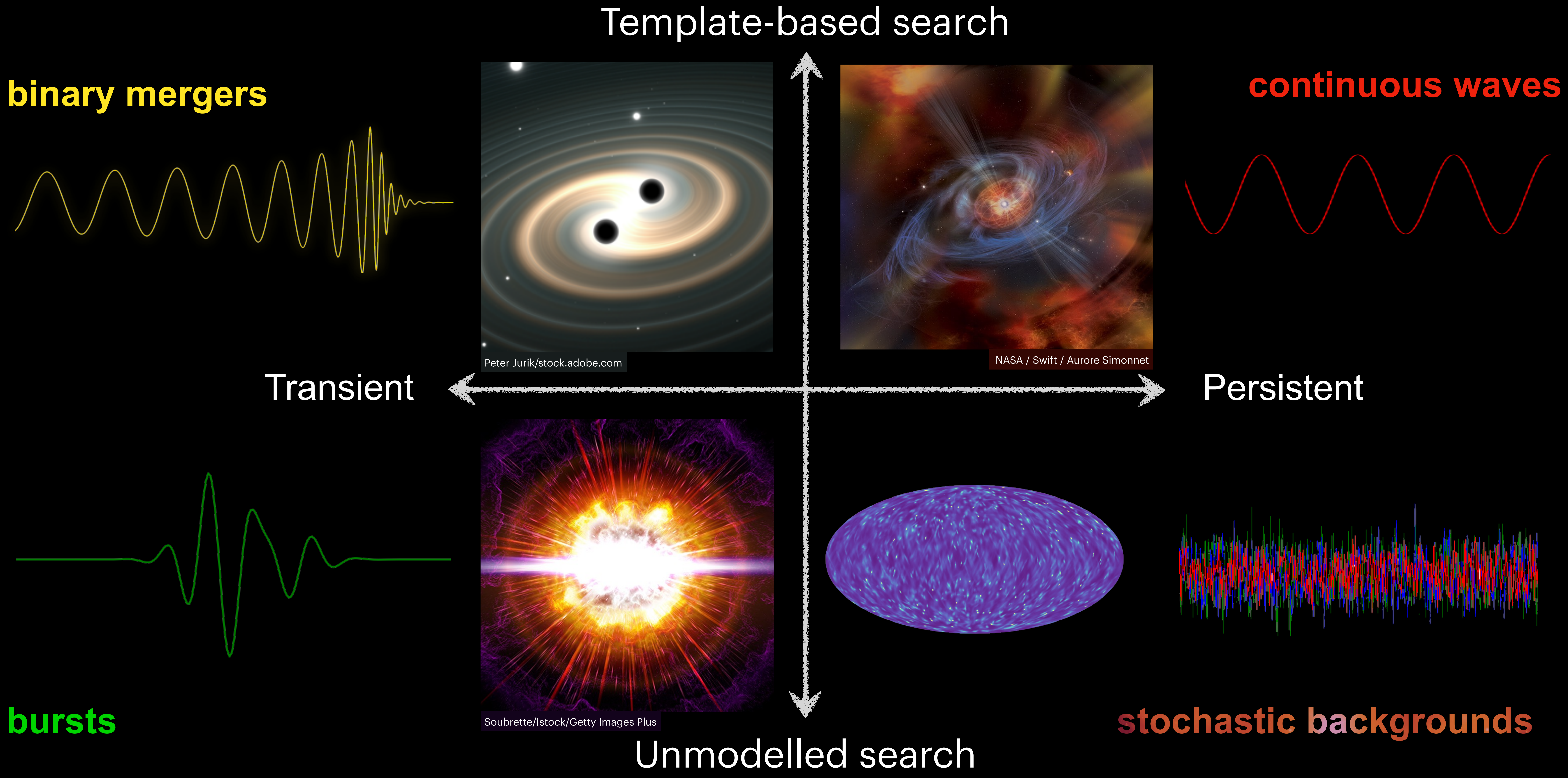
Cosmology with black hole binaries

dark sirens, anisotropies, & stochastic methods

