Model Independent Methods for Cosmological Inference

Arman Shafieloo,

Korea Astronomy and Space Science Institute (KASI)

University of Science and Technology (UST)

ModIC 2024 - Model-Independent Cosmology with gravitational waves, large-scale structure, and high-energy surveys

13-18 May 2024,

IFUP, Trieste, Italy

On model selection, validation and reconstruction in the context of physical cosmology

Arman Shafieloo,

Korea Astronomy and Space Science Institute (KASI)

University of Science and Technology (UST)

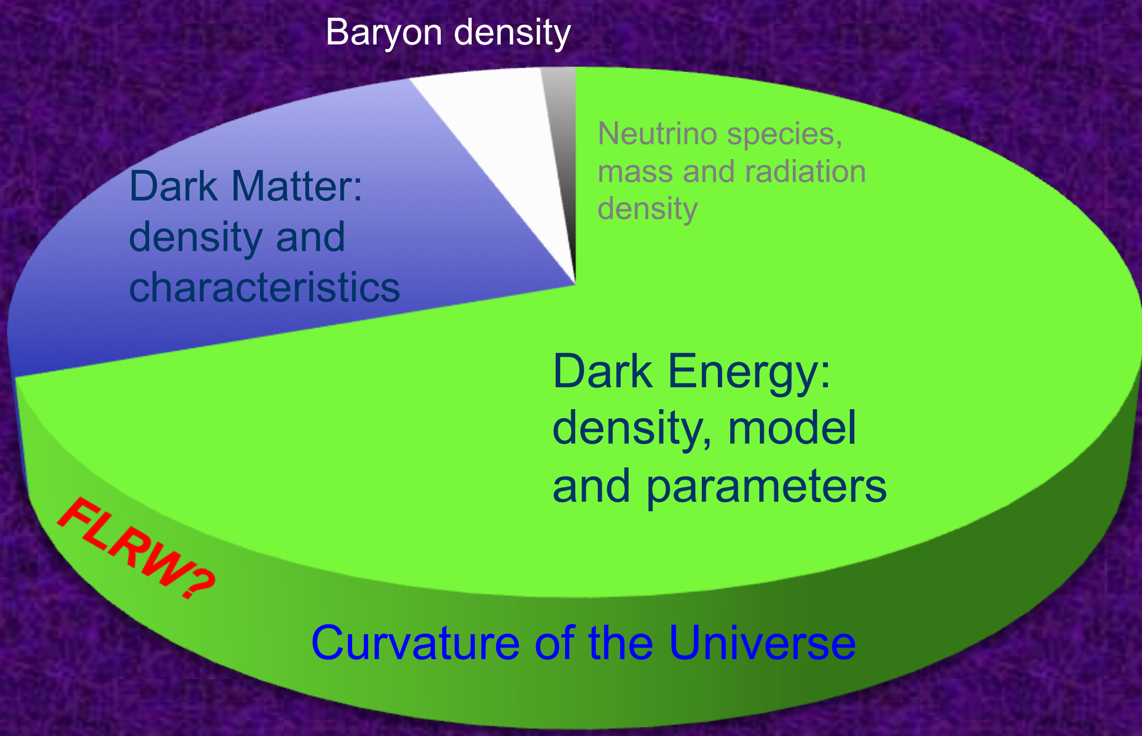
ModIC 2024 - Model-Independent Cosmology with gravitational waves, large-scale structure, and high-energy surveys

13-18 May 2024,

IFUP, Trieste, Italy

Era of Precision Cosmology

We try to reconstruct and understand the dynamics of the universe and properties of its constituents using various measurements and statistical techniques. Phenomenological and then theoretical works can follow to place constraints on suggested models and their parameters.



Initial Conditions:
Form of the Primordial Spectrum and Model of Inflation and its Parameters

Epoch of reionization

Hubble Parameter and the Rate of Expansion

What do we do?

- There are various reconstruction approaches, parametric and non-parametric.

- There have been many phenomenological and theoretical models proposed (recently, to alleviate tensions).

Reconstruction → Phenomenology → Theory

- There have been continuous attempts looking for systematics in various data.

- These models/reconstructions can be very different.
How do we compare them?

Consistency of a proposed model and the data:

Frequentist Approach:

Assuming a proposed model, the probability of the observed data must not be insignificant. Best is to do large number of careful simulations based on a well defined covariance error-matrix.

Bayesian Approach:

Priors and simplicity of the proposed model *also* matters (in model comparison)

Chi square analysis plays a crucial role in calculation of the likelihood in both approaches

Why things are more complicated than what we think…

Likelihood

We are interested to calculate the probability of the observed data given the model.

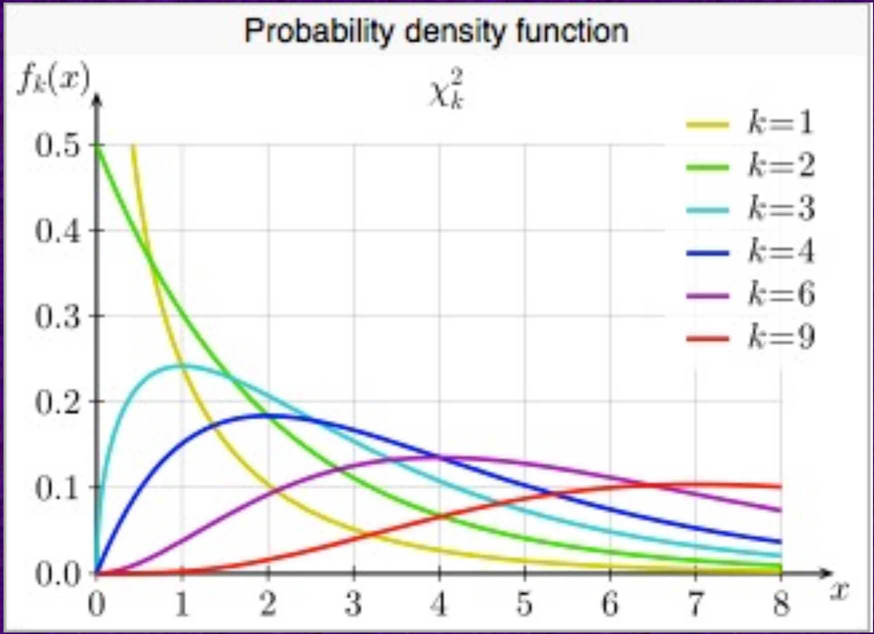
$$\chi^2 = \sum_i^N (\mu_i^t - \mu_i^e)^T Cov^{-1} (\mu_i^t - \mu_i^e)$$

$$\chi^2 = \sum_i^N \frac{(\mu_i^t - \mu_i^e)^2}{\sigma_i^2},$$

When data is uncorrelated

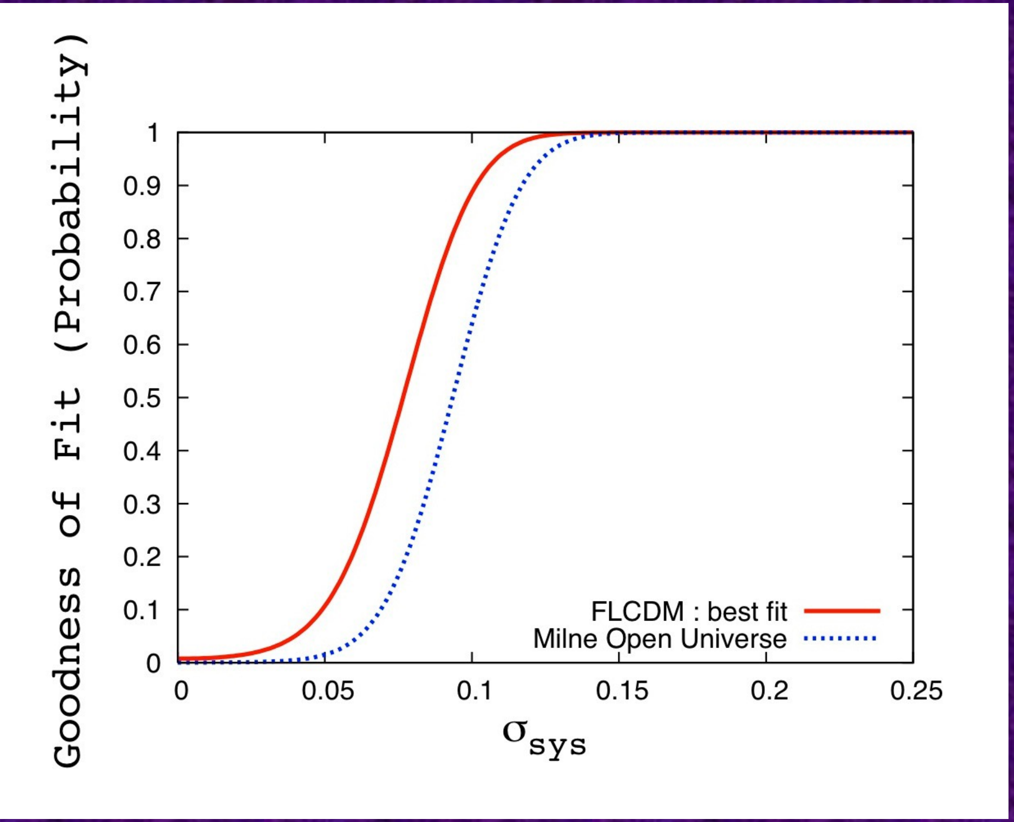
$$P(\chi^2; N) = \frac{2^{-N/2}}{\Gamma(N/2)} \chi^{N-2} e^{-\chi^2/2}$$

$$Prob(\chi^2; N) = \int_{\chi^2}^{\infty} P(\chi^2; N) d\chi'^2.$$



What if the exact form of the error matrix is not known?

Point 1



$$\chi^2 = \sum_i^N (\mu_i^t - \mu_i^e)^T Cov^{-1} (\mu_i^t - \mu_i^e)$$

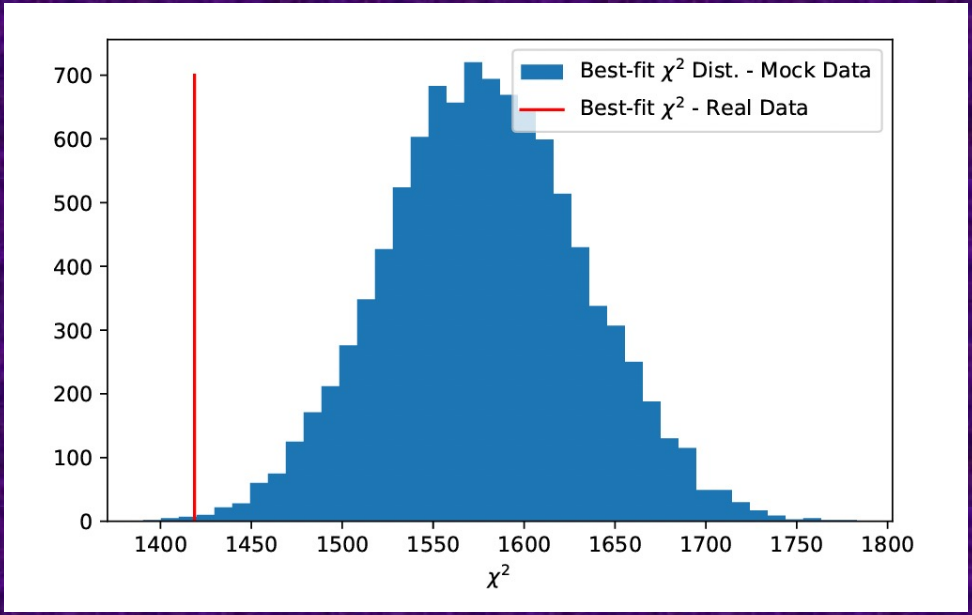
$$\sigma_i^2 = \sigma_{i(data)}^2 + \sigma_{(sys)}^2$$

$$P(\chi^2; N) = \frac{2^{-N/2}}{\Gamma(N/2)} \chi^{N-2} e^{-\chi^2/2}$$

$$Prob(\chi^2; N) = \int_{\chi^2}^{\infty} P(\chi^2; N) d\chi'^2.$$

e.g. *The case of Type Ia supernovae*

This can still happen!



$$\chi^2 = \sum_i^N (\mu_i^t - \mu_i^e)^T Cov^{-1} (\mu_i^t - \mu_i^e)$$

$$\sigma_i^2 = \sigma_{i(data)}^2 + \sigma_{(sys)}^2$$

$$P(\chi^2; N) = \frac{2^{-N/2}}{\Gamma(N/2)} \chi^{N-2} e^{-\chi^2/2}$$

$$Prob(\chi^2; N) = \int_{\chi^2}^{\infty} P(\chi^2; N) d\chi'^2.$$

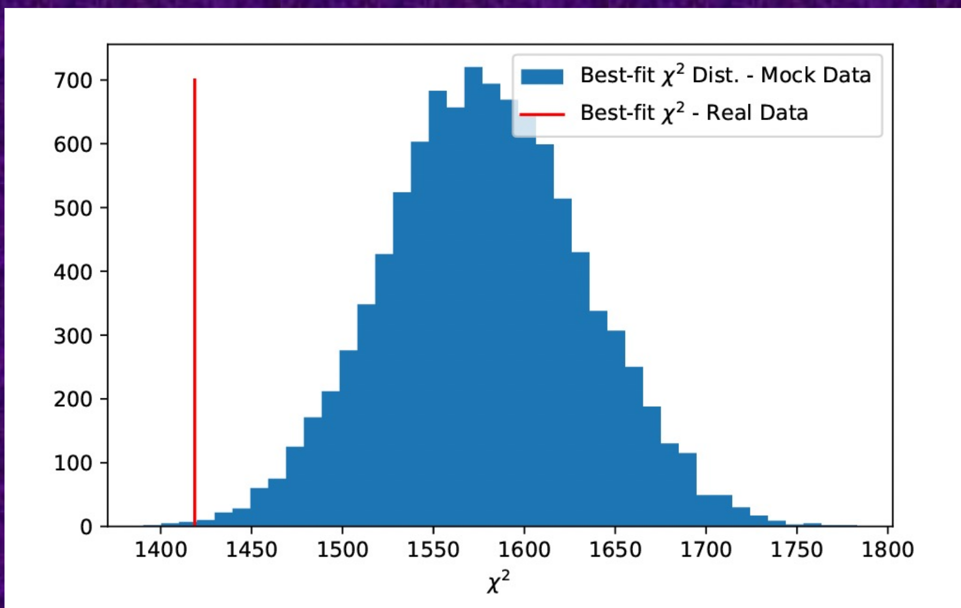
Pantheon+ data

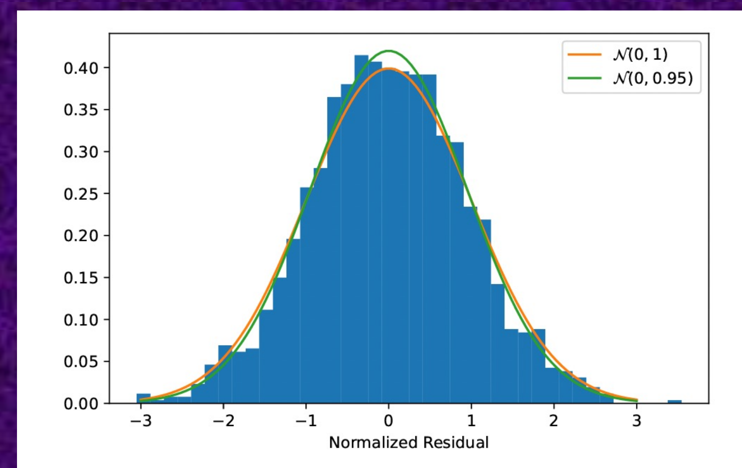
Keeley, Shafieloo, L'Huillier, arXiv:2212.07917

This can still happen!

Point 1

$$\chi^2 = \sum_{i}^{N} (\mu_i^t - \mu_i^e)^T Cov^{-1} (\mu_i^t - \mu_i^e)$$





Pantheon+ data

Keeley, Shafieloo, L'Huillier, arXiv:2212.07917

$$Prob(\chi^2; N) = \int_{\chi^2}^{\infty} P(\chi^2; N) d\chi'^2.$$

Likelihood and Model Fitting

When number of data points is more than ~30 one can use relative chi square for likelihood analysis and N, number of free parameters of the fitting function, will become the degrees of freedom.

$$\chi^2 = \sum_i^N (\mu_i^t - \mu_i^e)^T Cov^{-1} (\mu_i^t - \mu_i^e)$$

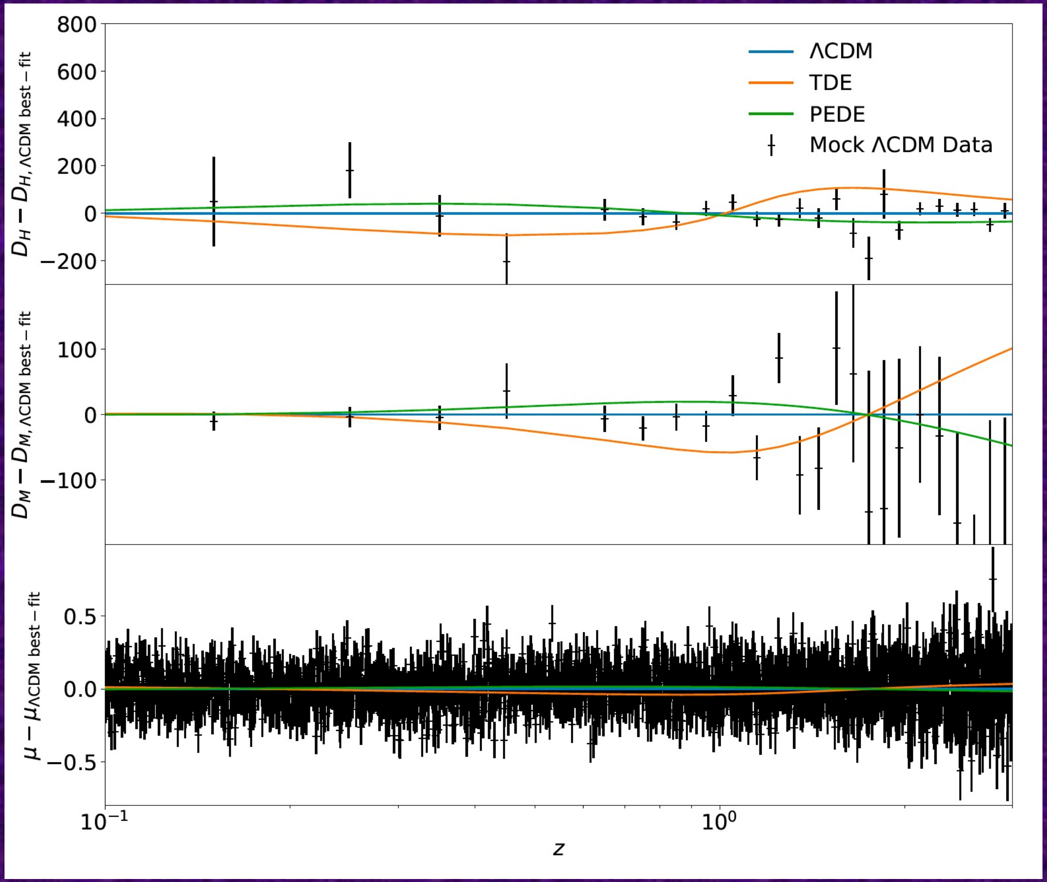
In likelihood estimation:

$$\chi^2 \longrightarrow \Delta\chi^2$$

$$\Delta\chi^2 = \chi^2 - \chi^2_{best}$$

$$P(\chi^2; N) = \frac{2^{-N/2}}{\Gamma(N/2)} \chi^{N-2} e^{-\chi^2/2}$$

$$Prob(\chi^2; N) = \int_{\chi^2}^{\infty} P(\chi^2; N) d\chi'^2.$$



Bayesian Analysis

- Bayesian approach provides the means to incorporate ***prior*** knowledge in data analysis.
- Bayes's law states that the posterior probability is proportional to the product of the likelihood and the prior probability.

Posterior probability and the priors:

Likelihood

Prior probability

$$p(x|d) = \frac{p(d|x)p(x)}{p(d)}$$

Posterior probability

Normalization factor

Model fitting has Bayesian essence since we assume that we are considering a correct model.

Point 2

Posterior probability and the priors:

Likelihood

Prior probability

$$p(x|d) = \frac{p(d|x)p(x)}{p(d)}$$

Posterior probability

Normalization factor

Model fitting has Bayesian essence since we assume that we are considering a correct model.

Point 2

Bayesian Evidence and Model Selection

- Bayesian evidence: Integral of (likelihood)x(prior) over the parameter space: $Z = \int L(\theta)\pi(\theta)d\theta$

- Bayes factor: Ratio of the evidence of the two models:

$$\Delta\log Z = \log Z(\mathrm{M}_1) - \log Z(\mathrm{M}_2)$$

Supports Model 1 over Model 2 when $\Delta\log Z$ have a positive value

Jeffreys scale Z_i/Z_j	Kass-Rafferty scale Z_i/Z_j	Interpretation
1 to 3.2	1 to 3	Not worth mentioning
3.2 to 10	3 to 20	Positive
10 to 100	20 to 150	Strong
> 100	>150	Very Strong

Bayesian Evidence and Model Selection

- Bayesian evidence: Integral of (likelihood)x(prior) over the parameter space: $Z = \int L(\theta)\pi(\theta)d\theta$

- Bayes factor: Ratio of the evidence of the two models:

$$\Delta \log Z = \log Z(\mathrm{M}_1) - \log Z(\mathrm{M}_2)$$

Supports Model 1 over Model 2 when $\Delta \log Z$ have a positive value

Jeffreys scale Z_i/Z_j	Kass-Rafferty scale Z_i/Z_j	Interpretation
1 to 3.2	1 to 3	Not worth mentioning
3.2 to 10	3 to 20	Positive
10 to 100	20 to 150	Strong
> 100	>150	Very Strong

Point 3

How reliable are these scales?

(Present)

Standard Model of Cosmology

Universe is Flat

Universe is Isotropic

Universe is Homogeneous

Dark Energy is Lambda (w=-1)

Power-Law primordial spectrum (n_s=const)

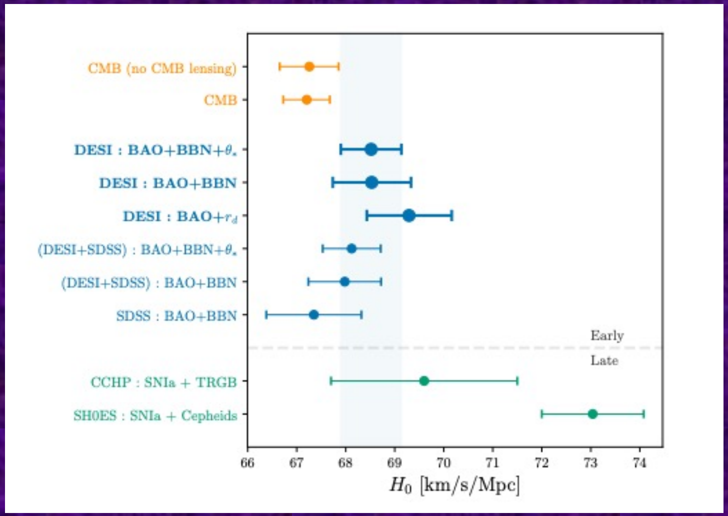
Dark Matter is cold

All within framework of FLRW

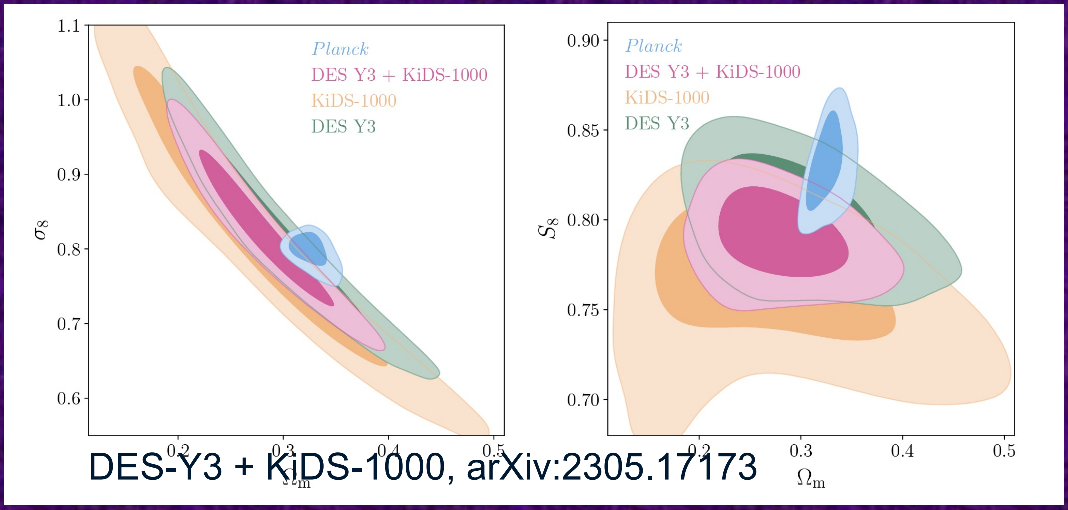
Let’s solve Hubble tension with evolving DE!

