Generalized 5-vector resampling methodology for neutron stars in a binary system

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ROMA1 - INFN

Outline

- Introduction
- Modulation
- Method
- Robustness
- Search
- Results

Goal

- To test and understand the robustness of the algorithm (Time domain correction), for the uncertainties in source parameters

- In case of no detection, set upper limits on the intrinsic gravitational wave strain

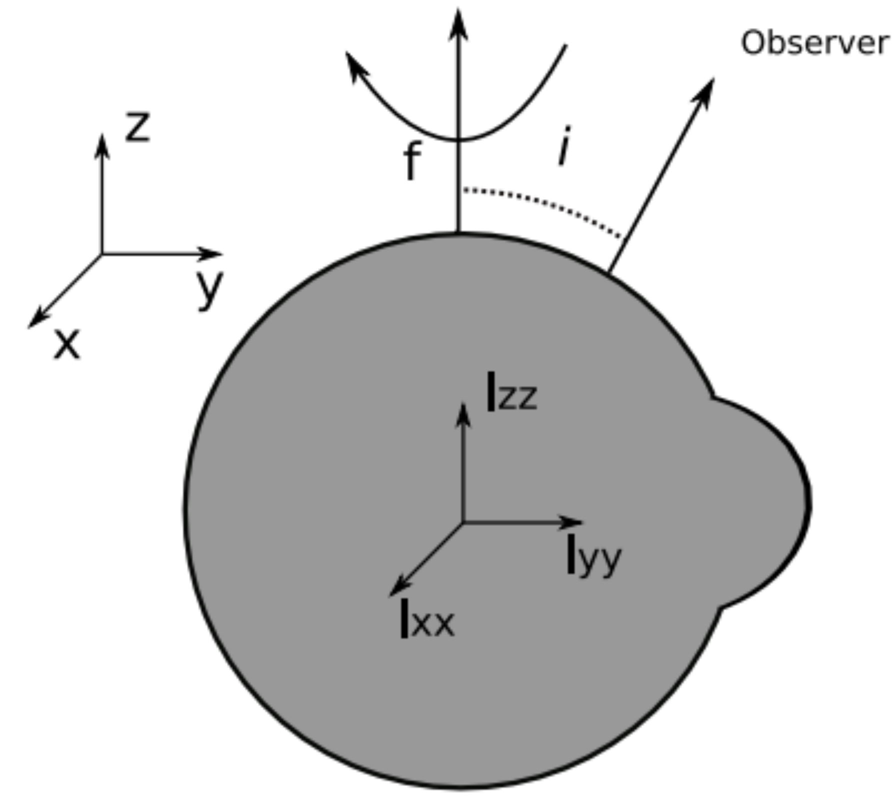
Basics

- Continuous Waves (CW):
  - Emitted by sources with quasi-periodically varying mass quadrupole moment
  - duration of signal >> observation time
- Signals are very weak (of the order of 10^{-25} or less)
- Hence, have to be integrated over a long time, to increase SNR ~ $O(\sqrt{t})$

Basics

[P. Patel, 2011]

- Neutron Star (NS) is rotating around the z axis and the observer is at an inclination angle (i) off the z-axis



$$h_0 = 1.05 \cdot 10^{-27} \left(\frac{I_{zz}}{10^{38} kg \cdot m^2} \right) \left(\frac{10kpc}{r} \right) \left(\frac{\nu}{100Hz} \right)^2 \left(\frac{\epsilon}{10^{-6}} \right)$$

$$\epsilon = \frac{I_{xx} - I_{yy}}{I_{zz}}$$

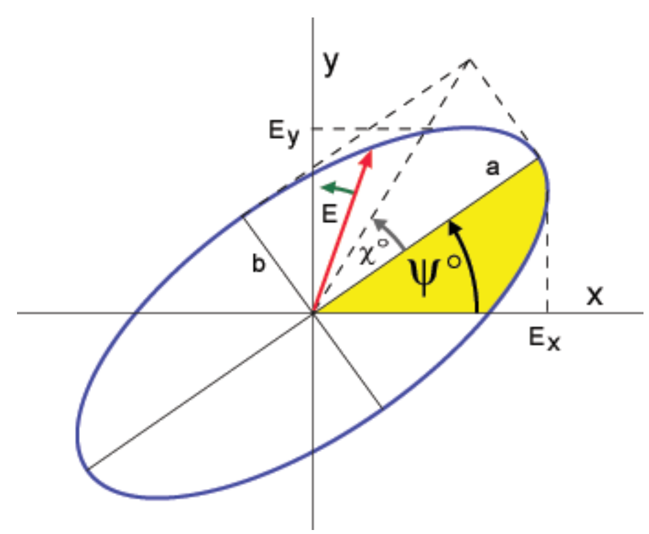
Basics

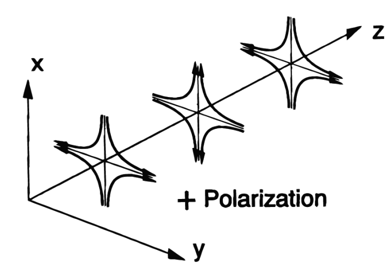
- Continuous Gravitational waves are described by polarization ellipse [P Astone et al., CQG, 194016 (2010)]

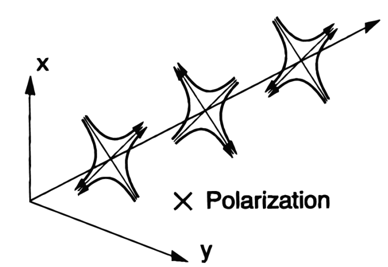
$$\mathfrak{H}(t) = h_0 \left(H_+ \mathfrak{e}_\oplus + H_\times \mathfrak{e}_\otimes \right) \exp^{j\omega(t)t}$$

$$H_+ = \frac{\cos 2\psi - j\eta \sin 2\psi}{\sqrt{1+\eta^2}}$$

$$H_\times = \frac{\sin 2\psi + j\eta \cos 2\psi}{\sqrt{1+\eta^2}}$$

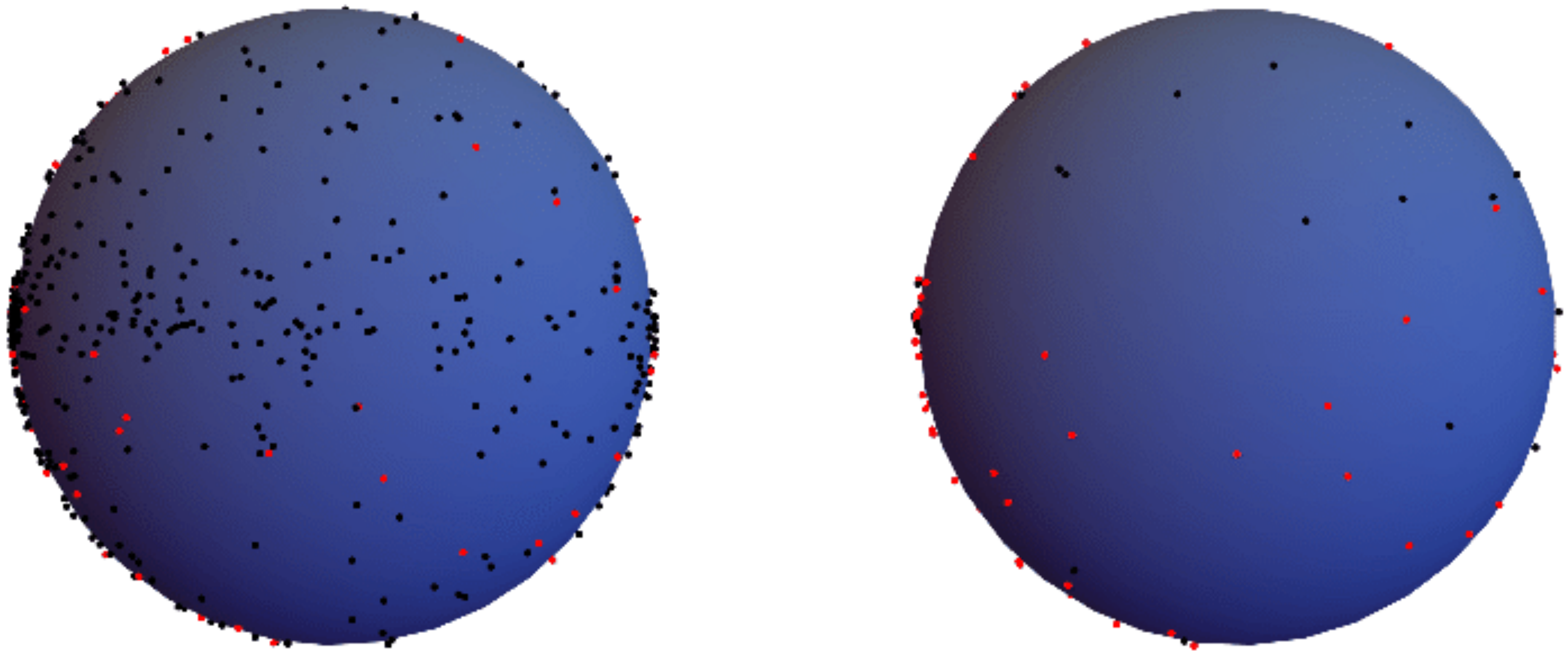






$$\eta = -\frac{2\cos\iota}{1+\cos^2\iota}$$

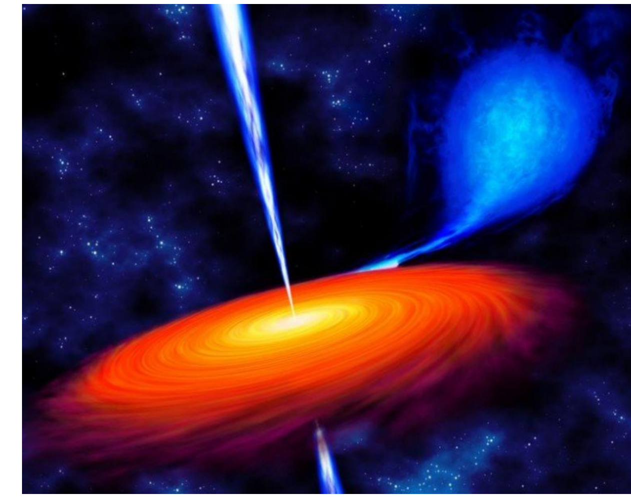
Binary population



	All Freq	LIGO/Virgo band [15-2000] Hz
Total Pulsars	2567	349
Binary Pulsars	**264**	**222**
Fractional Binary	10%	64%

Manchester, R. N. et al., 129, 1993-2006 (2005)
http://www.atnf.csiro.au/research/pulsar/psrcat

Low Mass X-ray binary



- Low Mass X-ray binaries (LMXBs) in a state of torque balance, could be a strong gravitational-wave source.

  - Scorpius X-1 (Sco X-1) is the brightest known LMXB

$$h \propto \sqrt{\mathscr{F}_{\mathrm{Xray}}}$$

L. Bildsten, ApJ L89, (1998)

    - Location is known, source parameters unknown

- The signal from the NSs get modulated due to the relative motions between the detector and the source

$$h(t_{\mathrm{arr}}) = h_0(H_+A_+ + H_\times A_\times)\exp\{j[\phi(t_{\mathrm{arr}}) + \phi_0]\}$$

$$\phi = 2\pi f_{\mathrm{gw}} \quad t_{\mathrm{arr}} = \mathrm{time\ at\ detector}$$

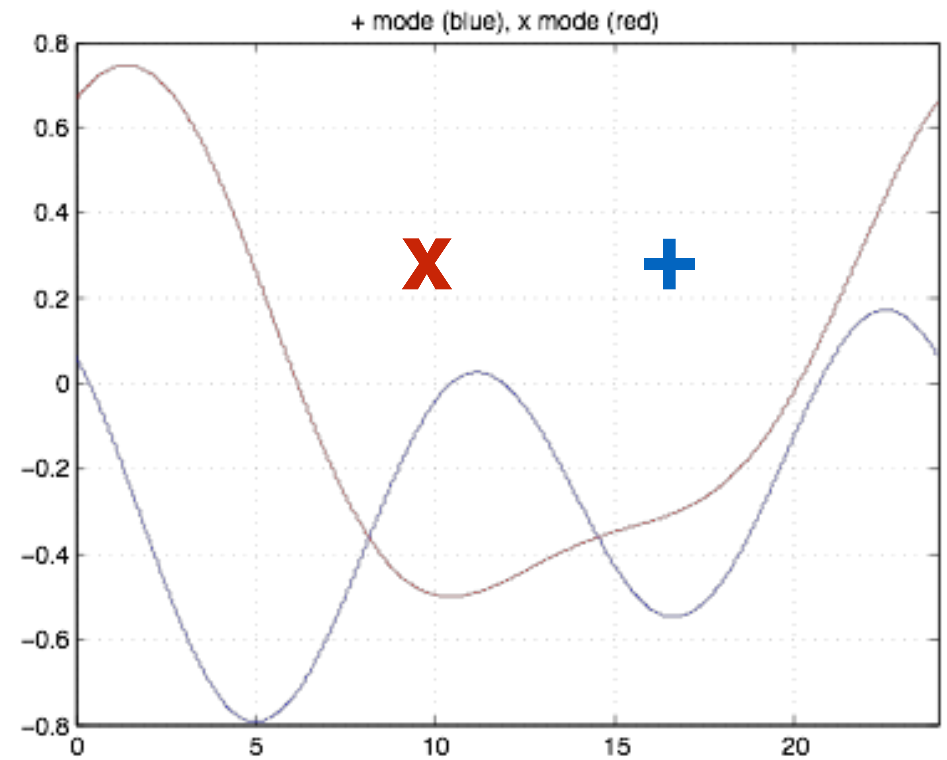
Signal at detector

- Gravitational strain at the detector will be [S D'Antonio et al. CQG ,204012 (2009)]

$$H(t_{arr}) = H_0\,(A_+ H_+ + A_\times H_\times)\,e^{\iota(\phi(t_{arr})+\phi_0)}$$
$$A_+(t_{arr}) = a_0 + a_{1c}\cos\Omega t_{arr} + a_{1s}\sin\Omega t_{arr} + a_{2c}\cos 2\Omega t_{arr} + a_{2s}\sin 2\Omega t_{arr}$$
$$A_\times(t_{arr}) = b_{1c}\cos\Omega t_{arr} + b_{1s}\sin\Omega t_{arr} + b_{2c}\cos 2\Omega t_{arr} + b_{2s}\sin 2\Omega t_{arr}$$
$$t_{arr} = \text{detector time}$$

- The detector response function depends on source location and detector geometry and latitude (for perpendicular arms). Hence can be used as template.

Scorpius X-1 at LIGO-H

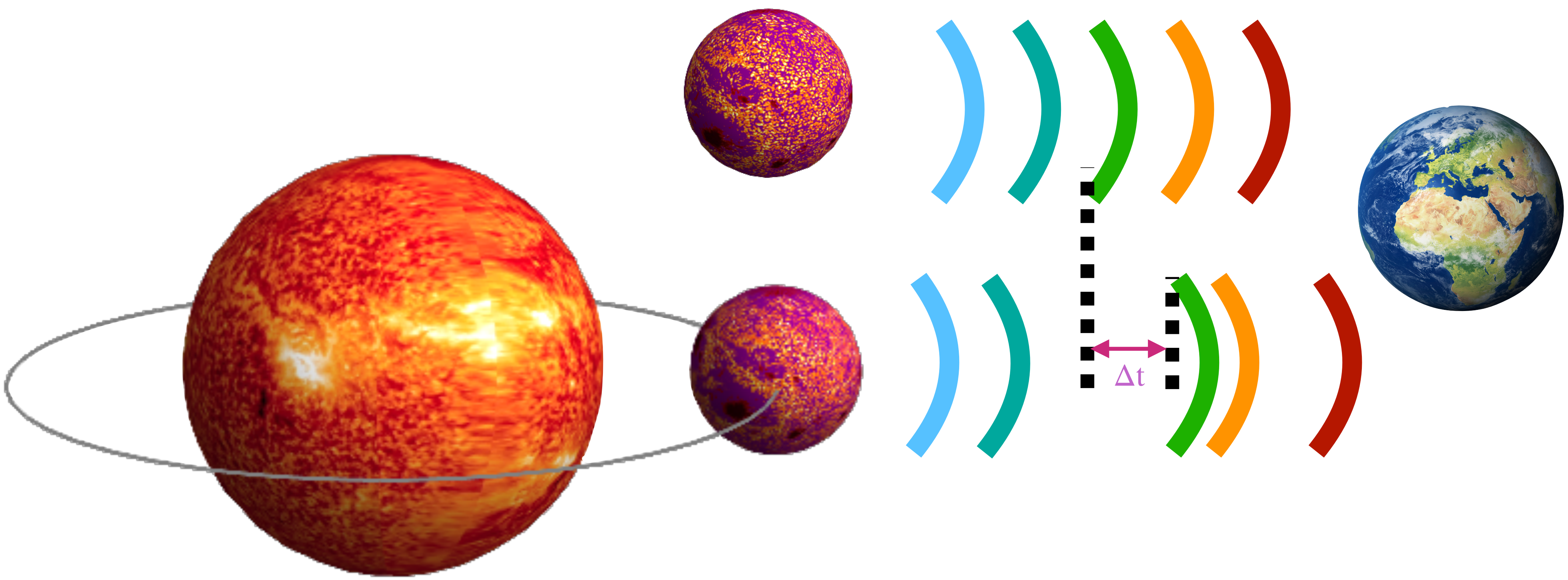


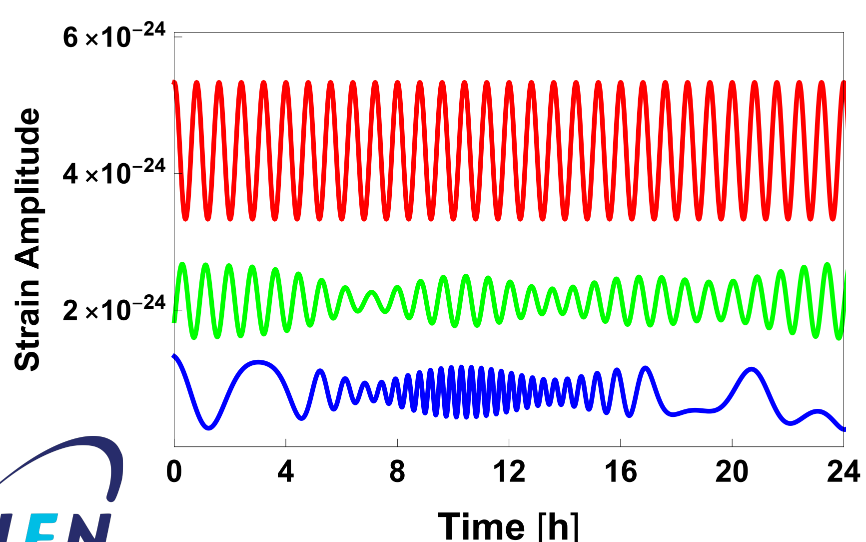
Sidereal time

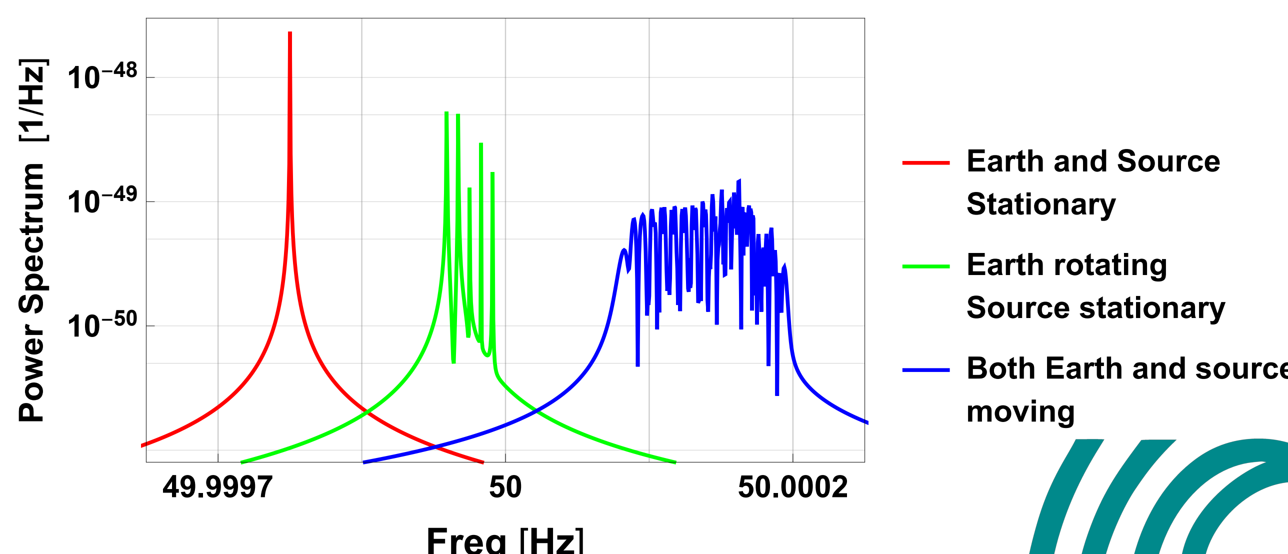
- 5 Fourier components can describe the signal. In frequency domain the a monochromatic signal splits into 5 peaks (sidereal frequency apart).