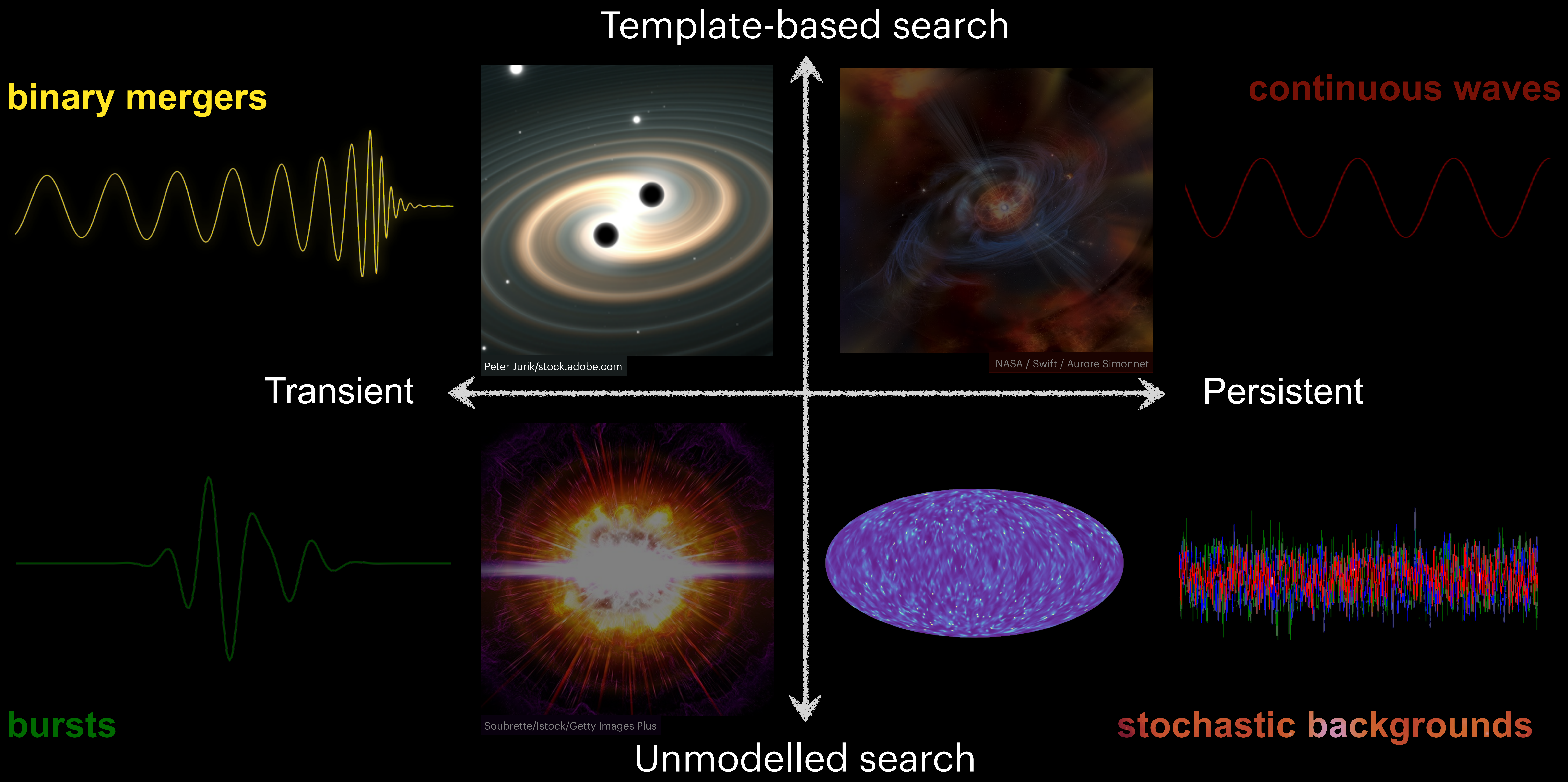
Arianna Renzini

ModIC — Trieste, IFPU — 14 May 2024

Gravitational-wave zoology



Gravitational-wave zoology