Tensions in the Standard Model

It is not only about H0 and CMB

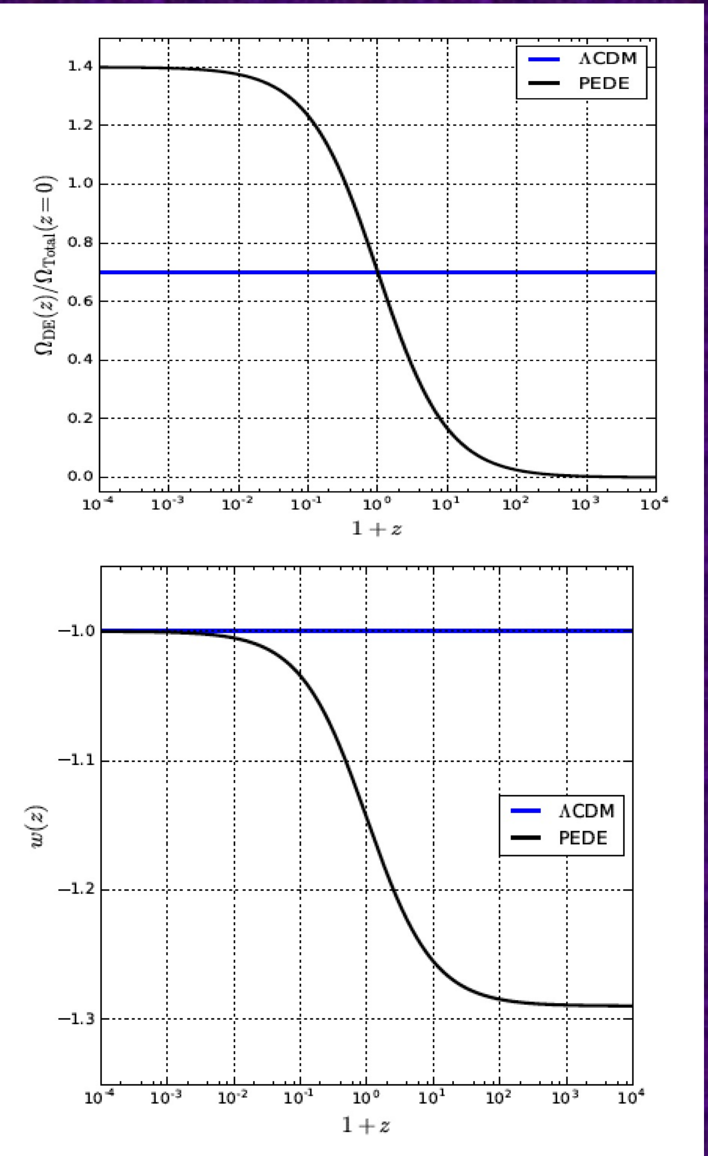


DESI-Y1 (2024), arXiv:2404.03002



DES-Y3 + KiDS-1000, arXiv:2305.17173

Phenomenologically Emergent Dark Energy (PEDE)



No Dark Energy in the past and it acts as an emergent phenomena:

Allows lower rate of expansion in the past and higher rate of expansion at late times

$$\Omega_{\rm DE}(z) = \Omega_{\rm DE,0} \times [1 - \tanh(\log_{10}(1+z))]$$

$$\begin{aligned} w(z) &= -\frac{1}{3\ln 10} \times \frac{1 - \tanh^2[\log_{10}(1+z)]}{1 - \tanh[\log_{10}(1+z)]} - 1 \\ &= -\frac{1}{3\ln 10} \times (1 + \tanh[\log_{10}(1+z)]) - 1. \end{aligned}$$

Li and Shafieloo, ApJ Lett 2019

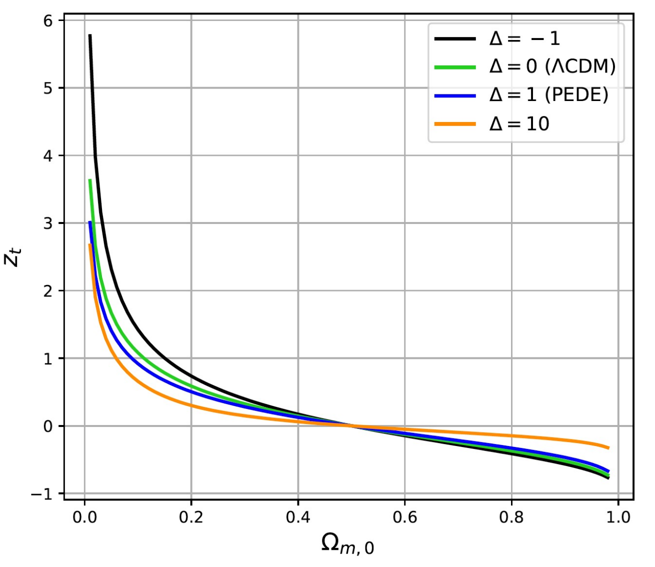
Generalized Emergent Dark Energy (GEDE)

$$\widetilde{\Omega}_{\rm DE}(z) = \Omega_{\rm DE,0}\frac{1-\tanh\left(\Delta\times\log_{10}(\frac{1+z}{1+z_t})\right)}{1+\tanh\left(\Delta\times\log_{10}(1+z_t)\right)}$$

$$w(z) = -\frac{\Delta}{3\ln 10}\times\left(1+\tanh\left(\Delta\times\log_{10}(\frac{1+z}{1+z_t})\right)\right)-1.$$

-Has one degree of freedom for DE sector

-LCDM and PEDE are both included at special limits



$$\Omega_{\rm DE}(z_t) = \Omega_{m,0}(1+z_t)^3$$

$\Delta = 0$ LCDM

$\Delta = 1$ PEDE

Li and Shafieloo, ApJ 2020 (arXiv:2001.05103)

Generalized Emergent Dark Energy (GEDE)

Data	$\ln B_{ij}$
Planck 2018	2.9
Planck 2018+BAO	0.8
Planck 2018+R19	12.1
Planck 2018+BAO+R19	7.9
Planck 2018+JLA	−0.2
Planck 2018+Pantheon	−0.9
Planck 2018+BAO+JLA+R19	6.1
Planck 2018+BAO+Pantheon+R19	5.8

Full analysis using various combination of the data

$\Delta \log Z$	Evidence against M_1
0 to 1	Negligible
1 to 3	Positive
3 to 5	Strong
> 5	Very strong

Model Comparison:
Bayesian evidence analysis in strong support of emergent dark energy

W. Yang, et al, PRD 2021 [arXiv:2103.03815]

Generalized Emergent Dark Energy (GEDE)

Data	$\ln B_{ij}$
Planck 2018	2.9
Planck 2018+BAO	0.8
Planck 2018+R19	12.1
Planck 2018+BAO+R19	7.9
Planck 2018+JLA	−0.2
Planck 2018+Pantheon	−0.9
Planck 2018+BAO+JLA+R19	6.1
Planck 2018+BAO+Pantheon+R19	5.8

Full analysis using various combination of the data

Current tensions allow us to find models statistically better (?) than LCDM but are all tensions resolved?

Model Comparison:
Bayesian evidence analysis in strong support of emergent dark energy

No!

True for any successful evolving DE model!

W. Yang,et al, PRD 2021 [arXiv:2103.03815]

Distribution of Bayesian Evidence:

- Be cautious about Jeffery's scale!

Jeffreys scale Z_i/Z_j	Kass-Rafferty scale Z_i/Z_j	Interpretation
1 to 3.2	1 to 3	Not worth mentioning
3.2 to 10	3 to 20	Positive
10 to 100	20 to 150	Strong
> 100	>150	Very Strong

Distribution of Bayes factors can greatly depend on the models and the data!

On The Distribution of Bayesian Evidences

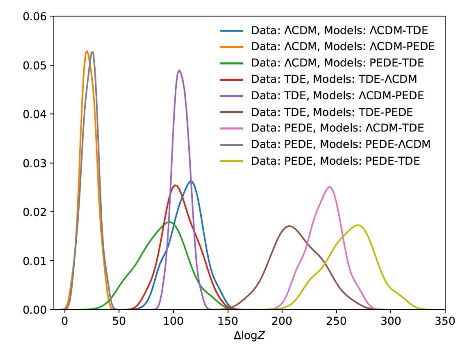
Ryan E. Keeley,[1,2]★ Arman Shafieloo,[2,3]†
[1]Department of Physics, University of California Merced, 5200 North Lake Road, Merced, CA 95343, USA
[2]Korea Astronomy and Space Science Institute (KASI), 776 Daedeok-daero, Yuseong-gu, Daejeon 34055, Korea
[3]KASI Campus, University of Science and Technology, 217 Gajeong-ro, Yuseong-gu, Daejeon 34113, Korea

Accepted XXX. Received YYY; in original form ZZZ

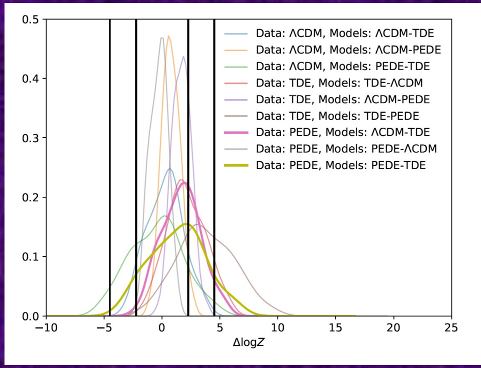
ABSTRACT
We look at the distribution of the Bayesian evidence for mock realizations of supernova and baryon acoustic oscillation data. The ratios of Bayesian evidences of different models are often used to perform model selection. The significance of these Bayes factors are then interpreted using scales such as the Jeffreys or Kass & Raftery scale. First, we demonstrate how to use the evidence itself to validate the model, that is to say how well a model fits the data, regardless of how well other models perform. The basic idea is that if, for some real dataset a model's evidence lies outside the distribution of evidences that result when the same fiducial model that generates the datasets is used for the analysis, then the model in question is robustly ruled out. Further, we show how to assess the significance of a hypothetically computed Bayes factor. We show that the range of the distribution of Bayes factors can greatly depend on the models in question and also the number of data points in the dataset. Thus, we have demonstrated that the significance of Bayes factors needs to be calculated for each unique dataset.

Key words: dark energy – cosmological parameters – methods: statistical

Keeley and Shafieloo, MNRAS 2022



Data with OK quality



Data with worse quality

Data with OK quality

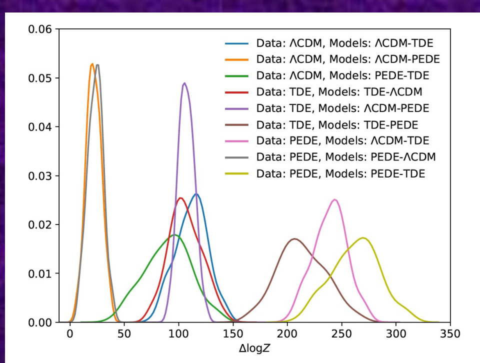
Bayes Factor:

Point 3

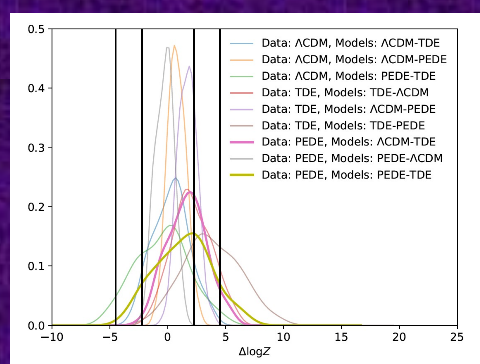
- Be cautious about Jeffery's scale!

Jeffreys scale Z_i/Z_j	Kass-Rafferty scale Z_i/Z_j	Interpretation
1 to 3.2	1 to 3	Not worth mentioning
3.2 to 10	3 to 20	Positive
10 to 100	20 to 150	Strong
> 100	>150	Very Strong

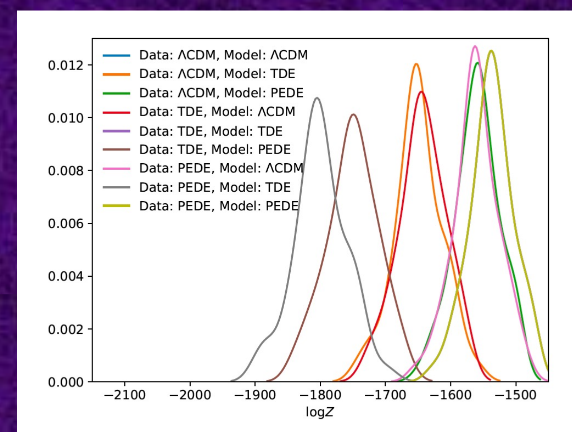
Distribution of Bayes factors can greatly depend on the models and the data!



Data with OK quality



Data with worse quality



Keeley and Shafieloo, MNRAS 2022

See also:

Starkman et al, arXiv:0811.2415
Jenkins & Peacock, MNRAS 2011;
Nesseris & Garcia-Bellido, JCAP 2013;
Joachimi et al., A&A 2021;

Data with OK quality

Model Validation

Bayesian evidence approach is solid but only can find the better model among the candidates (or less wrong model/ranking models)

Importance of Model Validation

$\Delta \log Z > 3$	PEDE consistent	PEDE ruled-out
ΛCDM consistent	6	994
ΛCDM ruled-out	0	0
$\Delta \log Z > 5$	PEDE consistent	PEDE ruled-out
ΛCDM consistent	89	911
ΛCDM ruled-out	0	0

Conventional Bayesian Evidence Approach

→When true model is unknown, finding a statistical anchor is not trivial

→One can attempt using reliable non-parametric/model independent reconstructions

Both models are wrong!

Point 4

Koo, Keeley, Shafieloo, L'Huillier, JCAP 2022

Iterative Smoothing Method

- The non-parametric method to reconstruct the distance modulus and expansion history of the universe
  Shafieloo et al. 2006, 2018; Shafieloo. 2007; Shafieloo & Clarkson 2010

- Starts from initial guess of distance modulus, but generates model-independent reconstruction of distance modulus with lower χ^2 value after numerous iterations

$$\hat{\mu}_{n+1}(z) = \hat{\mu}_n(z) + \frac{\boldsymbol{\delta\mu_n}^T \cdot \mathbf{C}^{-1} \cdot \boldsymbol{W}(z)}{\mathbf{1}^T \cdot \mathbf{C}^{-1} \cdot \boldsymbol{W}(z)}$$ ($\mathbf{C}$: Covariance matrix of the data)

$$\mathbf{1}^T = (1, \cdots, 1), \boldsymbol{W}_i(z) = \exp\left(-\frac{\ln^2\left(\frac{1+z}{1+z_i}\right)}{2\Delta^2}\right), \boldsymbol{\delta\mu_n}|_i = \mu_i - \hat{\mu}_n(z_i)$$ ($\boldsymbol{\Delta}$: Smoothing width)

$$\chi_n^2 = \boldsymbol{\delta\mu_n}^T \cdot \mathbf{C}^{-1} \cdot \boldsymbol{\delta\mu_n}$$

- Derive the **likelihood distribution** function $P(\Delta\chi^2)$ (for a large number of data realizations), where $\Delta\chi^2 = \chi^2_{\text{smooth}} - \chi^2_{\text{best-fit}}$, when the true model is assumed
  Koo et al. 2021, JCAP, 03, 034

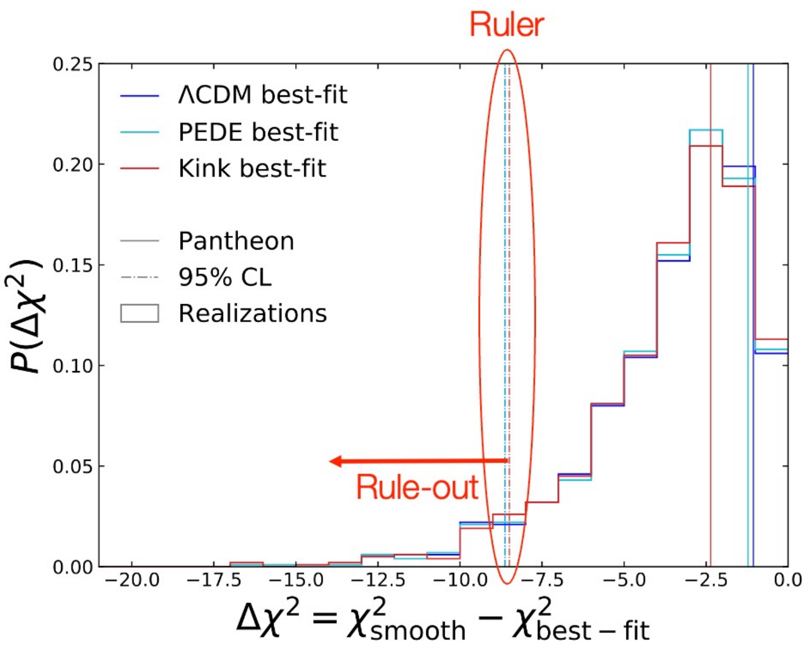
- χ^2_{smooth}: χ^2 of the converged reconstruction using smoothing method

- $\chi^2_{\text{best-fit}}$: Best-fit χ^2 of the correct model fits the data

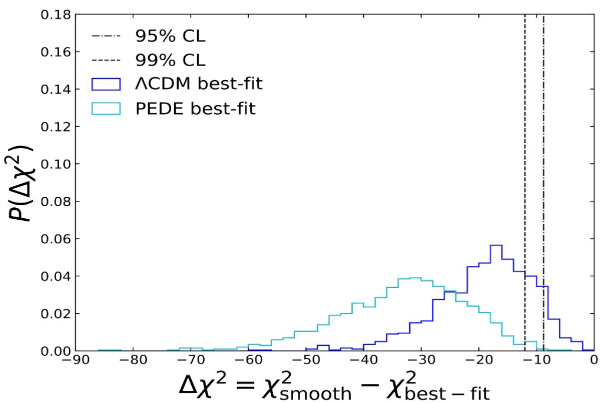
Testing Models based on Likelihood Distribution

- $P(\Delta\chi^2)$ have no dependence on the true model and depends only on the covariance matrix of the data
  → One $\Delta\chi^2$ for given confidence (Ruler)
  Koo et al. 2021, JCAP, 03, 034

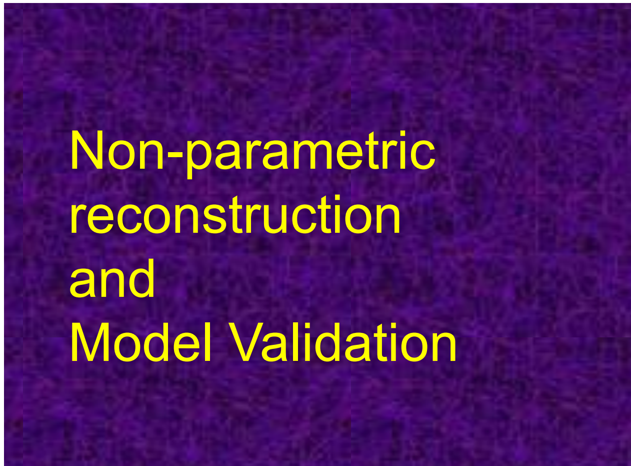
- The model being tested is ruled out if the $\Delta\chi^2$ value is lower than the ruler



- Likelihood distributions exclude both models



95% CL	PEDE consistent	PEDE ruled-out
ΛCDM consistent	2	82
ΛCDM ruled-out	0	916
99% CL	**PEDE consistent**	**PEDE ruled-out**
ΛCDM consistent	14	193
ΛCDM ruled-out	0	793



Model Validation

Bayesian evidence approach is solid but only can find the better model among the candidates (or less wrong model/ranking models)

Importance of Model Validation

One can design robust statistical approaches for model validation

$\Delta \log Z > 3$	PEDE consistent	PEDE ruled-out
ΛCDM consistent	6	994
ΛCDM ruled-out	0	0
$\Delta \log Z > 5$	PEDE consistent	PEDE ruled-out
ΛCDM consistent	89	911
ΛCDM ruled-out	0	0

Conventional Bayesian Evidence Approach

95% CL	PEDE consistent	PEDE ruled-out
ΛCDM consistent	2	82
ΛCDM ruled-out	0	916
99% CL	PEDE consistent	PEDE ruled-out
ΛCDM consistent	14	193
ΛCDM ruled-out	0	793

Iterative smoothing validation approach

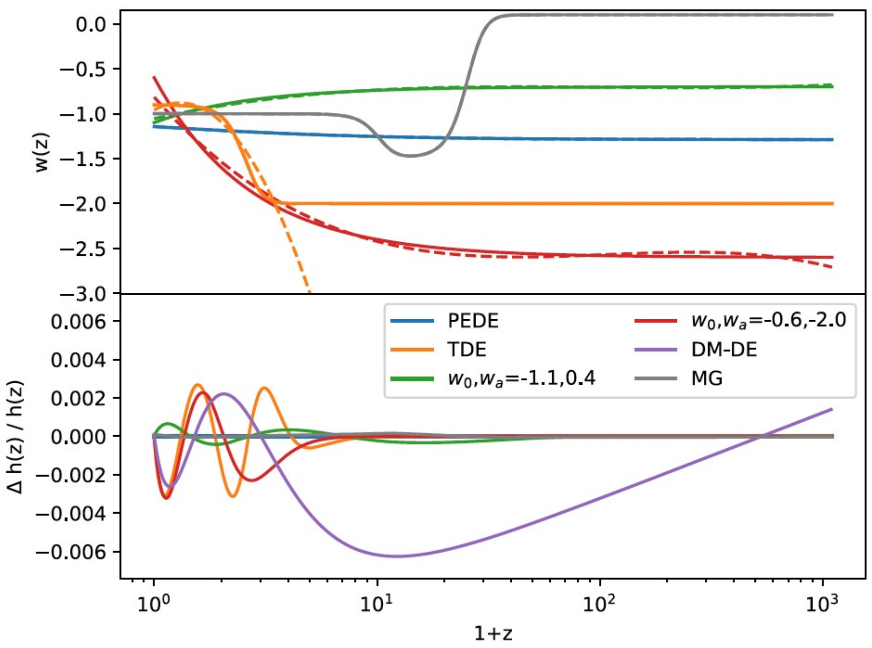
Both models are wrong!

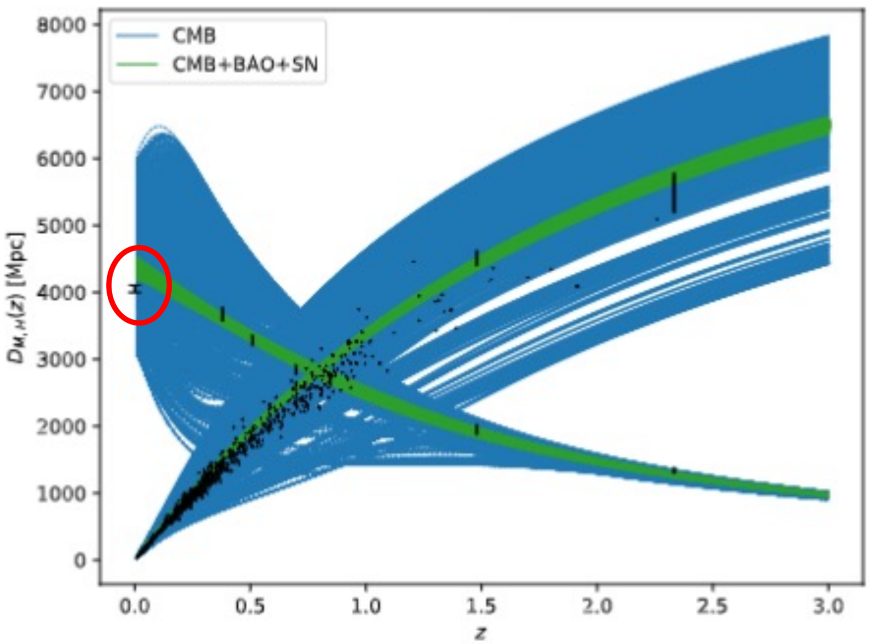
Point 4

Koo, Keeley, Shafieloo, L'Huillier, JCAP 2022

Application of model validation

Ruling Out New Physics at Low Redshift as a solution to the H0 Tension



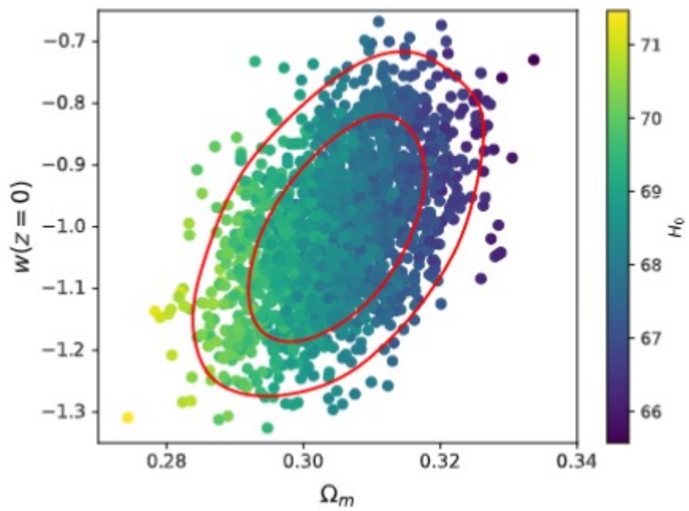


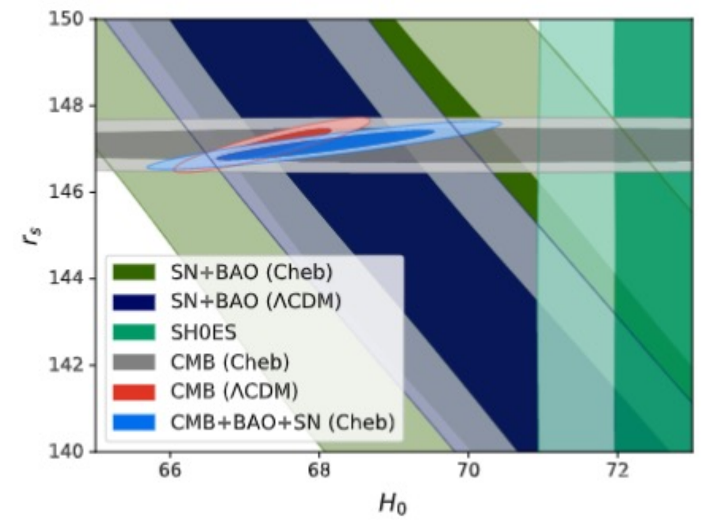
Exploring an **extensive** physical space with Crossing functions for validation (Chebyshev polynomials)

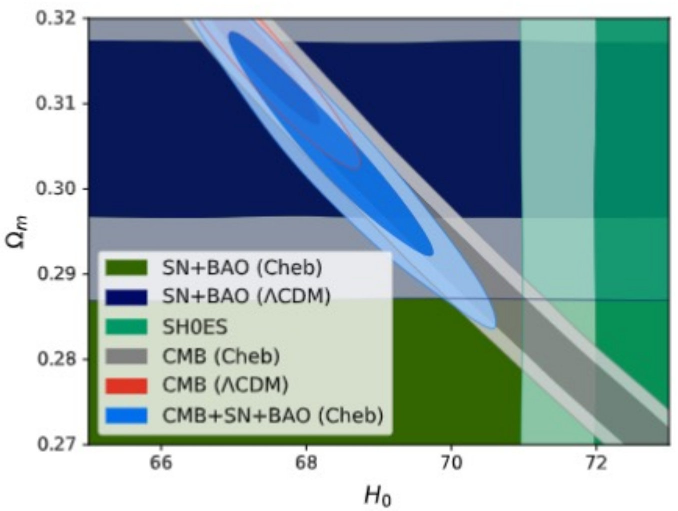
Keeley and Shafieloo, Phys. Rev. Lett, 2023

Application of model validation

Ruling Out New Physics at Low Redshift as a solution to the H0 Tension





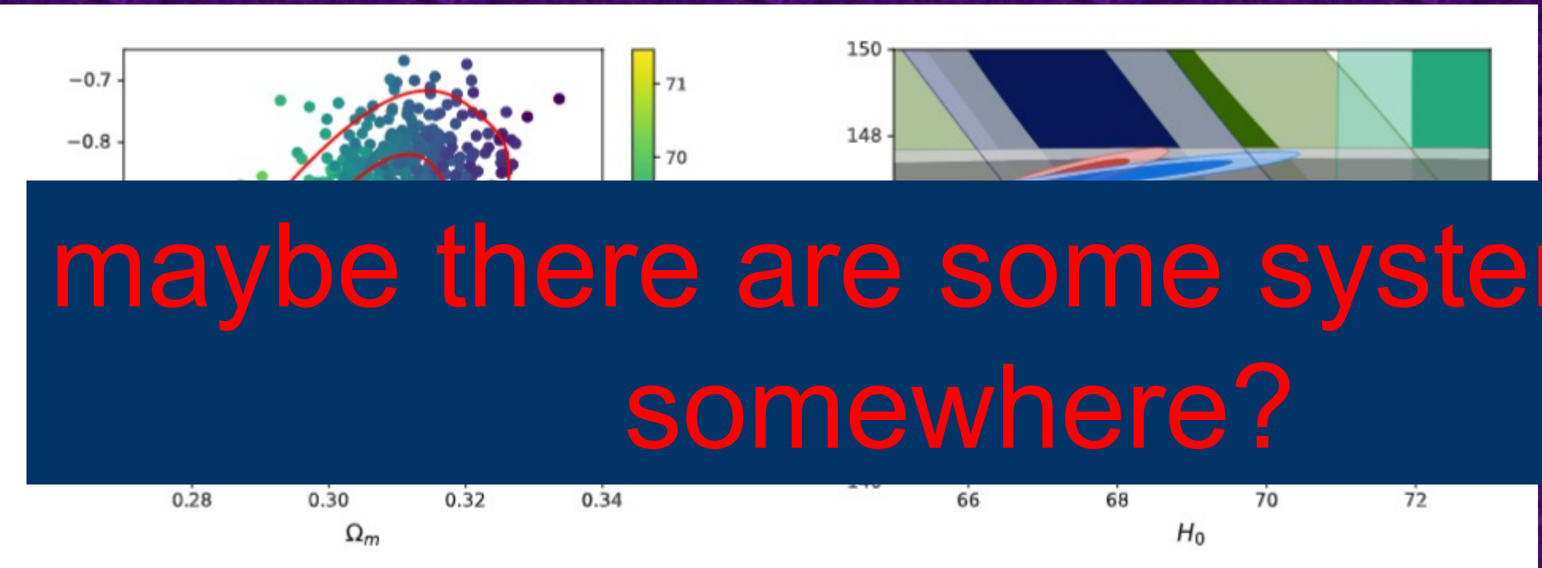


Even in such extensive physical space, inference on H0 is not consistent with SH0ES.