Modulation

$$\tau = t_{arr} + \underbrace{\Delta t_{\rm Bin} + \Delta t_{R\odot} + \Delta t_{S\odot} + \cdots}_{\Delta t}$$







Search for CWs from NSs in binary systems

- Targeted/Directed searches are computationally cheaper (compared to All Sky search) as the search parameter space is smaller
  - Targeted: all information is known [with uncertainty]
  - Directed: only sky location is known
- Astronomers do their best in providing source parameters, however with big* uncertainties; [* For our purpose]

Binary parameters

P. Leaci et al., PRD, 102003 (2015)

$$R = y\sin(i)\sin(\omega+\nu)$$

$$y(\nu) = \frac{a(1-e^2}{1+e\cos(\nu)}$$

$$y(E) = a(1-e\cos(E))$$

$$\tau - t_p = \frac{P}{2\pi}(E - e\sin(E))$$ — 1

$$\frac{R}{c} = \frac{a\sin(i)}{c}\left[\sin(\omega)\,(\cos(E)-e) + \sqrt{1-e^2}\cos(\omega)\sin(E)\right]$$ — 2

Transcendental equation

3 — $$\frac{R}{c} = \frac{a\sin(i)}{c}\left[\sin(\psi(t)) + \frac{\kappa}{2}\sin(2\psi(t)) - \frac{\eta}{2}\cos(2\psi(t))\right]$$

where,

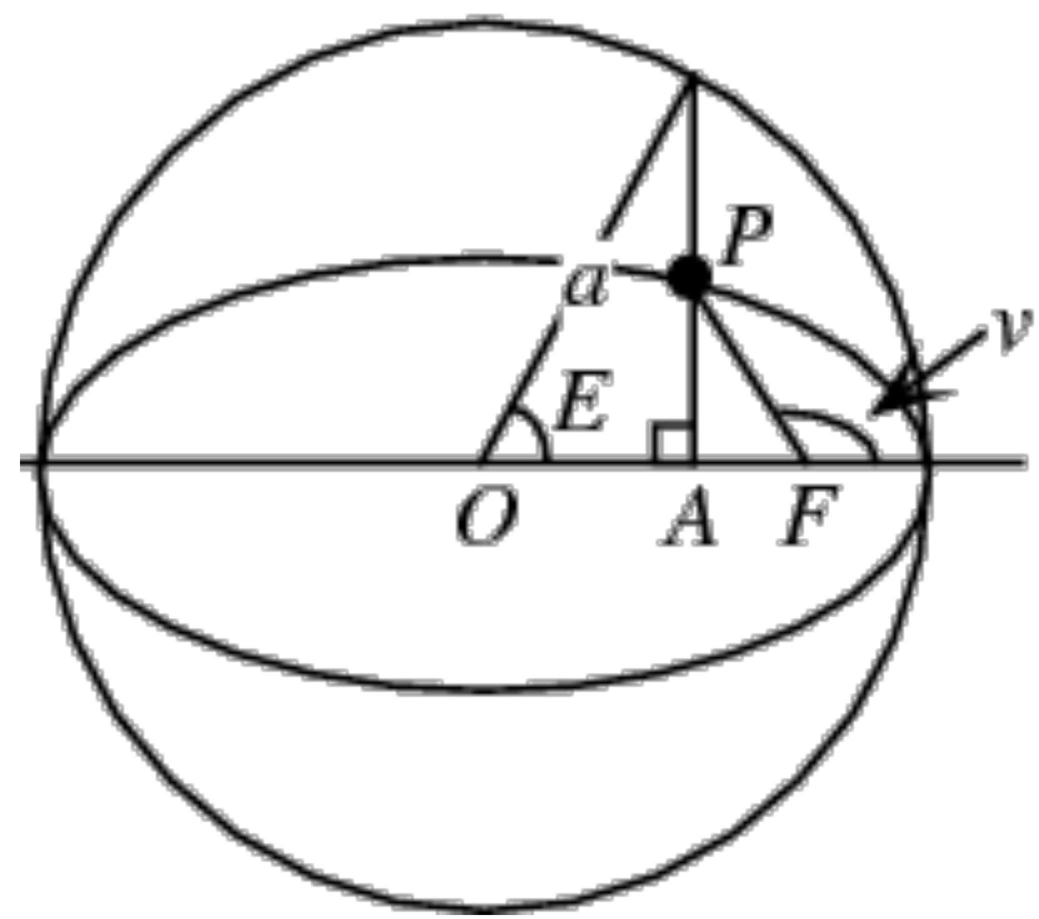
$\kappa = e\cos(\omega)$ and $\eta = e\sin(\omega)$

$$\psi(t) = \Omega(t - t_{asc})$$

$\Omega = \frac{2\pi}{P}$ and $t_{asc} = t_p - \frac{\omega}{\Omega}$

E: Eccentric Anomaly

ν : True Anomaly



Resampling

$$\tau = t_{arr} + \underbrace{\Delta t_{\rm Bin} + \Delta t_{R\odot} + \Delta t_{S\odot} + \cdots}_{\Delta t}$$ **Eq 1**

$$\Delta t_{Bin} = a_p \left[\sin(\psi_t) + \frac{\kappa}{2}\sin(2\psi_t) - \frac{\eta}{2}\sin(\psi_t) \right]$$ **Eq 2**

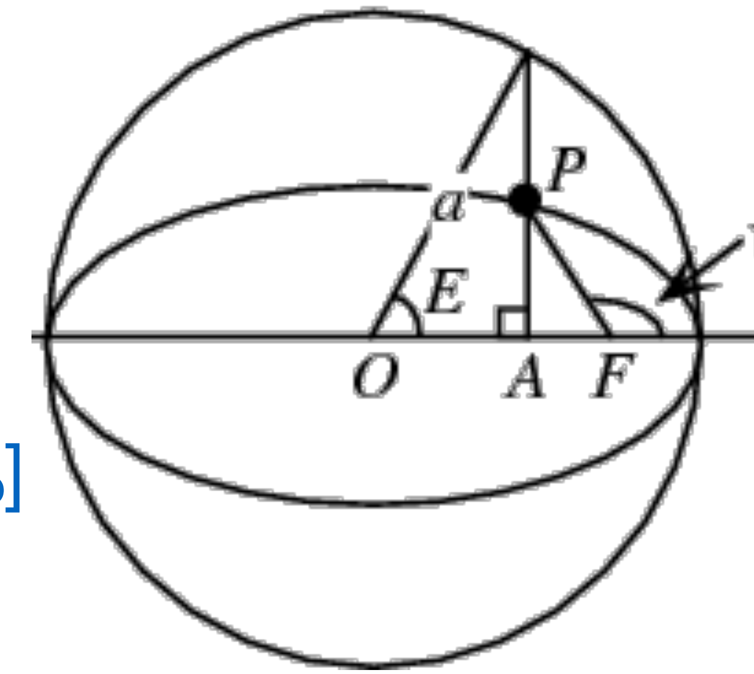
$$\kappa = \frac{e}{2}\cos\omega; \frac{\eta}{2}\sin\omega$$

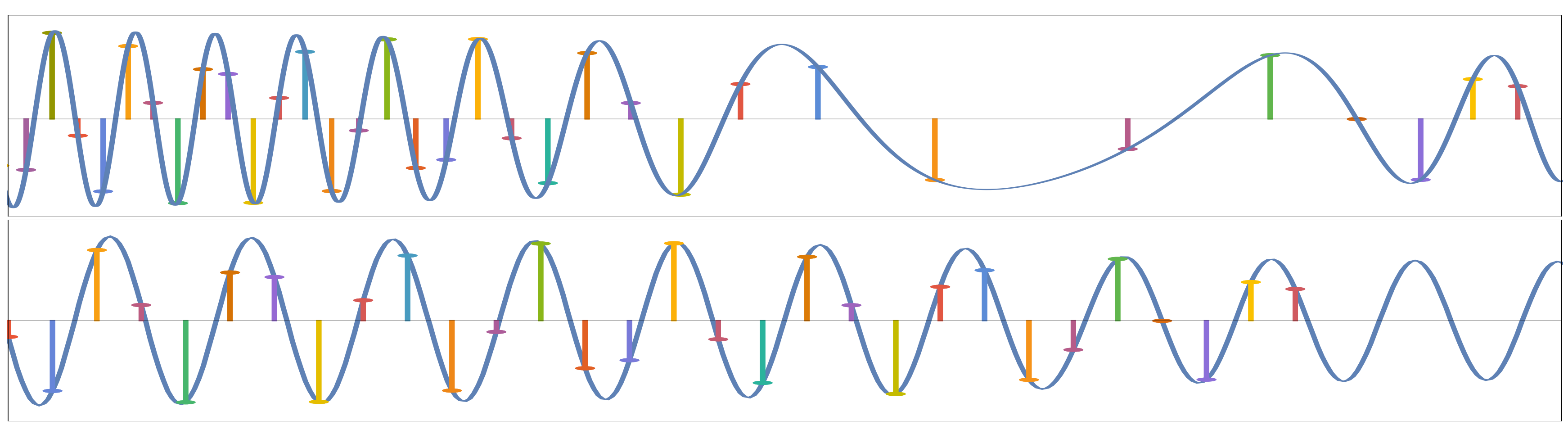
$$\psi_t = \frac{2\pi}{P}(t - t_p) + \omega$$

$$\Delta t_{R\odot} = \frac{\vec{r}(t)\cdot n}{c}$$

[P, a_p, e, ω, t_p]

$$\dot{f} = 0$$



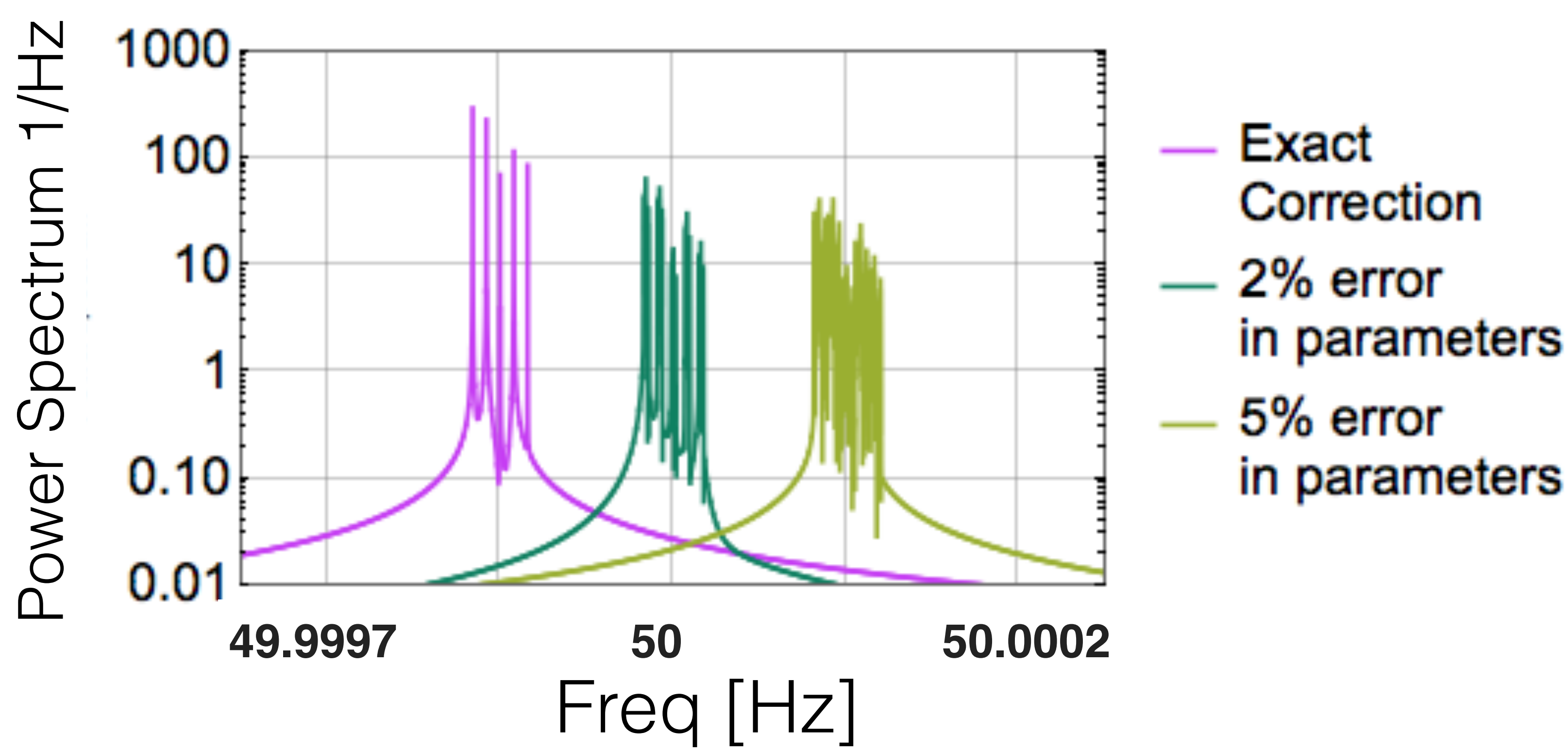


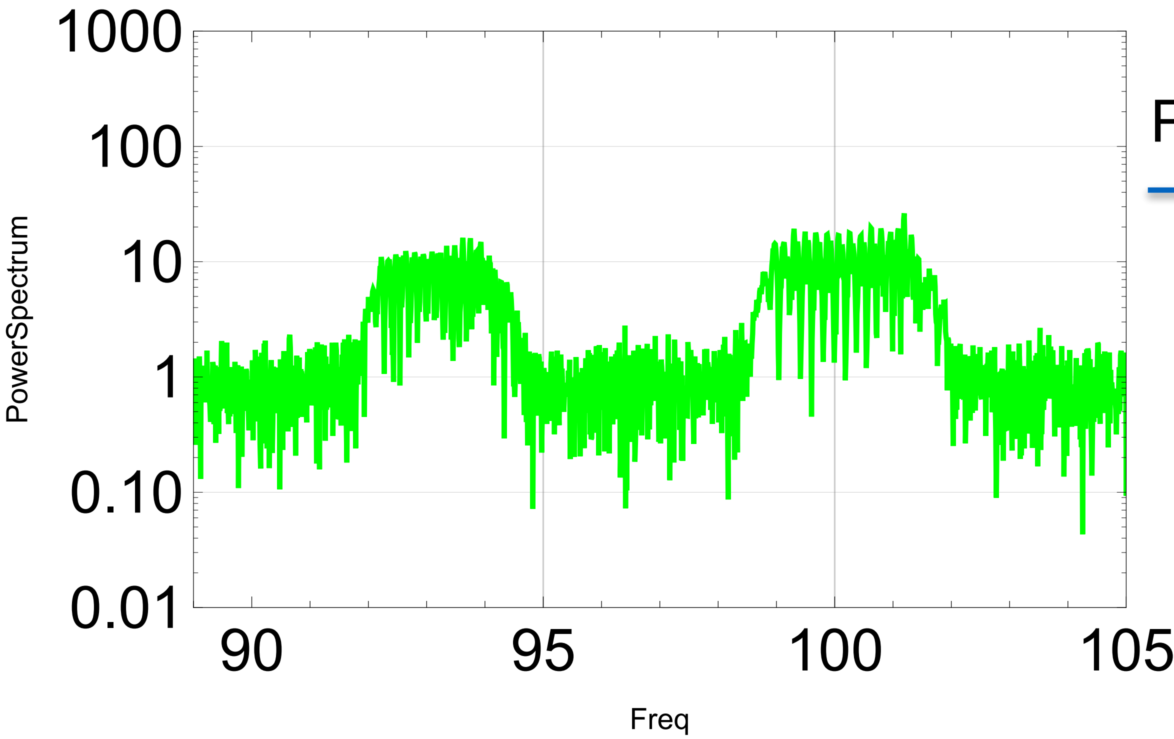
Problems

- To correct the shift in signal, we need to know the exact orbital parameters
    - Even for directed/narrowband searches, parameter space is huge

- The tolerance for uncertainties of orbital parameters is very small; we wish to understand what are the limiting uncertainties [for resampling] for various parameters

Simulated data

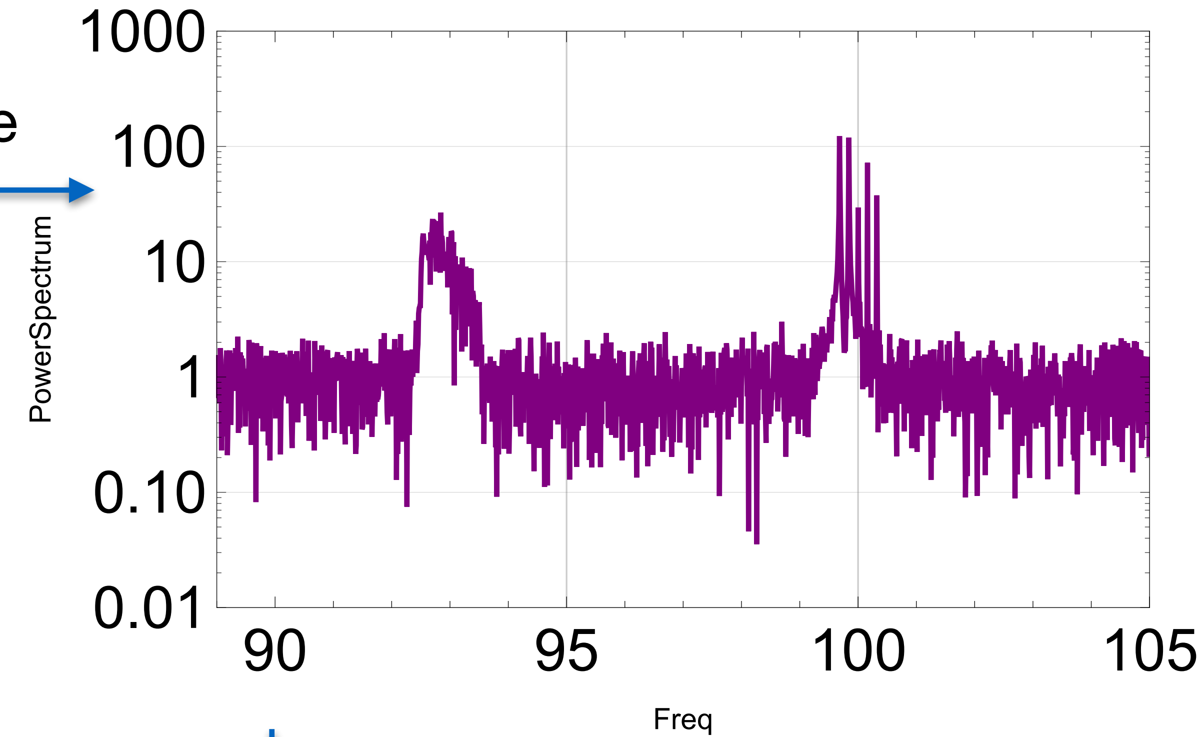




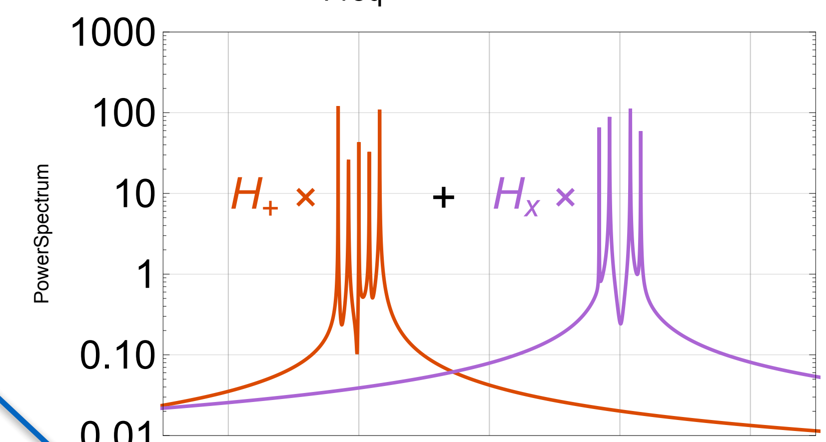
2 ~identical signals

Resample

1 signal



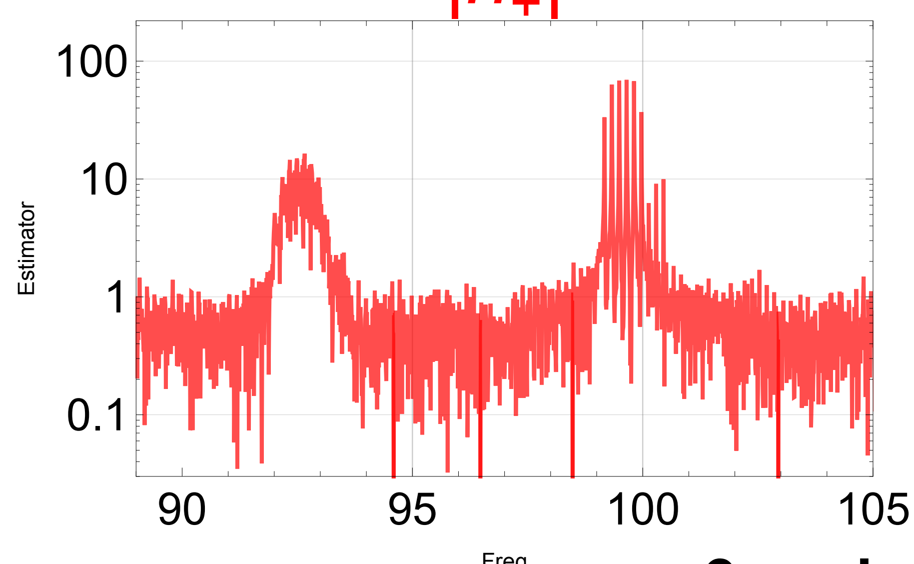
Match filtering



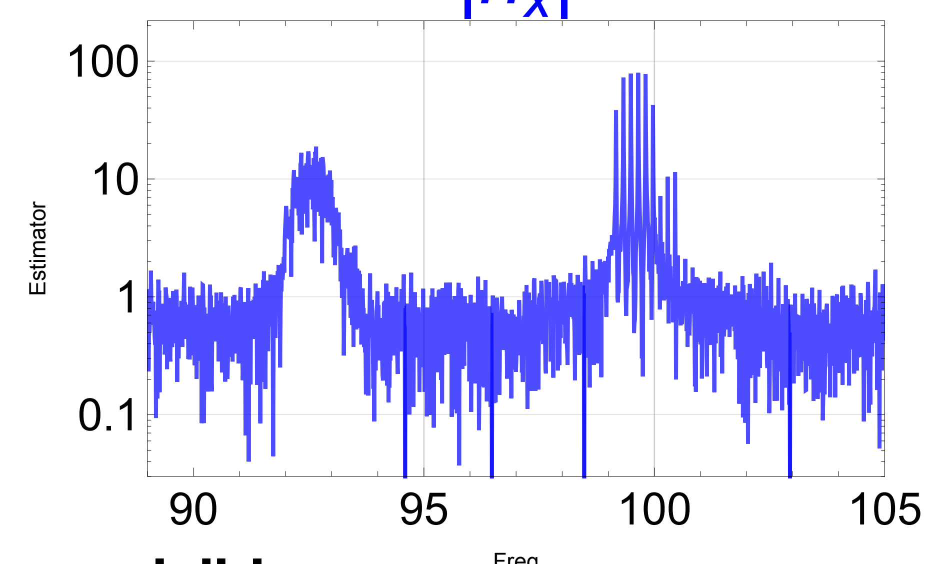
$$\mathcal{S} = |A^{+}|^{4}|\hat{h}_{+}|^{2} + |A^{\times}|^{4}|\hat{h}_{\times}|^{2}$$

Detection Statistics

$|\hat{h}_{+}|$

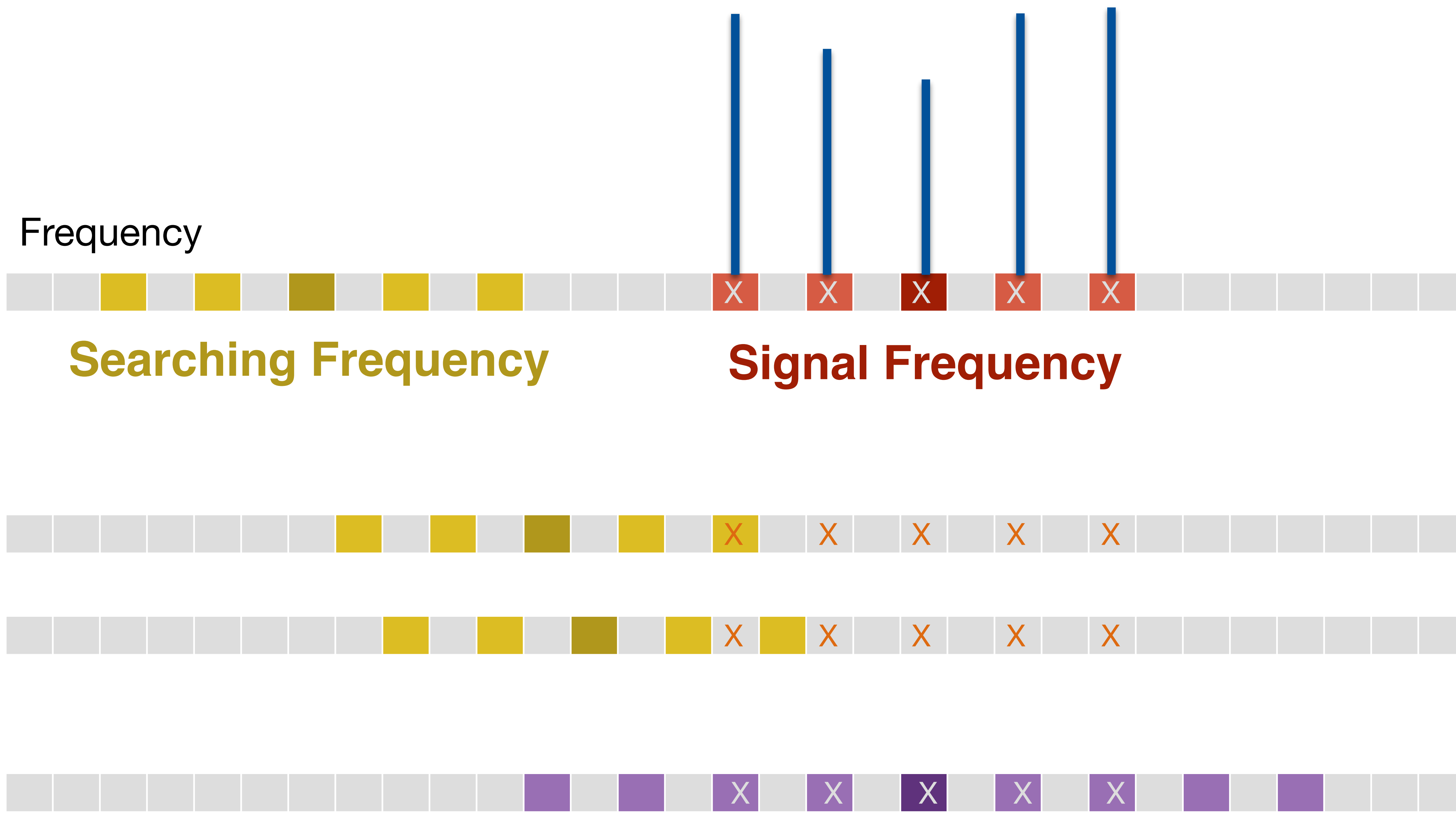


$|\hat{h}_{x}|$



9 peaks are visible

9- peaks


Frequency
X
X
X
X
X
Searching Frequency
Signal Frequency
X
X
X
X
X
X
X
X
X
X
X
X
X
X
X

