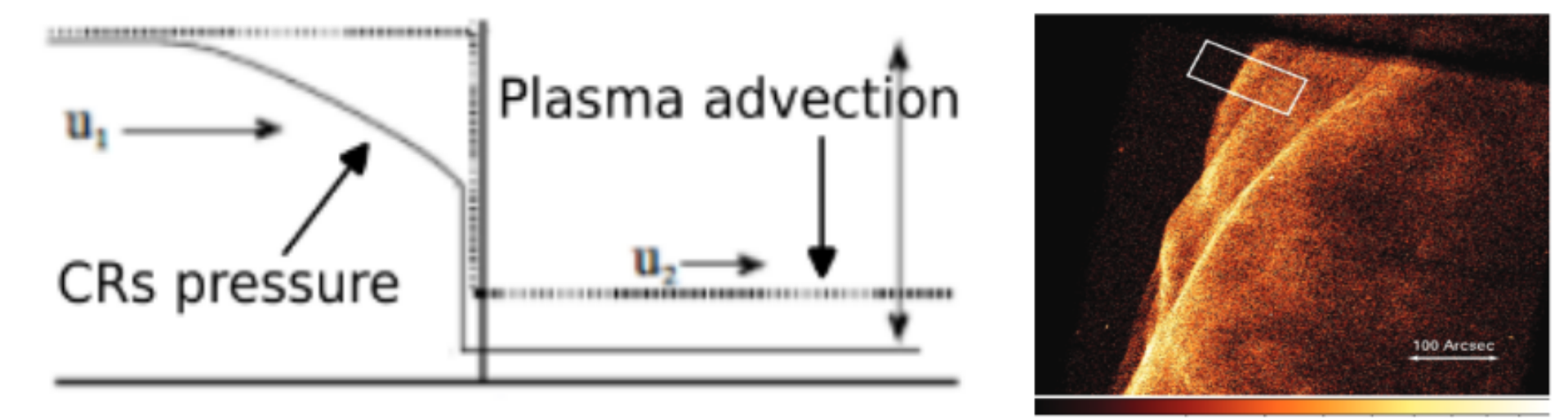


Figure: Morlino, G. and Amato, E. and Blasi, P. and Caprioli, D. "Spatial structure of X-ray filaments in SN 1006" (2010), MNRAS: Letters.