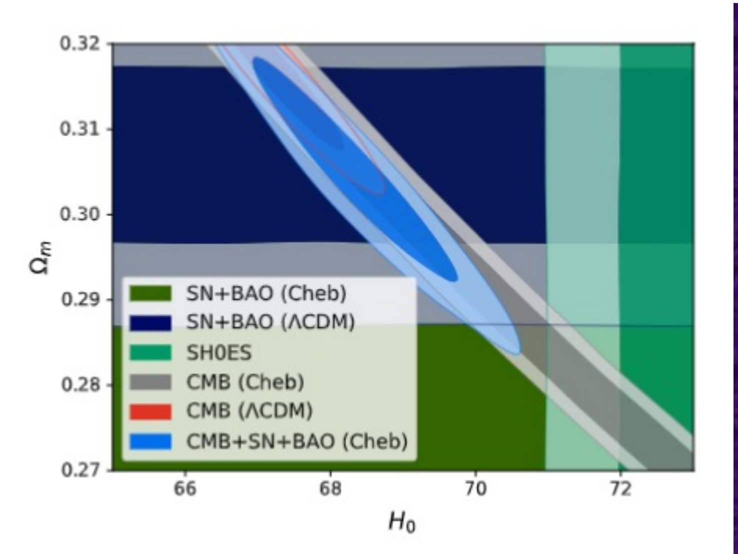
Keeley and Shafieloo, Phys. Rev. Lett, 2023

Application of model validation

Isn’t it suspicious that nothing works?!



maybe there are some systematics somewhere?



Even in such extensive physical space, inference on H0 is not consistent with SH0ES.

Validation of a large number of models can hints towards systematic

(Present)

Lets talk about tensions again…

Standard Model of Cosmology

Universe is Flat

Universe is Isotropic

Universe is Homogeneous

Dark Energy is Lambda (w=-1)

Power-Law primordial spectrum (n_s=const)

Dark Matter is cold

All within framework of FLRW

On Importance of non-parametric and Model Independent Reconstruction

When we don't know what to look for!

Point 5

Let's Reconstruction Leads the way! **Point 5**

Model Independent Reconstruction of Primordial Spectrum

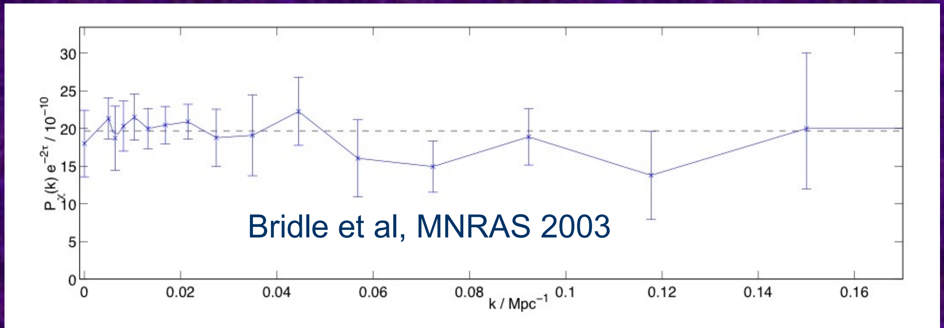
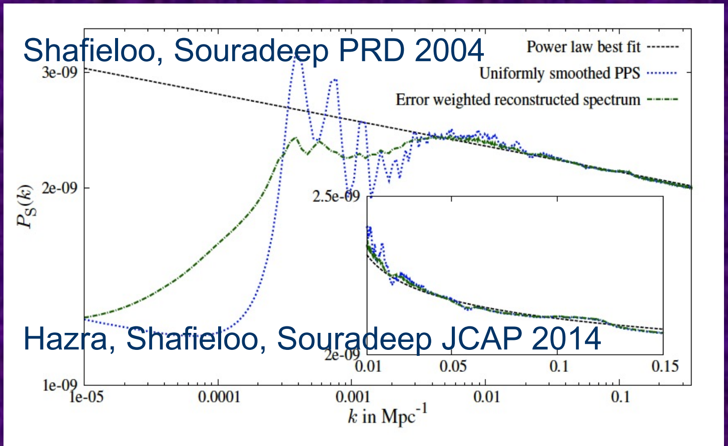
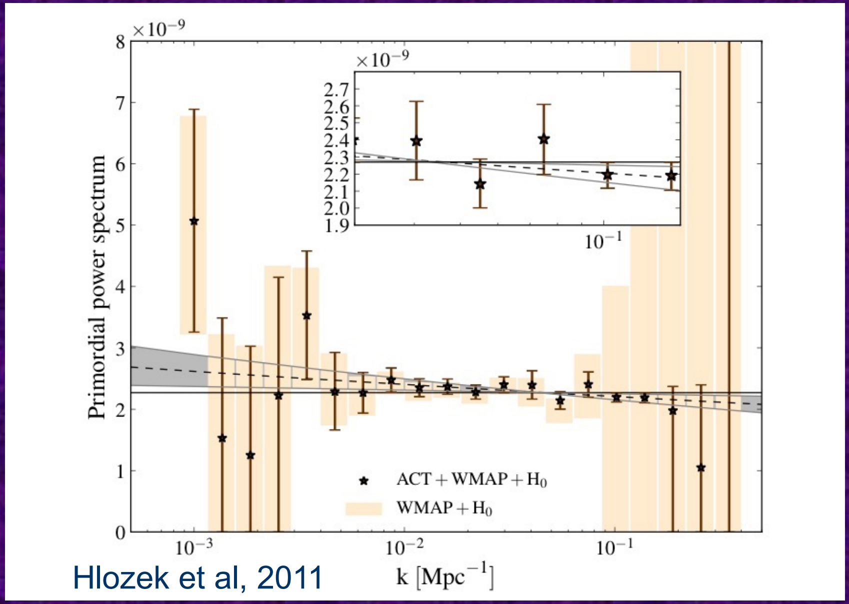
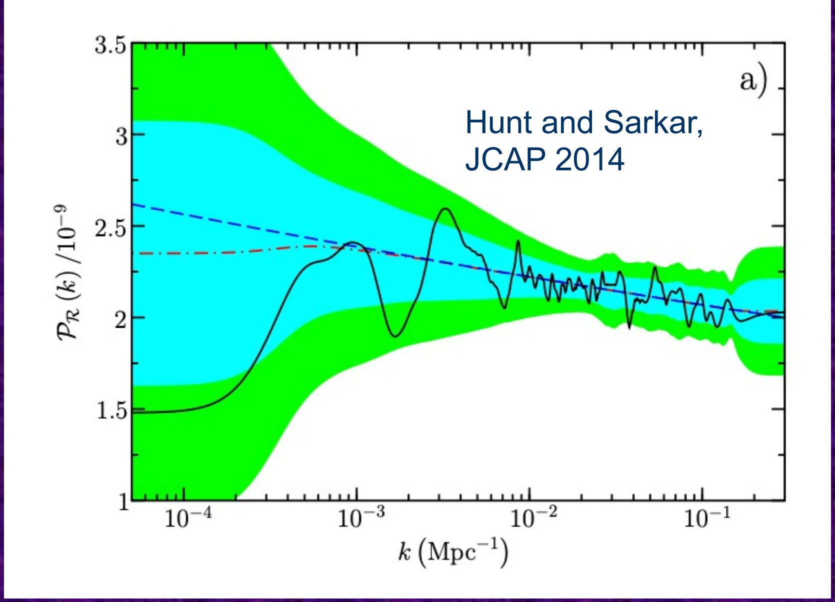


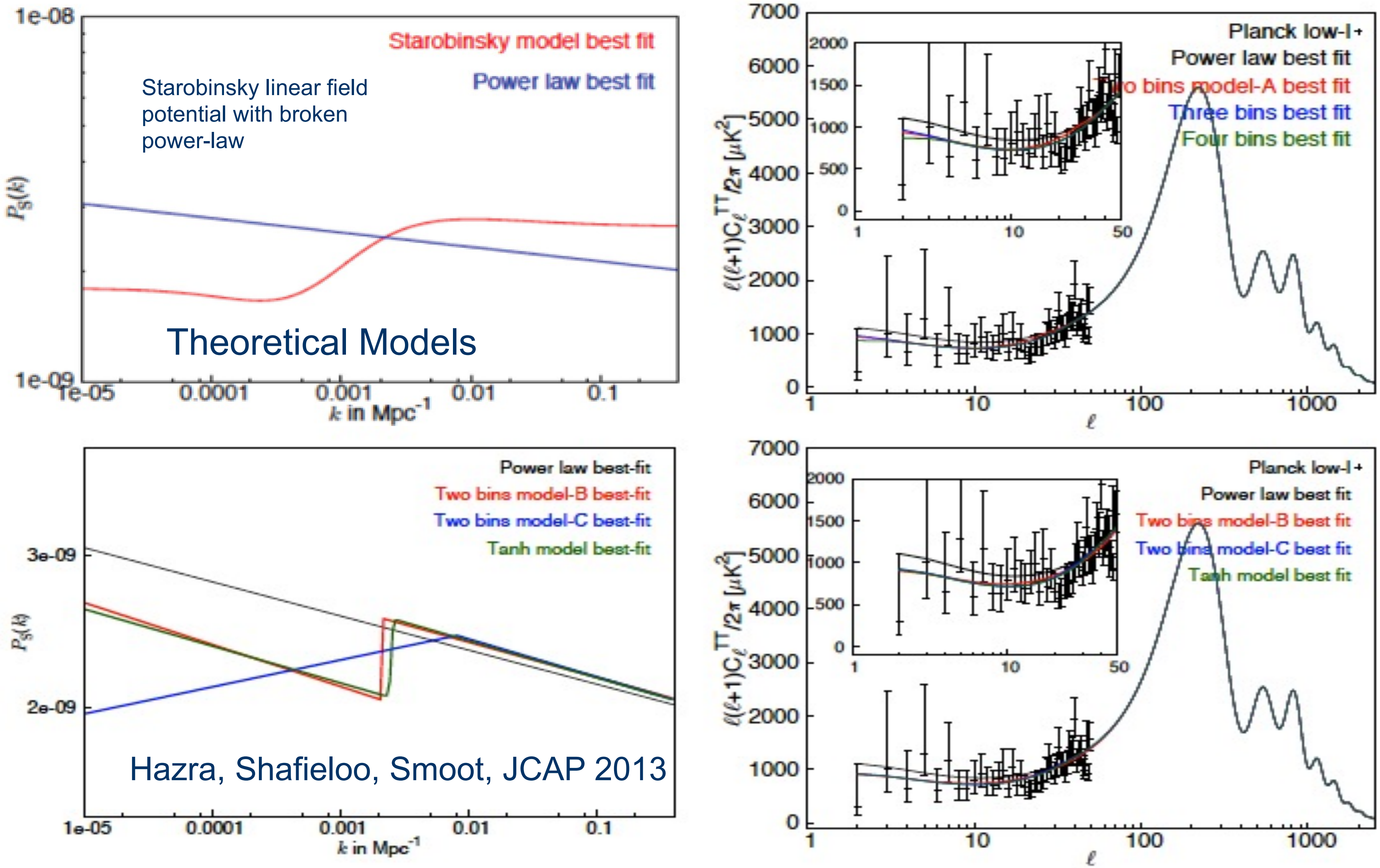
Figure 4. Reconstruction of the shape of the primordial power spectrum in 16 bands after marginalising over the Hubble constant, baryon and dark matter densities, and the redshift of reionization.







Beyond Power-Law: there are some other models consistent to the data.

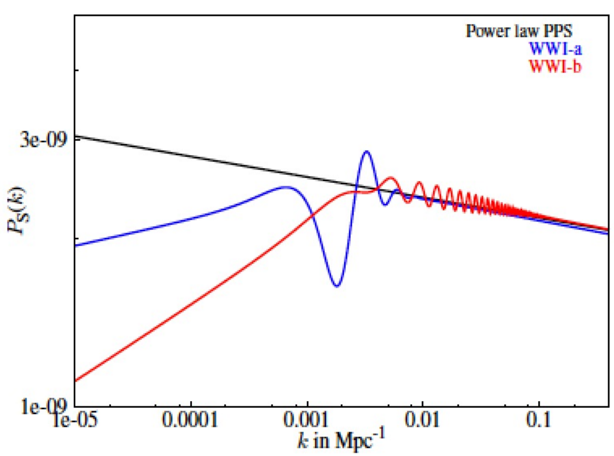


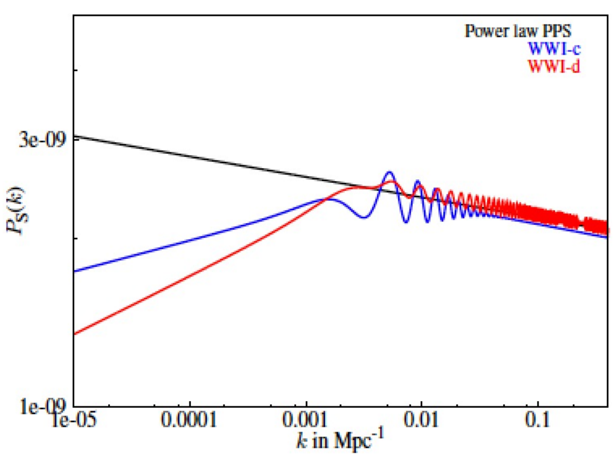
Individual likelihoods comparison						
Individual likelihood	Baseline	WWI-a $\Delta_{\rm DOF}=4$	WWI-b $\Delta_{\rm DOF}=4$	WWI-c $\Delta_{\rm DOF}=4$	WWI-d $\Delta_{\rm DOF}=4$	WWI′ $\Delta_{\rm DOF}=2$
TT	761.1	762	761.9	762.8	762.8	762.4
lowT	15.4	8.2	13.4	12.1	13	10.2
Total	778.1	772.1 (-6)	777 (-1.1)	777 (-1.1)	778.4 (0.3)	775 (-3.1)
EE	751.2	748.8	747.2	748.6	750.2	746.8
lowTEB	10493.6	10490	10495.6	10492.4	10495.7	10492.2
Total	11248.8	11241.8 (-7)	11246.2 (-2.6)	11244.5 (-4.3)	11249.3 (0.5)	11242.3 (-6.5)
TTTEEE	2431.7	2432.7	2422.6	2427.8	2421.7	2426.5
lowTEB	10497	10490.8	10495.1	10493.4	10495.3	10492.7
Total	12935.6	12929.5 (-6.1)	12924.2 (-11.4)	12927.6 (-8)	12923.4 (-12.2)	12925.2 (-10.4)
TT	764.5	763.6	762.2	764.4	762.9	762.8
EE	753.9	754.8	750.5	750.8	750.8	751
TE	932	933.4	928.7	929.2	927	928.8
lowTEB	10498.4	10490.4	10495.8	10493.7	10495.6	10492.4
BKP	41.6	42	42	42.6	41.8	42.9
Total	12997	12991 (-6)	12985.9 (-11.1)	12987.2 (-9.8)	12985 (-12)	12985.1 (-11.9)
TTTEEE	2431.7	2432.8	2421.4	2426.7	2421	2425.7
lowTEB	10498.5	10490.5	10495.5	10493.6	10495.8	10492.6
BKP	41.6	42	42.7	42	41.9	42.5
Total	12978.3	12971.3 (-7)	12967.3 (-11)	12968.6 (-9.7)	12965 (-13.3)	12968.6 (-9.7)
TT (bin1)	8402.1	8404.1	8403.9	8405.2	8402.1	8401.9
lowT	15.4	8.3	13.3	11.9	13.2	10.3
Total	8419.6	8414.7 (-4.9)	8419.5 (-0.1)	8419.8 (0.2)	8418.1 (-1.5)	8414.4 (-5.2)
TTTEEE (bin1)	24158.2	24158.6	24149	24155	24148.4	24151.5
lowTEB	10497.6	10490.3	10493.4	10493.6	10495.3	10492.7
Total	34661.9	34655.3 (-6.6)	34650.5 (-11.4)	34654.4 (-7.5)	34649.5 (-12.4)	34650.6 (-11.3)

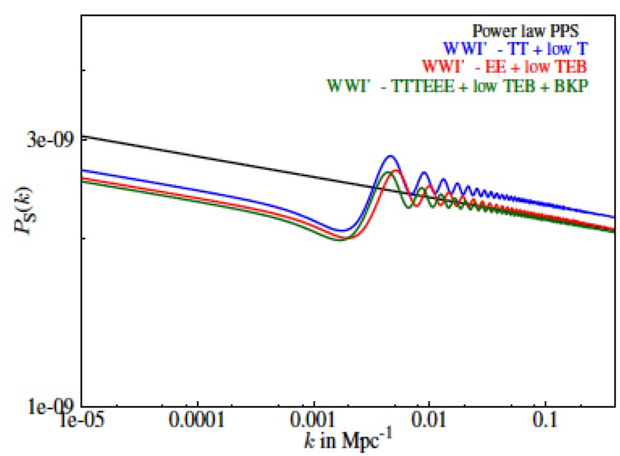
Beyond Power-Law: ***there are some other models consistent to the data.***

Whipped Inflation

Hazra, Shafieloo, Smoot, JCAP 2013
Hazra, Shafieloo, Smoot, Starobinsky, JCAP 2014A
Hazra, Shafieloo, Smoot, Starobinsky, JCAP 2014B
Hazra, Shafieloo, Smoot, Starobinsky, Phys. Rev. Lett 2014
Hazra, Shafieloo, Smoot, Starobinsky, JCAP 2016
Hazra et al, JCAP 2018
Debono, Hazra, Shafieloo, Smoot, Starobinsky, MNRAS 2020
Hazra, Paoletti, Debono, Shafieloo, Smoot, Starobinsky, JCAP 2021







Forms of PPS and Effects on the Background Cosmology

- Flat Lambda Cold Dark Matter Universe (LCDM) with power–law form of the primordial spectrum
- It has 6 main parameters.

Ω_b

Ω_m

H_0

τ

1

2

G(l,k)

$$C_l = \sum G(l,k)P(k)$$

3

$$C_l^{obs}$$

2

1

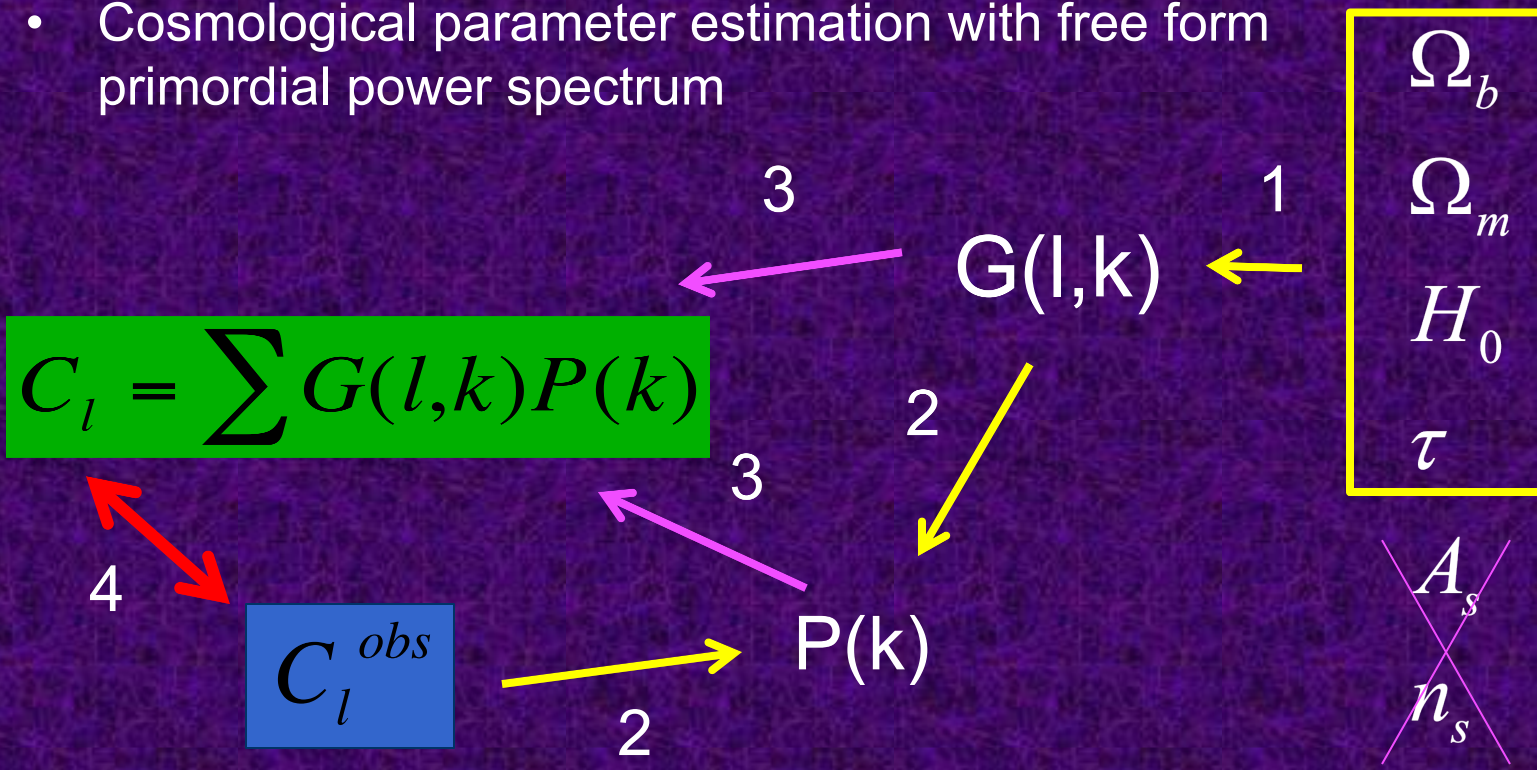
A_s

n_s

$$P(k) = A_\mathrm{s}[\frac{k}{k_*}]^{n_\mathrm{s}-1}$$

Forms of PPS and Effects on the Background Cosmology

- Cosmological parameter estimation with free form primordial power spectrum



Modified Richardson-Lucy Deconvolution

- ➔ Iterative algorithm.
- ➔ Not sensitive to the initial guess.
- ➔ Enforce positivity of P(k).

[$G(l,k)$ is positive definite and C_l is positive]

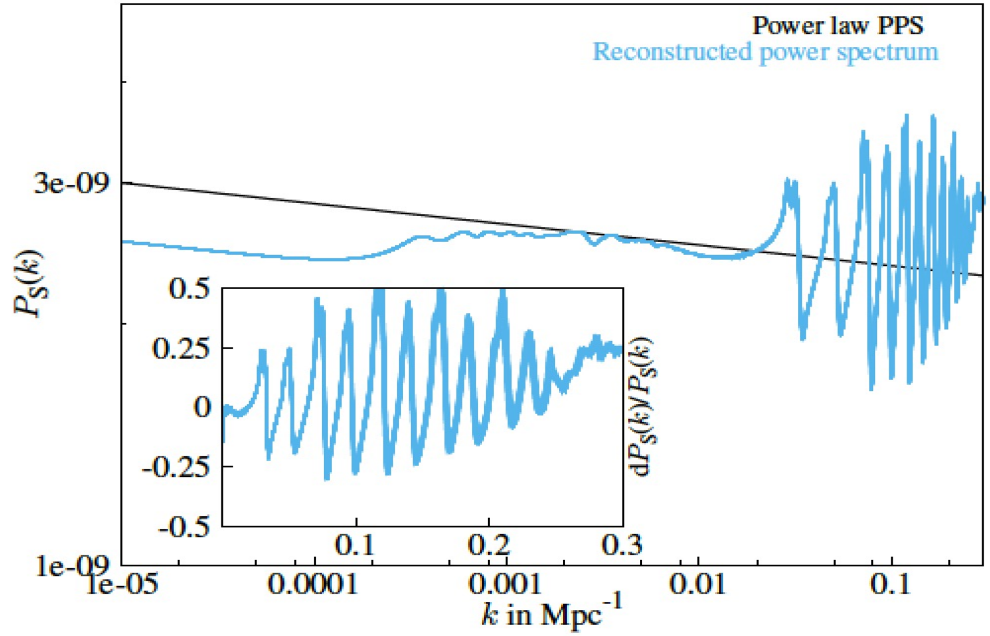
$$C_\ell = \sum_i G_{\ell k_i} P_{k_i}$$

$$\begin{aligned} P_k^{(i+1)} - P_k^{(i)} &= P_k^{(i)} \times \left[\sum_{\ell=2}^{\ell=900} \widetilde{G}_{\ell k}^{\text{un-binned}} \left\{ \left(\frac{C_\ell^{\rm D} - C_\ell^{\rm T(i)}}{C_\ell^{\rm T(i)}} \right) \tanh^2 \left[Q_\ell (C_\ell^{\rm D} - C_\ell^{\rm T(i)}) \right] \right\}_{\text{un-binned}} \right. \\ &+ \left. \sum_{\ell_{\text{binned}} > 900} \widetilde{G}_{\ell k}^{\text{binned}} \left\{ \left(\frac{C_\ell^{\rm D} - C_\ell^{\rm T(i)}}{C_\ell^{\rm T(i)}} \right) \tanh^2 \left[\frac{C_\ell^{\rm D} - C_\ell^{\rm T(i)}}{\sigma_\ell^{\rm D}} \right]^2 \right\}_{\text{binned}} \right], \end{aligned} \quad (1)$$

Shafieloo & Souradeep PRD 2004 ;
Shafieloo et al, PRD 2007;
Shafieloo & Souradeep, PRD 2008;
Nicholson & Contaldi JCAP 2009
Hamann, Shafieloo & Souradeep JCAP 2010
Hazra, Shafieloo & Souradeep PRD 2013
Hazra, Shafieloo & Souradeep JCAP 2013
Hazra, Shafieloo & Souradeep JCAP 2014
....
Sohn, Shafieloo, Hazra, JCAP 2024

$$Q_\ell = \sum_{\ell'} (C_{\ell'}^{\rm D} - C_{\ell'}^{\rm T(i)}) COV^{-1}(\ell, \ell'),$$

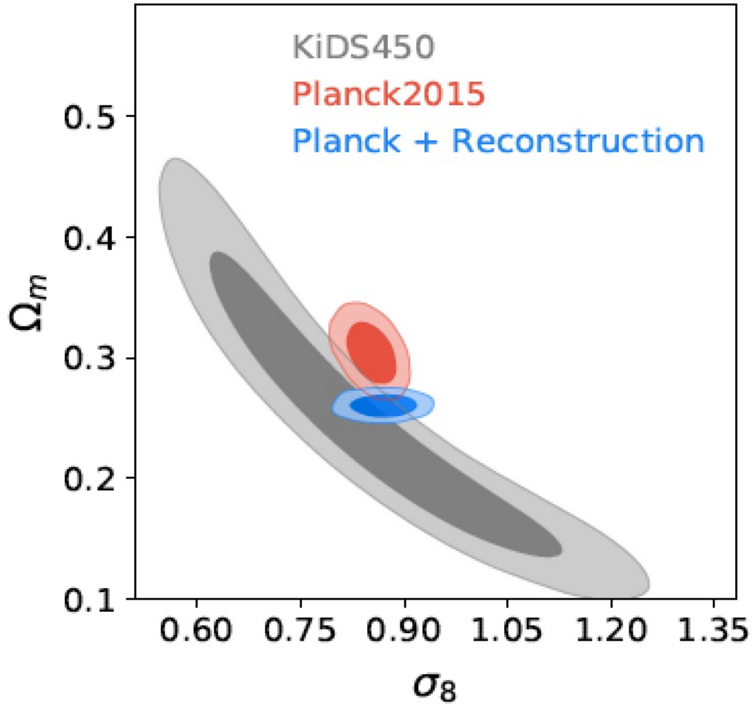
Theoretical Implication: Importance of the Features in the primordial spectrum

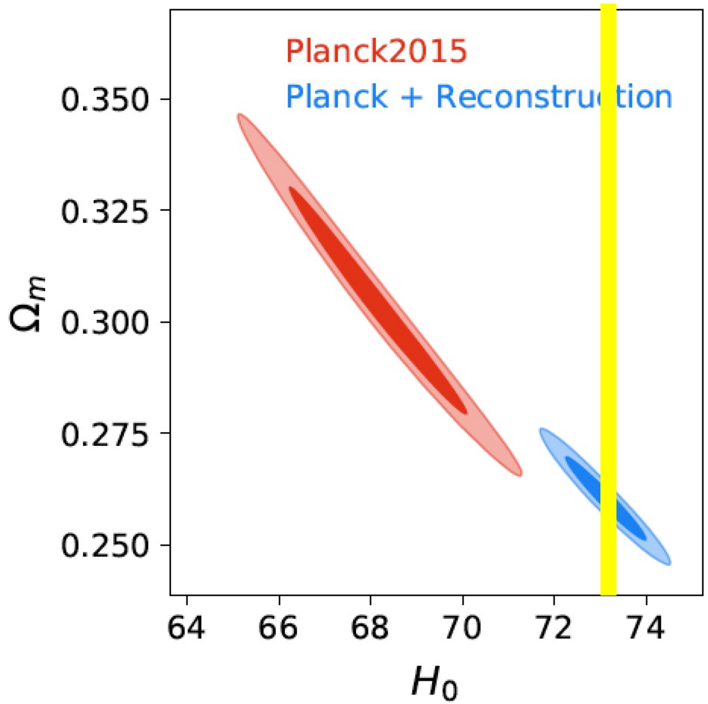


Hazra, Shafieloo, Souradeep, JCAP 2019
Keeley et al, MNRAS 2020

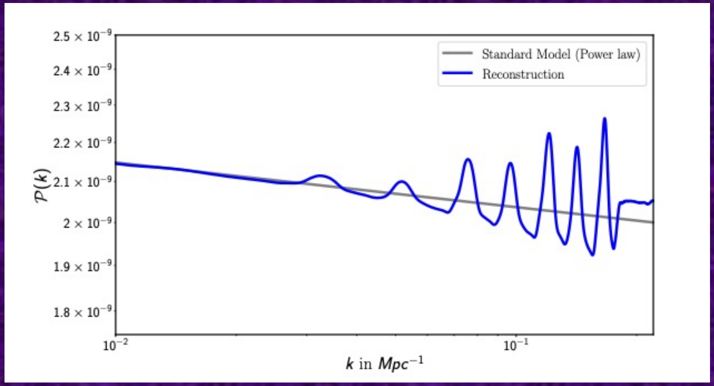
Background Cosmological Parameters and PPS

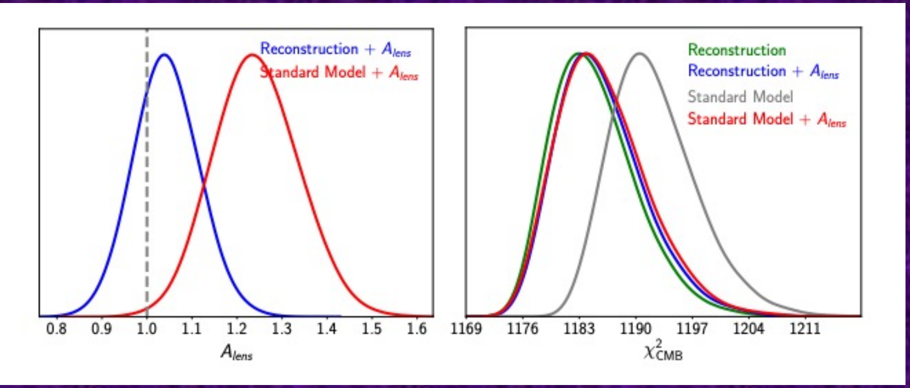
We use the reconstructed PPS for parameter estimation, similar to what we do with PL.