Detection statistics

- Probability density function for the detection statistics (assuming gaussian noise)

$$f(\mathcal{S}) = \frac{e^{-\frac{\mathcal{S}}{\sigma_X^2 |\mathbf{A}^\times|^2}} - e^{-\frac{\mathcal{S}}{\sigma_X^2 |\mathbf{A}^+|^2}}}{\sigma_X^2 \left(|\mathbf{A}^\times|^2 - |\mathbf{A}^+|^2\right)}$$

$$f(\mathcal{S}) = Ae^{-bx}$$

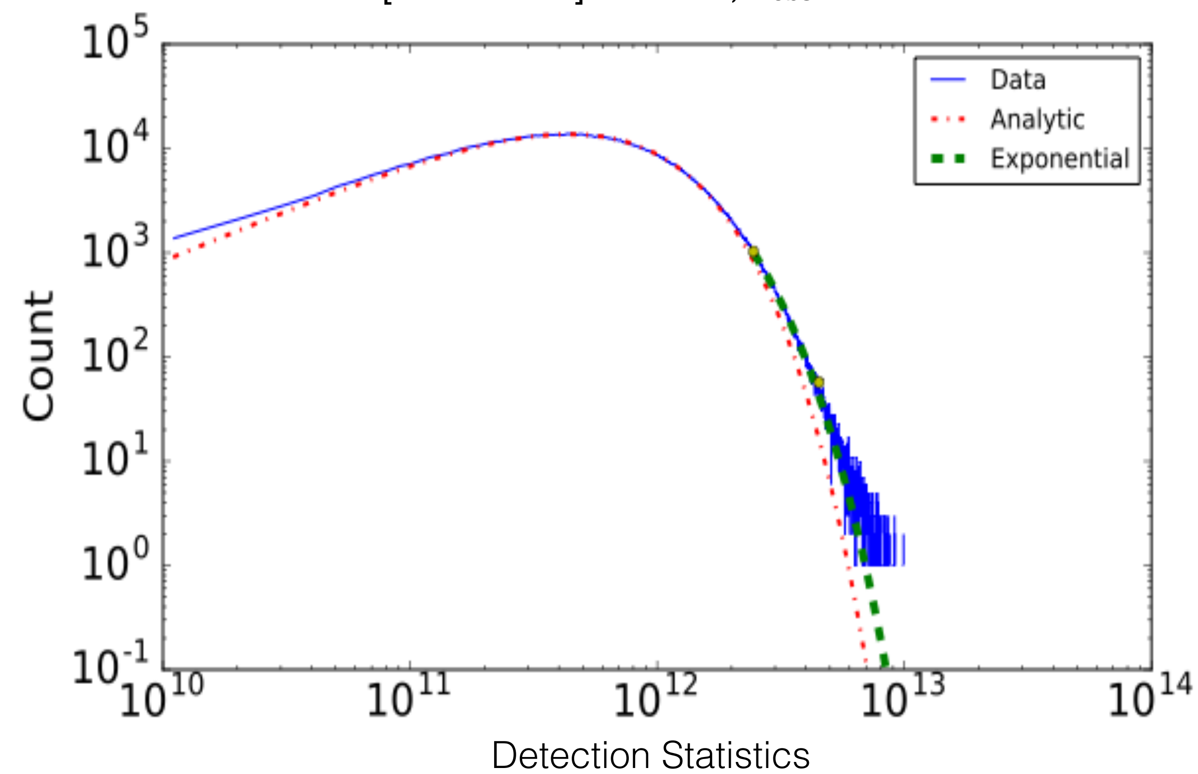
$$P(\mathcal{S} > \mathcal{S}^*) = \frac{|\mathbf{A}^\times|^2 e^{-\frac{\mathcal{S}^*}{\sigma_X^2 |\mathbf{A}^\times|^2}} - |\mathbf{A}^+|^2 e^{-\frac{\mathcal{S}^*}{\sigma_X^2 |\mathbf{A}^+|^2}}}{|\mathbf{A}^\times|^2 - |\mathbf{A}^+|^2}$$

[P. Astone, et al PRD. 062008 (2014)]

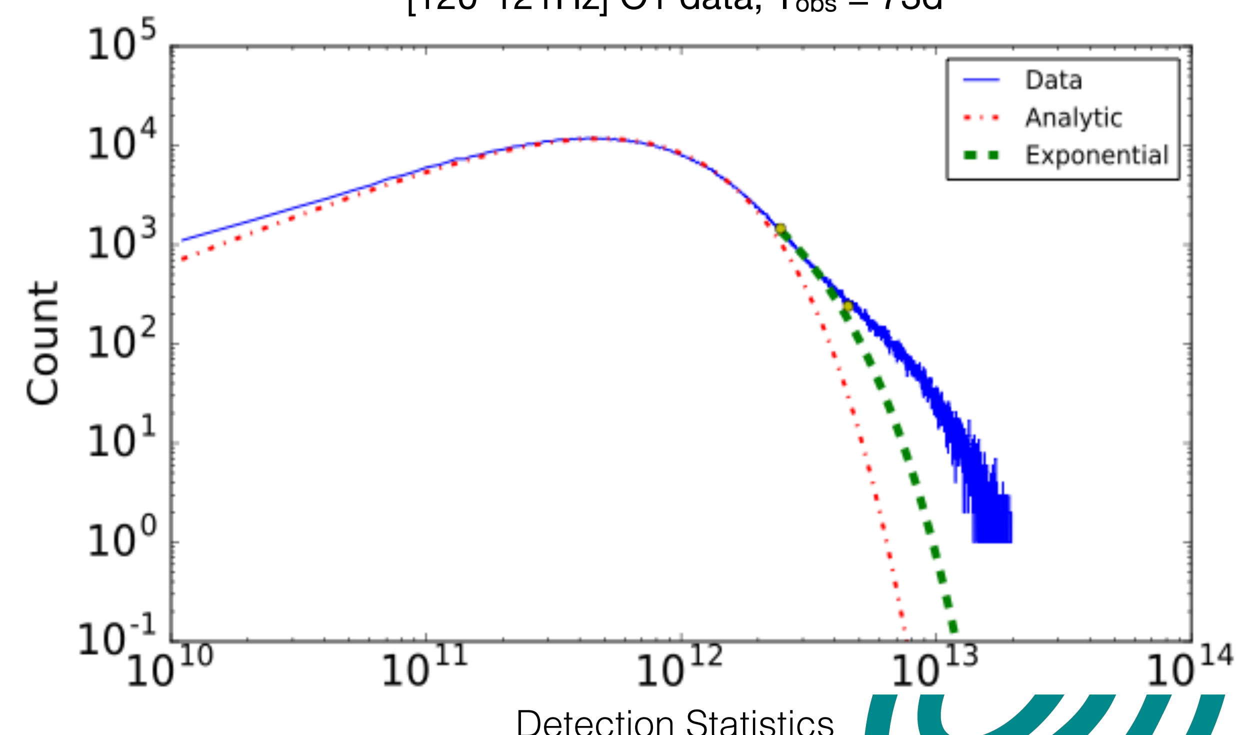
- Threshold is chosen based on given Normalised **F**alse **A**larm **R**ate

$$\mathrm{NFAR} = \frac{\mathrm{FAR}}{\mathrm{N_{trials}}}$$

[125-126Hz] O1 data, T_{obs} = 73d



[120-121Hz] O1 data, T_{obs} = 73d



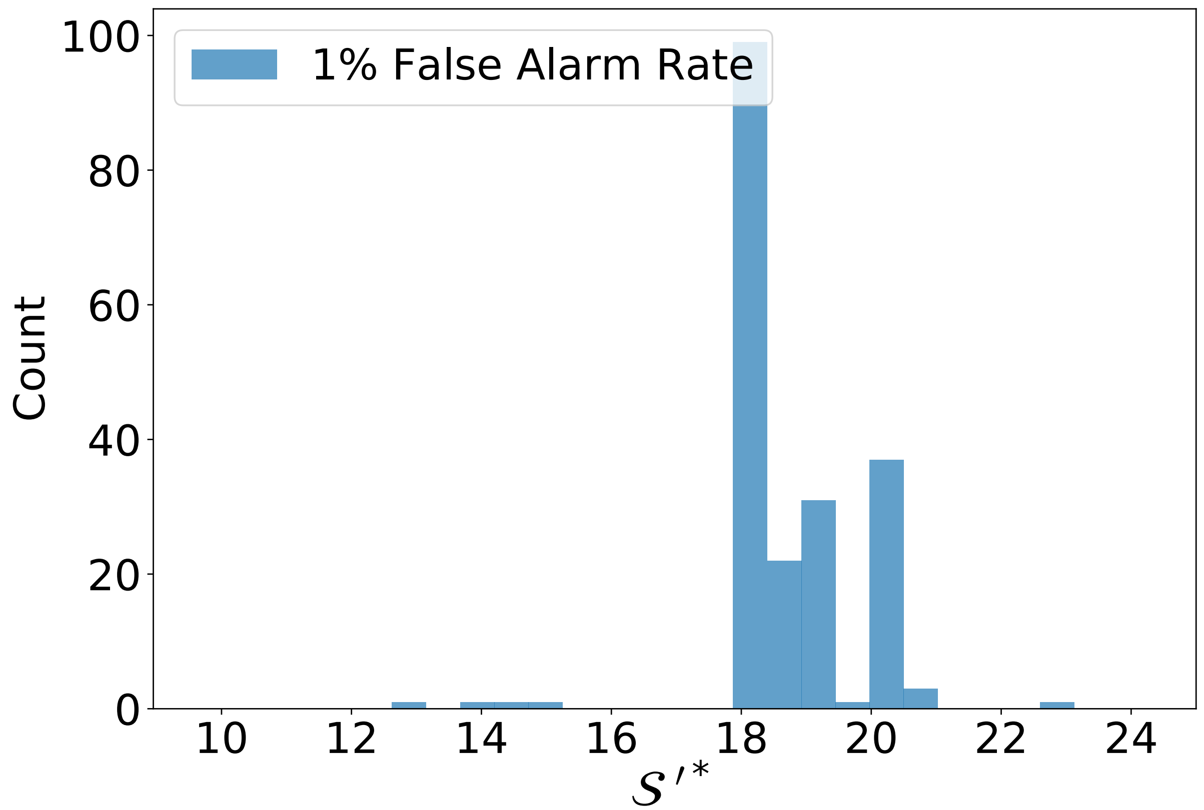
Detection Statistics

- A new linear function of detection statistics is used for O2 search, which is more robust to noise

$$\mathcal{S} = |A^{+}|^{4}|\hat{h}_{+}|^{2} + |A^{\times}|^{4}|\hat{h}_{\times}|^{2}$$

$$\mathcal{S}' = \frac{\mathcal{S} - \mu(\mathcal{S})}{\sigma(\mathcal{S})}$$

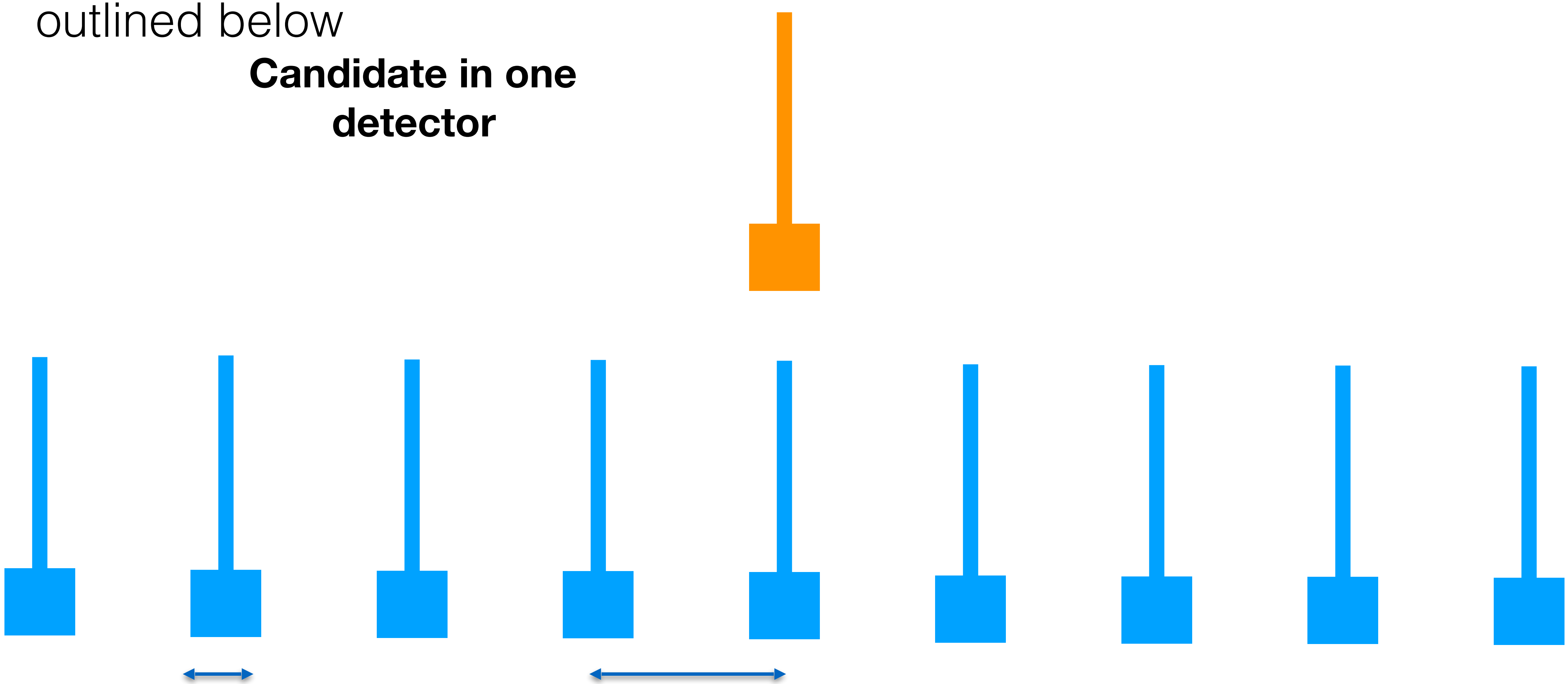
Threshold value at 1% NFAR ~ S`* = 18 (conservative) for HL detectors (~minimum among both detectors)



- For a given candidate above threshold, a coincidence will be considered only if at least **3** candidates are within the 9-peak interval (slide 20)

Coincidence veto

- For every candidate above **threshold** in a given detector, **coincidence** candidates are searched in 9 intervals in the other detector having widths outlined below



$3/T_{obs}=1.5\times10^{-7}$ Hz

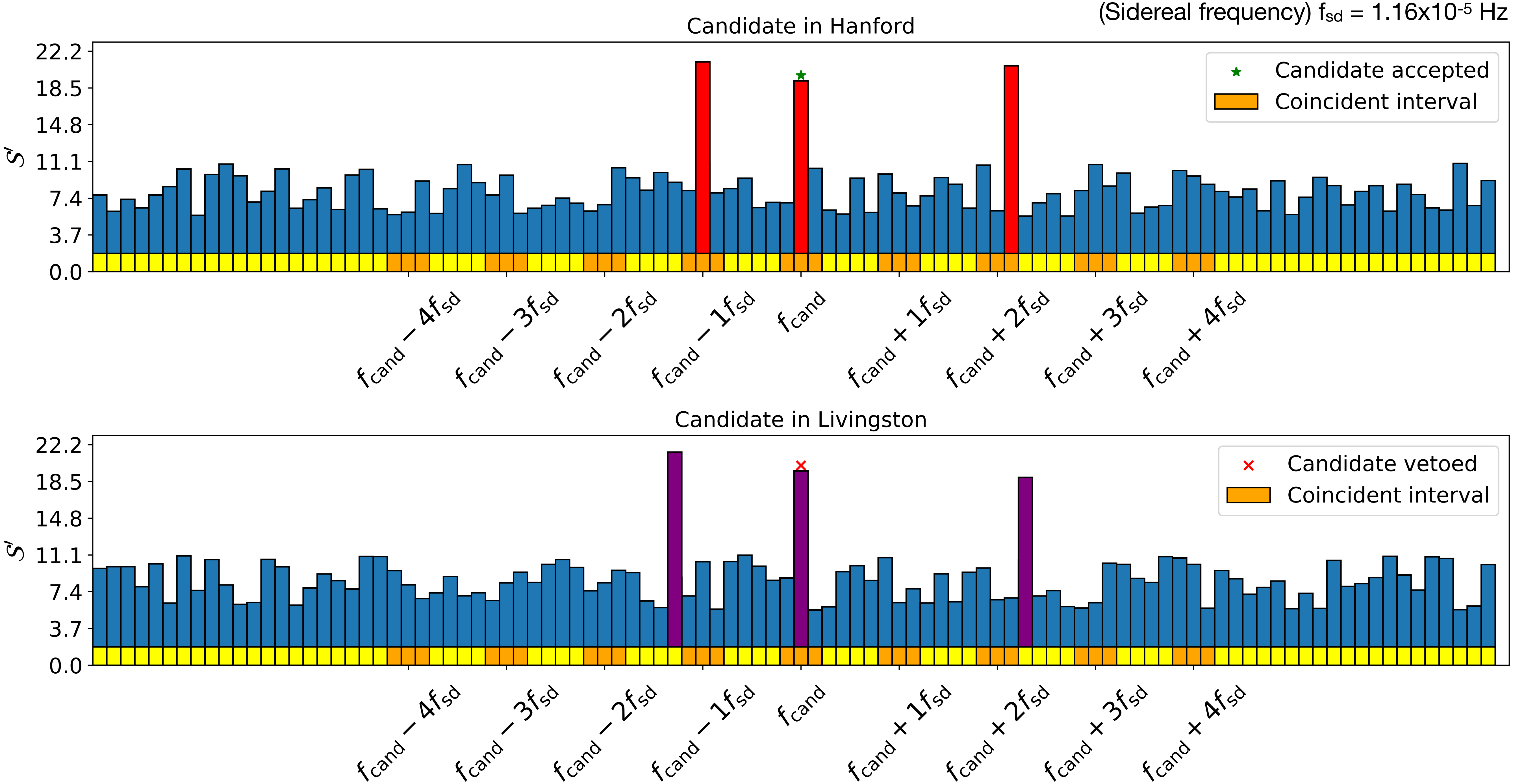
$1/T_{sidereal}=1.16\times10^{-5}$ Hz

T_{obs} = 243 days

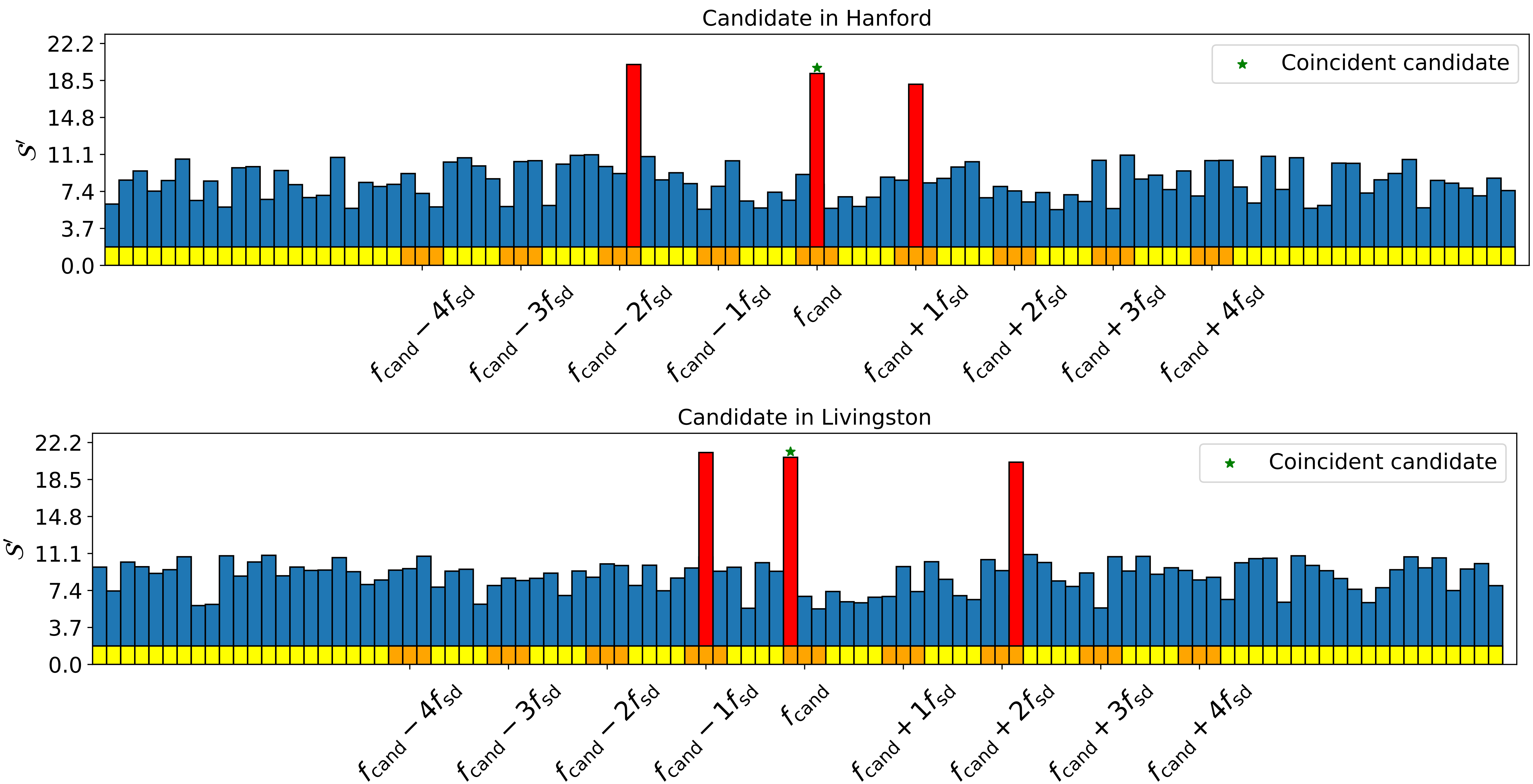
must coincide with at least 3 coincidence in each detector; [slide 20]

3-coincident veto

(Sidereal frequency) f_{sd} = 1.16x10^{-5} Hz



3-coincident veto



Software injections

- Injected one binary signal in detector data in every 1 Hz band ranging from [121-169] Hz

- Applying the time domain resampling technique (for each source in both detectors), using different combinations (7168) of parameter inaccuracies

    - 1.5 million simulations

- Compute Detection statistics and apply veto

    - See if signal is recovered

- Find for which parameter mismatches 90% signal is recovered

Results

~90% signals were recovered with amplitude $\mathbf{H_0 = 3.0x10^{-25}}$ and the following offset in binary

#Simulations: 40(sources)*8961(#parameter sets)*2(detectors) = ~7x10^5 per amplitude

ΔP [ms]	Δa_p [ms]	Δe	$\Delta\omega$ [deg]	Δt_p [s]	No. of Sources in ATNF*	Comment
24ms	14ms	10^{-4}	0.01 deg	0.1s	12	max Δa_p
300ms	7ms	10^{-4}	0.05 deg	10s	25	max ΔP
300ms	4ms	10^{-3}	0.05 deg	1s	18	max Δe,$\Delta\omega$
56ms	4ms	10^{-4}	0.03 deg	50s	28	max Δt_p
56ms	4ms	10^{-4}	0.02 deg	50s	28	max sources**
24ms	4ms	10^{-3}	0.05 deg	50s	22	max vol.***
40ms	**1800ms**	**<0.033**	**NA**	**~100s**	**Sco X-1 EM measurement**	

* All binary sources in ATNF where uncertainty is less than the one in table
** Uncertainty parameter set for which maximum number of sources qualify
***Uncertainty parameter set which has the maximum volume of grid

Sensitivity Simulation (O2)

- 12000 Software injections of Sco X-1 like signal in different frequency range of O2 data (one at a time per 1Hz band)
  - [120-121Hz], [154-155Hz] [190-191Hz],

H_0	freq	Confidence
3.0×10^{-25}	120	96%
3.0×10^{-25}	154	95%
5.1×10^{-25}	154	98%
4.5×10^{-25}	120	99%
3.1×10^{-25}	190	96%

- **Goal:**
  - Min amplitude at which 90% of signals are recovered using offset parameters. We iteratively reduce the amplitude in above table till we reach 90% confidence.