Galactic Halo

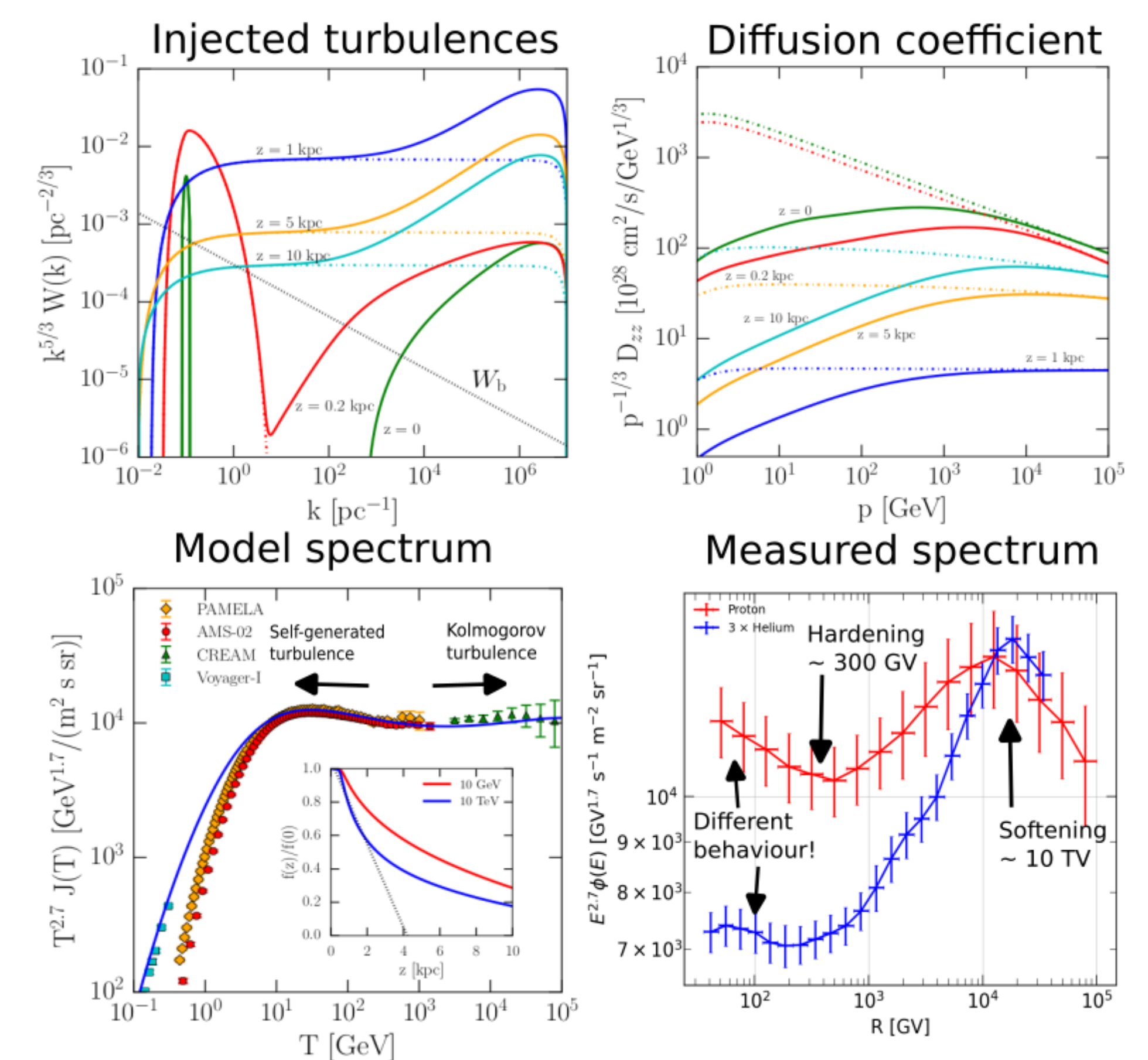


Figure: Evoli, C. and Blasi, P. and Morlino, G. and Aloisio, R. "Origin of the Cosmic Ray Galactic Halo Driven by Advected Turbulence and Self-Generated Waves" (2018), PRL. An, Q. et al. "Measurement of the cosmic ray proton spectrum from 40 GeV to 100 TeV with the DAMPE satellite" (2019), Science Advances.

Low-diffusivity bubbles

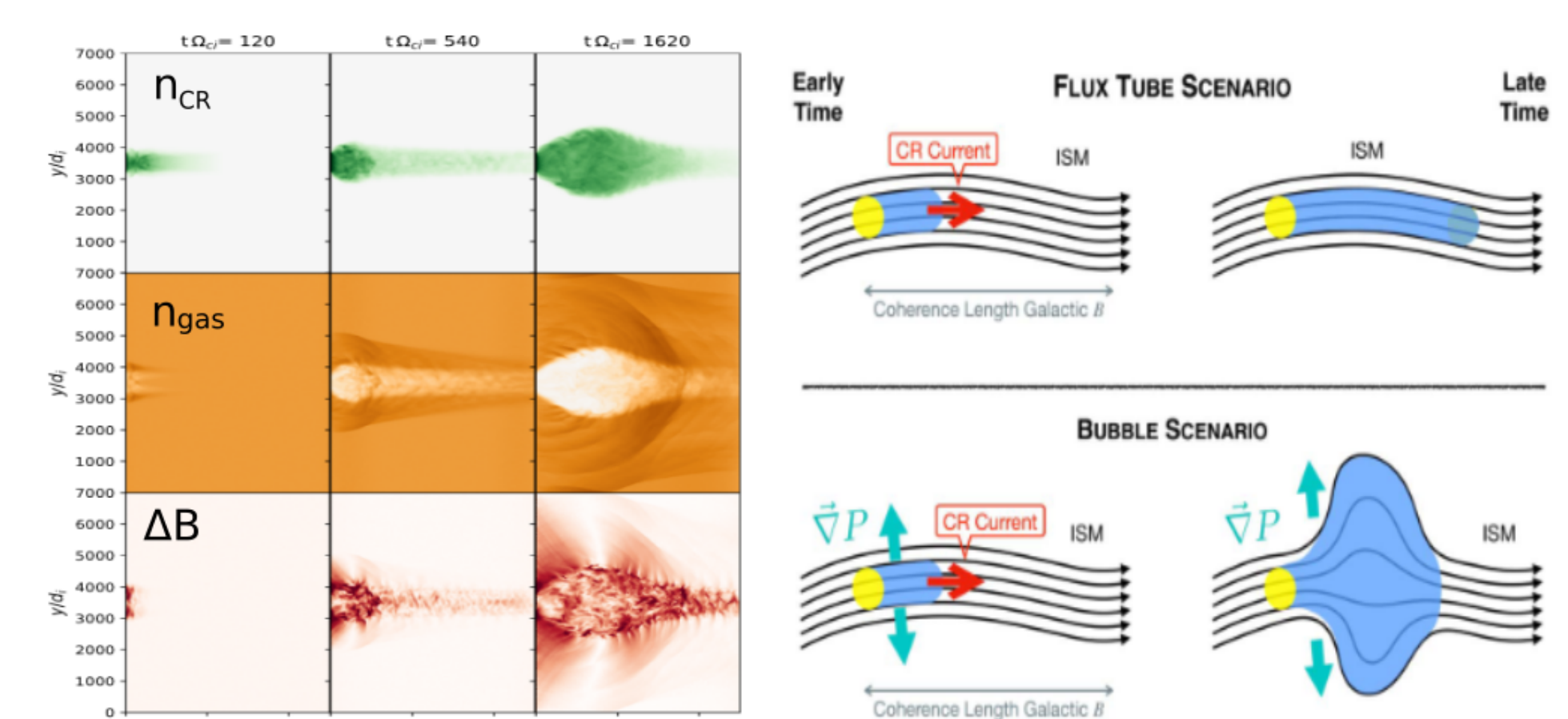


Figure: Schroer, B. et al. "Dynamical Effects of Cosmic Rays on the Medium Surrounding Their Sources" (2021), AAS.