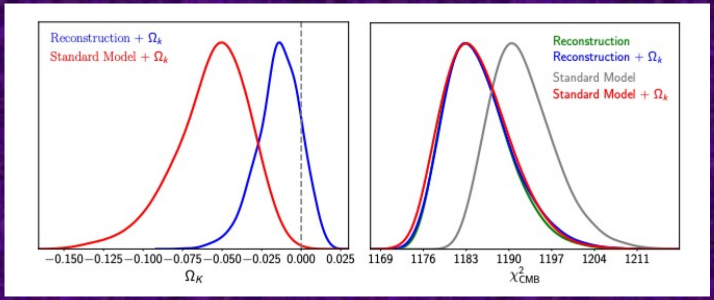


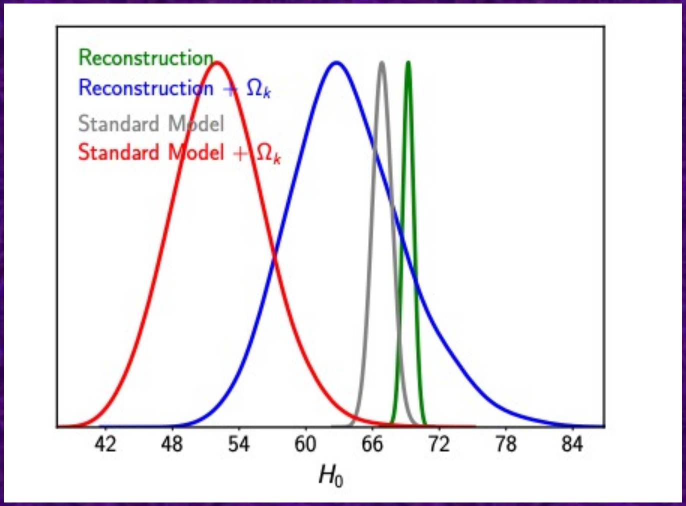
One spectrum to cure them all: looking for signature from early Universe to solve major anomalies and tensions in cosmology





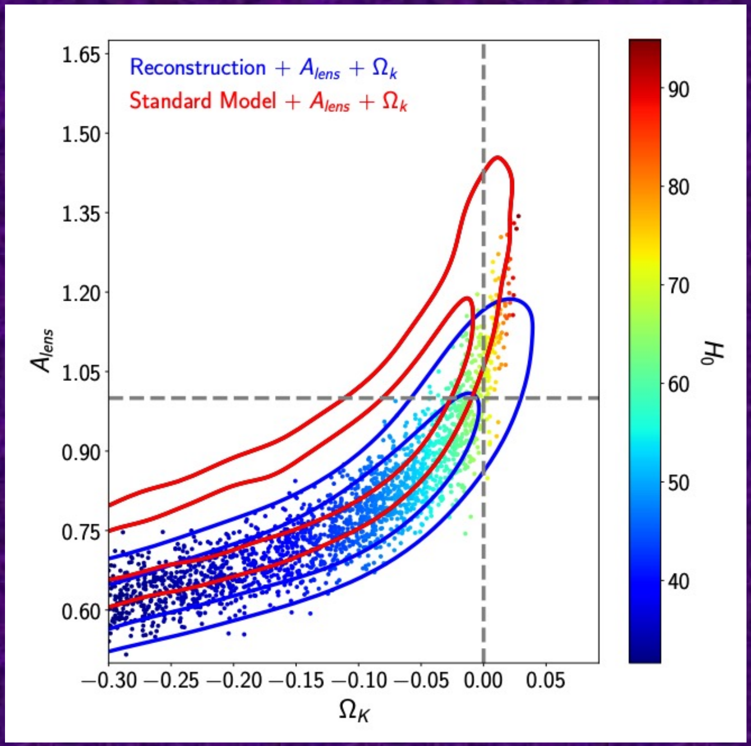
Curvature and A_lens anomalies

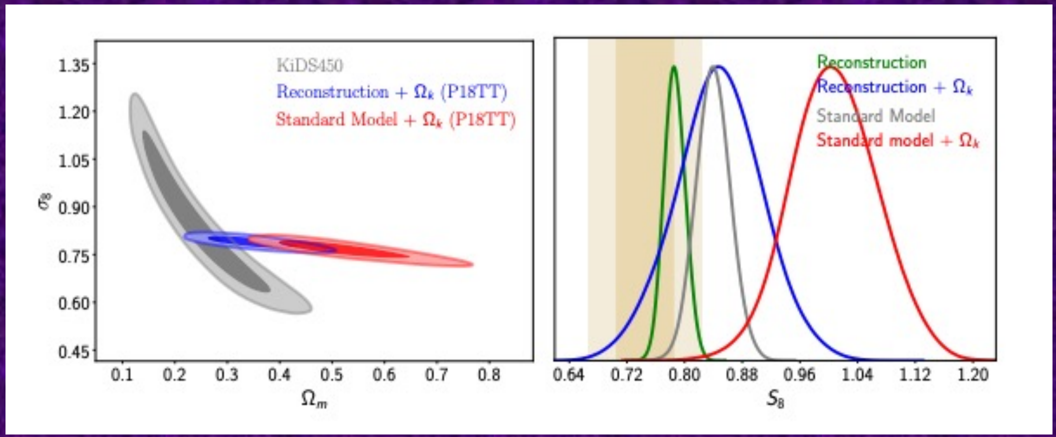




Hazra, Antony, Shafieloo : JCAP 2022

One spectrum to cure them all: Signature from early Universe solves major anomalies and tensions in cosmology





Hazra, Antony, Shafieloo :JCAP 2022

Addressing Majour Anomalies and tensions

Now we know what to look for!

Point 5

Reconstruction → Phenomenology → Theory

See Antony, Finelli, Hazra, Shafieloo, Phys Rev Lett 2023, for theoretical implication

Reconstructing Dark Energy

To find cosmological quantities and parameters there are two general approaches:

1. Parametric methods

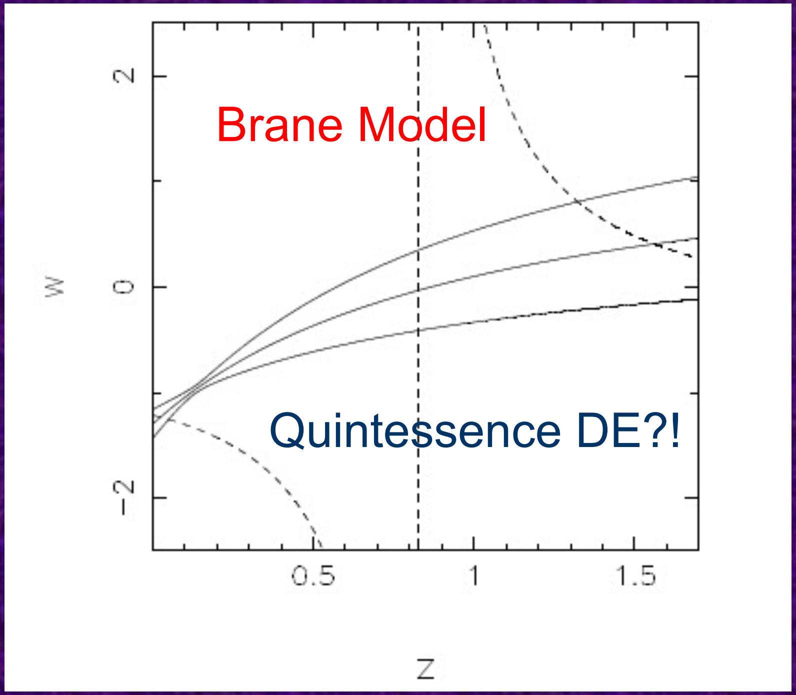
   Easy to confront with cosmological observations to put constrains on the parameters, but the results are highly biased by the assumed models and parametric forms.

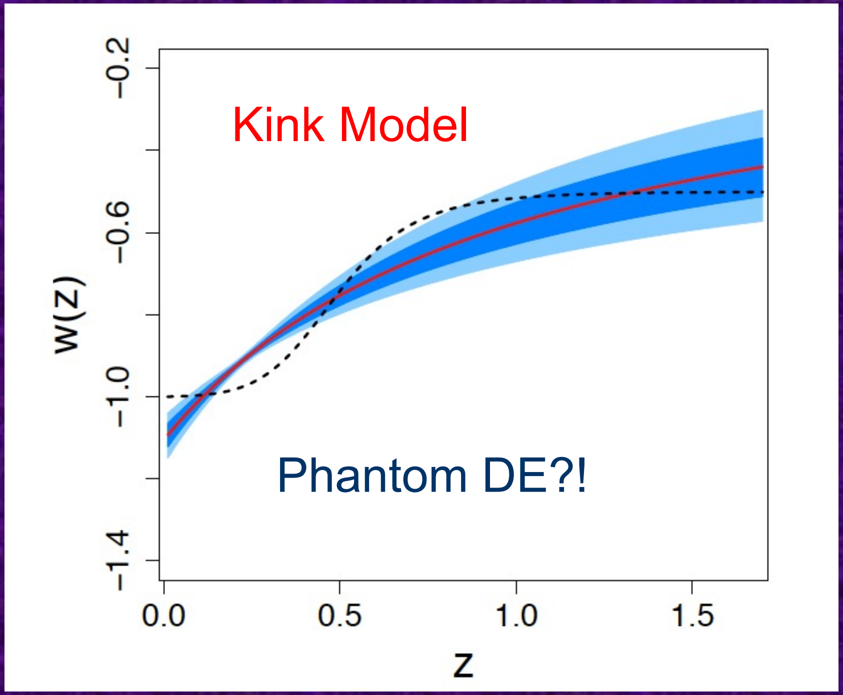
2. Non Parametric methods

   Difficult to apply ***properly*** on the raw data, but the results will be less biased and more reliable and independent of theoretical models or parametric forms.

.

Problems of Dark Energy Parameterizations (model fitting)





Shafieloo, Alam, Sahni & Starobinsky, MNRAS 2006

Holsclaw et al, PRD 2011

$$w(z) = w_0 - w_a \frac{z}{1+z}.$$

Chevallier-Polarski-Linder ansatz (CPL).

Testing deviations from an assumed model

(without comparing different models)

Gaussian Processes:

Modeling of the data around a mean function searching for likely features by looking at the the likelihood space of the hyperparameters.

Bayesian Interpretation of Crossing Statistic:

Comparing a model with its own possible variations.

Gaussian Process

- ➔Efficient in statistical modeling of stochastic variables
- ➔Derivatives of Gaussian Processes are Gaussian Processes
- ➔Provides us with all covariance matrices

Shafieloo, Kim & Linder, PRD 2012
Shafieloo, Kim & Linder, PRD 2013

Data **Mean Function**

$$\begin{bmatrix} \mathbf{y} \\ \mathbf{f} \\ \mathbf{f'} \\ \mathbf{f''} \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mathbf{m}(\mathbf{Z}) \\ \mathbf{m}(\mathbf{Z_1}) \\ \mathbf{m'}(\mathbf{Z_1}) \\ \mathbf{m''}(\mathbf{Z_1}) \end{bmatrix}, \begin{bmatrix} \Sigma_{00}(Z,Z) & \Sigma_{00}(Z,Z_1) & \Sigma_{01}(Z,Z_1) & \Sigma_{02}(Z,Z_1) \\ \Sigma_{00}(Z_1,Z) & \Sigma_{00}(Z_1,Z_1) & \Sigma_{01}(Z_1,Z_1) & \Sigma_{02}(Z_1,Z_1) \\ \Sigma_{10}(Z_1,Z) & \Sigma_{10}(Z_1,Z_1) & \Sigma_{11}(Z_1,Z_1) & \Sigma_{12}(Z_1,Z_1) \\ \Sigma_{20}(Z_1,Z) & \Sigma_{20}(Z_1,Z_1) & \Sigma_{21}(Z_1,Z_1) & \Sigma_{22}(Z_1,Z_1) \end{bmatrix} \right),$$

$$\Sigma_{\alpha\beta} = \frac{d^{(\alpha+\beta)}K}{dz_i^{\alpha} dz_j^{\beta}},$$

$$\begin{bmatrix} \bar{\mathbf{f}} \\ \bar{\mathbf{f}}' \\ \bar{\mathbf{f}}'' \end{bmatrix} = \begin{bmatrix} \mathbf{m}(\mathbf{Z_1}) \\ \mathbf{m'}(\mathbf{Z_1}) \\ \mathbf{m''}(\mathbf{Z_1}) \end{bmatrix} + \begin{bmatrix} \Sigma_{00}(Z_1,Z) \\ \Sigma_{10}(Z_1,Z) \\ \Sigma_{20}(Z_1,Z) \end{bmatrix} \Sigma_{00}^{-1}(Z,Z)\,\mathbf{y}$$

Kernel →

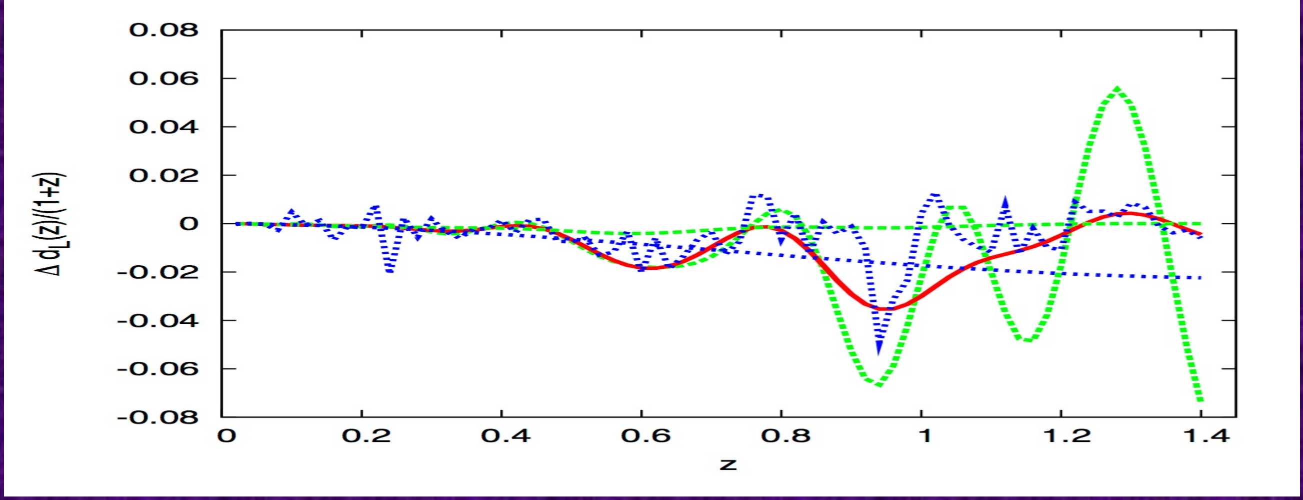
$$k(z,z') = \sigma_f^2 \exp\left(-\frac{|z-z'|^2}{2l^2}\right),$$

GP Hyper-parameters

$$\mathrm{Cov}\left(\begin{bmatrix} \mathbf{f} \\ \mathbf{f'} \\ \mathbf{f''} \end{bmatrix}\right) = \begin{bmatrix} \Sigma_{00}(Z_1,Z_1) & \Sigma_{01}(Z_1,Z_1) & \Sigma_{02}(Z_1,Z_1) \\ \Sigma_{10}(Z_1,Z_1) & \Sigma_{11}(Z_1,Z_1) & \Sigma_{12}(Z_1,Z_1) \\ \Sigma_{20}(Z_1,Z_1) & \Sigma_{21}(Z_1,Z_1) & \Sigma_{22}(Z_1,Z_1) \end{bmatrix} - \begin{bmatrix} \Sigma_{00}(Z_1,Z) \\ \Sigma_{10}(Z_1,Z) \\ \Sigma_{20}(Z_1,Z) \end{bmatrix} \Sigma_{00}^{-1}(Z,Z)\left[\Sigma_{00}(Z,Z_1), \Sigma_{01}(Z,Z_1), \Sigma_{02}(Z,Z_1)\right].$$

$$2\ln p(y|f) = -y^T \Sigma_{00}(Z,Z)^{-1} y - \ln\det\Sigma_{00}(Z,Z) - n\ln(2\pi),$$

GP Likelihood



Red : l=0.1, sig_f=0.001
Blue : l=0.01, sig_f=0.001
l=1.0, sig_f=0.001
Green : l=0.1, sig_f=0.1
l=0.1, sig_f=0.00001

Shafieloo, Kim & Linder, PRD 2012
Shafieloo, Kim & Linder, PRD 2013
....
Hwang et al, JCAP 2023

Data → $\mathbf{y}$ **Mean Function** → $\mathbf{m}(\mathbf{Z})$

$$\begin{bmatrix}\mathbf{y}\\ \mathbf{f}\\ \mathbf{f}'\\ \mathbf{f}''\end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix}\mathbf{m}(\mathbf{Z})\\ \mathbf{m}(\mathbf{Z_1})\\ \mathbf{m}'(\mathbf{Z_1})\\ \mathbf{m}''(\mathbf{Z_1})\end{bmatrix}, \begin{bmatrix}\Sigma_{00}(Z,Z) & \Sigma_{00}(Z,Z_1) & \Sigma_{01}(Z,Z_1) & \Sigma_{02}(Z,Z_1)\\ \Sigma_{00}(Z_1,Z) & \Sigma_{00}(Z_1,Z_1) & \Sigma_{01}(Z_1,Z_1) & \Sigma_{02}(Z_1,Z_1)\\ \Sigma_{10}(Z_1,Z) & \Sigma_{10}(Z_1,Z_1) & \Sigma_{11}(Z_1,Z_1) & \Sigma_{12}(Z_1,Z_1)\\ \Sigma_{20}(Z_1,Z) & \Sigma_{20}(Z_1,Z_1) & \Sigma_{21}(Z_1,Z_1) & \Sigma_{22}(Z_1,Z_1)\end{bmatrix}\right),$$

$$\Sigma_{\alpha\beta} = \frac{d^{(\alpha+\beta)}K}{dz_i^{\alpha} dz_j^{\beta}},$$

$$\begin{bmatrix}\overline{\mathbf{f}}\\ \overline{\mathbf{f}'}\\ \overline{\mathbf{f}''}\end{bmatrix} = \begin{bmatrix}\mathbf{m}(\mathbf{Z_1})\\ \mathbf{m}'(\mathbf{Z_1})\\ \mathbf{m}''(\mathbf{Z_1})\end{bmatrix} + \begin{bmatrix}\Sigma_{00}(Z_1,Z)\\ \Sigma_{10}(Z_1,Z)\\ \Sigma_{20}(Z_1,Z)\end{bmatrix} \Sigma_{00}^{-1}(Z,Z)\,\mathbf{y}$$

Kernel →

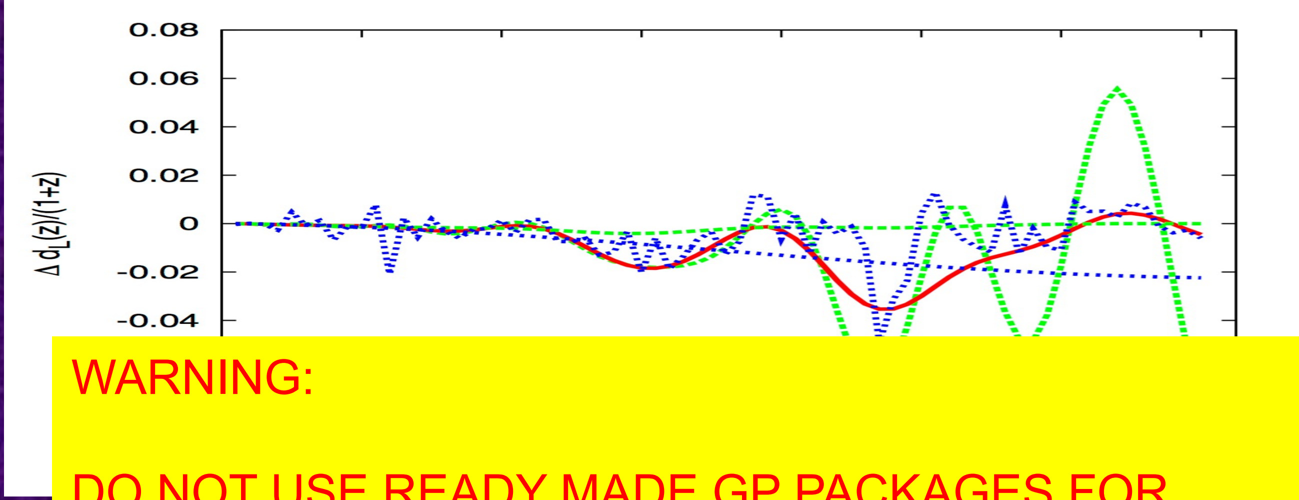
$$k(z,z') = \sigma_f^2 \exp\left(-\frac{|z-z'|^2}{2l^2}\right),$$

GP Hyper-parameters

$$\mathrm{Cov}\left(\begin{bmatrix}\mathbf{f}\\ \mathbf{f}'\\ \mathbf{f}''\end{bmatrix}\right) = \begin{bmatrix}\Sigma_{00}(Z_1,Z_1) & \Sigma_{01}(Z_1,Z_1) & \Sigma_{02}(Z_1,Z_1)\\ \Sigma_{10}(Z_1,Z_1) & \Sigma_{11}(Z_1,Z_1) & \Sigma_{12}(Z_1,Z_1)\\ \Sigma_{20}(Z_1,Z_1) & \Sigma_{21}(Z_1,Z_1) & \Sigma_{22}(Z_1,Z_1)\end{bmatrix} - \begin{bmatrix}\Sigma_{00}(Z_1,Z)\\ \Sigma_{10}(Z_1,Z)\\ \Sigma_{20}(Z_1,Z)\end{bmatrix} \Sigma_{00}^{-1}(Z,Z)\left[\Sigma_{00}(Z,Z_1), \Sigma_{01}(Z,Z_1), \Sigma_{02}(Z,Z_1)\right].$$

$$2\ln p(y|f) = -y^T\Sigma_{00}(Z,Z)^{-1}y - \ln\det\Sigma_{00}(Z,Z) - n\ln(2\pi),$$

GP Likelihood



WARNING:

DO NOT USE READY MADE GP PACKAGES FOR RECONSTRUCTION PURPOSES UNLESS YOU KNOW HOW GP WORKS IN DETAILS.

MOST IMPORTANTLY SINCE GP RECONSTRUCTIONS ARE SENSITIVE TO THE CHOICE OF THE MEAN FUNCTION.