Sensitivity O2 (approx)

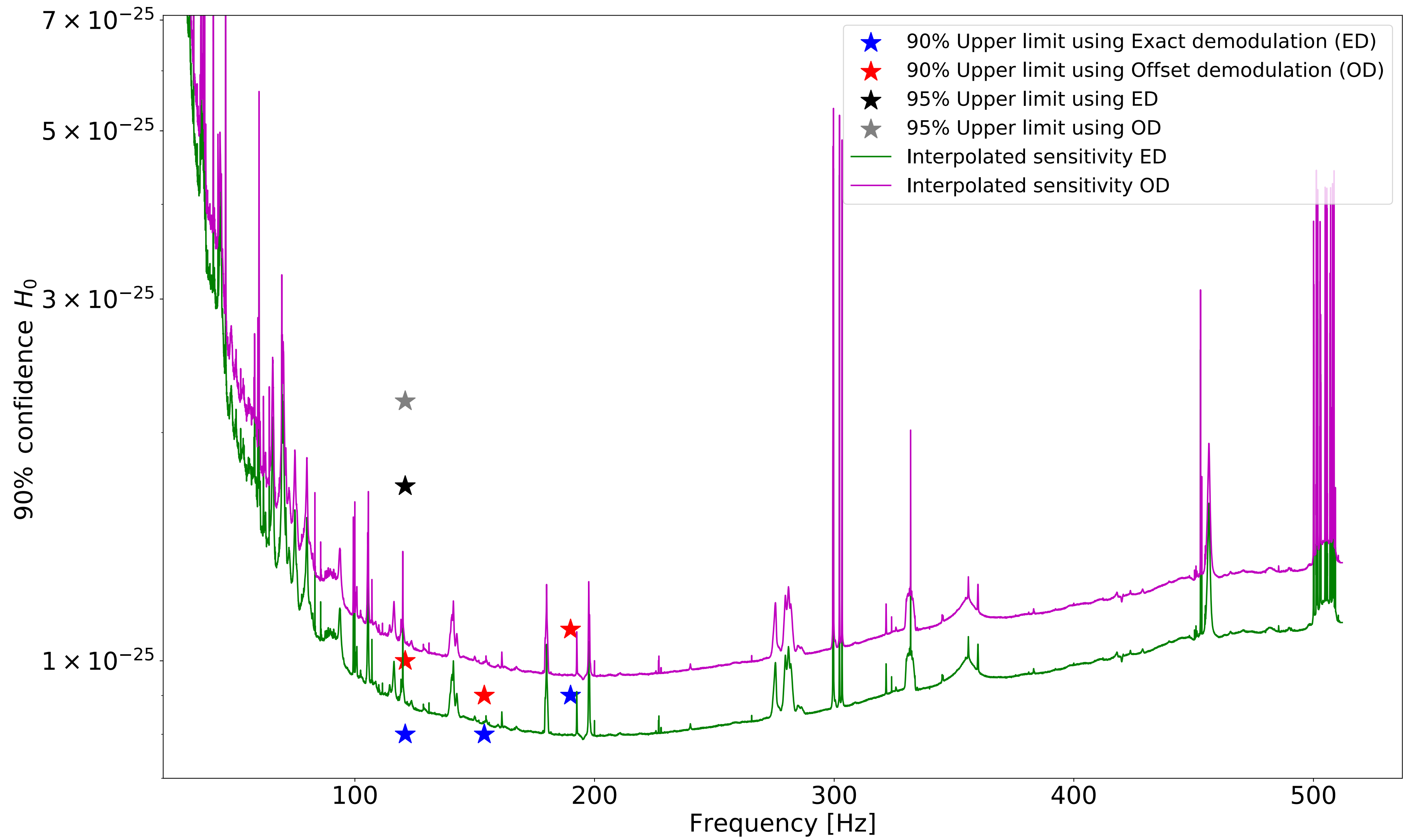
- Sensitivity of the methodology is proportional to the O2 sensitivity.
- To estimate the proportionality constant, the ratio between

$$\frac{u_f}{s_f} = \frac{90\,\%\ \text{sensitivity of method}}{\text{sensitivity of O2}}$$

(in this example s_f for [121-122] Hz, [154,155] Hz, [190,191] Hz)

[P. Astone et al. *PRD* 042002 (2014)]

- **Methodological paper will be soon will send out to CQG**



ΔP [ms]	Δa_p [ms]	Δe	Δω [deg]	Δt_p [s]
24ms	14ms	10^{-4}	0.01 deg	0.1s

Computational cost:
Total (parallel) CPU time: 6×10^7 s
Time taken: 3 weeks
Disk Space: ~90 TB

Future steps

- Border effects and candidates near instrumental lines to be investigated
- Further veto of coincidence candidates
- Simulations for different amplitudes, wider frequency band (from few Hz to few kHz), different T_{obs}
- Analytical function for uncertainty and detection statistics

Future for the method

- If uncertainties of a binary source is outside simulated range, a general empirical formula will be more useful; e.g.

	Mean	uncertainty	acceptable (simulations; Sco X-1)
Period	**187.3 hour**	59.8 µs	56000 µs
sma	**16.7 s**	1.2 µs	4000 µs
ecc	9.7×10^{-5}	NA	10^{-4}
omega	97.8°	**0.08**	0.03°
t_periapsis	49778.4 MJD	**154s**	50 s

- It will be difficult to **guess** the sensitivity of the above source. Limited computational resources didn't allow to repeat the tolerance estimation. Will soon send out a methodological paper to CQG

  - Formulation for more dynamic range will require more simulation and therefore more disk space and CPU time

$$\mathcal{S} = F\begin{pmatrix} \Delta P, & \Delta a_p, & \Delta e, & \Delta\omega, & \Delta t_p, \\ P, & a_p, & e, & \omega, & t_p, \\ h_0, & T_{obs}, & \mathrm{f}_{gw}, & \text{Noise}\cdots & \end{pmatrix}$$

Thank you for your attention

Questions?

Bonus slides

CW ROMA1

Summary

- We have generalised the 5-vector method to search for CWs from NSs in binary systems
- We have checked the robustness of this generalized method against the tolerance in the orbital parameter uncertainties
- Semi-major axis is the most critical parameter
- In absence of any frequency knowledge, the search slows down by a couple of order of magnitude however, the real search will run only on small parameter space
- In absence of signal we present approximate sensitivity to the method

LVC Collaboration

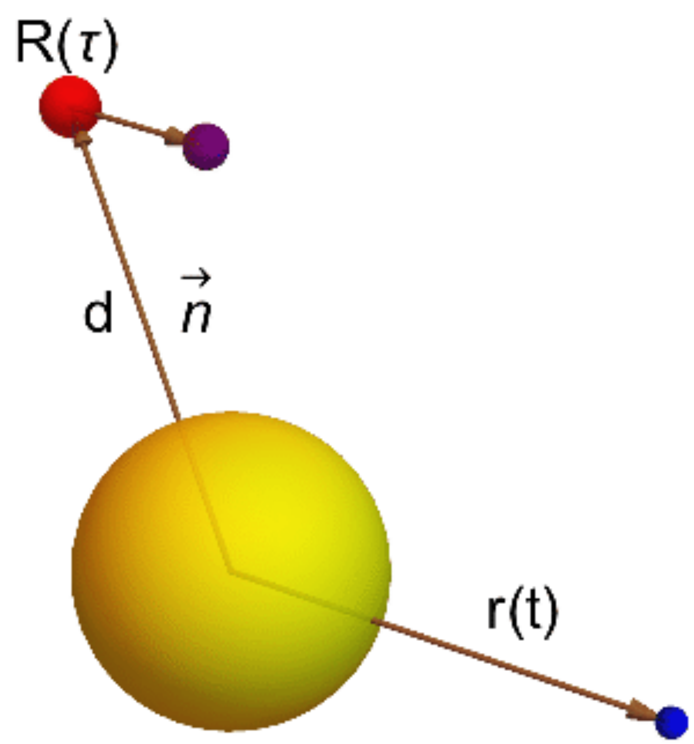
Citations summary

Generated on 2019-05-08

92 papers found, 90 of them citeable (published or arXiv)

Citation summary results	Citeable papers	Published only
Total number of papers analyzed:	90	77
Total number of citations:	18,052	17,711
Average citations per paper:	200.6	230.0
Breakdown of papers by citations:		
Renowned papers (500+)	10	10
Famous papers (250-499)	3	3
Very well-known papers (100-249)	13	12
Well-known papers (50-99)	9	8
Known papers (10-49)	30	29
Less known papers (1-9)	20	13
Unknown papers (0)	5	2
h_{HEP} index [?]	38	37

Resampling (Time Domain)



$$\Phi(\tau) = \Phi_0 + 2\pi \sum \frac{f^{(l)}(\tau - \tau_0)^{l+1}}{(l+1)!}$$

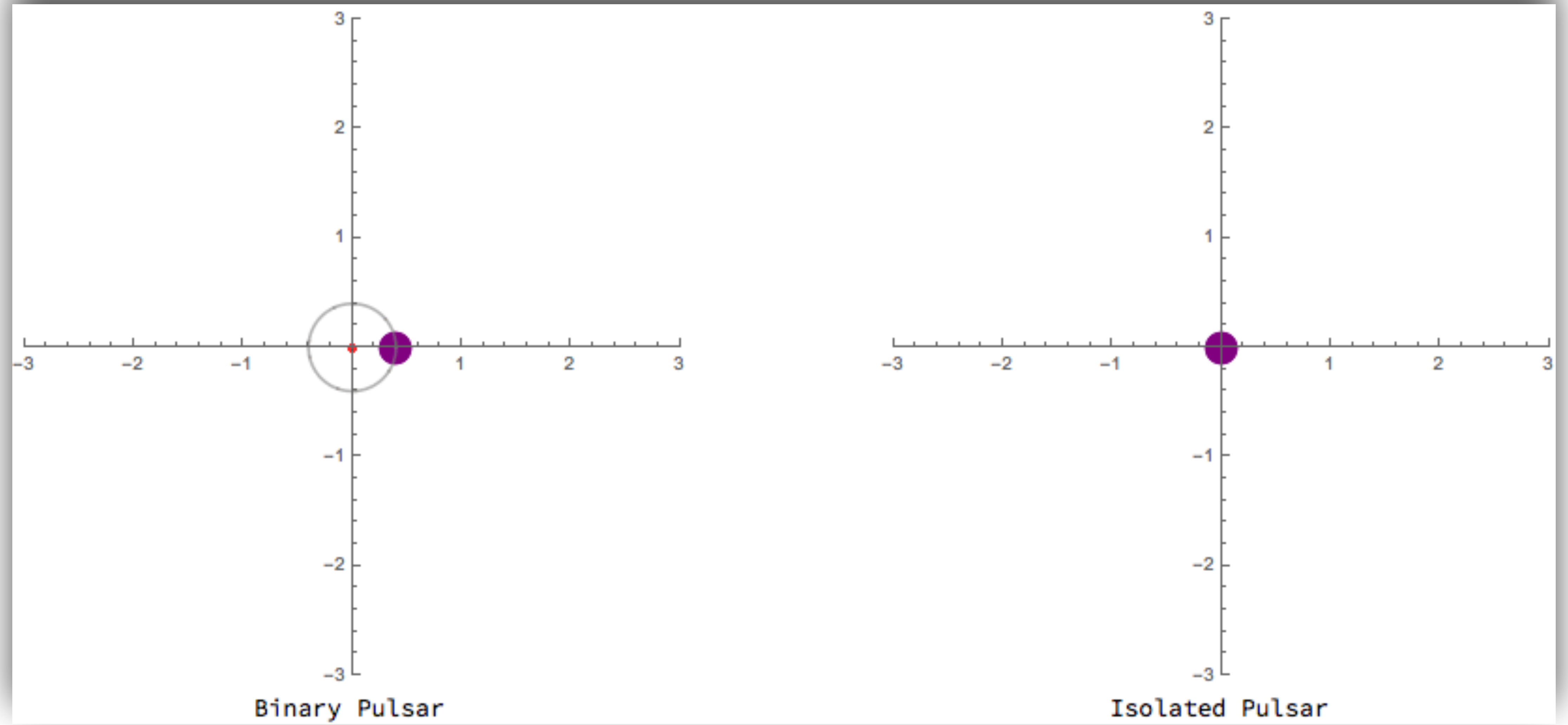
$$\Phi(t_{arr}) = \phi_0 + 2\pi \sum \frac{f^{(l)}(t_{arr} + \delta(t_{arr}) - t_0)^{l+1}}{(l+1)!}$$

$$\tau(t_{arr}) = t_{arr} + \frac{\vec{r}(t_{arr}) \cdot \vec{n}}{c} - \frac{d}{c} - \frac{R(\tau)}{c}$$

Paola Leaci et al ,PRD, 102003 (2015)

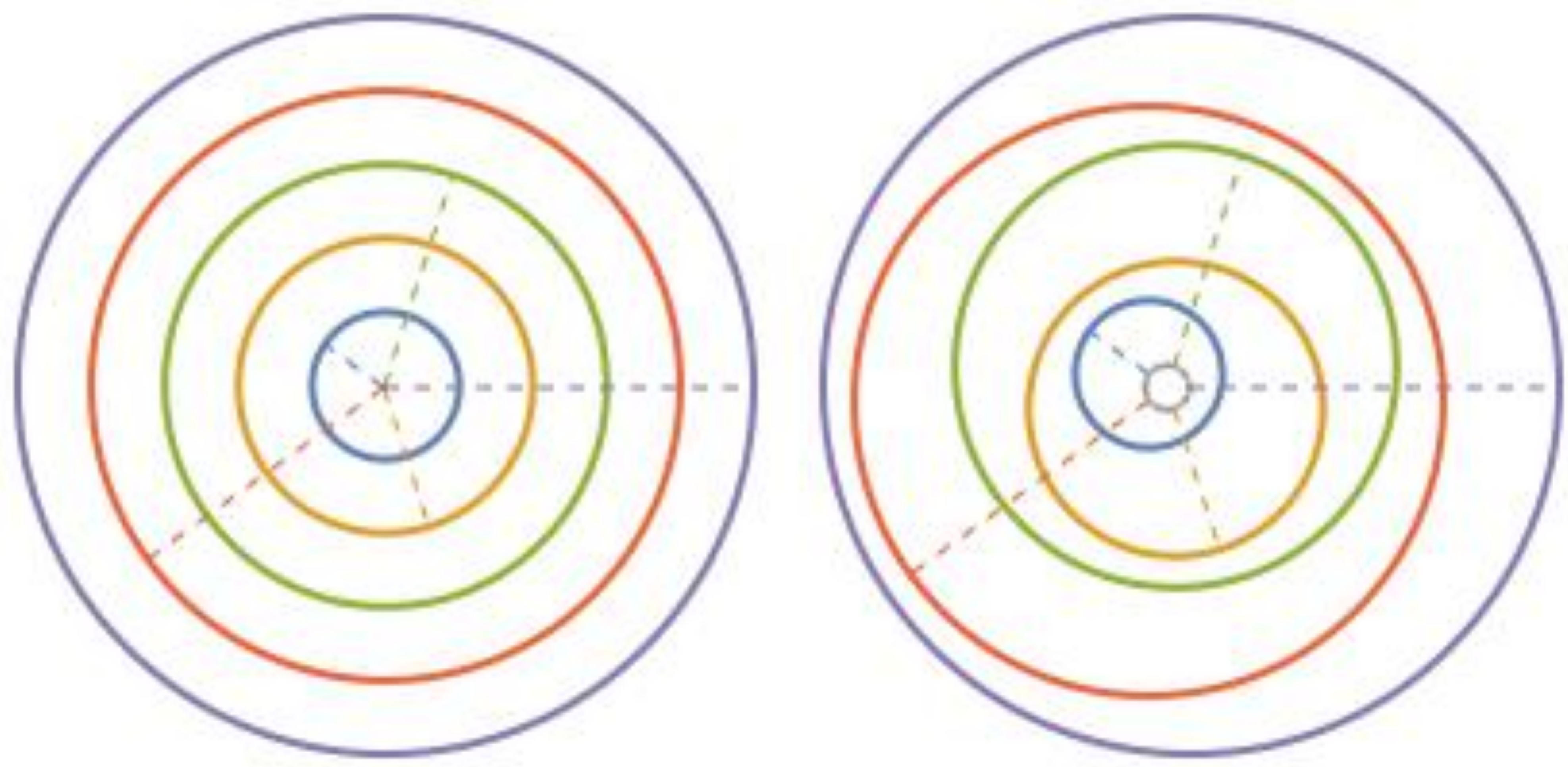
Φ_0 is the initial phase at reference time

Binary modulation



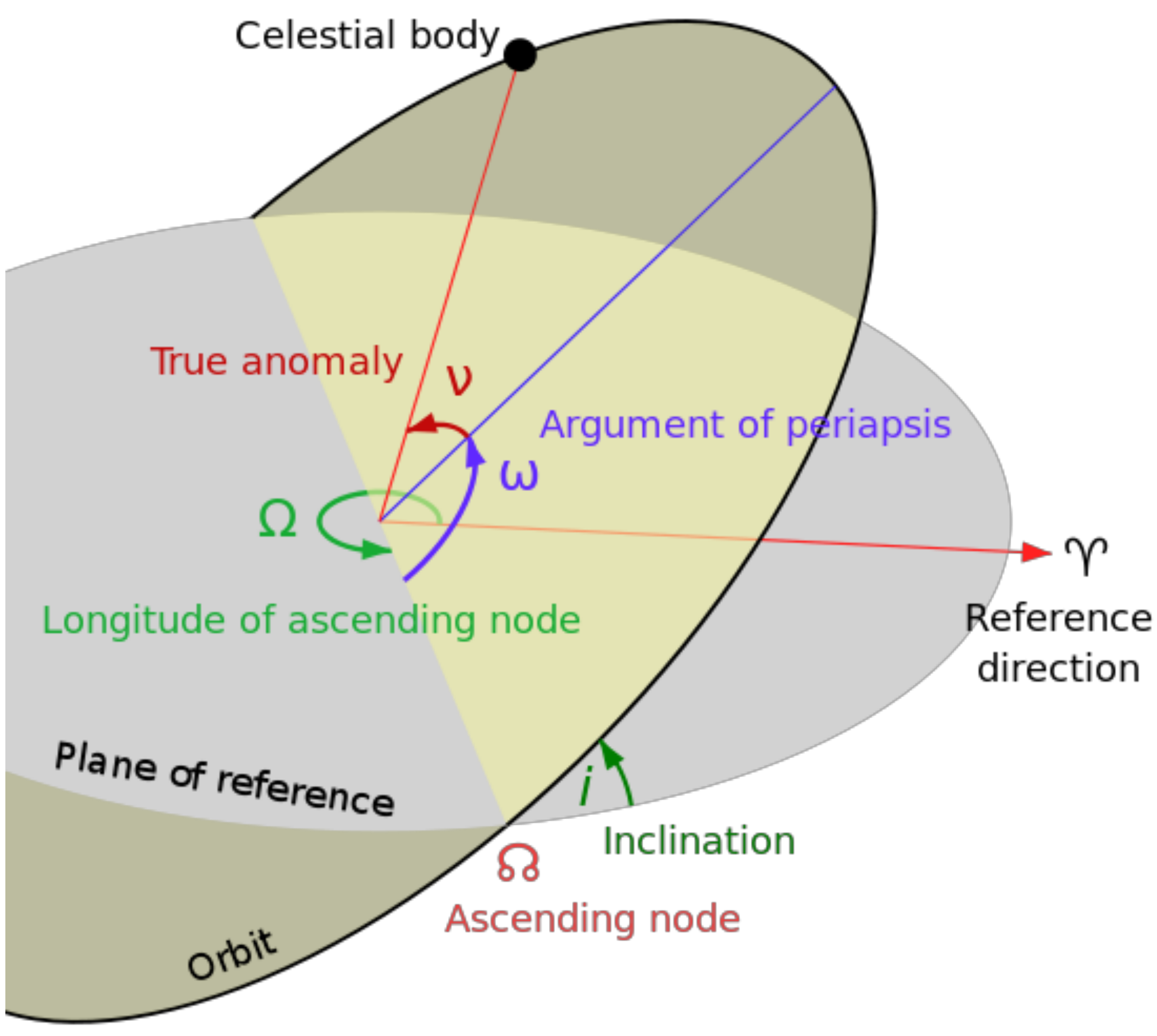
Binary

- Signals from a Pulsar in a binary system will be Doppler shifted due to orbital motion

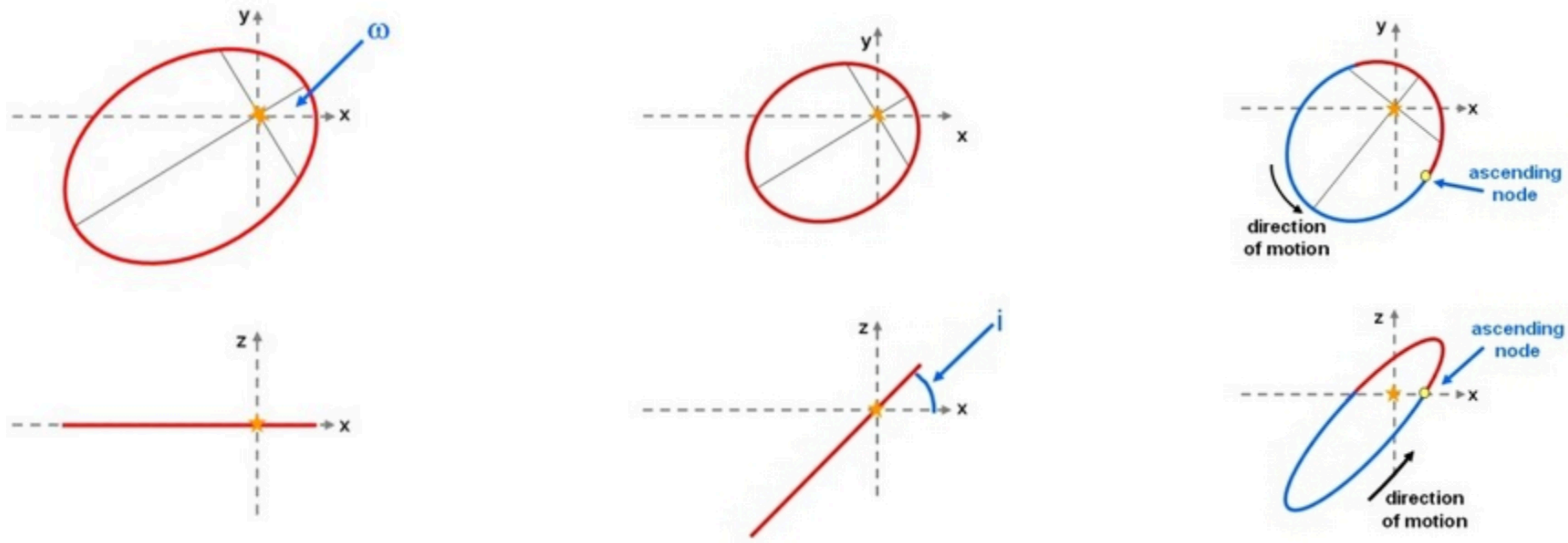


Parameters

- **Binary:**
  - Period : P
  - Semi-major axis : a
    - a_p Projection (time) : a sin i /c
  - Inclination : i
  - eccentricity : e
  - Argument of periapsis : ω
  - Time of periapsis : t_p
- **Other:**
  - Spindown*, Location, Frequency…



Binary angles



Scorpius X-1

TABLE I. Scorpius X-1: system parameters.