Ultra-High Energy Cosmic Rays

For more details, see **R. Aloisio short course, HE-7: UHECR theory**.

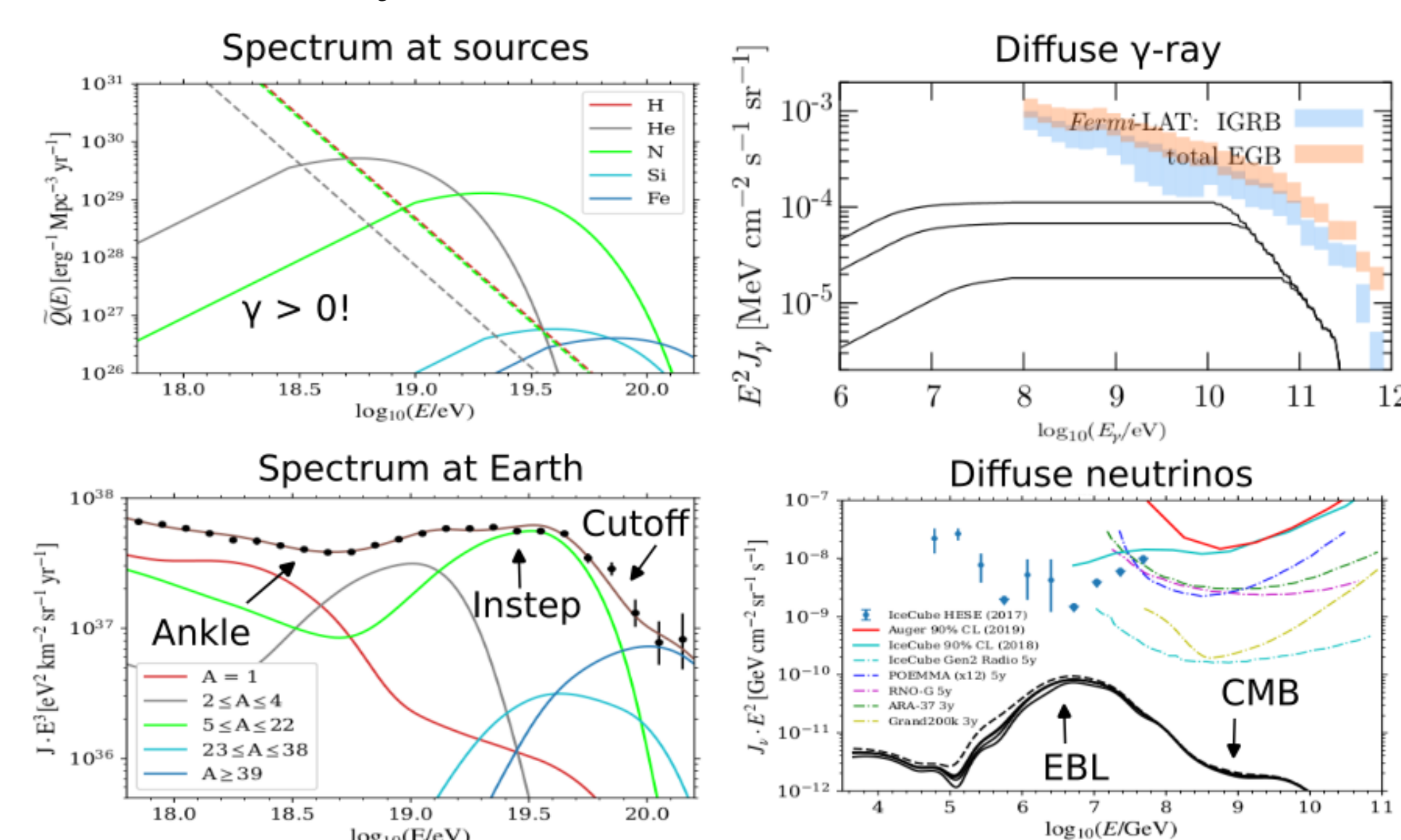


Figure: Halim, A. et al. "Constraining the sources of ultra-high-energy cosmic rays across and above the ankle with the spectrum and composition data measured at the Pierre Auger Observatory" (2023), JCAP

What we measure:

- Anisotropy** due to local sources distribution.
- Energy-spectrum features** due to mixed mass-composition and interactions.
- Highest proton-fraction** at the ankle.

(some) Open questions:

- Which are the **sources**?
- Which is the **acceleration mechanism** at play? (DSA, relativistic shocks, unipolar inductor...)
- Magnetic horizon**:
 - Self-induced** or **pre-existing** turbulence? (the cosmological magnetic field is poorly known...)
 - Where the **transition between Galactic and extra-galactic flux** takes place? How the **mass composition in the transition region** is affected?
- cosmogenic neutrinos**:
 - ν 's produced on the **CMB** still below observational limits.
 - neutral particles produced only by UHE hadrons \Rightarrow they can **point-back to the sources!**