Kim & Linder, PRD 2012
Kim & Linder, PRD 2013

Dat

$$\begin{bmatrix} \mathbf{y} \\ \mathbf{f} \\ \mathbf{f}' \\ \mathbf{f}'' \end{bmatrix} \sim \mathcal{N}$$

$$\Sigma_{\alpha\beta} = \frac{d^{(\alpha+\beta)}K}{dz_i^\alpha dz_j^\beta},$$

$$\begin{bmatrix} \overline{\mathbf{f}} \\ \overline{\mathbf{f}'} \\ \overline{\mathbf{f}''} \end{bmatrix} = \begin{bmatrix} \mathbf{m}(\mathbf{Z}_1) \\ \mathbf{m}'(\mathbf{Z}_1) \\ \mathbf{m}''(\mathbf{Z}_1) \end{bmatrix} + \begin{bmatrix} \Sigma_{00}(Z_1, Z) \\ \Sigma_{10}(Z_1, Z) \\ \Sigma_{20}(Z_1, Z) \end{bmatrix} \Sigma_{00}^{-1}(Z, Z)\,\mathbf{y}$$

Kernel → $k(z, z') = \sigma_f^2 \exp\left(-\frac{|z - z'|^2}{2l^2}\right),$

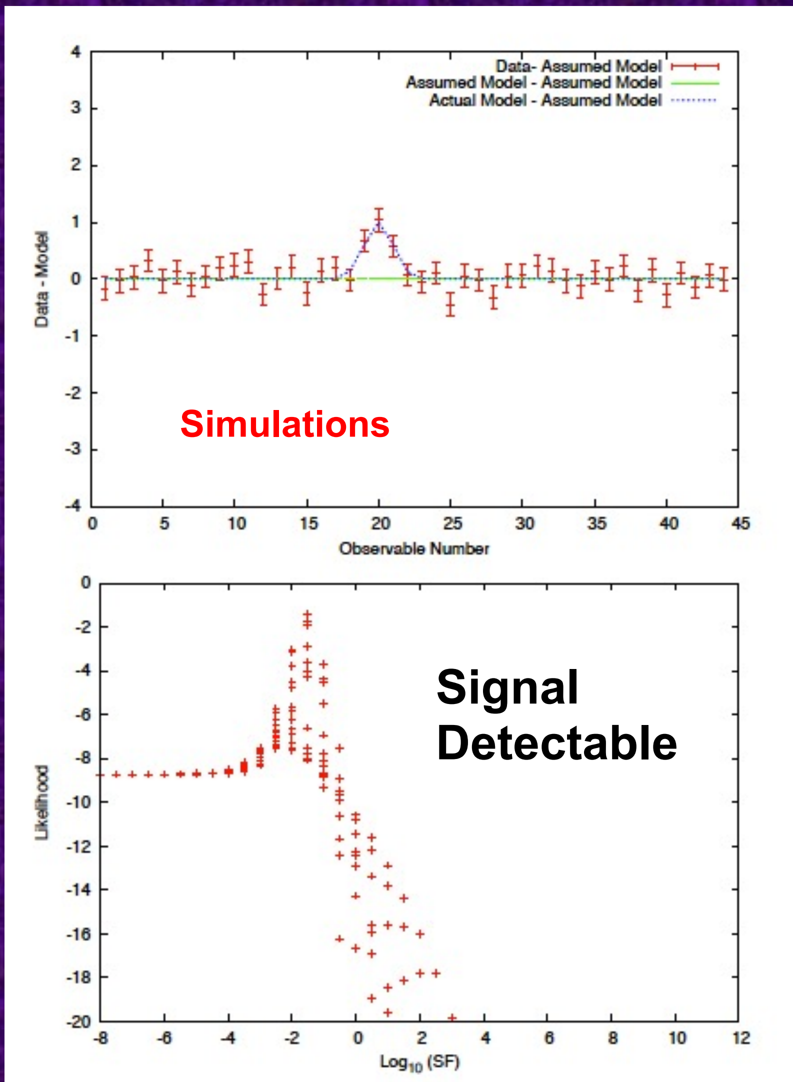
GP Hyper-parameters

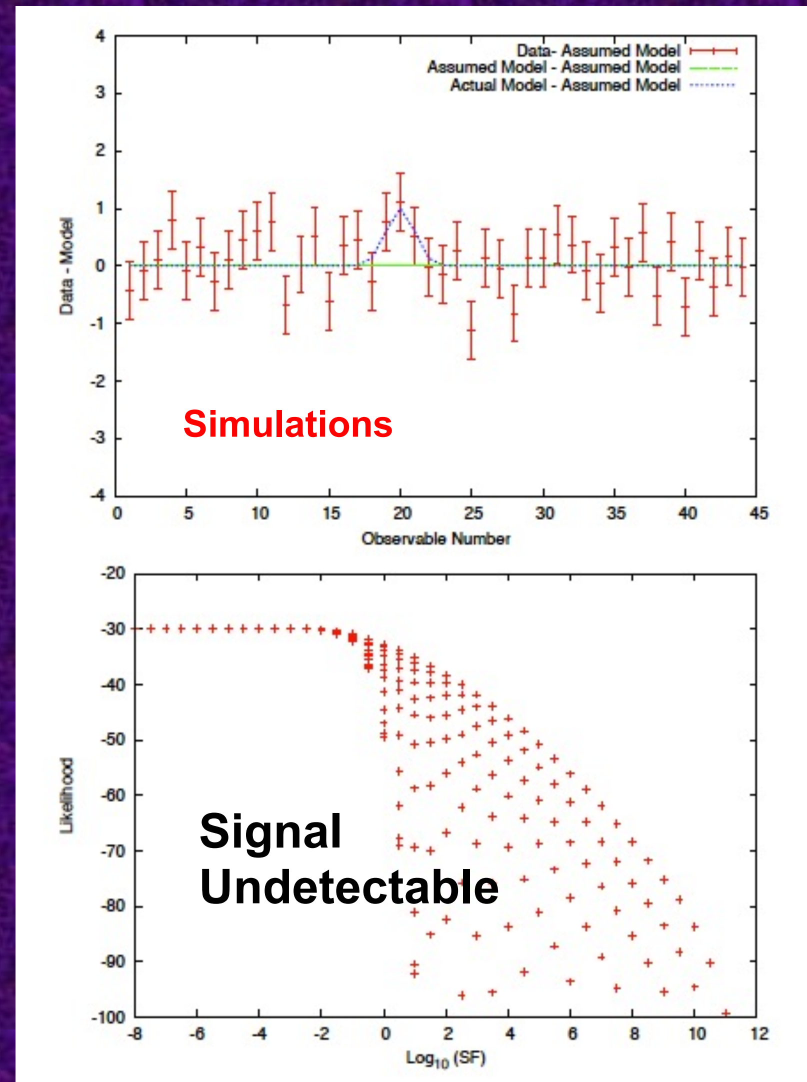
$$\mathrm{Cov}\left(\begin{bmatrix} \mathbf{f} \\ \mathbf{f}' \\ \mathbf{f}'' \end{bmatrix}\right) = \begin{bmatrix} \Sigma_{00}(Z_1, Z_1) & \Sigma_{01}(Z_1, Z_1) & \Sigma_{02}(Z_1, Z_1) \\ \Sigma_{10}(Z_1, Z_1) & \Sigma_{11}(Z_1, Z_1) & \Sigma_{12}(Z_1, Z_1) \\ \Sigma_{20}(Z_1, Z_1) & \Sigma_{21}(Z_1, Z_1) & \Sigma_{22}(Z_1, Z_1) \end{bmatrix} - \begin{bmatrix} \Sigma_{00}(Z_1, Z) \\ \Sigma_{10}(Z_1, Z) \\ \Sigma_{20}(Z_1, Z) \end{bmatrix} \Sigma_{00}^{-1}(Z, Z)\left[\Sigma_{00}(Z, Z_1), \Sigma_{01}(Z, Z_1), \Sigma_{02}(Z, Z_1)\right].$$

$$2\ln p(y|f) = -y^T \Sigma_{00}(Z, Z)^{-1} y - \ln\det\Sigma_{00}(Z, Z) - n\ln(2\pi),$$

GP Likelihood

Detection of the features in the residuals

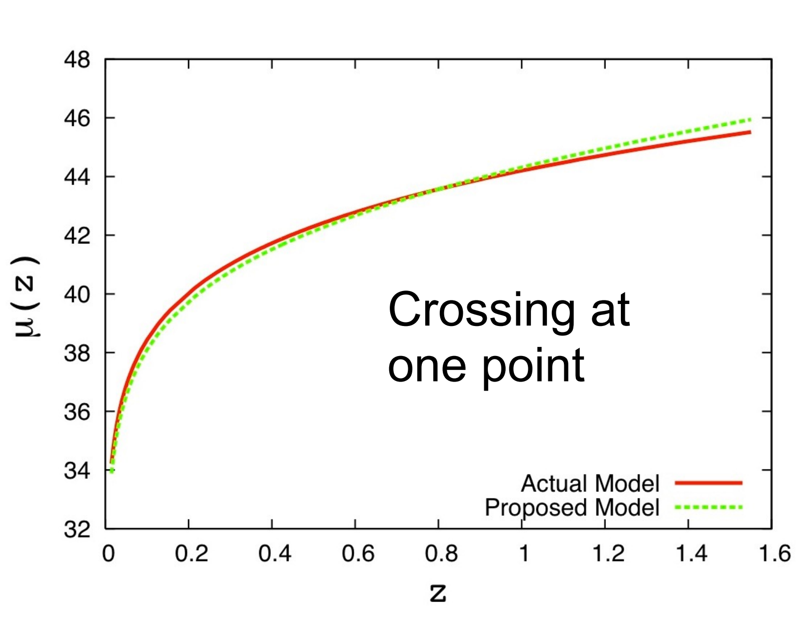


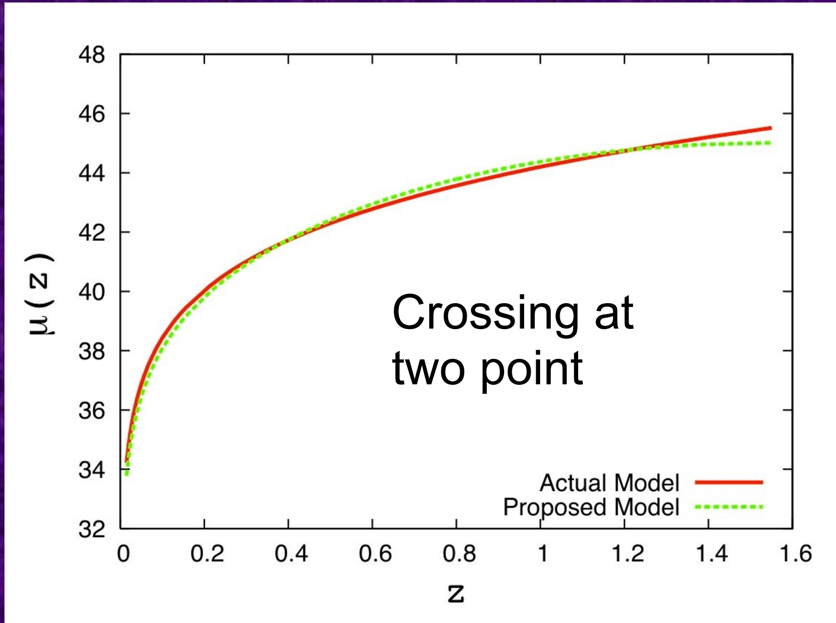


Crossing Statistic

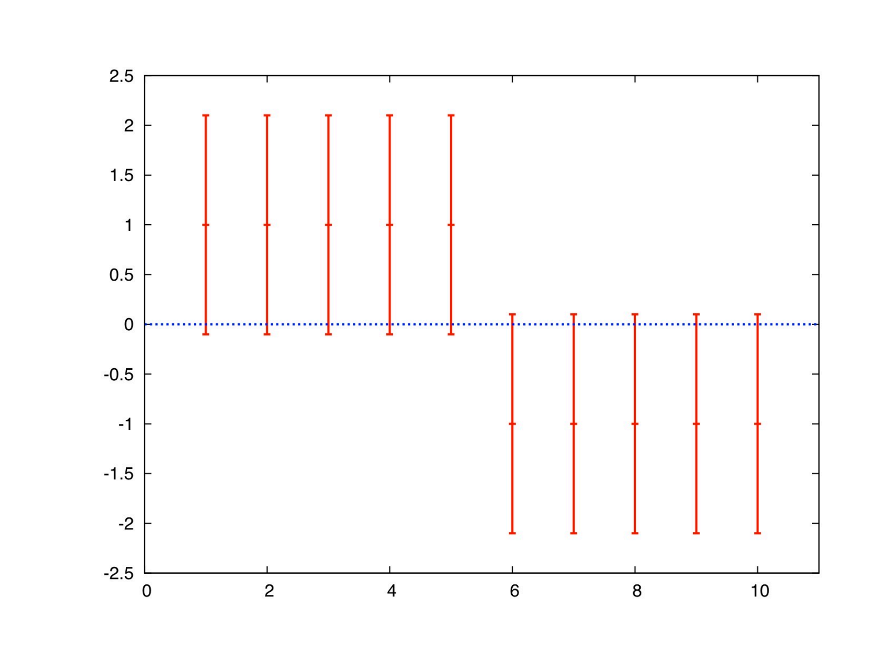
If a proposed model is different than the actual model, then they cross each other at one or two or three or … N points.

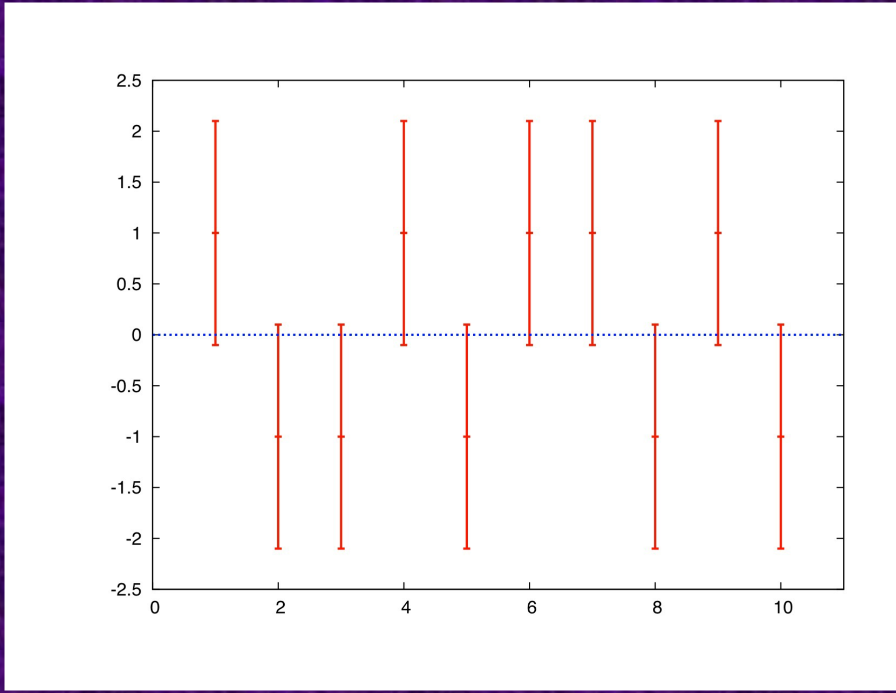
A. Shafieloo, T. Clifton & P. Ferreira, JCAP 2010





Equal in being probable?!





$$\chi^2 = \sum_i^N \frac{(\mu_i^t - \mu_i^e)^2}{\sigma_i^2},$$

One point Crossing: T1

1. Assume a model
2. Construct the normalized residuals
3. Finding the crossing point and calculating T1 by maximizing T(n1):
4. Comparing the results with Monte Carlo simulations.

$$q_i(z_i) = \frac{\mu_i(z_i) - \bar{\mu}(z_i)}{\sigma_i(z_i)}.$$

$$T(n_1) = Q_1(n_1)^2 + Q_2(n_1)^2,$$

$$Q_1(n_1) = \sum_{i=1}^{n_1} q_i(z_i)$$

$$Q_2(n_1) = \sum_{i=n_1+1}^{N} q_i(z_i),$$

Two points Crossing: T2

1-2.....

3. Finding the crossing points and calculating T2 by maximizing T(n1,n2):

$$q_i(z_i) = \frac{\mu_i(z_i) - \bar{\mu}(z_i)}{\sigma_i(z_i)}.$$

$$T(n_1, n_2) = Q_1(n_1, n_2)^2 + Q_2(n_1, n_2)^2 + Q_3(n_1, n_2)^2,$$

4. Comparing the results with Monte Carlo simulations.

$$\begin{aligned} Q_1(n_1, n_2) &= \sum_{i=1}^{n_1} q_i(z_i) \\ Q_2(n_1, n_2) &= \sum_{i=n_1+1}^{n_2} q_i(z_i) \\ Q_3(n_1, n_2) &= \sum_{i=n_2+1}^{N} q_i(z_i). \end{aligned}$$

And so on we can derive T3, T4,…

Important Features:

For N data points, **the last mode of Crossing Statistic** is T(N-1) which **is identical to Chi Square Statistic**

$$T_{N-1} = \sum_{i}^{N} (q_i)^2 = \chi^2$$

The zero mode of Crossing Statistic is similar to Median Statistic

not only should the whole sample of residuals have a Gaussian distribution around the mean, but so should any continuous subsample.

$$T_0 = (\sum_{i}^{N} q_i)^2$$

Comparing Two Statistics

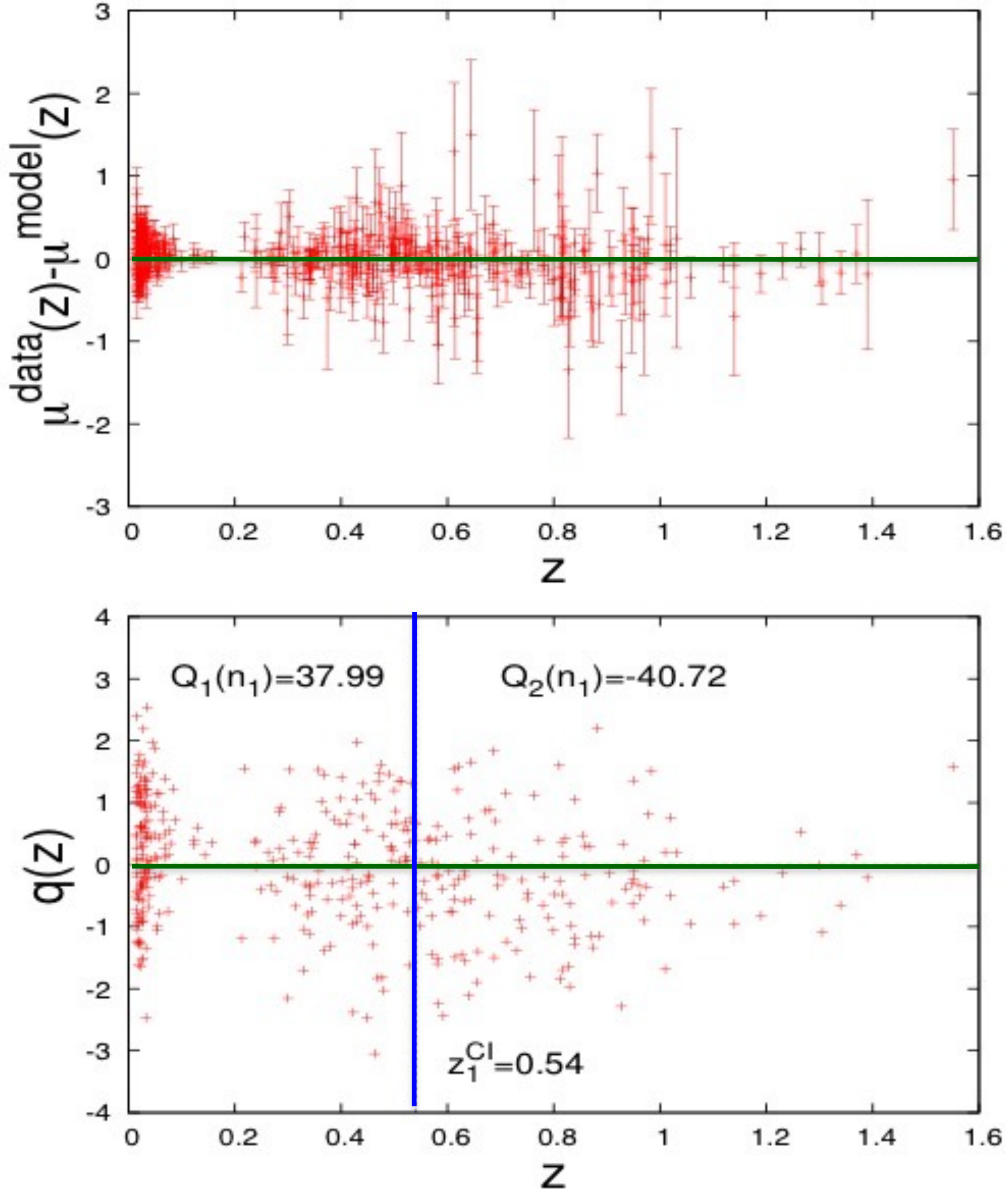
	T1	Chi Square
Ruling out by 99% CL	1% (Correct Model) 28.5% (Incorrect Model)	1% (Correct Model) 1.9% (Incorrect Model)
Ruling out by 99% CL Assuming extra (0.05) intrinsic dispersion	0.5% (Correct Model) 26.4% (Incorrect Model)	0% (Correct Model) 0% (Incorrect Model)

$$\sigma_i^2 = \sigma_{i(data)}^2 + \sigma_{(sys)}^2$$

Correct Model: Flat LCDM with $\Omega_{0m}^{true} = 0.27$

Incorrect Model: Flat LCDM with $\Omega_{0m}^{erroneous} = 0.22$

Simulated SN Ia data similar to Constitution compilation



Assumed model is consistent with the data using chi square

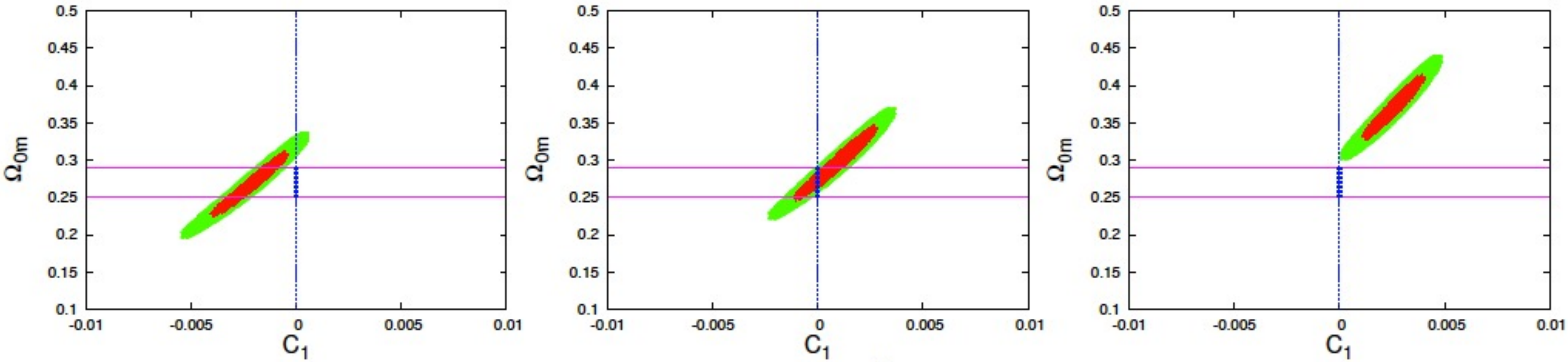
Data:
Flat LCDM $\Omega_{0m}^{true} = 0.27$

Assumed Model:
Flat LCDM $\Omega_{0m}^{erroneous} = 0.22$

Assumed model is ruled out at 99% CL using T1

Crossing Statistic (Bayesian Interpretation)

Theoretical model | Crossing function | Comparing a model with its own variations



$$T_I(C_1, z) = 1 + C_1(\frac{z}{z_{max}})$$

$$T_{II}(C_1, C_2, z) = 1 + C_1(\frac{z}{z_{max}}) + C_2[2(\frac{z}{z_{max}})^2 - 1],$$

Chebishev Polynomials as Crossing Functions

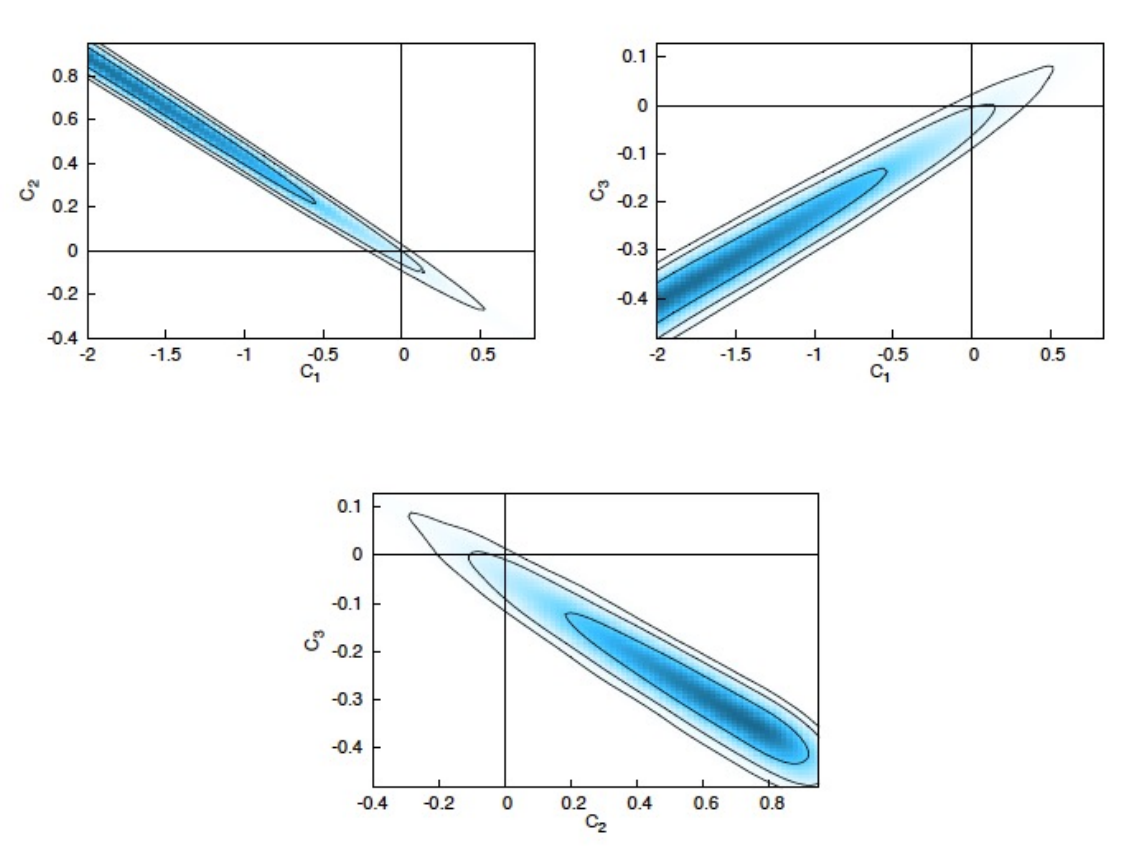
Shafieloo. JCAP 2012 (a)
Shafieloo, JCAP 2012 (b)

Theoretical Model Crossing Function

$$\mathcal{C}_\ell^{\rm TT}\,|^N_{\rm modified} = \mathcal{C}_\ell^{\rm TT}\,|_{\Omega_{\rm b},\Omega_{\rm CDM},{\rm H}_0,\tau,{\rm A_s},{\rm n_s}} \times T_N(C_0, C_1, C_2, ..., C_N, \ell).$$

$$T_{\rm III}(C_0, C_1, C_2, C_3, x) = C_0 + C_1\, x + C_2(2x^2-1) + C_3(4x^3-3x)$$