Sco X-1 parameter	Value	Uncertainty	
Period	68023.70 sec	0.04 sec	
Orbital semimajor axis	1.44 sec	0.18 sec	[
Time of ascension	897753994	100 sec	
Orbital eccentricity	< 0.068	3σ	[
Right Ascension	$16^h19^m55^s.067$	$0''.06$	
Declination	$-15°38'25''.02$	$0''.06$	
System inclination	44°	6°	
Companion mass	$0.42M_{Sol}$		
X-ray flux	3.9×10^{-10} Wm^{-2}		

Simulation

Search band in bucket region;
relatively cleaner region

$$\text{frequency}: \ f \in [121{,}169]\ \text{Hz},$$

$$\frac{asini}{c} \equiv a_p \in [1,5]\ \ \text{s},$$

$$\text{Period}: \ P \in [10,36]\ \ \text{hr},$$

$$\text{time of periapse}: \ t_p \in \left[-\frac{P}{2}, \frac{P}{2}\right]\ ,$$

$$\text{Log of eccentricity}: \ log_{10}e \in [-5, \text{-}2]\ ,$$

$$\text{argument of periapse}: \ \omega \in [0, 2\pi]\ \ \text{rad}$$

P. Leaci, et al., ,PRD, 102003, (2015)

Resampling

$$\tau(t_{arr}) = t_{arr} + \frac{\vec{r}(t_{arr}) \cdot \vec{n}}{c} - \frac{d}{c} - \frac{R(\tau)}{c}$$

Stroboscopic resampling:
We assume that the original data is a time series x_k (k = 1,...,N), k sampled at uniform intervals. In this method we obtain the value of the time series x_k at barycentric time τ by taking the value y_{k0} such that k_0 is the nearest integer to τ

It does not require frequency, hence useful for real search, but slow.

J. Livas. Gravitational Wave Data Analysis, B.F. Schutz (eds.) p. 217 (1985).

Pia Astone et al ,
Phys.Rev. D82 (2010) 022005

Heterodyne:
(phase correction) is achieved by multiplying the strain time series $[x(t_{arr}) = h(t_{arr}) + n(t_{arr})]$ by $\exp[-j\phi(t_{arr})]$ where $\phi(t_{arr}) = \Phi(t_{arr}) - \phi$ and t_{arr} is arival time at detector

Requires frequency, but is faster (used for simulations)

$$\Phi(\tau) = \Phi_0 + 2\pi \sum \frac{f^{(l)}(\tau - \tau_0)^{l+1}}{(l+1)!}$$

$$\Phi(t_{arr}) = \phi_0 + 2\pi \sum \frac{f^{(l)}(t_{arr} + \delta(t_{arr}) - t_0)^{l+1}}{(l+1)!}$$

Dupuis & Woan, PRD 72 102002 2005

Detection statistics

- GW strain at the detector can be written as follows
  [P. Astone, Classical and Quantum Gravity, Volume 27, Number 19,(194016)]

$$
\begin{aligned}
H(t_{arr}) &= H_0\,(A_+ H_+ + A_\times H_\times)\, e^{\iota(\phi(t_{arr})+\phi_0)} \\
A_+(t_{arr}) &= a_0 + a_{1c}\cos\Omega t_{arr} + a_{1s}\sin\Omega t_{arr} \\
&\quad + a_{2c}\cos 2\Omega t_{arr} + a_{2s}\sin 2\Omega t_{arr} \\
A_\times(t_{arr}) &= b_{1c}\cos\Omega t_{arr} + b_{1s}\sin\Omega t_{arr} \\
&\quad + b_{2c}\cos 2\Omega t_{arr} + b_{2s}\sin 2\Omega t_{arr} \\
t_{arr} &= \text{detector time}
\end{aligned}
$$

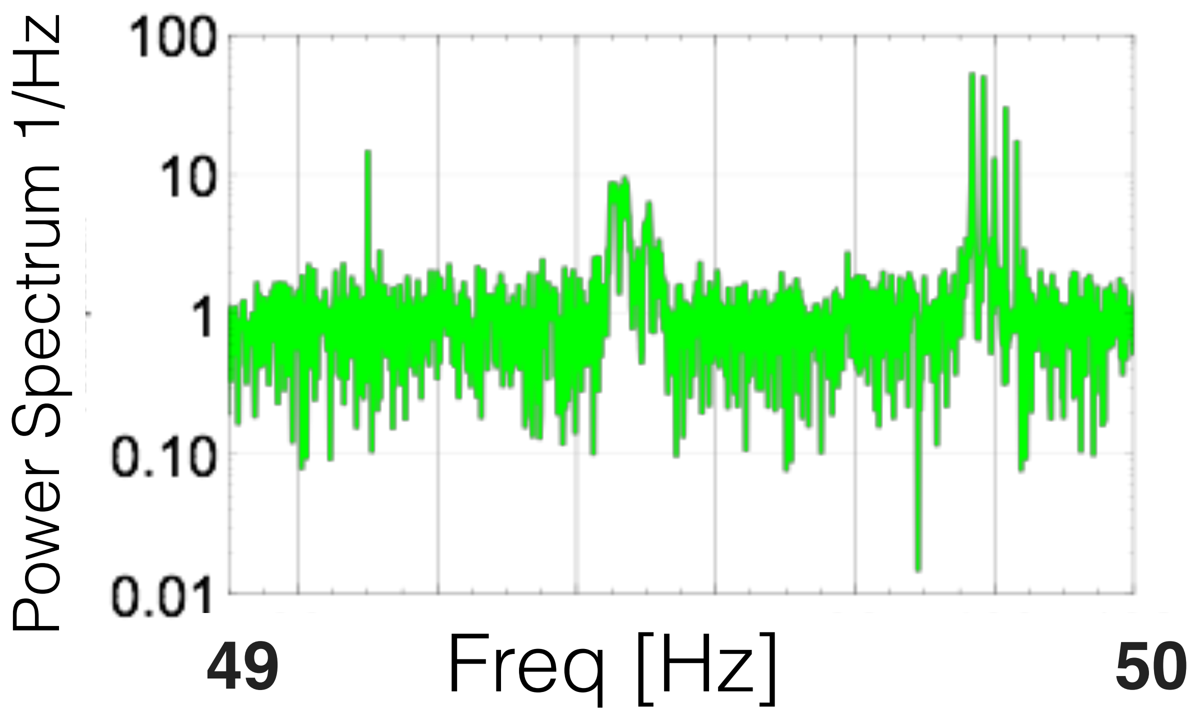
- signal in the detector is completely defined by its Fourier components at the five frequencies ω_0, $\omega_0 \pm \Omega$, $\omega_0 \pm 2\Omega$

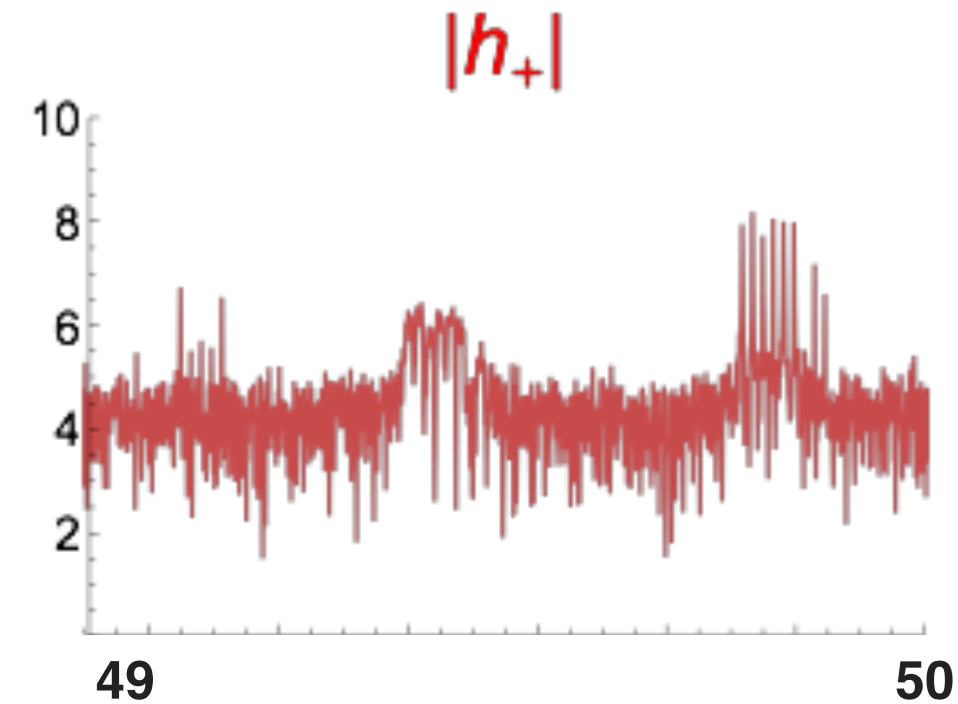
  - We ignore spin down terms for simplicity

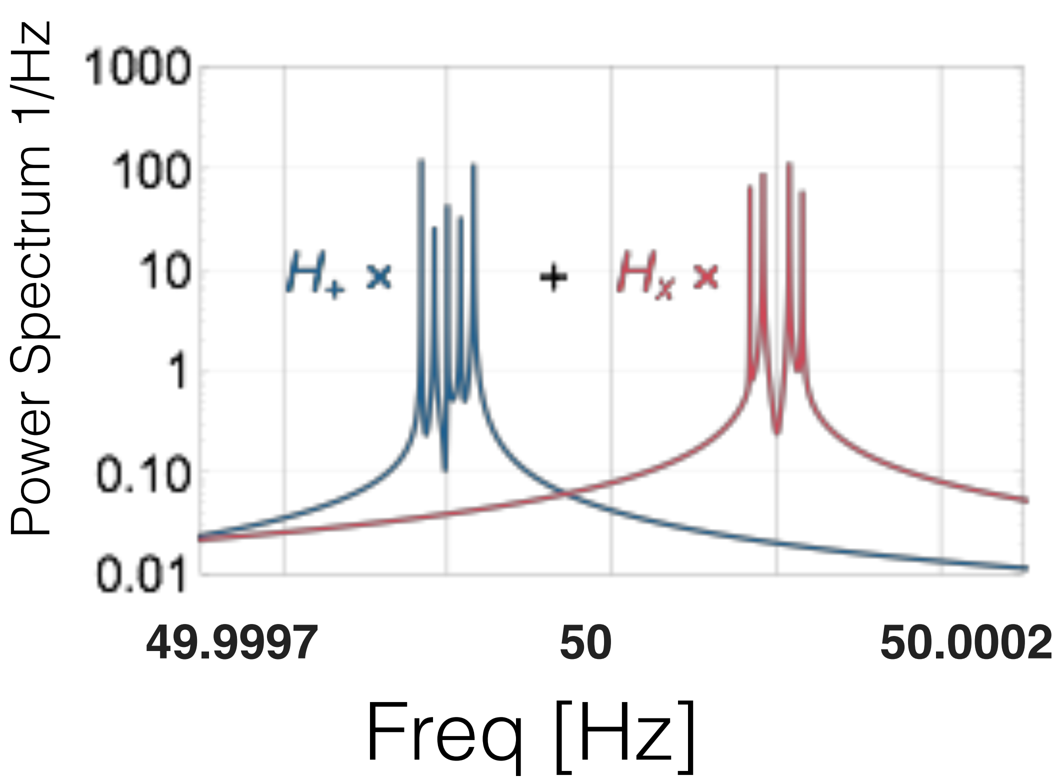
$$
H_0 = h_0 \sqrt{\frac{1 + 6\cos^2 i + \cos^4 i}{4}}
$$

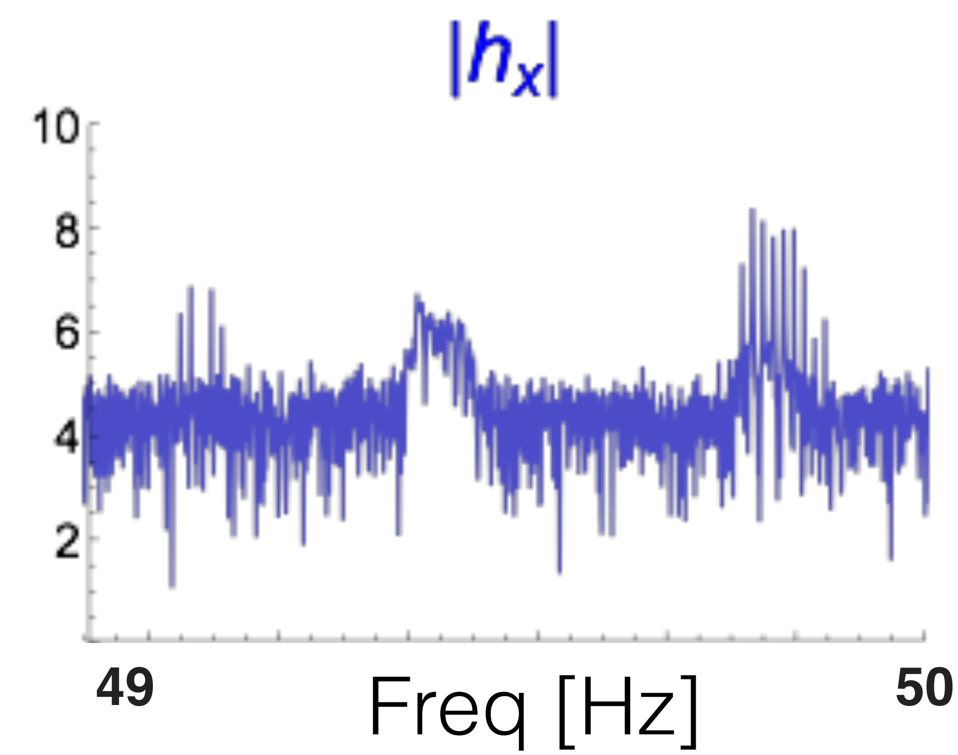
$$\hat{h}_+ = \frac{\mathbf{X}\cdot\mathbf{A}^+}{|\mathbf{A}^+|^2}; \qquad \hat{h}_\times = \frac{\mathbf{X}\cdot\mathbf{A}^\times}{|\mathbf{A}^\times|^2}$$

Simulated data in Fourier domain









$$A = H_+ A^+ + H_\times A^\times$$

$$S = c_+|\hat{h}_+|^2 + c_\times|\hat{h}_\times|^2.$$

$$c_+ = |\mathbf{A}^+|^4, \qquad c_\times = |\mathbf{A}^\times|^4.$$

Search procedure (based on software injection)

- Injected one binary signal in O1 data in every 1 Hz band ranging from 120Hz-169Hz in **both** Hanford and Livingston detectors. i.e. A binary injection in 120-121Hz (in the middle, around 120.53Hz) for the duration of 20 days
  # Sources: 50

- Applying the time domain resampling technique using, for every source in both detectors, different combinations (7168) of parameter inaccuracies/mismatches for two amplitude-signal values
  #Simulations: 50(sources)*7168(#parameter sets)*2(detectors) = ~7×10^5 per amplitude

- Detection statistics calculated for each frequency bin (5.78×10^{-7} Hz) across 1 Hz band (for all simulations). If demodulated correctly, detection statistics at the frequency bin containing the injection frequency will be significant.

- Setting a **threshold**

- **Coincidence** step

Tolerance in uncertainties

- We consider a set of mismatched parameters to be good enough to demodulate a signal with a good accuracy if and only if at least 90% of the injected sources are recovered in every 1-Hz wide frequency band.

- If after all the vetoes (**threshold** and **coincidence**), at least one candidate survives in every 1-Hz wide frequency band, we consider the related set of mismatched parameters to be good enough for our resampling methodology

Results for 20 days

Coincident search

Parameters where, at least one candidate in coincidence is found

B. P. Abbott et al
[https://dcc.ligo.org/DocDB/0138/P1600297/019/O1ScoX1CrossCorr.pdf]

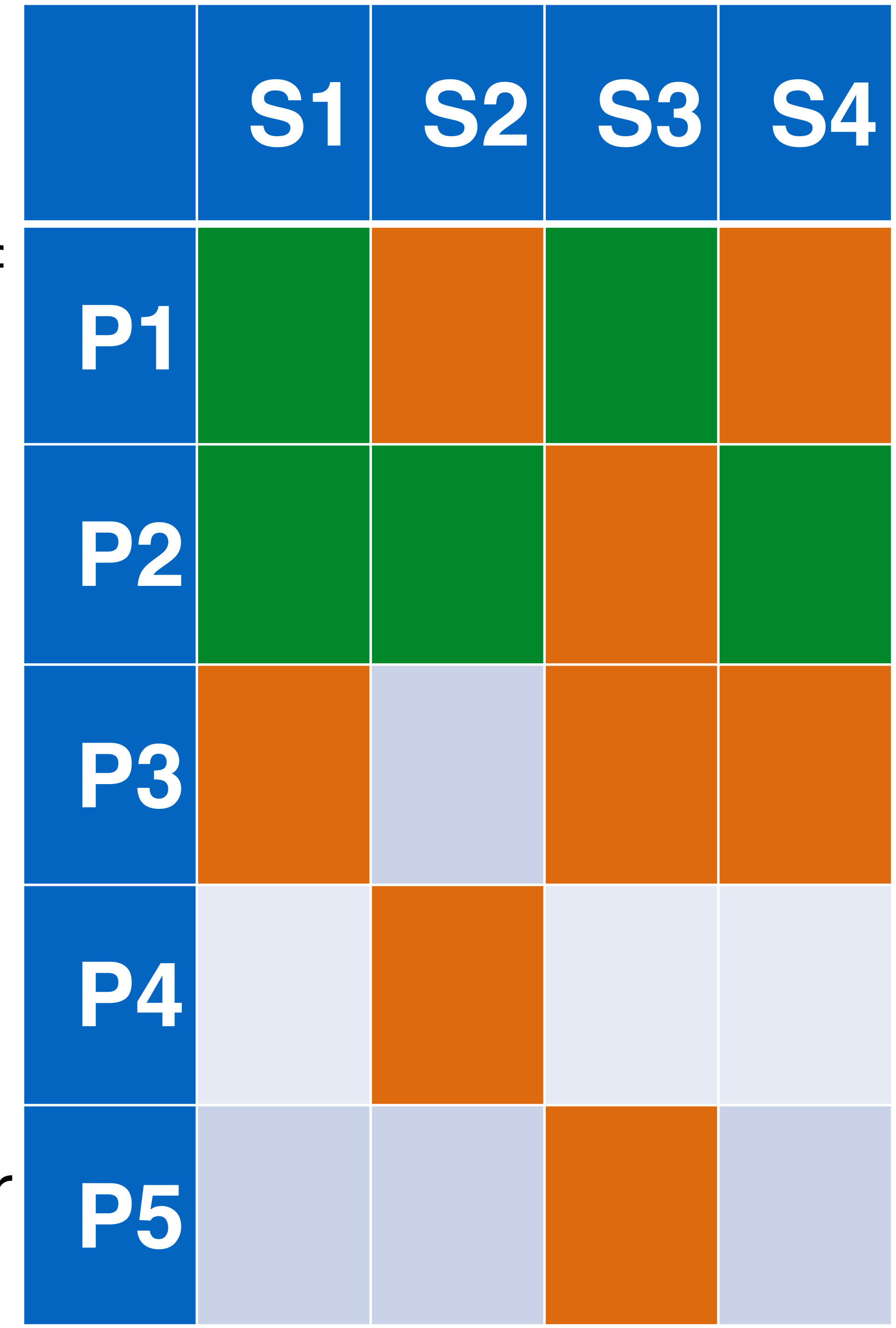
	$H_0=3.0\times10^{-25}$		$H_0=4.5\times10^{-25}$		EM uncertainties for Sco-X1	Median uncertain ties in ATNF (all data)
% of Sources detected	97%	90%	97%	90%		
Period	5ms	24ms	5ms	56ms	40ms	0.16ms
Semi-M-axis	14ms	14ms	53ms	53ms	180ms	0.003ms
Eccentricity	10^{-4}	10^{-4}	10^{-2}	10^{-2}	3σ: <0.03	1.6×10^{-7}
Arg. of P	0.01 deg	0.01 deg	0.03 deg	0.05 deg	NA	0.025 deg= 1.5”
Time of P	0.1s	0.1 s	1s	1s	~100s	NaN

Manchester, R. N., Hobbs, G. B., Teoh, A. & Hobbs, M., Astron. J., 129, 1993-2006 (2005) (astro-ph/0412641)

Tolerance in uncertainties

- We consider a set of mismatched parameters to be able to accurately to demodulate a signal with a good accuracy if and only if at least 90% of the injected sources are recovered in every 1-Hz wide frequency band.