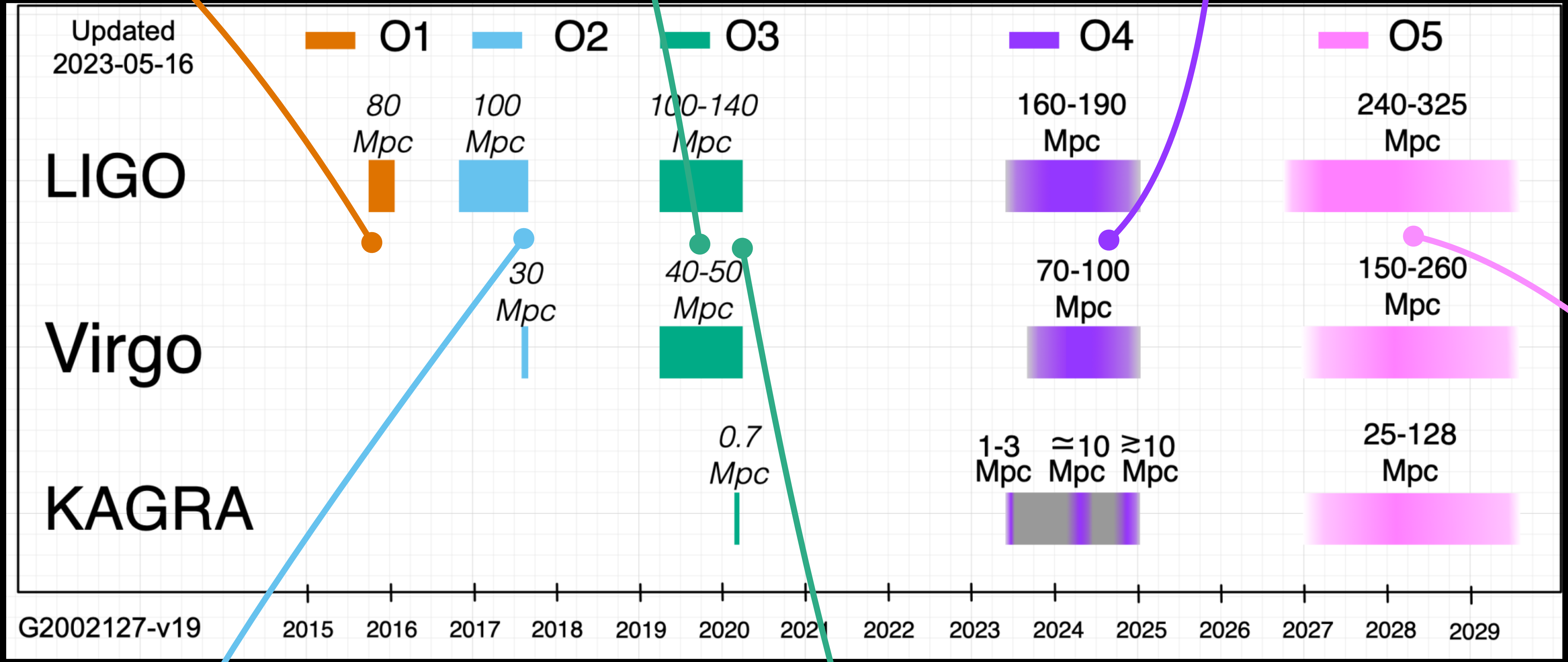


recap of LVK GW detection history/plans:

first NSBH!

first BBH!

probably more population studies...



...GWB?