Planck 2013

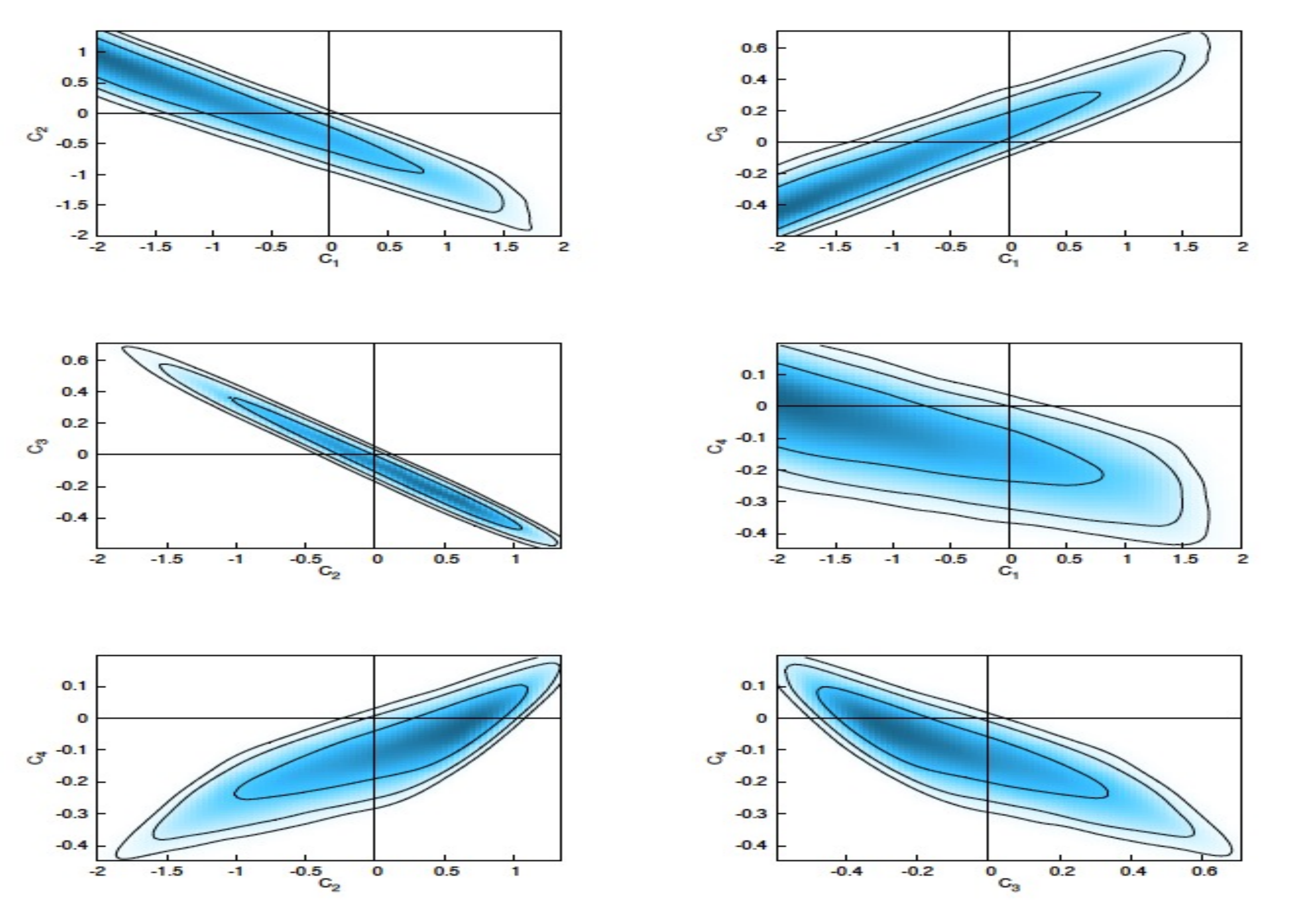


Parametric Bayesian Interpretation

Theoretical Model Crossing Function

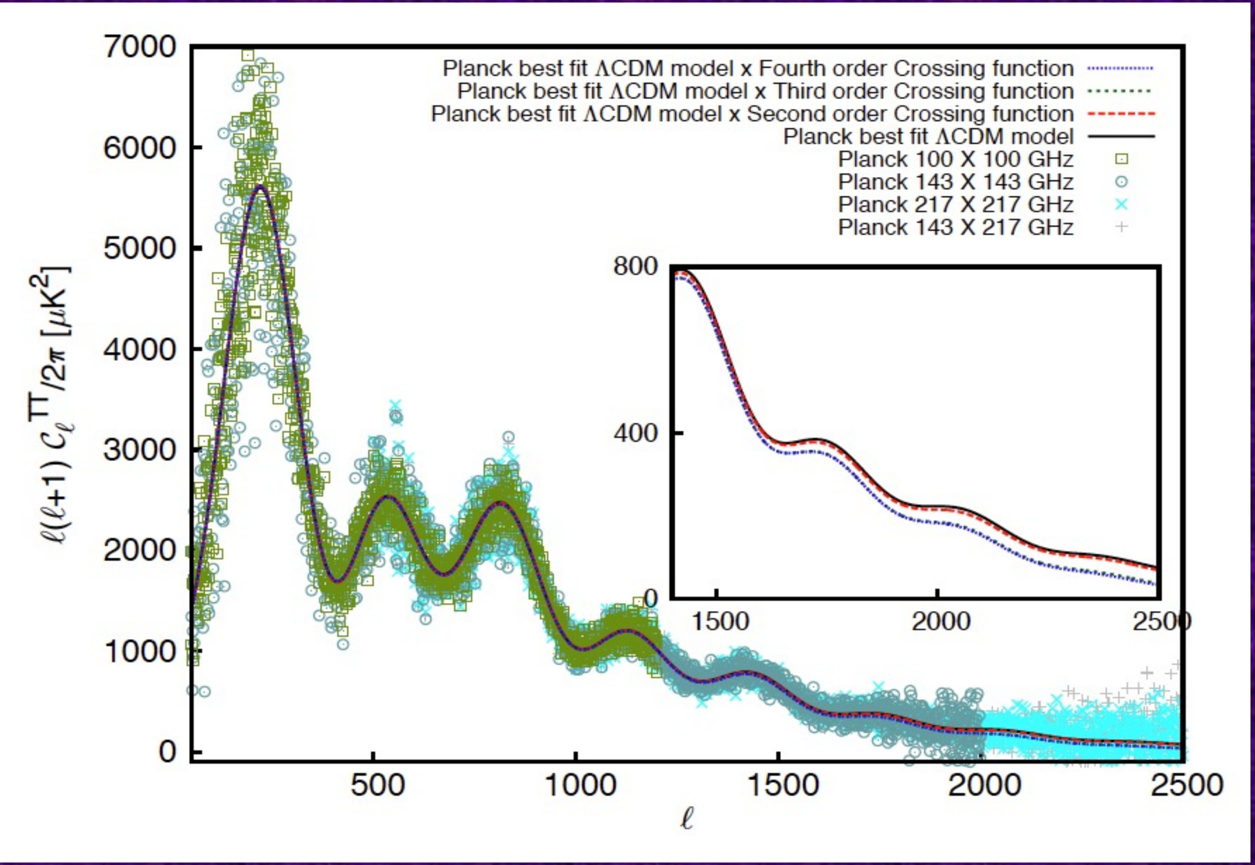
$$\mathcal{C}_\ell^{\rm TT}\,|^N_{\rm modified} = \mathcal{C}_\ell^{\rm TT}\,|_{\Omega_{\rm b},\Omega_{\rm CDM},{\rm H}_0,\tau,{\rm A_s},{\rm n_s}} \times T_N(C_0, C_1, C_2, ..., C_N, \ell).$$

$$T_{\rm IV}(C_0, C_1, C_2, C_3, C_4, x) = C_0 + C_1\, x + C_2(2x^2 - 1) + C_3(4x^3 - 3x) + C_4(8x^4 - 8x^2 + 1)$$



Theoretical Model   Crossing Function

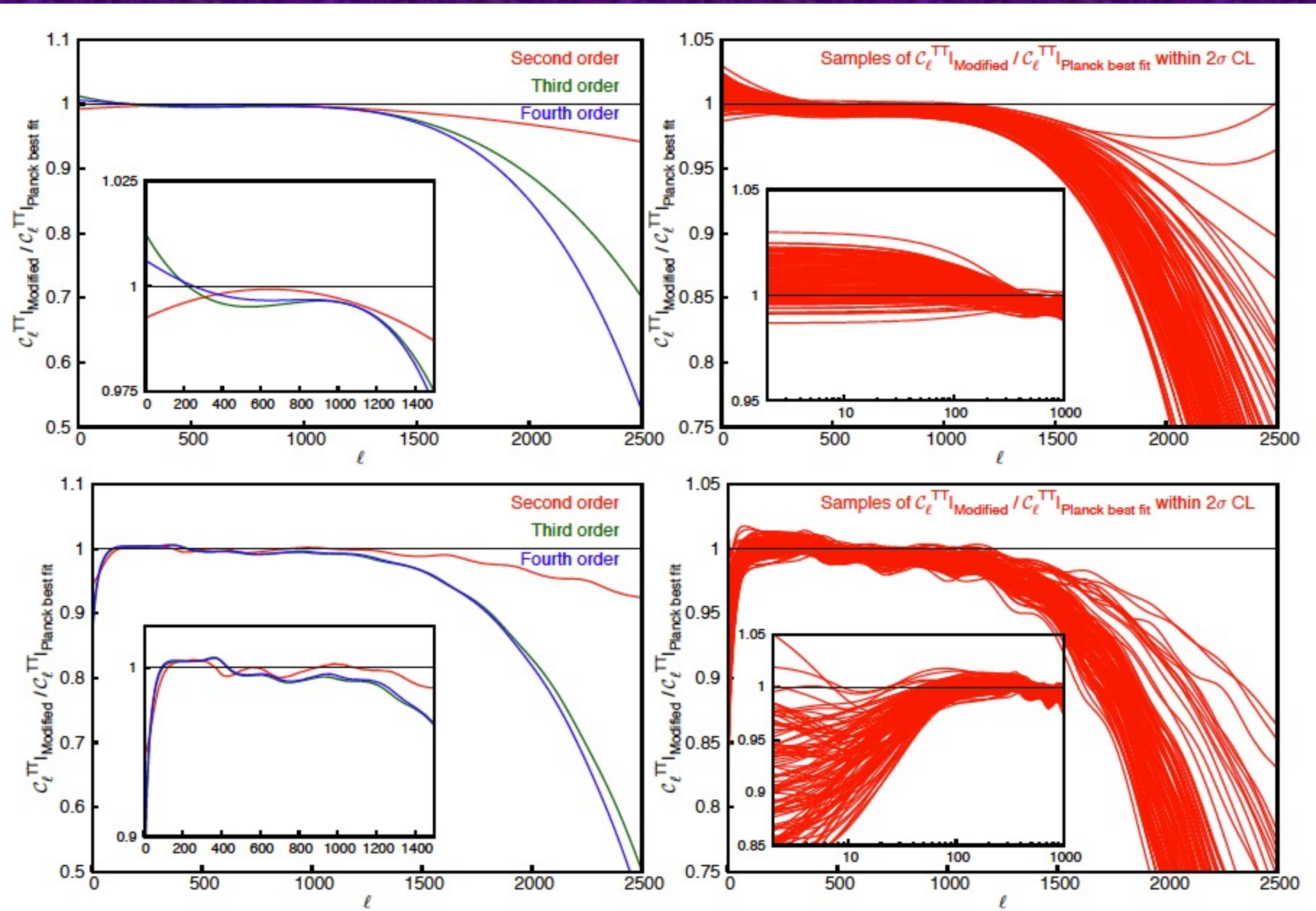
$$\mathcal{C}_\ell^{\rm TT}\,\Big|^{N}_{\rm modified} = \mathcal{C}_\ell^{\rm TT}\,\Big|_{\Omega_{\rm b},\Omega_{\rm CDM},{\rm H_0},\tau,{\rm A_s},{\rm n_s}} \times T_N(C_0, C_1, C_2, ..., C_N, \ell).$$



Data	ΛCDM	T_0	$T_{\rm I}$	$T_{\rm II}$	$T_{\rm III}$	$T_{\rm IV}$	$T_{\rm V}$
Planck low-ℓ (ℓ=2-49)	-6.3	-7	-8.5	-8.6	-9.8	-9.7	-9.7
Planck high-ℓ (ℓ=50-2500)	7794.9	7793.8	7793.8	7789.6	7785.9	7785.7	7784.7
Total	7788.6	7786.8	7785.3	7781	7776.1	7776	7775
$\chi^2_{\rm Model}$ - $\chi^2_{\Lambda{\rm CDM}}$	-	-1.8	-3.3	-7.6	-12.5	-12.6	-13.6

Theoretical Model Crossing Function

$$\mathcal{C}_\ell^{\rm TT}\,\Big|^{N}_{\rm modified} = \mathcal{C}_\ell^{\rm TT}\,\Big|_{\Omega_{\rm b},\Omega_{\rm CDM},{\rm H}_0,\tau,{\rm A_s},{\rm n_s}} \times T_N(C_0, C_1, C_2, ..., C_N, \ell).$$



Data suggests substantial suppressions are required at both low and high multiples.

Hazra & Shafieloo, JCAP 2014

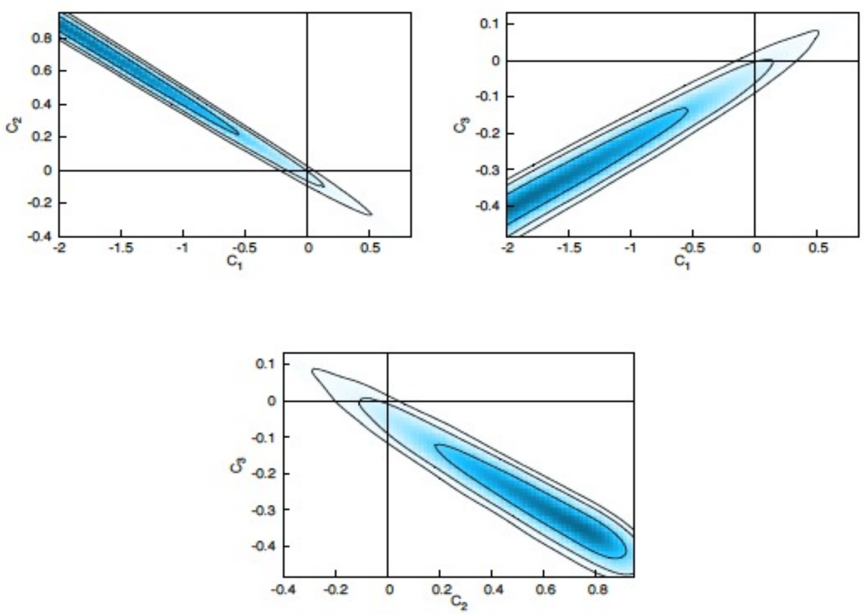
Crossing Statistic (Bayesian Interpretation)

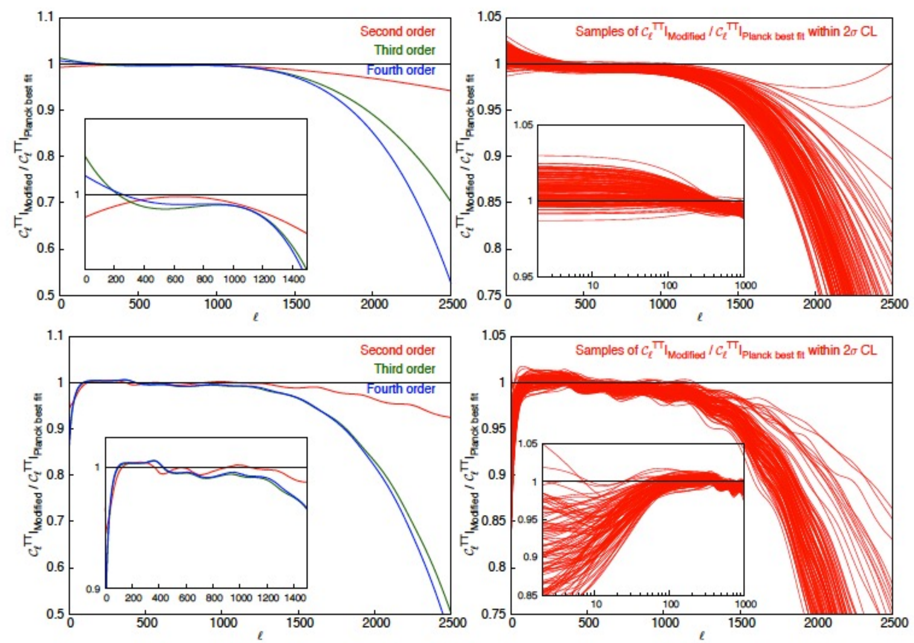
Theoretical model — Crossing function

$$C_\ell^{\rm TT}\,|^N_{\rm modified} = C_\ell^{\rm TT}\,|_{\Omega_{\rm b},\Omega_{\rm CDM},{\rm H}_0,\tau,{\rm A_S},{\rm n_S},\ell} \times T_i(C_0, C_1, C_2, ..., C_N, \ell).$$

Confronting the concordance model of cosmology with Planck data

Consistent only at 2~3 sigma CL





Dates

Issue 01 (January 2014)

Received 13 January 2014, accepted for publication 14 January 2014

Published 28 January 2014

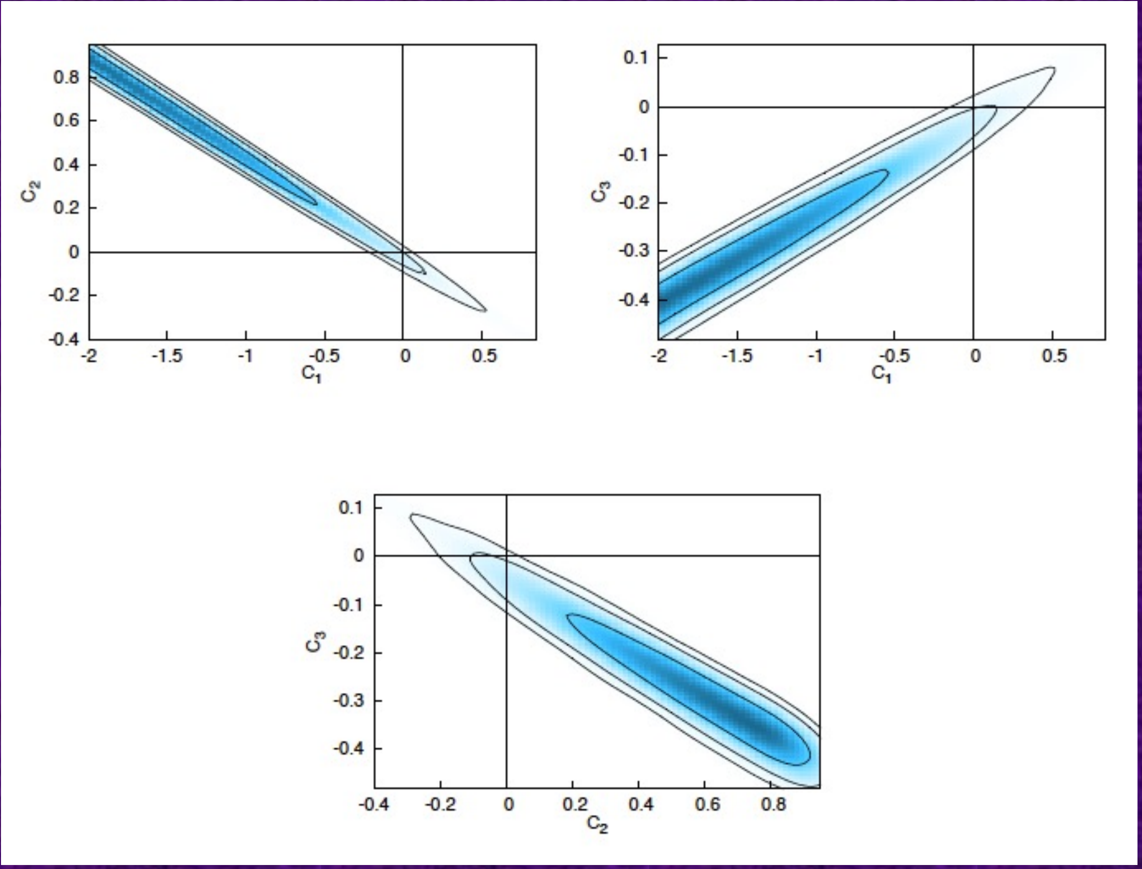
Hazra and Shafieloo, JCAP 2014

Theoretical Model Crossing Function

$$\mathcal{C}_\ell^{\mathrm{TT}}\,|^{N}_{\mathrm{modified}} = \mathcal{C}_\ell^{\mathrm{TT}}\,|_{\Omega_{\mathrm{b}},\Omega_{\mathrm{CDM}},\mathrm{H_0},\tau,\mathrm{A_s},\mathrm{n_s}} \times T_N(C_0, C_1, C_2, ..., C_N, \ell).$$

$$T_{\mathrm{III}}(C_0, C_1, C_2, C_3, x) = C_0 + C_1\,x + C_2(2x^2 - 1) + C_3(4x^3 - 3x)$$

With 217 GHz x 217 GHz

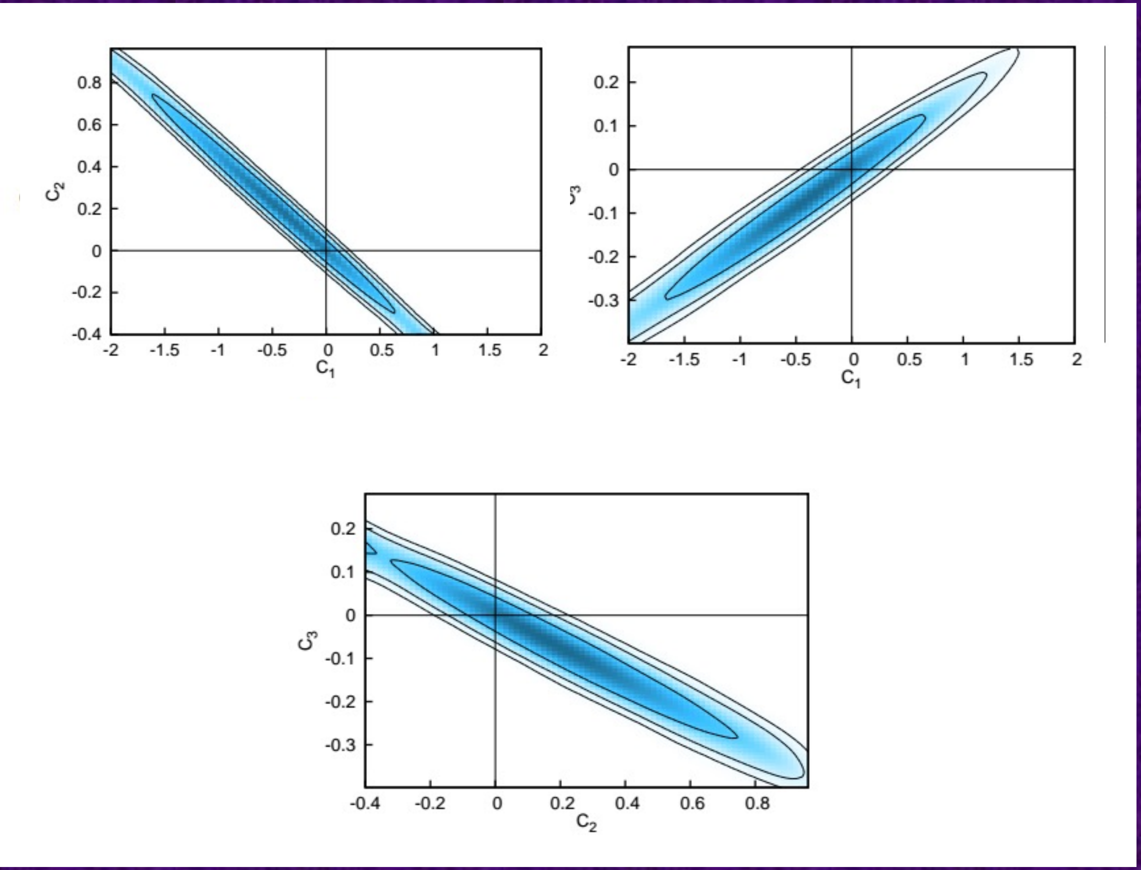


Theoretical Model Crossing Function

$$\mathcal{C}_\ell^{\rm TT}\,|^{N}_{\rm modified} = \mathcal{C}_\ell^{\rm TT}\,|_{\Omega_{\rm b},\Omega_{\rm CDM},{\rm H_0},\tau,{\rm A_s},{\rm n_s}} \times T_N(C_0, C_1, C_2, ..., C_N, \ell).$$

$$T_{\rm III}(C_0, C_1, C_2, C_3, x) = C_0 + C_1\, x + C_2(2x^2 - 1) + C_3(4x^3 - 3x)$$

Without 217 GHz x 217 GHz



Planck collaboration corrected systematics at 217 GHz chanel and problem was resolved analysing Planck 2015 data

Crossing Statistic (Bayesian Interpretation)

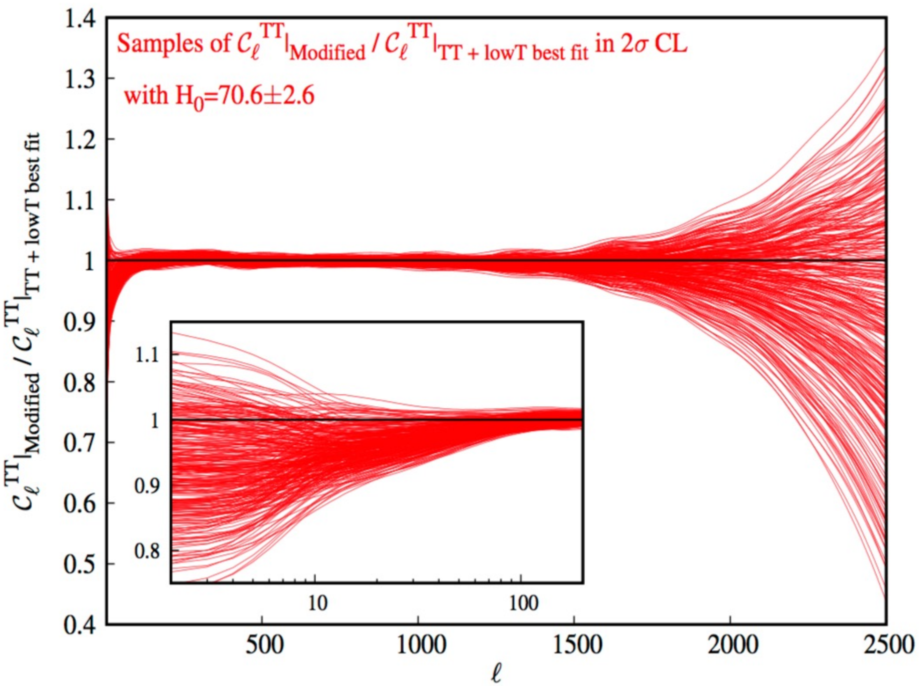
Theoretical model — Crossing function

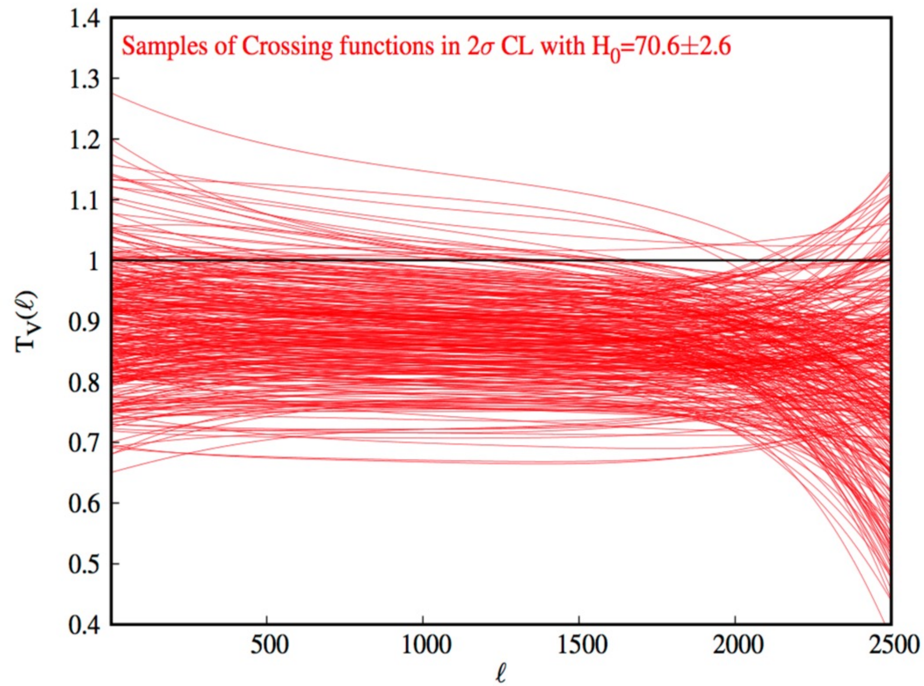
$$\mathcal{C}_\ell^{\mathrm{TT}}\,|^{N}_{\mathrm{modified}} = \mathcal{C}_\ell^{\mathrm{TT}}\,|_{\Omega_{\mathrm{b}},\Omega_{\mathrm{CDM}},\mathrm{H}_0,\tau,\mathrm{A_S},\mathrm{n_S},\ell} \times T_i(C_0, C_1, C_2, ..., C_N, \ell).$$