- If after all the vetoes (**threshold** and **coincidence**), at least one candidate survives in every 1-Hz wide frequency band, we consider the related set of mismatched parameters to be good enough or our resampling methodology



Binary parameters

Paola Leaci et al ,PRD 91, 102003 (2015)
[https://journals.aps.org/prd/abstract/10.1103/PhysRevD.91.102003]

$$R = y\sin(i)\sin(\omega+\nu)$$

$$y(\nu) = \frac{a(1-e^2}{1+e\cos(\nu)}$$

$$y(E) = a(1-e\cos(E))$$

$$\tau - t_p = \frac{P}{2\pi}(E - e\sin(E))$$ — 1

$$\frac{R}{c} = \frac{a\sin(i)}{c}\left[\sin(\omega)\left(\cos(E)-e\right)+\sqrt{1-e^2}\cos(\omega)\sin(E)\right]$$ — 2

Transcendental equation

3 — $$\frac{R}{c} = \frac{a\sin(i)}{c}\left[\sin(\psi(t)) + \frac{\kappa}{2}\sin(2\psi(t)) - \frac{\eta}{2}\cos(2\psi(t))\right]$$

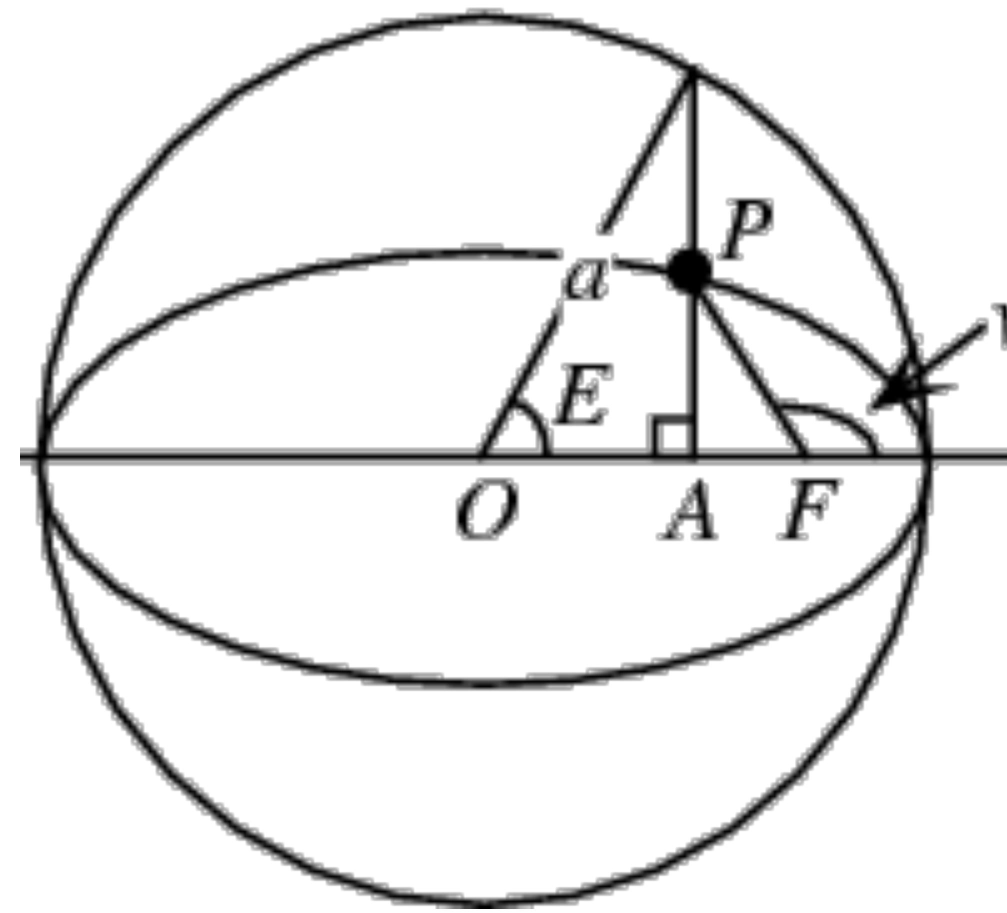
where,

$$\kappa = e\cos(\omega) \text{ and } \eta = e\sin(\omega)$$

$$\psi(t) = \Omega(t - t_{asc})$$

$$\Omega = \frac{2\pi}{P} \text{ and } t_{asc} = t_p - \frac{\omega}{\Omega}$$

E: Eccentric Anomaly
v : True Anomaly



Faster tool for "simulation"

- Current method is very time consuming
  - Why?
- It performs all necessary computation at higher sampling rate (~4kHz to ~16kHz) and down-samples at 1Hz sampling rate.

Faster tool for “simulation”

- Is it necessary to calculate (correction terms) at higher sampling rate?
  - Yes: Barycentric correction
    (as it uses interpolation method)
    [Have to be computed only once for directed sources]
  - No: Binary correction, Spin down*
    [Have to computed multiple times for different parameters]

Faster tool for “simulation”

- Hence,
  - We can make barycentric correction using stroboscopic resampling (time consuming but one time job)
  - Make binary correction using heterodyne method

Catch?

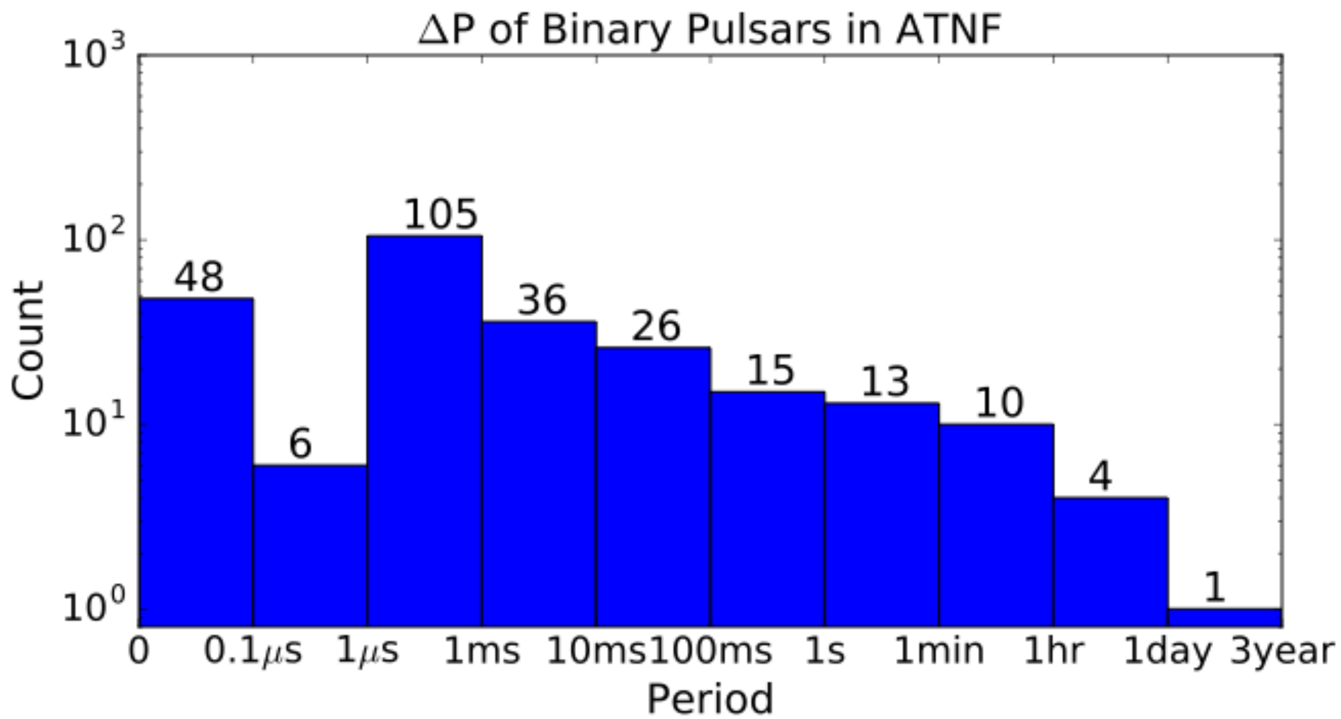
- Heterodyne method requires knowledge of frequency.
- By downsampling, we can make it 4-5 orders of magnitude faster,
- but introduce another unknown parameter (for narrow band searches)

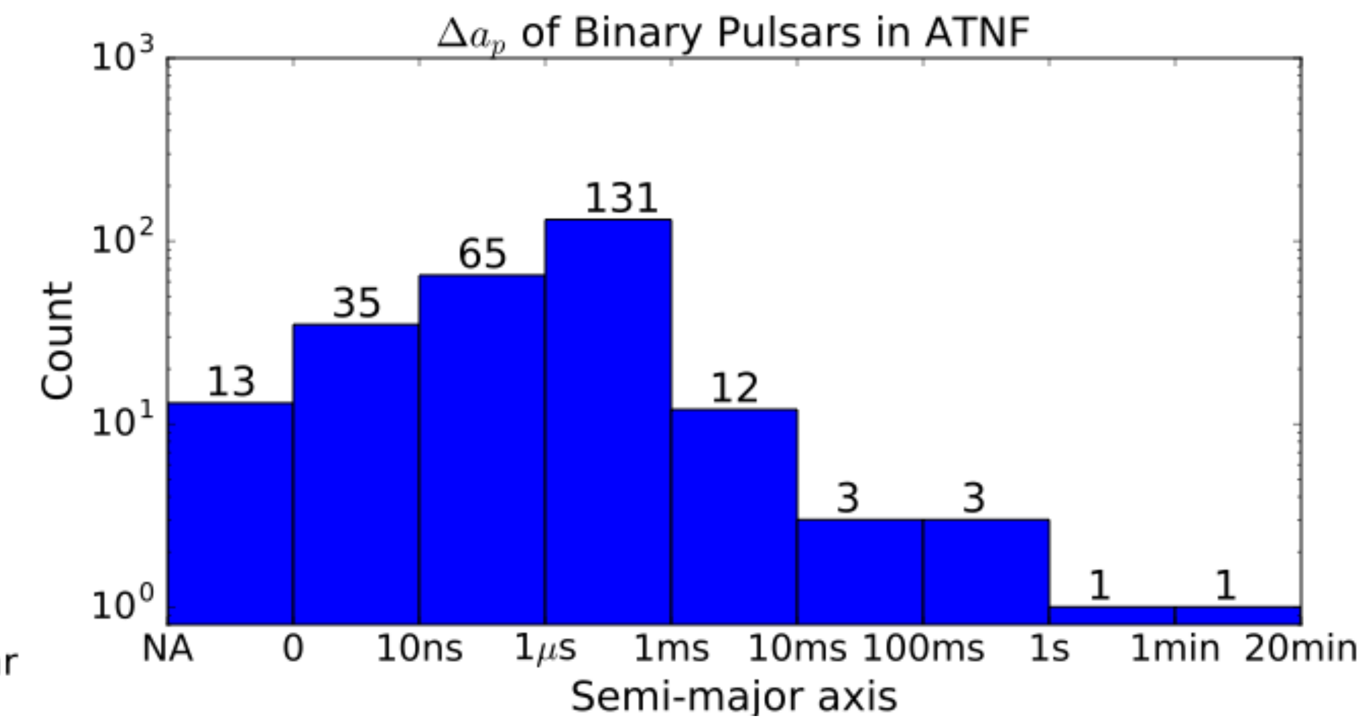
Catch?

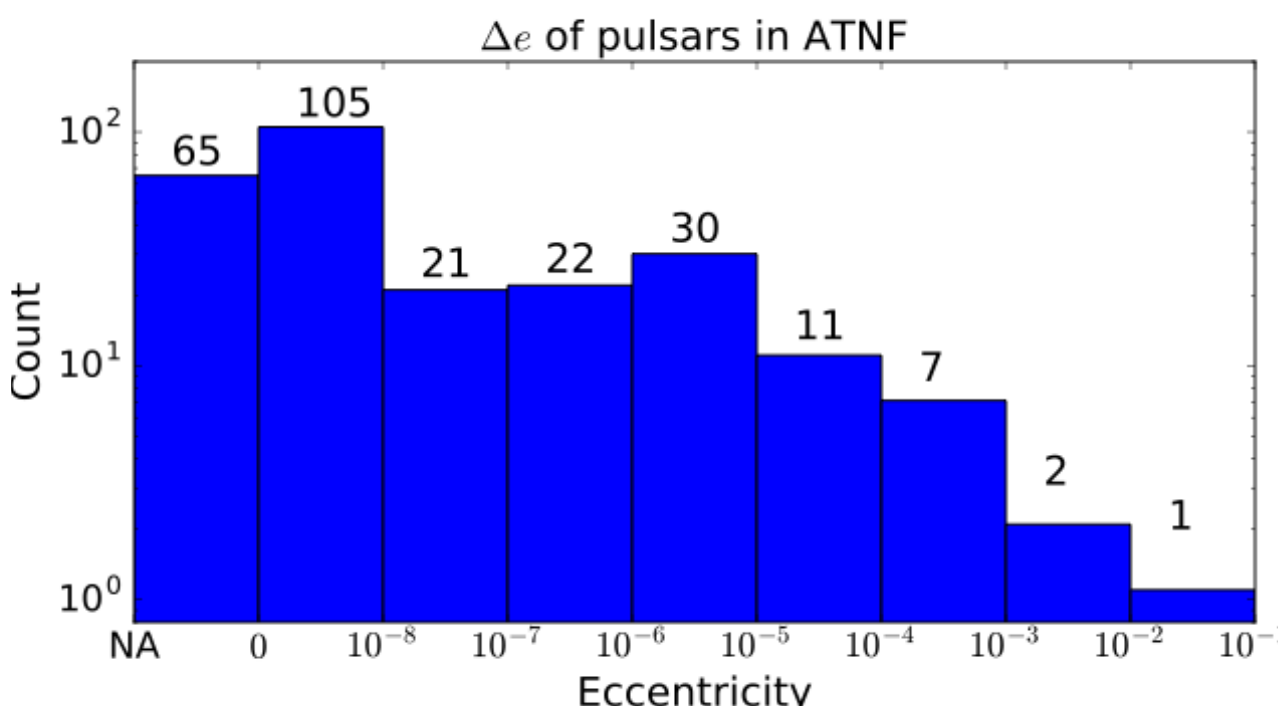
- We have demonstrated that
(assuming the frequency is known)
results were almost identical using the new tool to stroboscopic resampling
Mean : 0.04% Std. Dev.: 0.37%

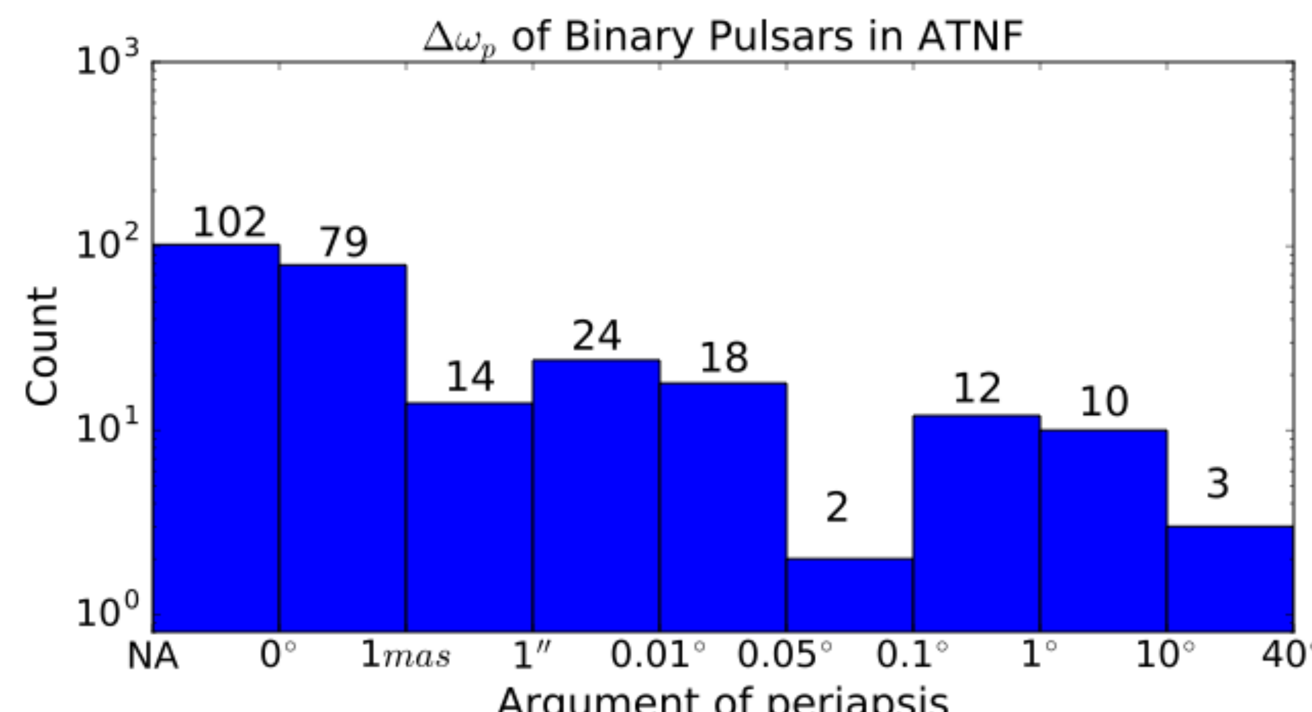
- Hence we can simulate effects of binary correction using faster method,
but as it requires frequency, it is not known, if it is fast enough for narrowband search pipeline
……….. **yet**

Uncertainty Binary parameter space









Manchester, R. N., Hobbs, G. B., Teoh, A. & Hobbs, M., Astron. J., 129, 1993-2006 (2005) (astro-ph/0412641)

http://www.atnf.csiro.au/research/pulsar/psrcat

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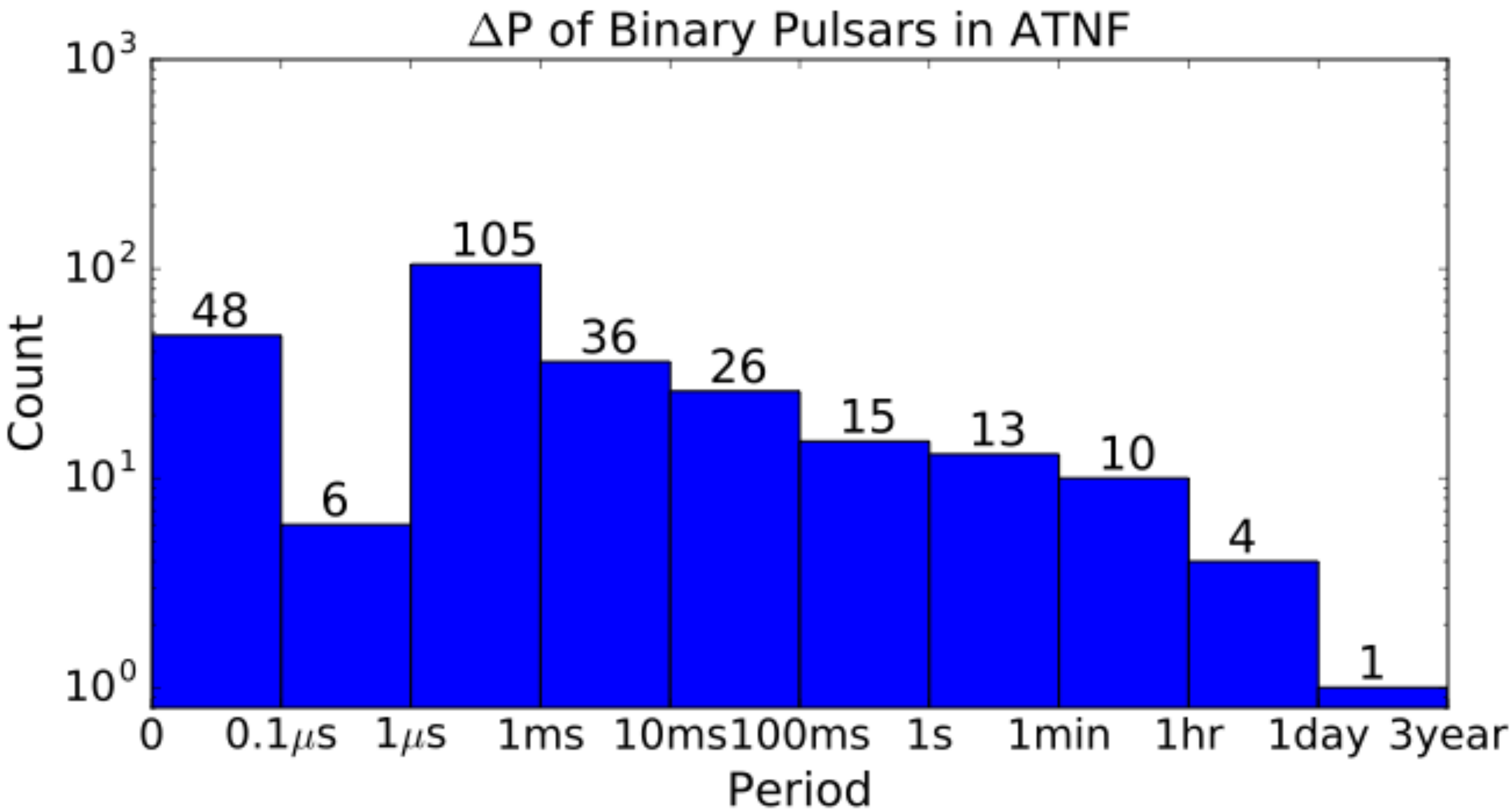

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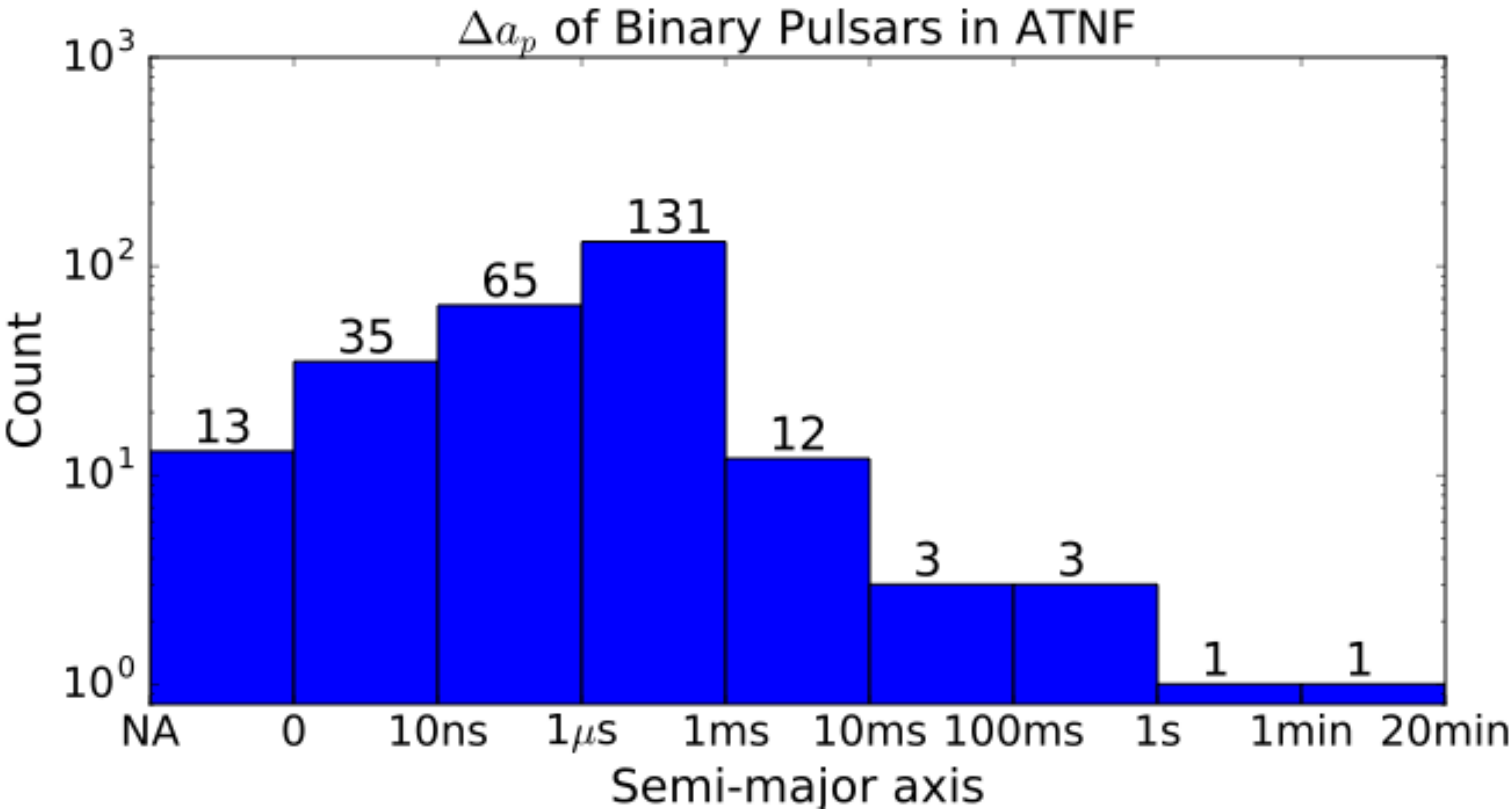