Confronting the concordance model of cosmology with Planck 2015 data

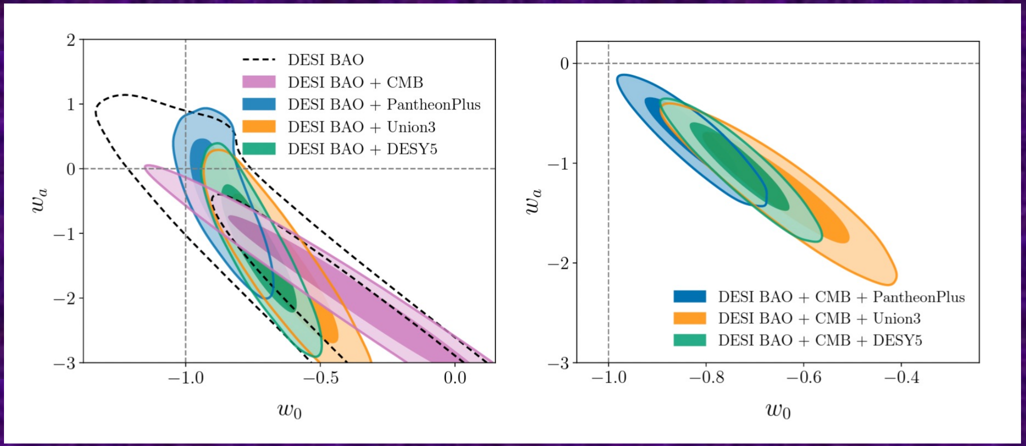
Shafieloo and Hazra, JCAP 2017

Completely Consistent





DESI-2024



DESI-Y1 (2024),
arXiv:2404.03002


동아사이언스

과학 인간 테크 환경·자연 생활 정책 하이테크 기업 사람들

암흑에너지 규명 위해 1년간 데이터 모아보니 "기존 우주론 맞나"

2024.04.15 12:25

DESI 프로젝트 첫 성과로 사상 최대 3D우주지도 공개

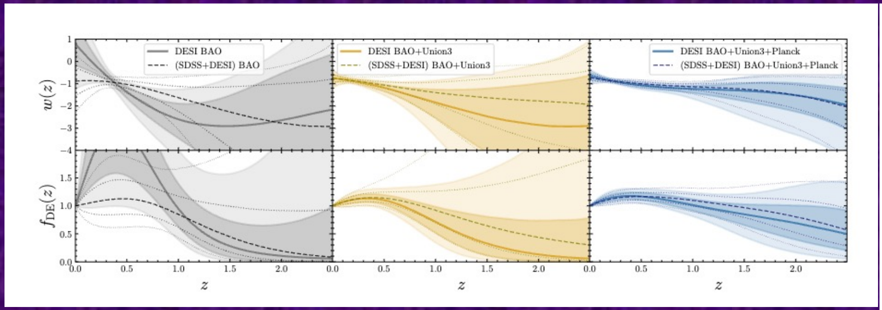
은하보다 밝은 '퀘이사'의 빛을 관측하는 상상도. DESI 제공

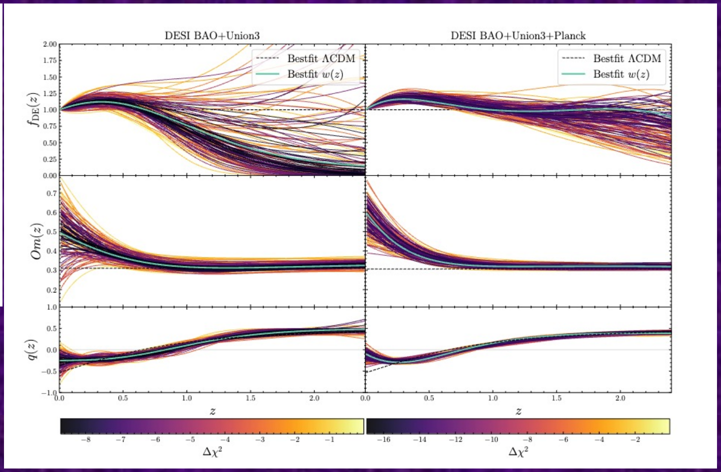
국제공동연구팀이 사상 최대 규모의 3차원(3D) 우주 지도를 공개하고 기존 우주론에 의문을 제기했다.


The New York Times

A Tantalizing 'Hint' That Astronomers Got Dark Energy All Wrong

Scientists may have discovered a major flaw in their understanding of that mysterious cosmic force. That could be good news for the fate of the universe.

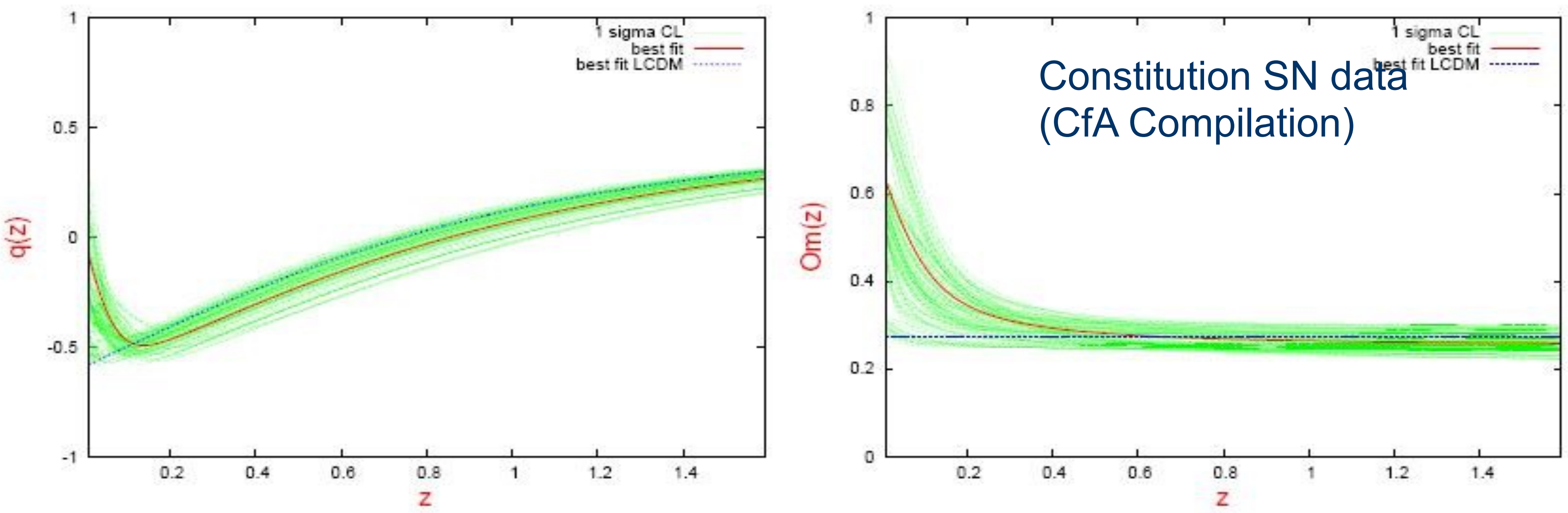




Reconstructing DE with Crossing Statistics
Calderon, Lodha, Shafieloo, Linder et al, arXiv:2405.04216

Theoretical application of direct reconstruction

IS COSMIC ACCELERATION SLOWING DOWN?

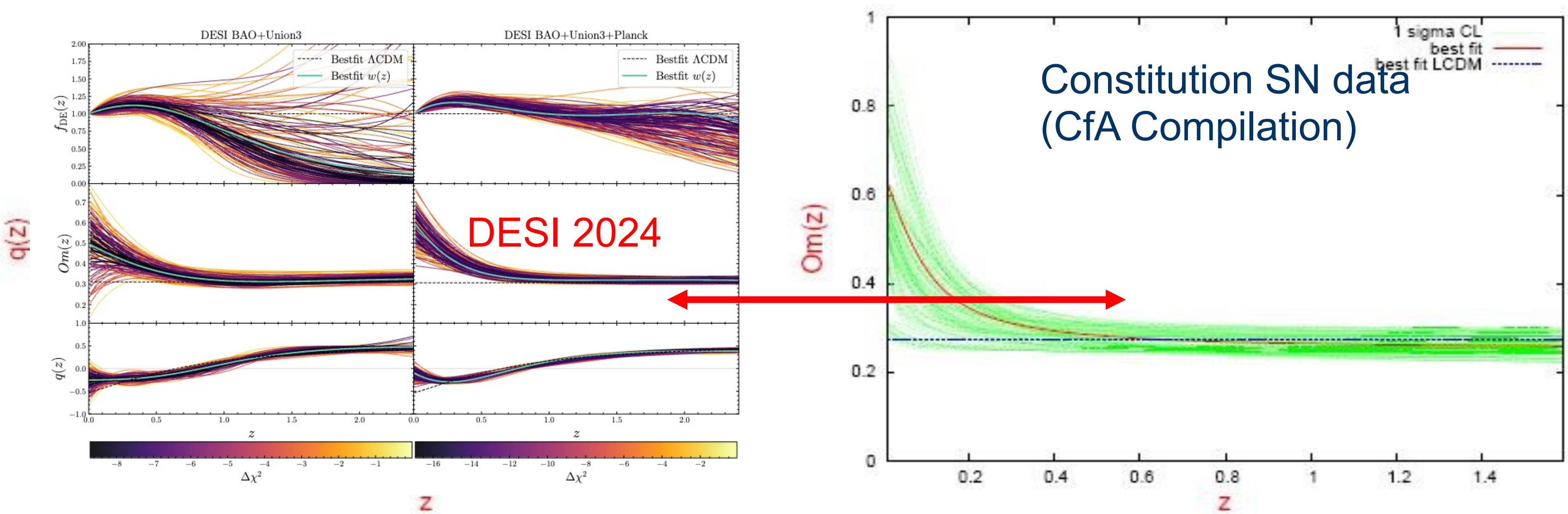


$$w(z) = -\frac{1+\tanh\left[\left(z-z_t\right)\Delta\right]}{2}$$

$\Delta\chi^2 = -0.6$ with respect to CPL

A. Shafieloo, V. Sahni, A. Starobinsky, Phys. Rev. D Rapid Communication 2009 [Reported in Nature and New Scientist]

IS COSMIC ACCELERATION SLOWING DOWN ***Again?***



$$w(z) = -\frac{1 + \tanh\left[\left(z - z_t\right)\Delta\right]}{2}$$

$\Delta\chi^2 = -0.6$ with respect to CPL

A. Shafieloo, V. Sahni, A. Starobinsky, Phys. Rev. D Rapid Communication 2009 [Reported in Nature and New Scientist]

Current Status

Open problem. Many tensions and hints for various systematics

Many theoretical/phenomenological models are proposed to ease the tensions. None is convincing so far (none can pass all validation tests).

Not possible to resolve all problems with minimal modification of the standard model. This has helped the standard model to survive so far.

Model independent consistency test between various data is essential to rule out systematics.

Point 6

Looking for systematics

Model independent consistency test between various data is essential to rule out systematics.

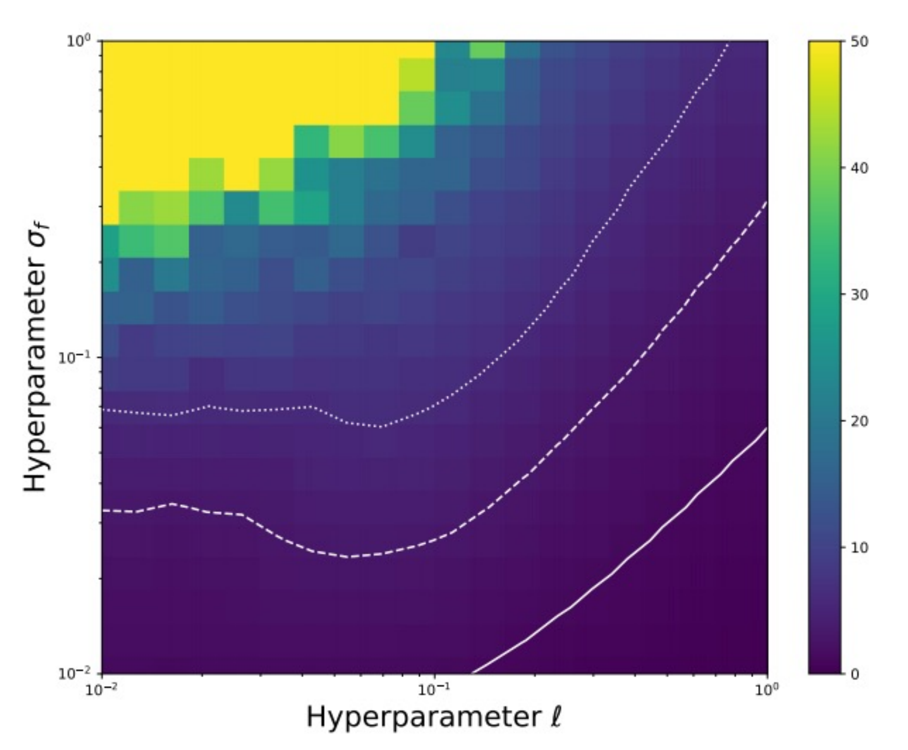
GP for Falsification

Shafieloo, Kim, Linder, PRD 2012
Shafieloo, Kim, Linder, PRD 2013
Hwang et al, JCAP 2023

Consistency of SDSS BAO and Pantheon SN Ia data
Keeley, Shafieloo, Zhao,…, MNRAS 2021 [arXiv:2010.03234] [SDSS IV paper]

$H0rd$ = 10040 ± 140 km/s and
Ωk = 0.02 ± 0.20

Point 6



Future Perspective

High possibilities for systematics in different data

Need for independent measurements

Two key questions:

Power-Law Primordial Power Spectrum?
Lambda Dark Energy?

Tip of the Red Giant Branch

Future Perspective

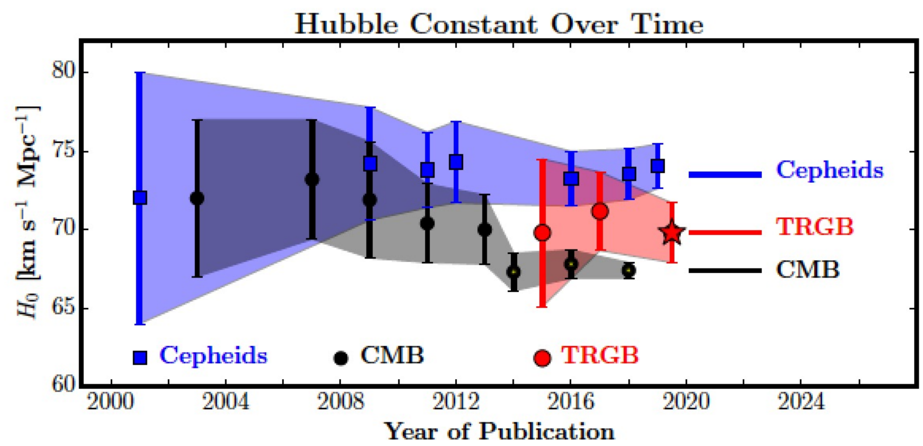
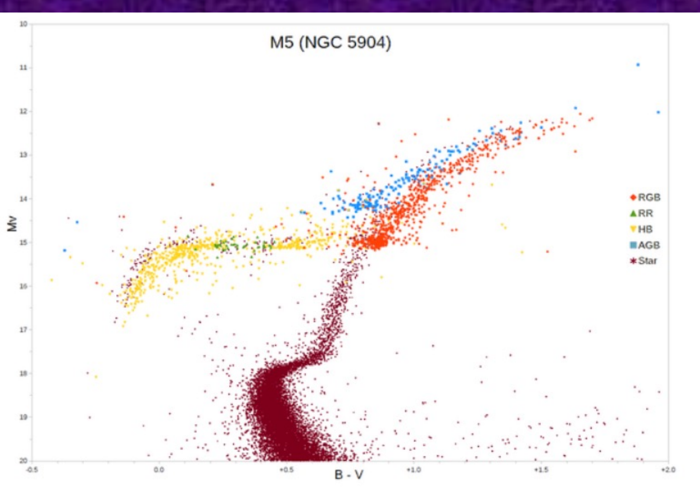


Figure 17. A plot of H_0 values as a function of time. The points and shaded region in black are those determined from measurements of the CMB; those in blue are Cepheid calibrations of the local value of H_0; and the red points are TRGB calibrations. The red star is the best-fit value obtained in this paper. Error bars are 1σ.

Freedman et al, arXiv:1907.05922



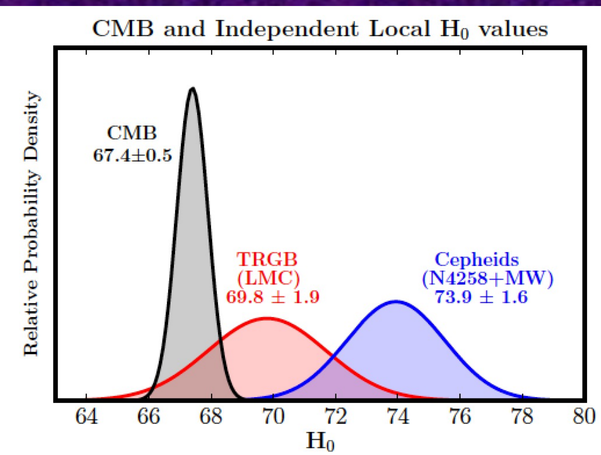
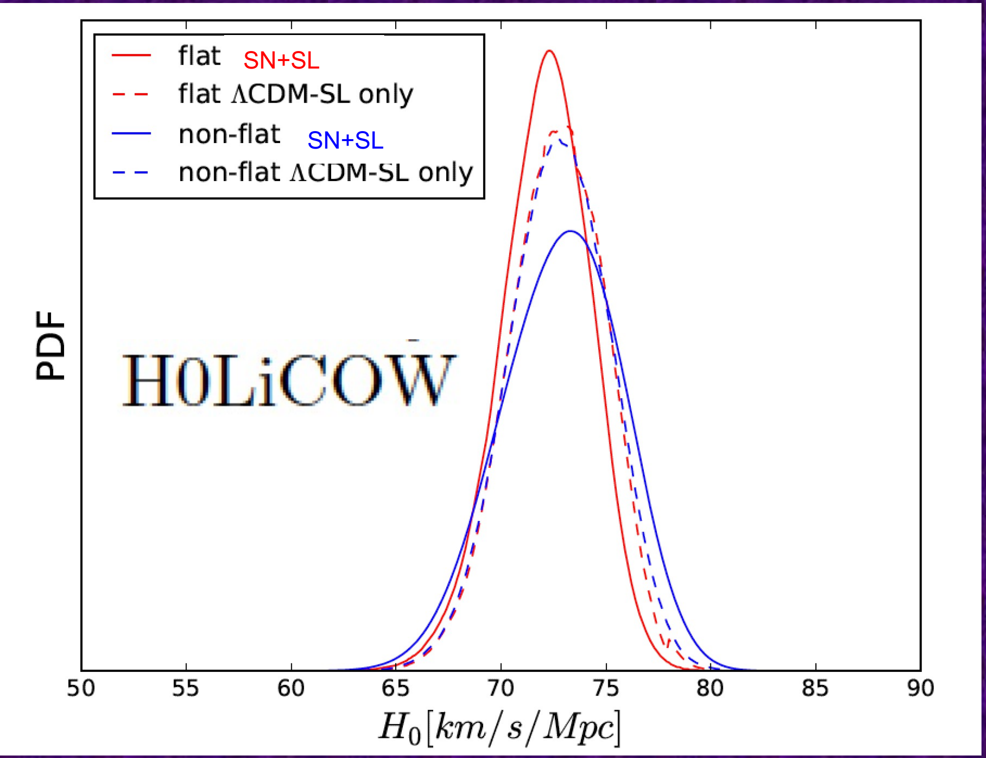


Figure 18. Completely independent calibrations of H_0. Shown in red is the probability density function based on our LMC CCHP TRGB calibration of CSP-I SNe Ia; in blue is the Cepheid calibration of H_0 (Riess et al. 2016), using the Milky Way parallaxes and the maser distance to NGC 4258 as anchors (excluding the LMC). The Planck value of H_0 is shown in black.

Cosmology with Strong Lens Systems: Has become already competative!



Liao, Shafieloo, Keeley, Linder, ApJ Letters 2019

H0 from Strongly Lensed systems

$$H_0 = 72.8^{+1.6}_{-1.7}\ \text{km/s/Mpc}$$

2.3% model-independent measurment of Hubble constant

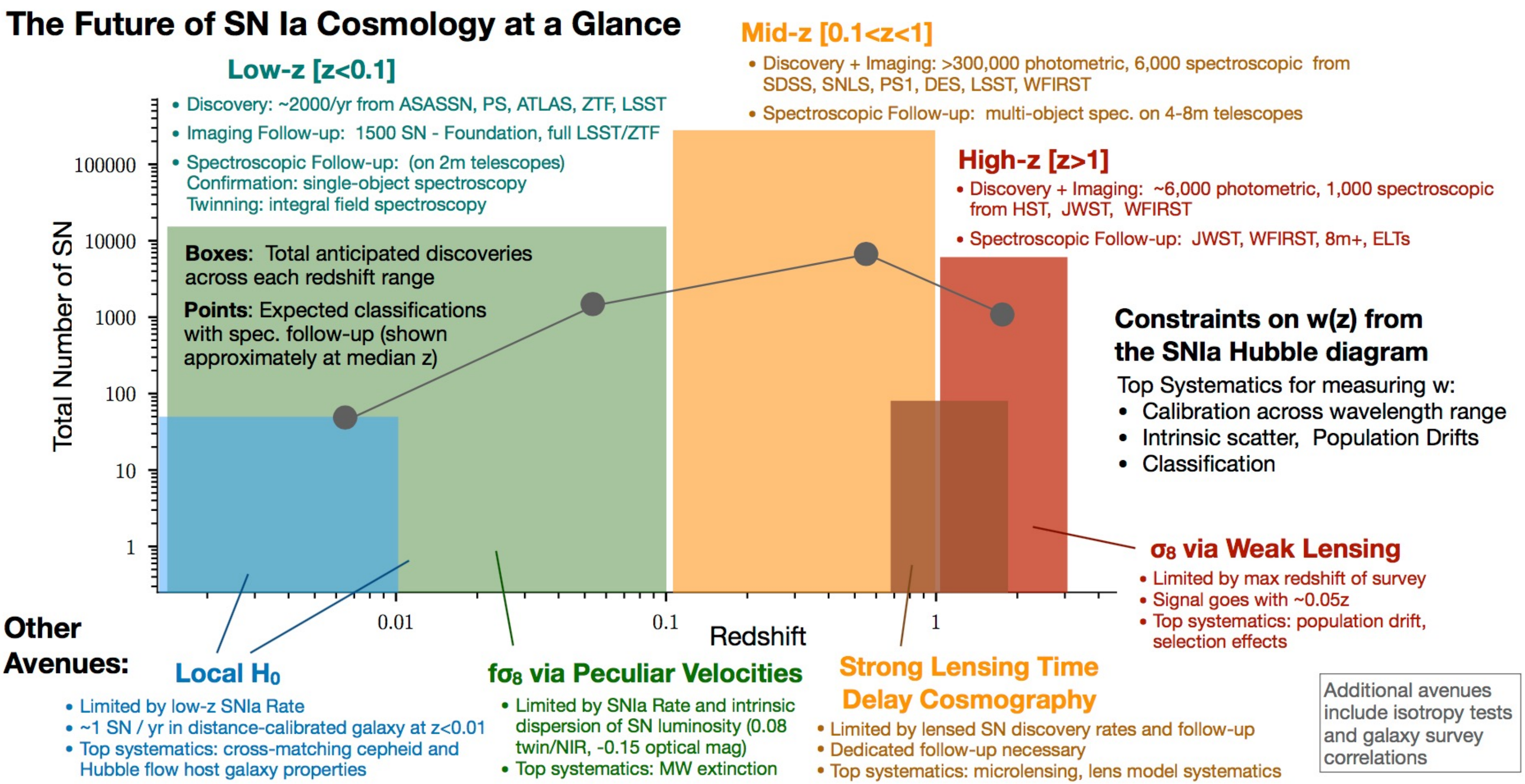
Liao, Shafieloo, Keeley, Linder, ApJ Letters 2020

H0LiCOW I. H0 Lenses in COSMOGRAIL's Wellspring

Suyu et al. MNRAS 2017

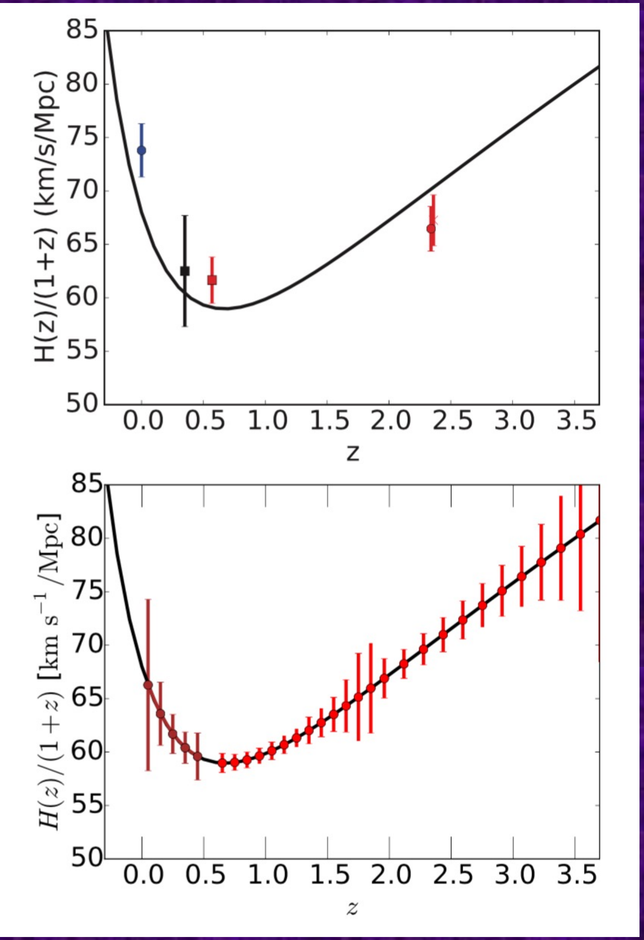
Order	Name	z_L	z_S
1	RXJ1131-1231	0.295	0.654
2	HE 0435-1223	0.4546	1.693
3	B1608+656	0.6304	1.394
4	SDSS 1206+4332	0.745	1.789

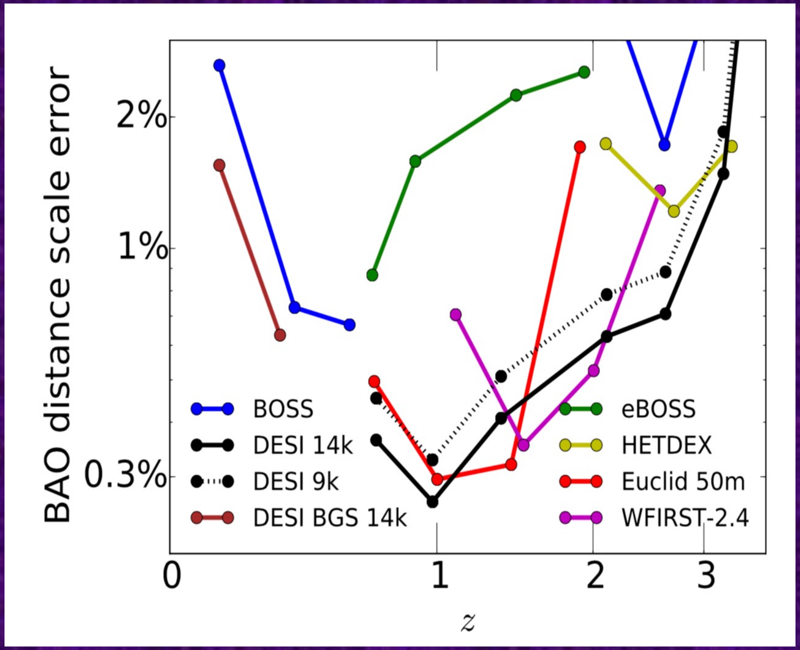
Future perspective (late universe, SN Ia)

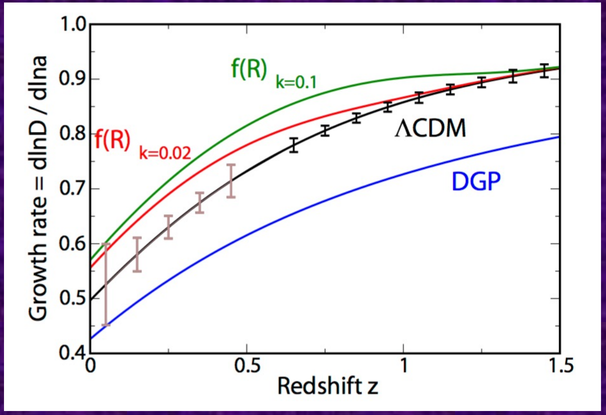


Scolnic,et al, arXiv:1903.05128

Future perspective (late universe; BAO & RSD)







Aghamousa et al, [arXiv:1611.00036]
DESI Collaboration

Future perspective [G-Waves and Standard Sirens]

Astro2020

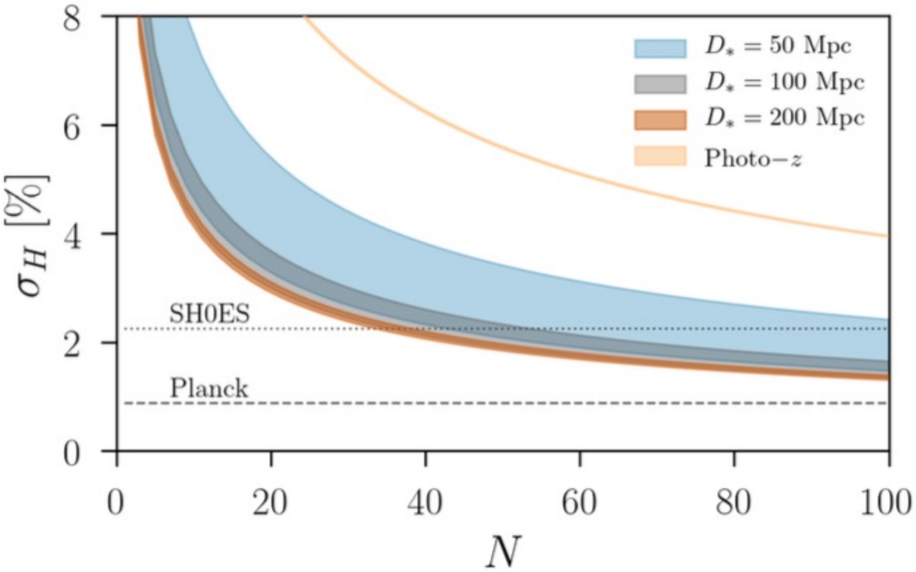


Figure 1: Hubble constant uncertainty (1σ) as a function of combined GW events with associated EM counterpart. The shaded regions show the impact of the peculiar velocity uncertainty between 100 and 400 km s^{-1} for different distance reaches D_*. The latest results from standard candles (SH0ES, [13]) and CMB (*Planck*, [14]) are also shown.

Palmese et al,
arXiv:1903.04730

Statistical dark sirens constraint

- Take every galaxy inside the sky localization $\Delta\Omega$ and marginalize over the probability that they hosted the GW event
- $P(\theta|d_{gal}, d_{GW}) \propto \sum_i \int dz_i \exp(-(d_{l,GW} - d_l(z_i,\theta)^2 / 2\sigma^2) P(z_i)$
- $P(z) = \delta(z - z_{gal})$ for spectroscopic redshifts

Statistical dark sirens

- sky localization is important for this technique
- can only be done if there are a handful candidate galaxies and can get individual modes of the posterior
- Thus currently can only work at lower redshifts
- But if there are biases as might be expected from photometric redshifts, result analysis will be biased too.

A simple and promising principle...

- Both galaxies and gravitational wave sources are biased tracers of the underlying matter density field.

- Biases can be scale-dependent and redshift-dependent.

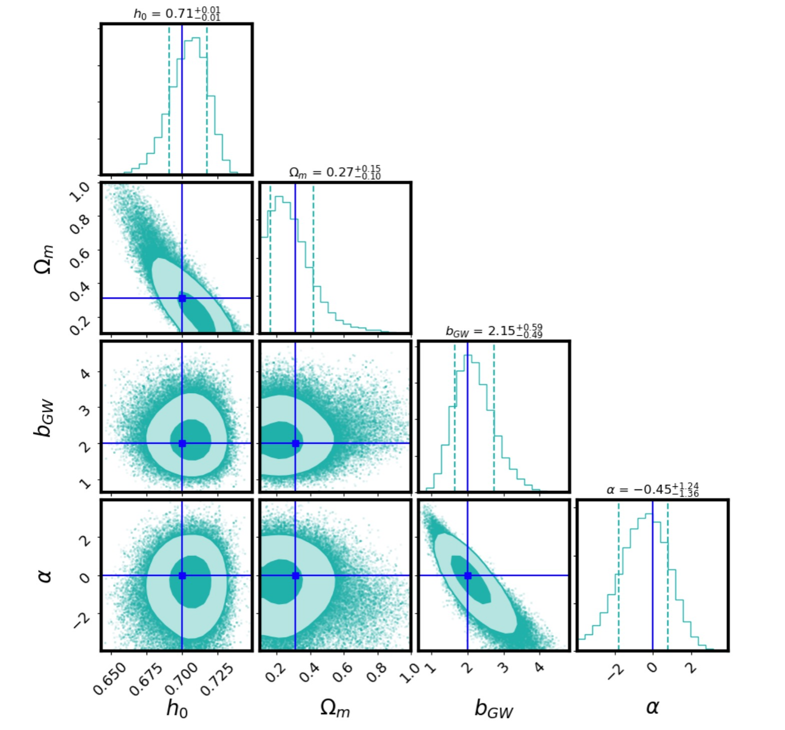
(density fields “delta” in Fourier space below)

$$\delta_{\rm gal}(\mathbf{k}, z) \equiv b_{\rm gal}(\mathbf{k}, z)\delta_{\rm m}(\mathbf{k}, z)$$
$$\delta_{\rm gw}(\mathbf{k}, z) \equiv b_{\rm gw}(\mathbf{k}, z)\delta_{\rm m}(\mathbf{k}, z)$$

- e.g. model $b_{\rm gw}(\mathbf{k}, z) \equiv b_{\rm gw}(z) = \boxed{b_0}(1+z)^{\boxed{\alpha}}$

Existing constraints

- 2% error on H0 for 200 BBH in 5 redshift bins from z~0.1 to 0.5
- Bias parameters b_{GW} and α uncorrelated with cosmological parameters
- Similar constraint from 50 BNS



Mukherjee et al PRD ‘21

Cross Correlating Dark Sirens with LSS data (using *lognormal mocks*)

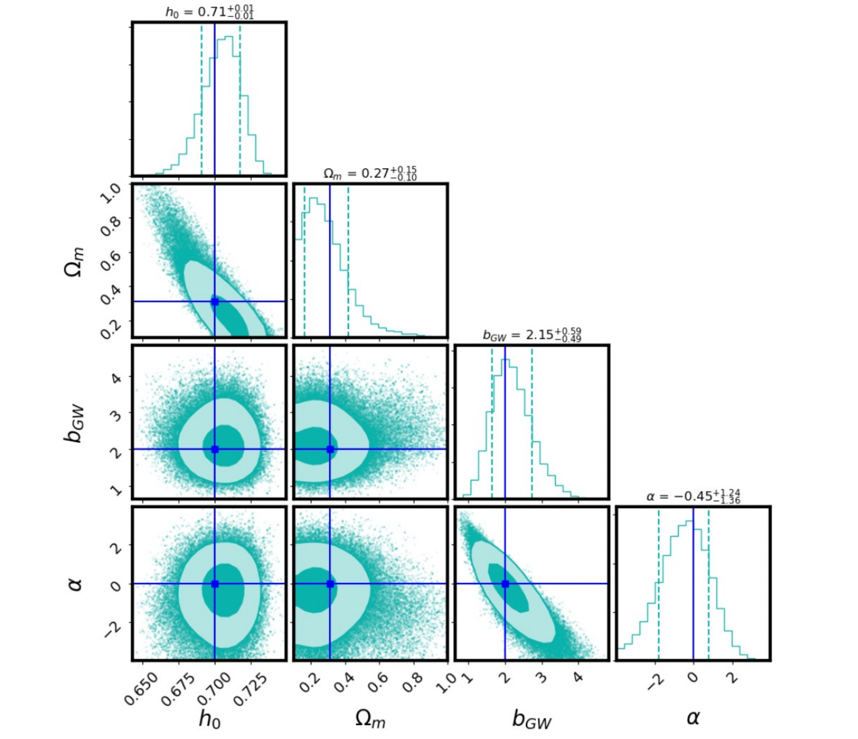
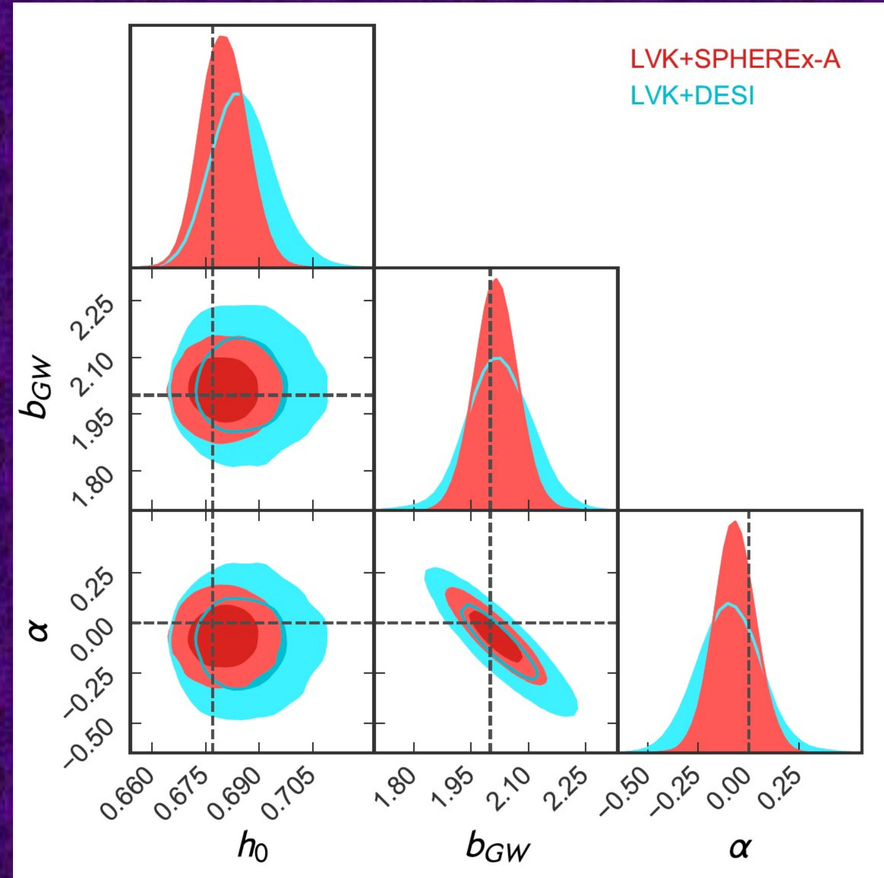


FIG. 2: We show the joint posterior of the cosmological parameters $H_0 = 100h_0$ km/s/Mpc and Ω_m along with the nuisance parameters related to the gravitational wave bias parameter $b_{GW}(z) = b_{GW}(1+z)^\alpha$ for number of gravitational wave sources $N_{GW}(z) = 40$ extended up to redshift $z = 0.5$, and sky localization error $\Delta\Omega_{GW} = 10$ sq. deg. The 68%, and 95% contours are shown in these plots along with the input values by the blue line. The mean value along with 1σ error-bar are mentioned in the title of the posterior distribution for all the parameters. Other cosmological parameters such as $w_0 = -1$ and $w_a - 0$ are kept fixed for these results.



Mukherjee et al, arXiv:2007.02943

Mukherjee et al, arXiv:2107.12787

Cross Correlating Dark Sirens with LSS data (using *lognormal mocks*)

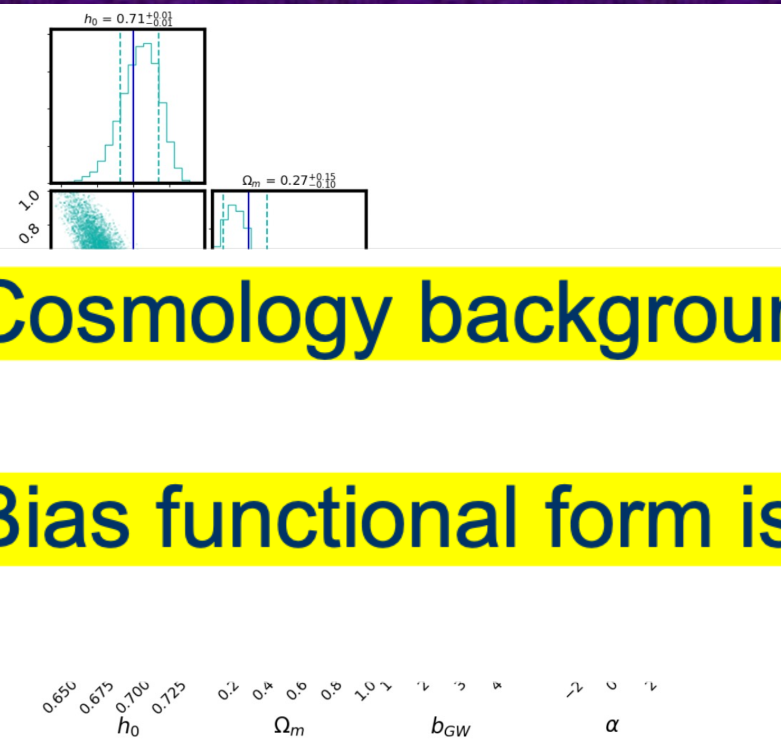
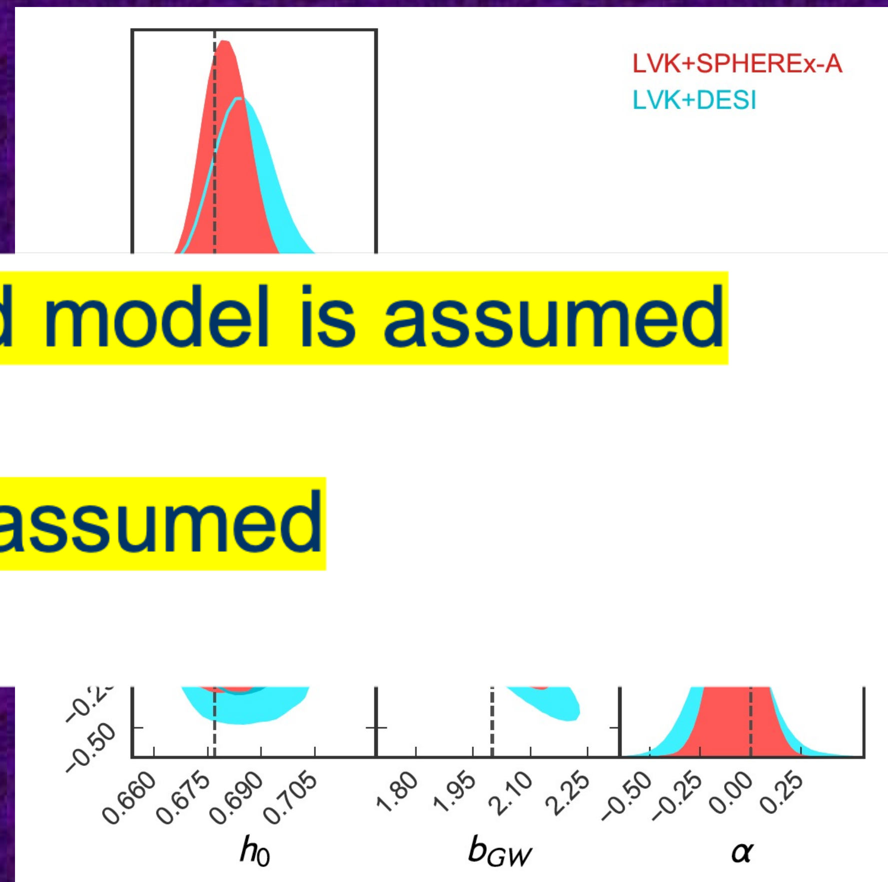


FIG. 2: We show the joint posterior of the cosmological parameters $H_0 = 100h_0$ km/s/Mpc and Ω_m along with the nuisance parameters related to the gravitational wave bias parameter $b_{GW}(z) = b_{GW}(1+z)^\alpha$ for number of gravitational wave sources $N_{GW}(z) = 40$ extended up to redshift $z = 0.5$, and sky localization error $\Delta\Omega_{GW} = 10$ sq. deg. The 68%, and 95% contours are shown in these plots along with the input values by the blue line. The mean value along with 1σ error-bar are mentioned in the title of the posterior distribution for all the parameters. Other cosmological parameters such as $w_0 = -1$ and $w_a - 0$ are kept fixed for these results.

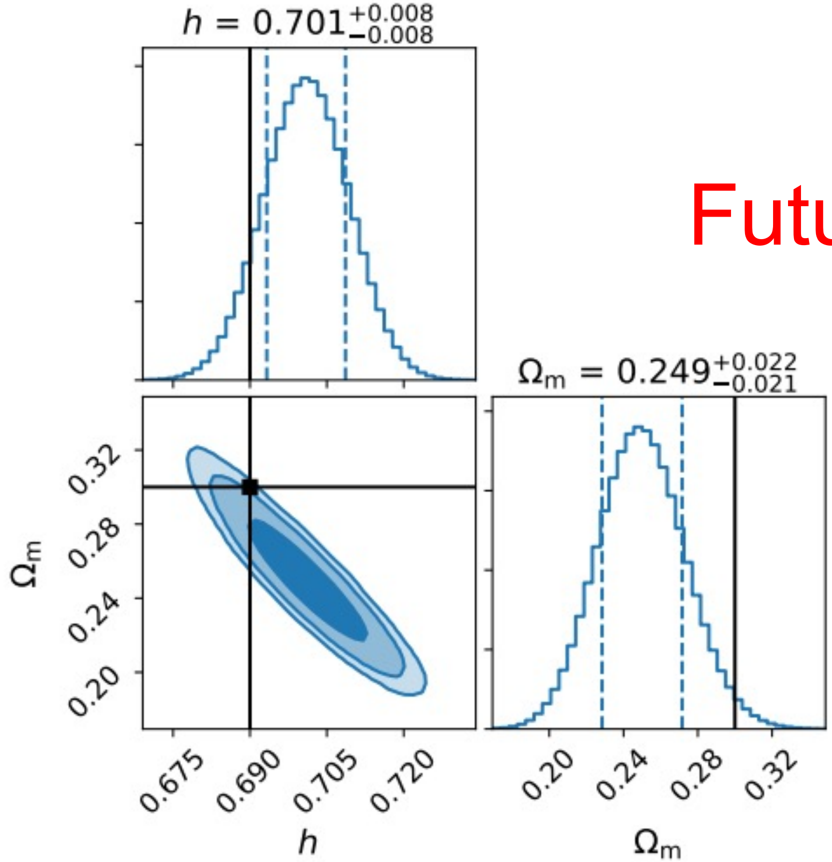


Cosmology background model is assumed

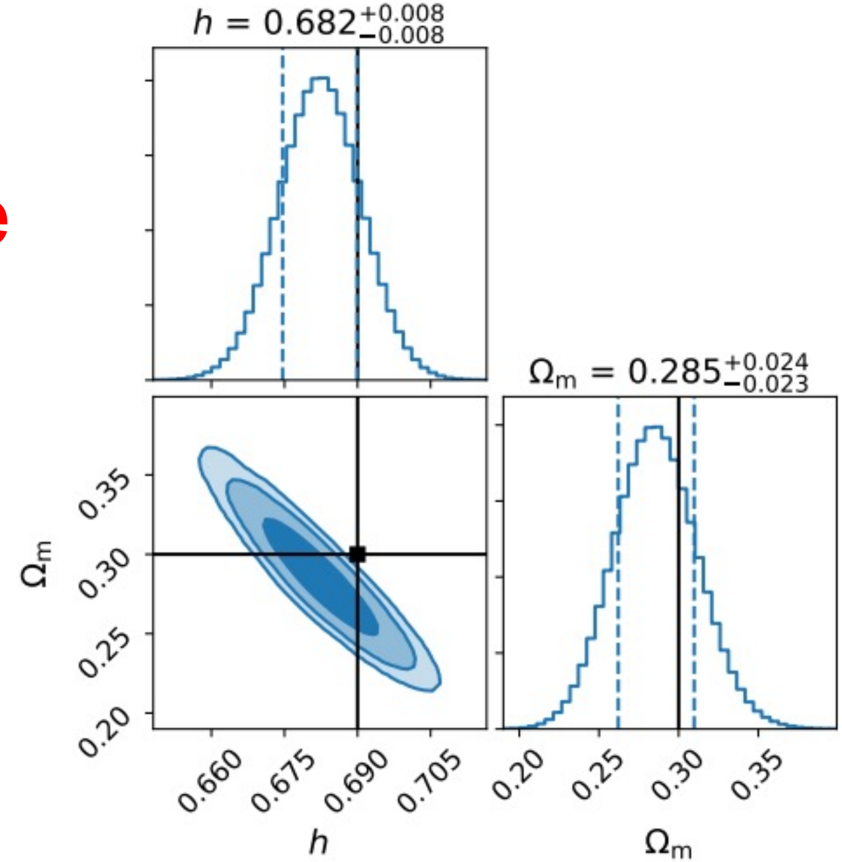
Bias functional form is assumed

Mukherjee et al, arXiv:2007.02943

Mukherjee et al, arXiv:2107.12787



Future



Shafieloo, Keeley, Linder, JCAP 2020

$(w_0, w_a) = (-0.90, -0.75)$

$\Omega_m = 0.3,\ h = 0.69$

$(w_0, w_a) = (-1.14, 0.35)$

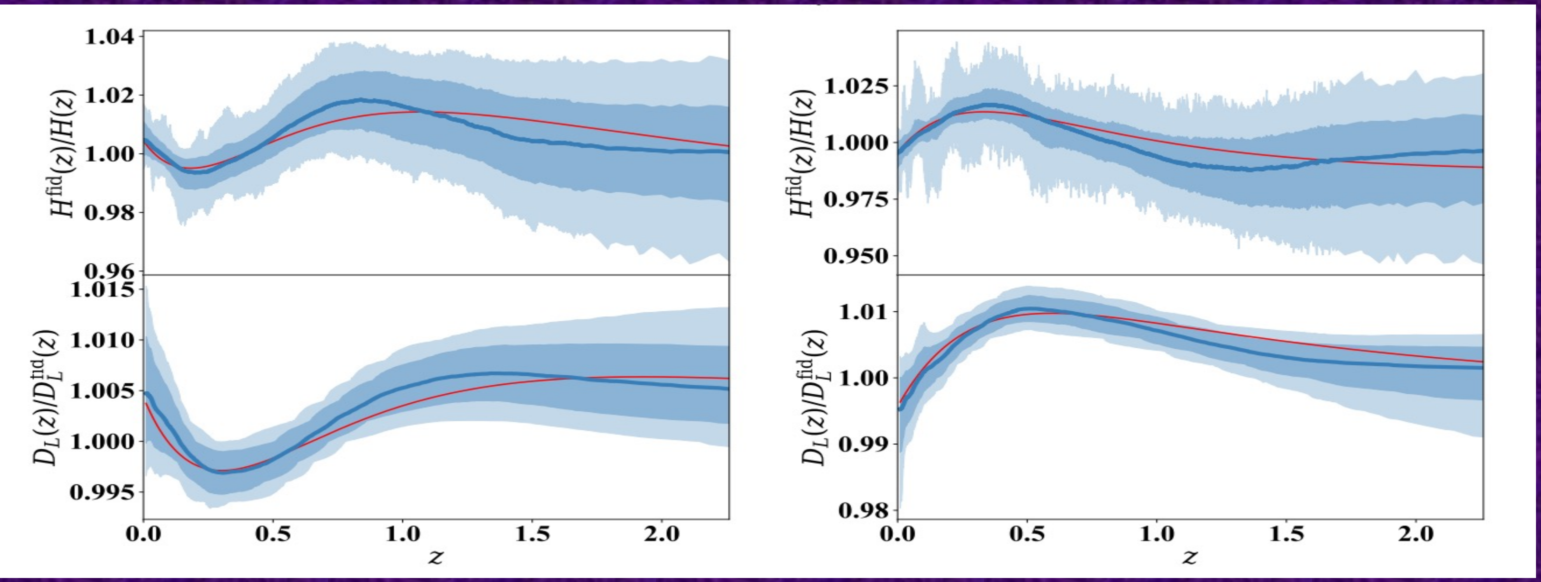
$\Omega_m = 0.3,\ h = 0.69$

What if the assumed model is wrong?!

Now with SN

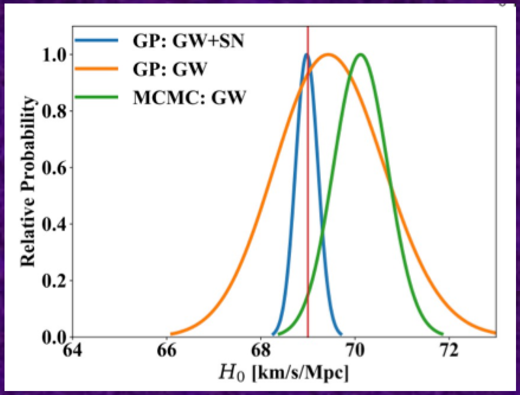
- Galaxies are observed in redshift space
- GW are observed in DL space
- Assume fiducial cosmology to convert between them
- Or use distances from SN

Reconstructing the expansion history using Gravitational Wave Sirens



Having reliable reconstruction of the expansion history we can attempt to measure H0 accurately

Keeley, Shafieloo, L'Huillier, Linder, MNRAS 2020



Data will be hugely better…but we have to be careful!

Cosmology vs Systematics

- With higher quality of the data the role of systematics will become more and more prominent.
- Higher precision may cost us uncontrollable bias if we make wrong assumptions.

Point 6

What we should be worried about!

Conclusion

- Many statistical tools are not used appropriately in cosmology and astrophysics and results can be strong but invalid conclusions.
- H0 tension (and some others) seems remaining persistent in the context of the LCDM model. This can open ways for competitive alternatives (GEDE?, EDE, features in PPS?) but we should not over-sell these models.
- ***Tensions are not resolved with minimal extensions of the standard model and there is no clear resolution.*** It is highly possible, from statistical point of view, that there are ***systematics*** in some of the data and we might need ***new physics*** too. ***It can be a combination of both!*** New independent measurements and observations can help to clear things up.
- With higher quality data, the effect of systematics and wrong assumptions are much more prominent in introducing substantial inaccuracies. This is a real challenge to avoid making big fake discoveries.