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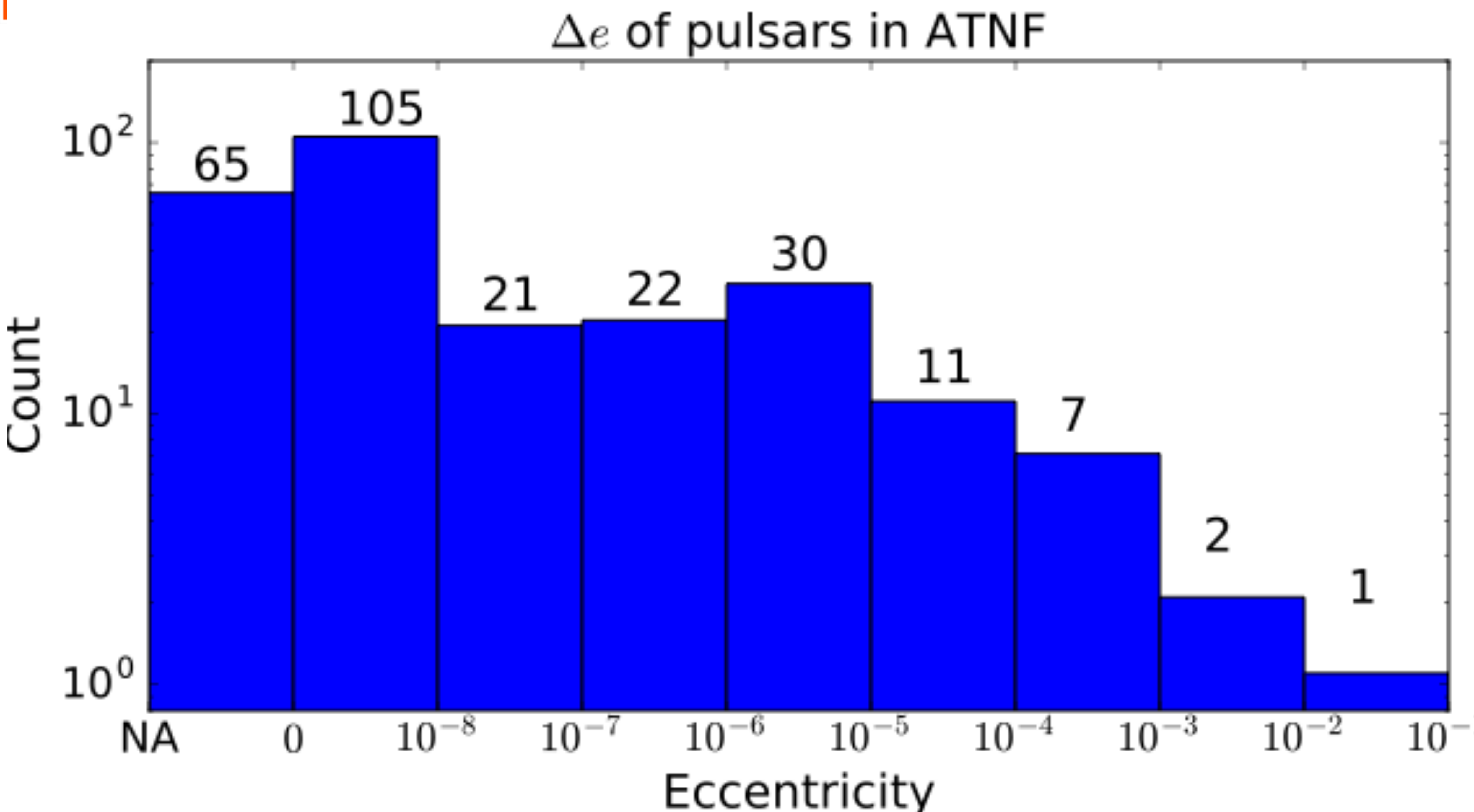

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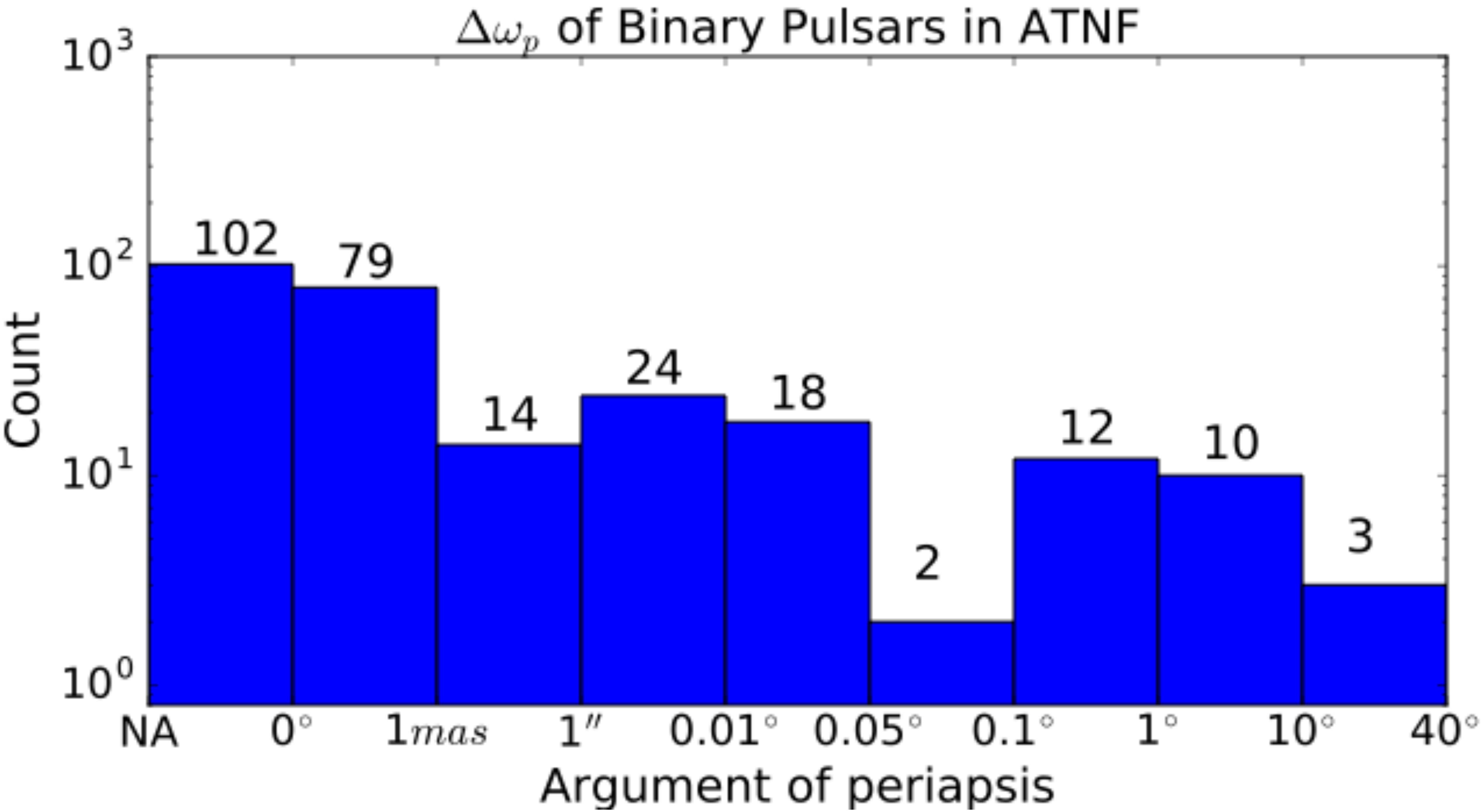

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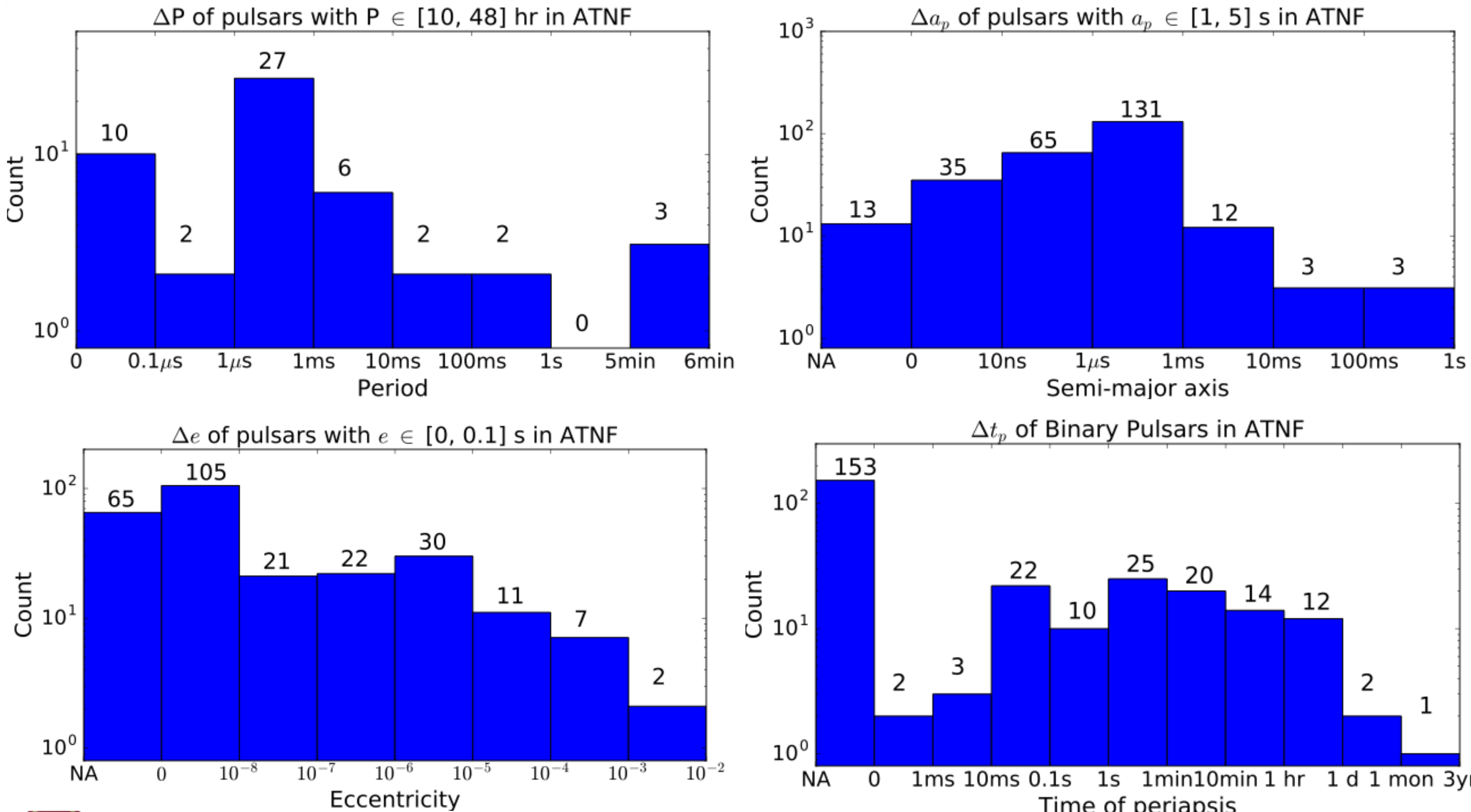

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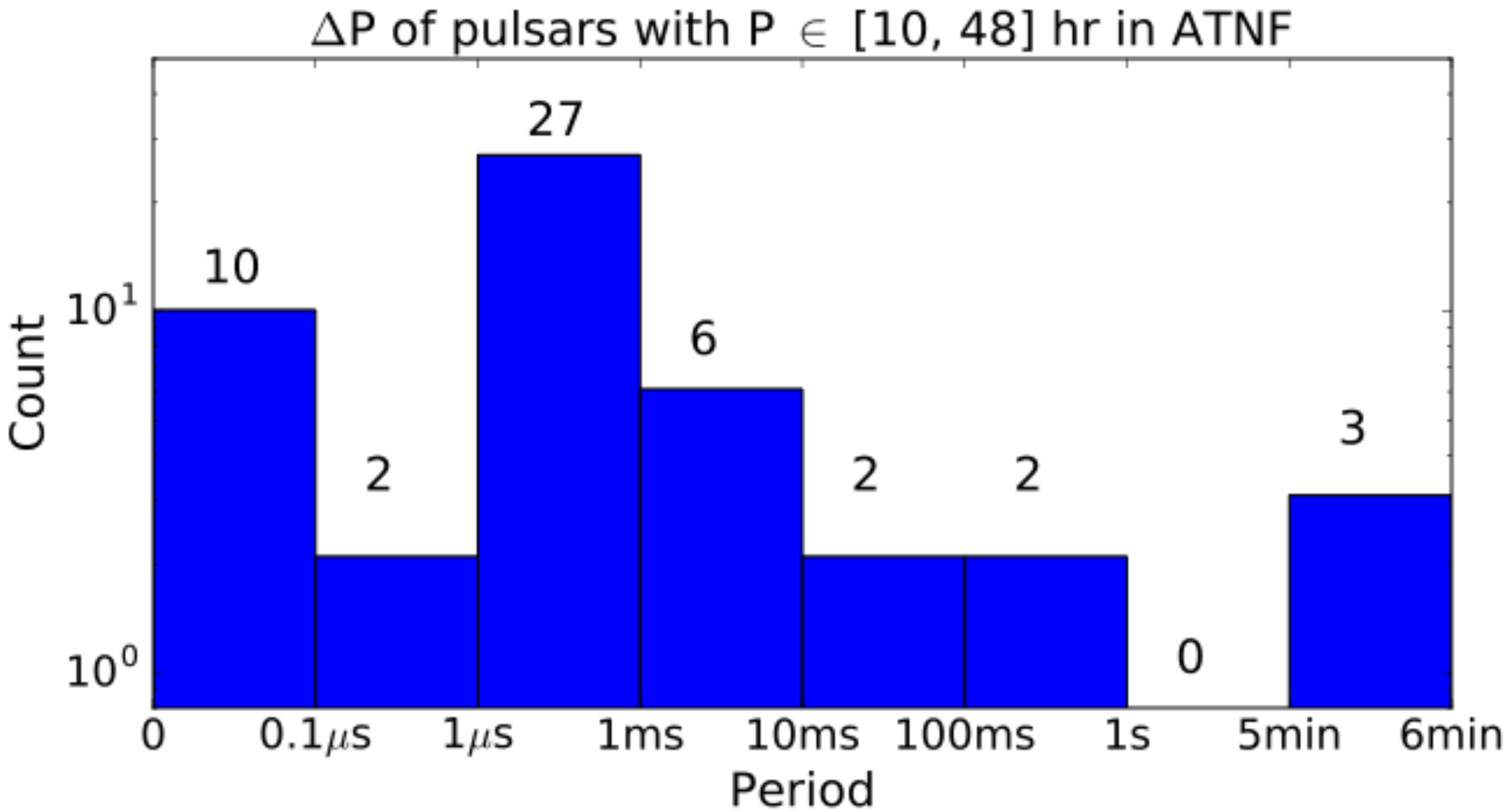

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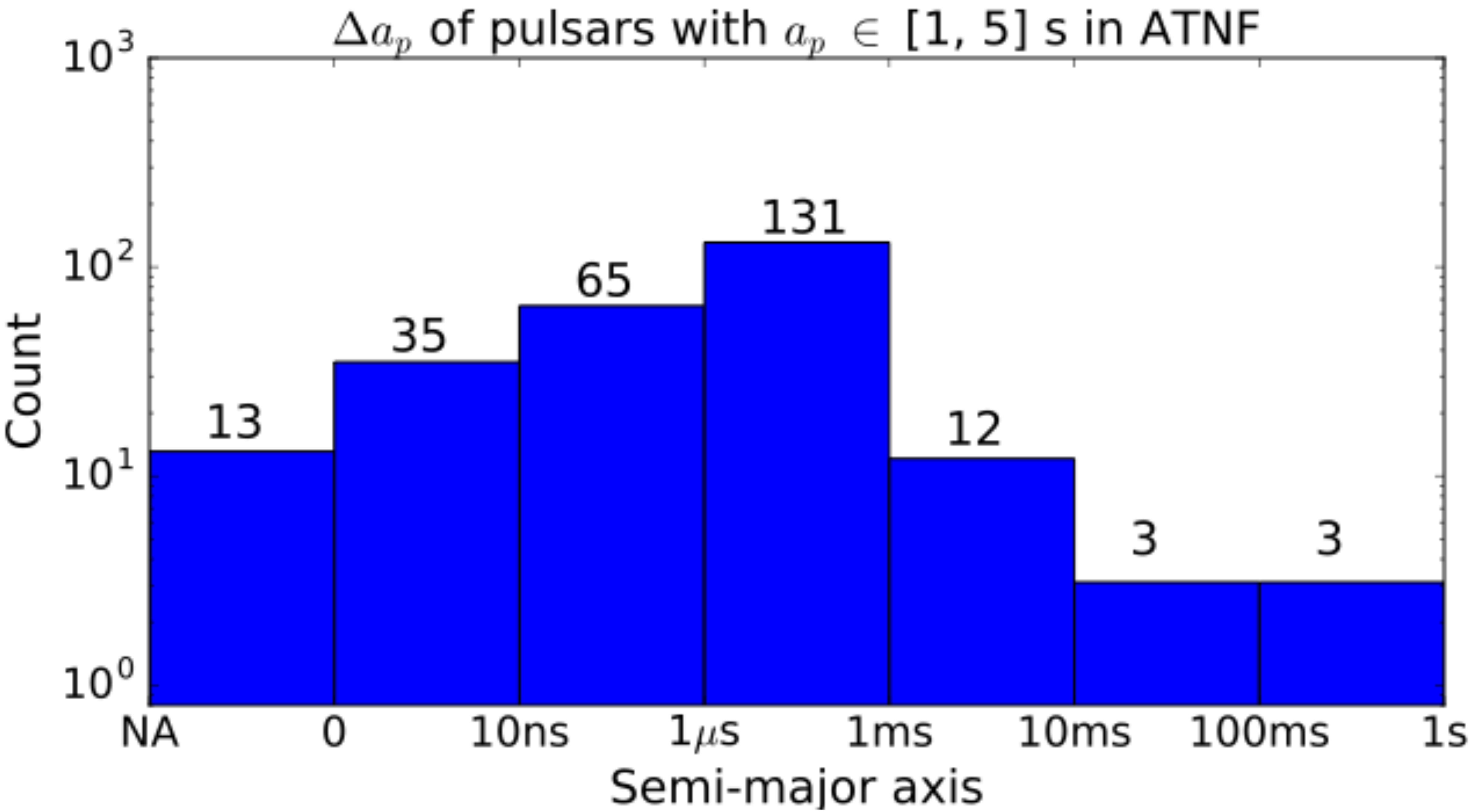

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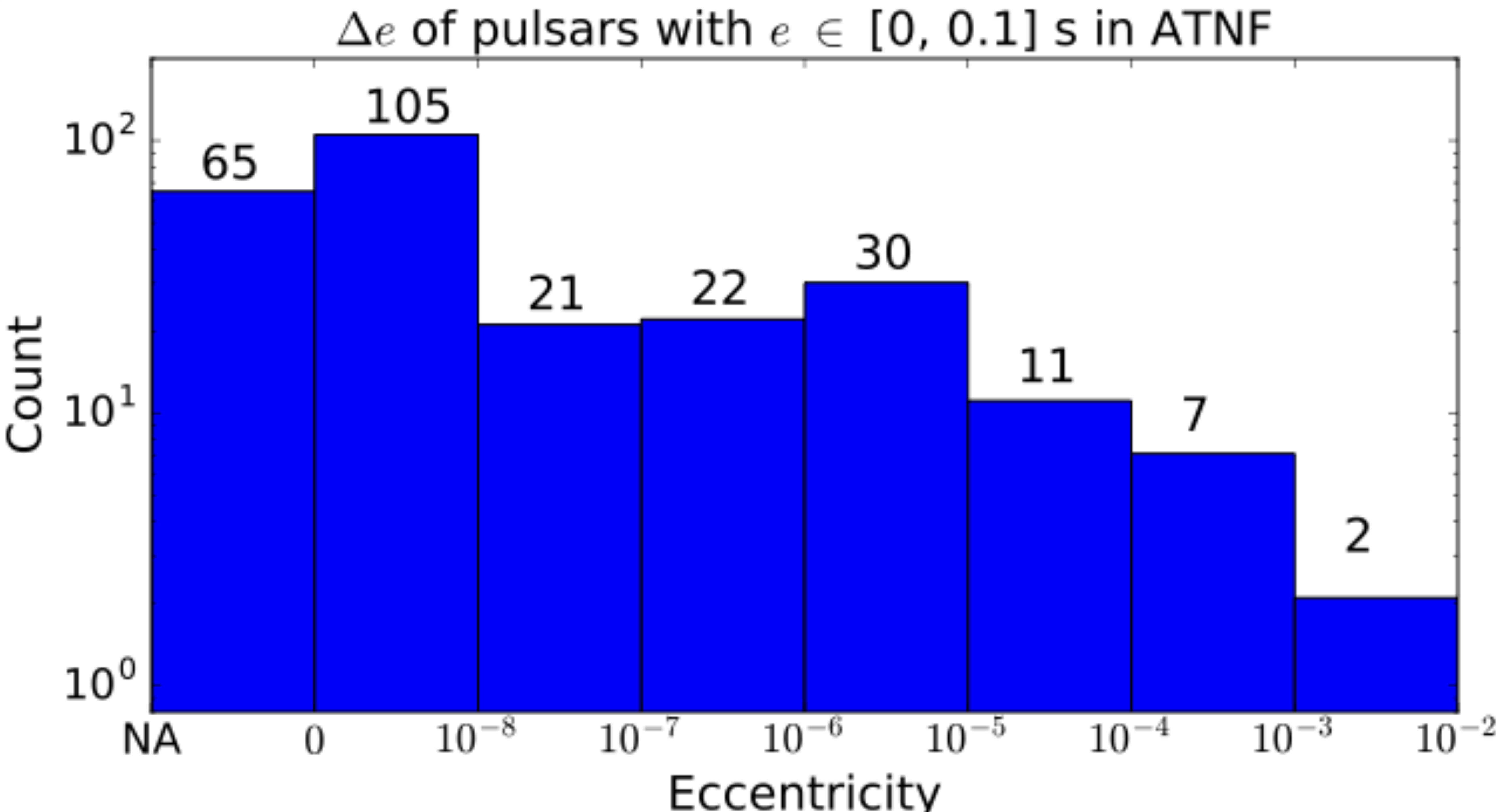

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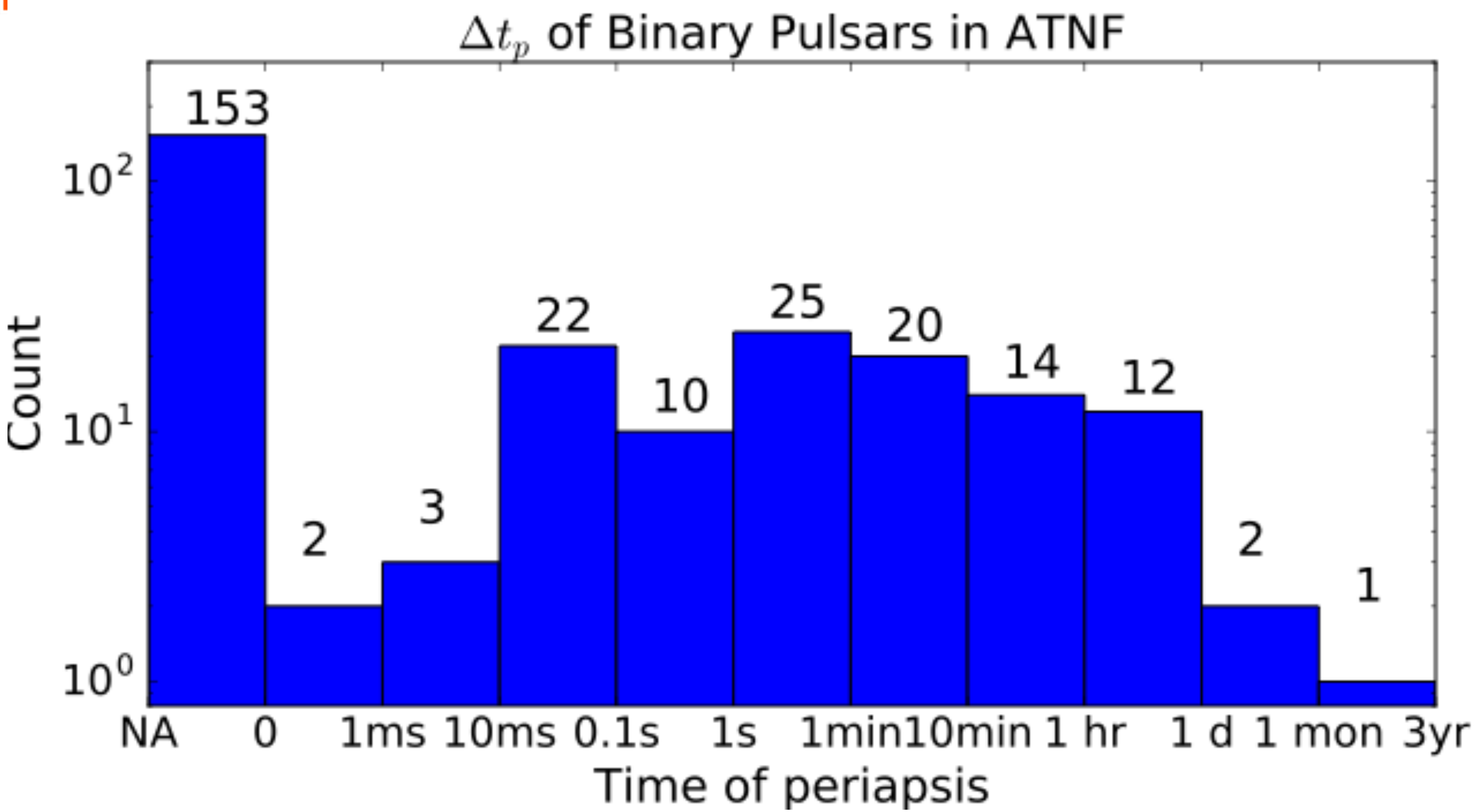

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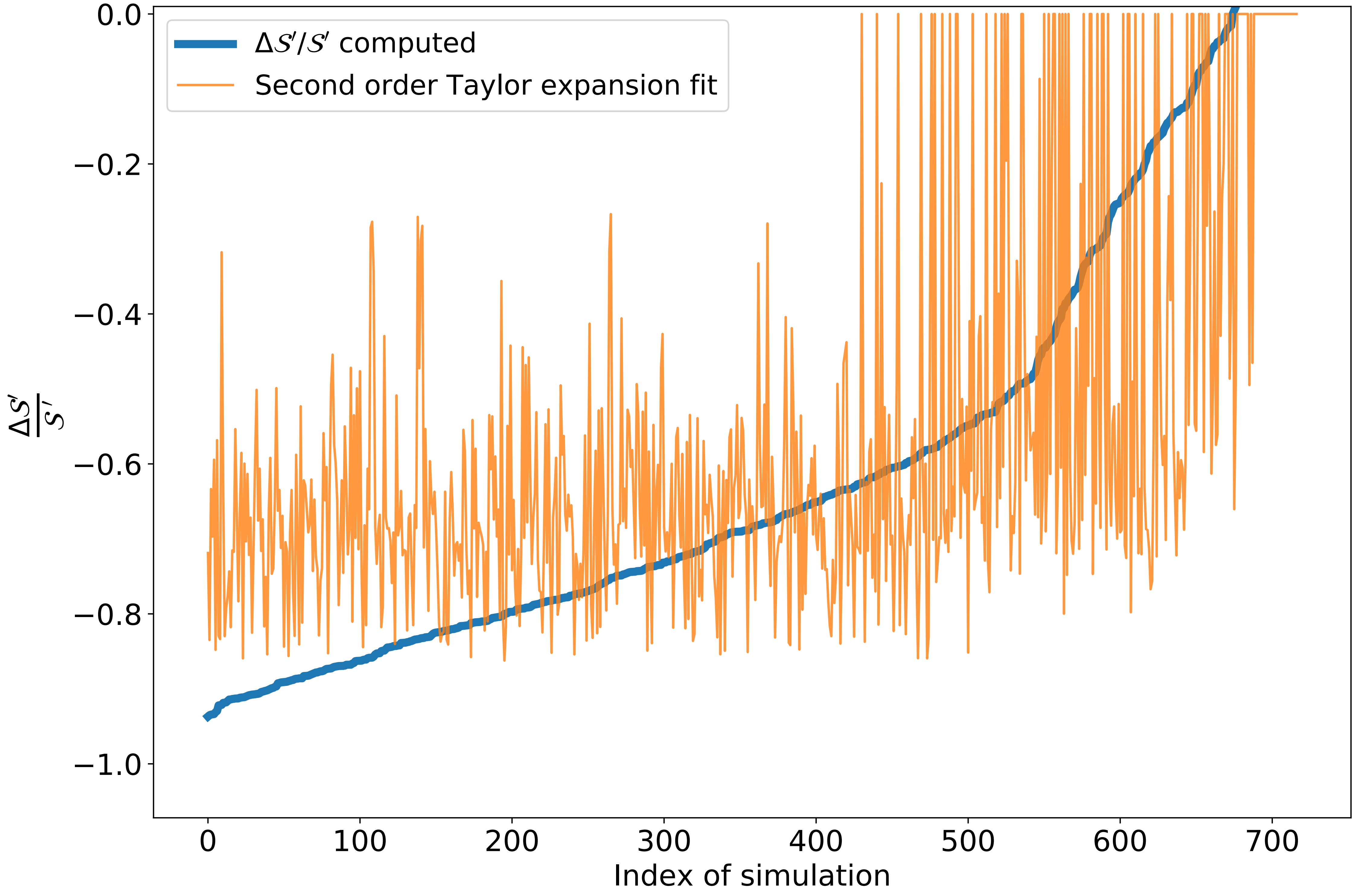
Uncertainties for Sco-X1 like sources

ΔP [ms]	Δa_p [ms]	Δω [deg]	Δe	Δt_p [s]
0.1728	0.004	0.0	0.0	0.0432
0.432	0.02	0.0	0.0	0.1728
0.012096	0.0005	0.0	0.0	0.0432
0.1728	0.002	—	1.5E-05	—
0.432	4E-05	0.0	0.0	0.0014688
0.0011232	0.0006	0.0	0.0	0.0013824
0.0864	0.005	9.0	1.1E-06	—
0.7776	0.003	40.0	3E-06	—
6.912	0.07	0.0	0.0	1.728
0.432	0.002	0.0	0.0	0.02592
0.06048	0.006	0.0	0.0	0.01728
0.0	0.0	—	0.0	—
0.0432	0.005	0.0	0.0	0.02592
0.000432	0.0005	0.0	0.0	0.0003456
0.0011232	4E-05	8.0	1E-08	—
0.012096	0.0002	0.0	0.0	0.000864

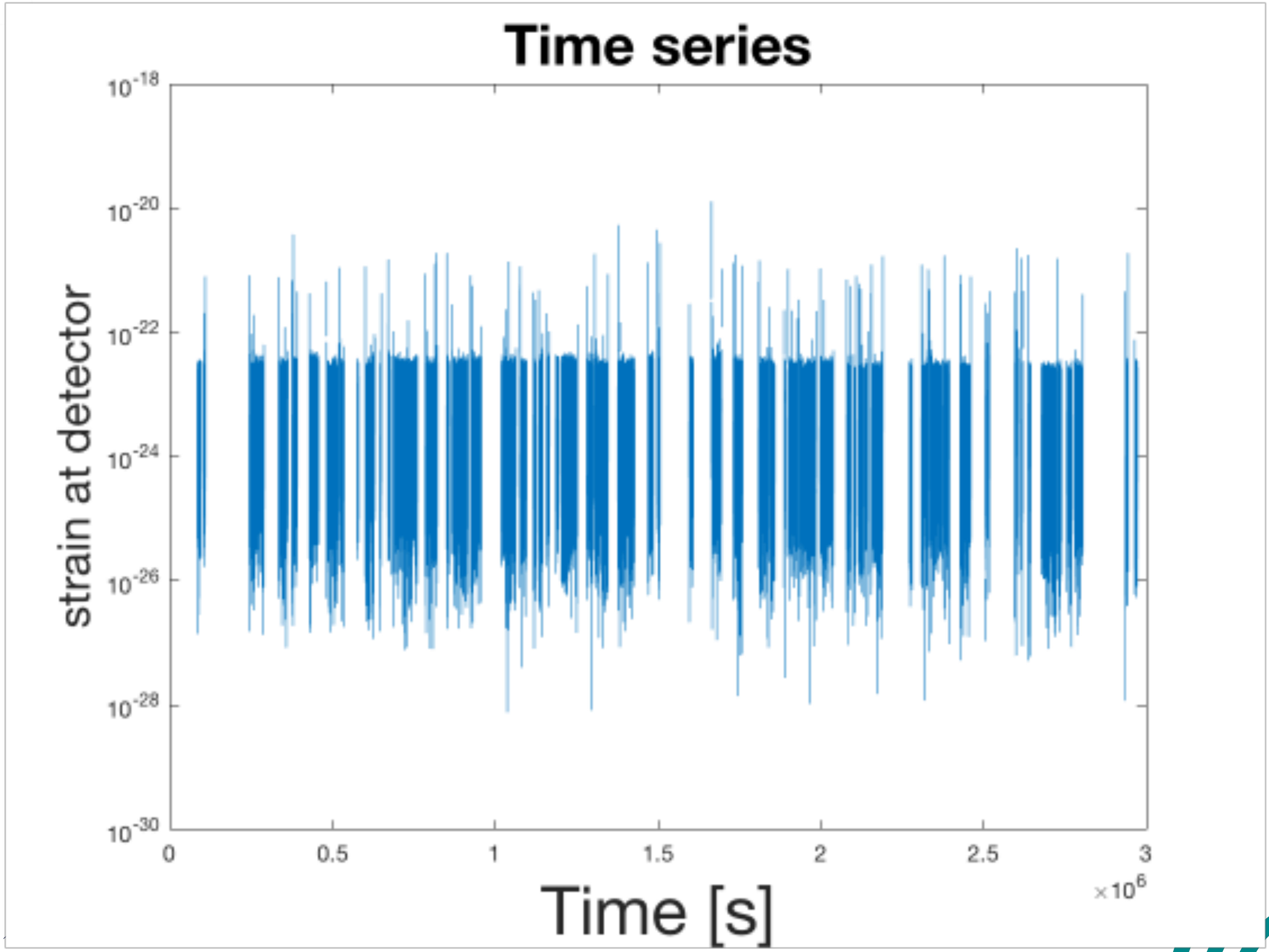


$\Delta\mathcal{S}'/\mathcal{S}'$ computed
Second order Taylor expansion fit
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
$\frac{\Delta\mathcal{S}'}{\mathcal{S}'}$
0
100
200
300
400
500
600
700
Index of simulation

Time residue

$$\frac{1}{2}\,(-3\,\Delta\, e\,\omega0\, a_P \cos(\omega) + 2\,\Delta\, e\,\psi0\, a_P \cos(-2\,\Delta\psi(t) - 2\,\psi(t) + \omega) - \Delta\, e\,\omega0\, a_P \cos(-2\,\Delta\psi(t) - 2\,\psi(t) + \omega) - e\, a_P \sin(-2\,\Delta\psi(t) - 2\,\psi(t) + \omega) - e\, a_P \sin(2\,\psi - \omega) +$$
$$2\,\Delta\,\psi0\, a_P \cos(\Delta\psi(t) + \psi(t)) + 2\, a_P \sin(\Delta\psi(t) + \psi(t)) - 2\, a_P \sin(\psi) - 3\,\mathrm{ap}\,\Delta\mathrm{e} \sin(\omega) - \mathrm{ap}\,\Delta\mathrm{e} \sin(-2\,\Delta\psi(t) - 2\,\psi(t) + \omega) - 3\,\Delta\mathrm{a}\, e \sin(\omega) - \Delta\mathrm{a}\, e \sin(-2\,\Delta\psi(t) - 2\,\psi(t) + \omega) + 2\,\Delta\mathrm{a} \sin(\Delta\psi(t) + \psi(t)))$$

$$\mathrm{ap}\, e\,\Delta\,\psi0 \cos(\omega)\cos^2(\psi(t))\cos^2(\Delta\psi(t)) - \frac{1}{2}\,\mathrm{ap}\, e\,\Delta\,\omega0 \cos(\omega)\cos^2(\psi(t))\cos^2(\Delta\psi(t)) - \mathrm{ap}\, e\,\Delta\,\psi0 \cos(\omega)\sin^2(\psi(t))\cos^2(\Delta\psi(t)) + \frac{1}{2}\,\mathrm{ap}\, e\,\Delta\,\omega0 \cos(\omega)\sin^2(\psi(t))\cos^2(\Delta\psi(t)) + \frac{1}{2}\,\mathrm{ap}\, e \sin(\omega)\sin^2(\psi(t))\cos^2(\Delta\psi(t)) +$$
$$\frac{1}{2}\, e\,\Delta\mathrm{a} \sin(\omega)\sin^2(\psi(t))\cos^2(\Delta\psi(t)) + \frac{1}{2}\,\mathrm{ap}\,\Delta\mathrm{e} \sin(\omega)\sin^2(\psi(t))\cos^2(\Delta\psi(t)) - \frac{1}{2}\,\mathrm{ap}\, e \cos^2(\psi(t))\sin(\omega)\cos^2(\Delta\psi(t)) - \frac{1}{2}\, e\,\Delta\mathrm{a} \cos^2(\psi(t))\sin(\omega)\cos^2(\Delta\psi(t)) - \frac{1}{2}\,\mathrm{ap}\,\Delta\mathrm{e} \cos^2(\psi(t))\sin(\omega)\cos^2(\Delta\psi(t)) +$$
$$\mathrm{ap}\, e \cos(\omega)\cos(\psi(t))\sin(\psi(t))\cos^2(\Delta\psi(t)) + e\,\Delta\mathrm{a} \cos(\omega)\cos(\psi(t))\sin(\psi(t))\cos^2(\Delta\psi(t)) + \mathrm{ap}\,\Delta\mathrm{e} \cos(\omega)\cos(\psi(t))\sin(\psi(t))\cos^2(\Delta\psi(t)) + 2\,\mathrm{ap}\, e\,\Delta\,\psi0 \cos(\psi(t))\sin(\omega)\sin(\psi(t))\cos^2(\Delta\psi(t)) -$$
$$\mathrm{ap}\, e\,\Delta\,\omega0 \cos(\psi(t))\sin(\omega)\sin(\psi(t))\cos^2(\Delta\psi(t)) - \mathrm{ap}\, e \cos(\omega)\sin(\Delta\psi(t))\sin^2(\psi(t))\cos(\Delta\psi(t)) - e\,\Delta\mathrm{a} \cos(\omega)\sin(\Delta\psi(t))\sin^2(\psi(t))\cos(\Delta\psi(t)) - \mathrm{ap}\,\Delta\mathrm{e} \cos(\omega)\sin(\Delta\psi(t))\sin^2(\psi(t))\cos(\Delta\psi(t)) -$$
$$2\,\mathrm{ap}\, e\,\Delta\,\psi0 \sin(\omega)\sin(\Delta\psi(t))\sin^2(\psi(t))\cos(\Delta\psi(t)) + \mathrm{ap}\, e\,\Delta\,\omega0 \sin(\omega)\sin(\Delta\psi(t))\sin^2(\psi(t))\cos(\Delta\psi(t)) + \mathrm{ap}\,\Delta\,\psi0 \cos(\psi(t))\cos(\Delta\psi(t)) + \mathrm{ap}\, e \cos(\omega)\cos^2(\psi(t))\sin(\Delta\psi(t))\cos(\Delta\psi(t)) +$$
$$e\,\Delta\mathrm{a} \cos(\omega)\cos^2(\psi(t))\sin(\Delta\psi(t))\cos(\Delta\psi(t)) + \mathrm{ap}\,\Delta\mathrm{e} \cos(\omega)\cos^2(\psi(t))\sin(\Delta\psi(t))\cos(\Delta\psi(t)) + 2\,\mathrm{ap}\, e\,\Delta\,\psi0 \cos^2(\psi(t))\sin(\omega)\sin(\Delta\psi(t))\cos(\Delta\psi(t)) - \mathrm{ap}\, e\,\Delta\,\omega0 \cos^2(\psi(t))\sin(\omega)\sin(\Delta\psi(t))\cos(\Delta\psi(t)) +$$
$$\mathrm{ap} \sin(\psi(t))\cos(\Delta\psi(t)) + \Delta\mathrm{a} \sin(\psi(t))\cos(\Delta\psi(t)) - 4\,\mathrm{ap}\, e\,\Delta\,\psi0 \cos(\omega)\cos(\psi(t))\sin(\Delta\psi(t))\sin(\psi(t))\cos(\Delta\psi(t)) + 2\,\mathrm{ap}\, e\,\Delta\,\omega0 \cos(\omega)\cos(\psi(t))\sin(\Delta\psi(t))\sin(\psi(t))\cos(\Delta\psi(t)) +$$
$$2\,\mathrm{ap}\, e \cos(\psi(t))\sin(\omega)\sin(\Delta\psi(t))\sin(\psi(t))\cos(\Delta\psi(t)) + 2\, e\,\Delta\mathrm{a} \cos(\psi(t))\sin(\omega)\sin(\Delta\psi(t))\sin(\psi(t))\cos(\Delta\psi(t)) + 2\,\mathrm{ap}\,\Delta\mathrm{e} \cos(\psi(t))\sin(\omega)\sin(\Delta\psi(t))\sin(\psi(t))\cos(\Delta\psi(t)) - \mathrm{ap}\, e\,\Delta\,\psi0 \cos(\omega)\cos^2(\psi(t))\sin^2(\Delta\psi(t)) +$$
$$\frac{1}{2}\,\mathrm{ap}\, e\,\Delta\,\omega0 \cos(\omega)\cos^2(\psi(t))\sin^2(\Delta\psi(t)) + \frac{1}{2}\,\mathrm{ap}\, e \cos^2(\psi(t))\sin(\omega)\sin^2(\Delta\psi(t)) + \frac{1}{2}\, e\,\Delta\mathrm{a} \cos^2(\psi(t))\sin(\omega)\sin^2(\Delta\psi(t)) + \frac{1}{2}\,\mathrm{ap}\,\Delta\mathrm{e} \cos^2(\psi(t))\sin(\omega)\sin^2(\Delta\psi(t)) + \mathrm{ap}\, e\,\Delta\,\psi0 \cos(\omega)\sin^2(\Delta\psi(t))\sin^2(\psi(t)) -$$
$$\frac{1}{2}\,\mathrm{ap}\, e\,\Delta\,\omega0 \cos(\omega)\sin^2(\Delta\psi(t))\sin^2(\psi(t)) - \frac{1}{2}\,\mathrm{ap}\, e \sin(\omega)\sin^2(\Delta\psi(t))\sin^2(\psi(t)) - \frac{1}{2}\, e\,\Delta\mathrm{a} \sin(\omega)\sin^2(\Delta\psi(t))\sin^2(\psi(t)) - \frac{1}{2}\,\mathrm{ap}\,\Delta\mathrm{e} \sin(\omega)\sin^2(\Delta\psi(t))\sin^2(\psi(t)) - \frac{3}{2}\,\mathrm{ap}\, e\,\Delta\,\omega0 \cos(\omega) - \frac{3}{2}\,\mathrm{ap}\, e \sin(\omega) -$$
$$\frac{3}{2}\, e\,\Delta\mathrm{a} \sin(\omega) - \frac{3}{2}\,\mathrm{ap}\,\Delta\mathrm{e} \sin(\omega) - \mathrm{ap}\left(\sin(\psi) + \frac{1}{2}\, e \cos(\omega)\sin(2\,\psi) - \frac{3}{2}\, e \sin(\omega) - \frac{1}{2}\, e \cos(2\,\psi)\sin(\omega)\right) + \mathrm{ap} \cos(\psi(t))\sin(\Delta\psi(t)) + \Delta\mathrm{a} \cos(\psi(t))\sin(\Delta\psi(t)) - \mathrm{ap}\, e \cos(\omega)\cos(\psi(t))\sin^2(\Delta\psi(t))\sin(\psi(t)) -$$
$$e\,\Delta\mathrm{a} \cos(\omega)\cos(\psi(t))\sin^2(\Delta\psi(t))\sin(\psi(t)) - \mathrm{ap}\,\Delta\mathrm{e} \cos(\omega)\cos(\psi(t))\sin^2(\Delta\psi(t))\sin(\psi(t)) - 2\,\mathrm{ap}\, e\,\Delta\,\psi0 \cos(\psi(t))\sin(\omega)\sin^2(\Delta\psi(t))\sin(\psi(t)) + \mathrm{ap}\, e\,\Delta\,\omega0 \cos(\psi(t))\sin(\omega)\sin^2(\Delta\psi(t))\sin(\psi(t)) - \mathrm{ap}\,\Delta\,\psi0 \sin(\Delta\psi(t))\sin(\psi(t))$$


Time series
strain at detector
10^-18
10^-20
10^-22
10^-24
10^-26
10^-28
10^-30
0
0.5
1
1.5
2
2.5
3
×10^6
Time [s]