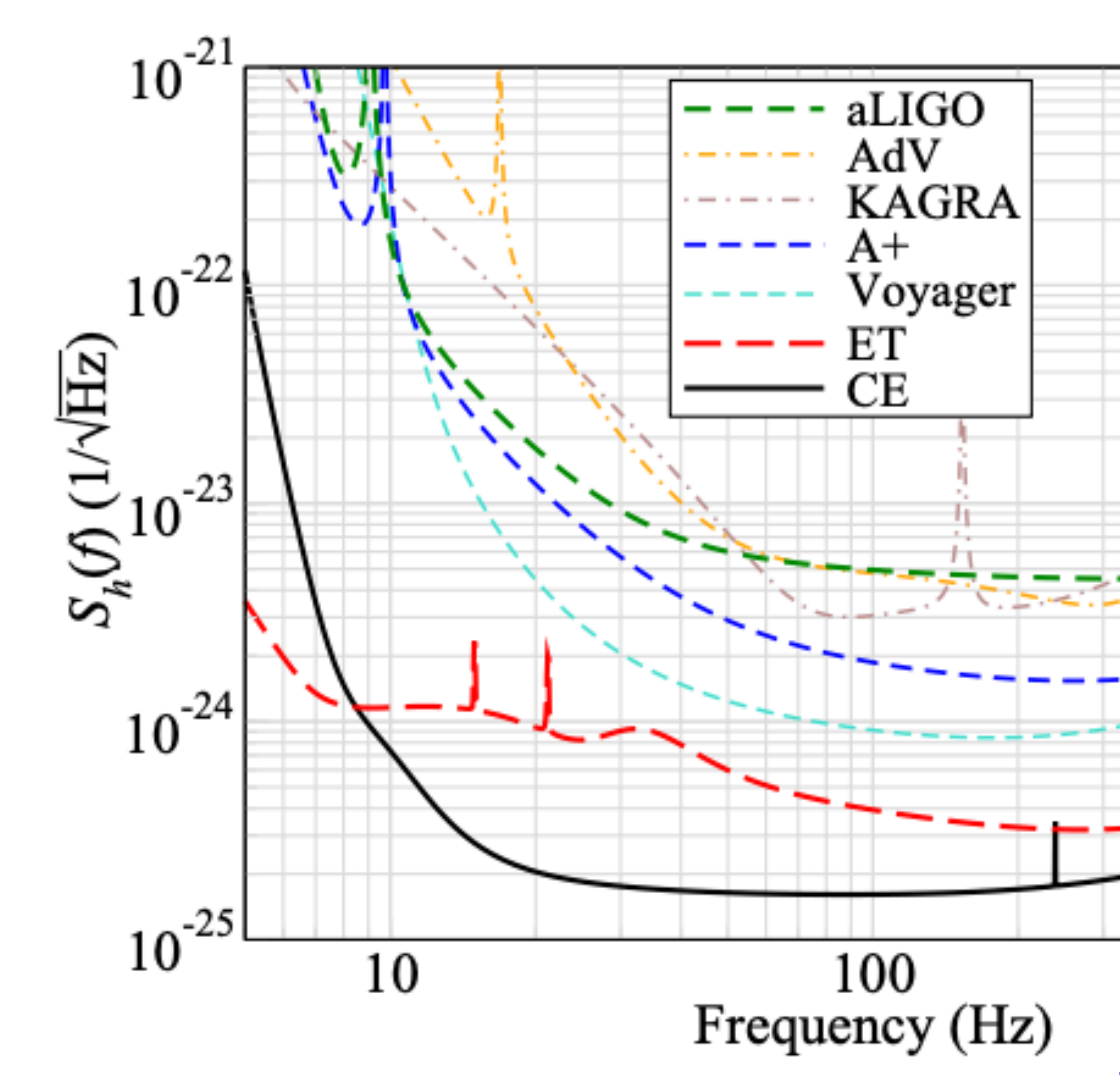
first BNS!

population studies!

The future (on the ground): 3G

Extend the depth of ground surveys up to $z \approx 20 \longrightarrow$ resolve all BBHs!

- BNS/BHNS foreground and background
- pop III / high z BBHs



Hall GitHub

- cosmological signals
- primordial BBHs
- ...

Cosmological information in GWs from BBHs

measured gravitational wave: $h_{\mathrm{GW}}\,(m_1, m_2, s_1, s_2, d_L, \theta, \phi, \varphi)$

distance

location

Cosmological information in GWs from BBHs

<u>measured</u> gravitational wave: $h_{\mathrm{GW}}\,(m_1, m_2, s_1, s_2, d_L, \theta, \phi, \varphi)$

distance

location

inherent degeneracy

$$m_{\mathrm{det}} = (1+z)\, m_{\mathrm{source}}$$

$$d_L = 1 + z \int_0^z dz' \frac{1}{H(z)}$$

$$H(z) = \sqrt{\Omega_{\mathrm{M},0}(1+z)^3 + \Omega_{k,0}(1+z)^2 + \Omega_{\Lambda,0}}$$

can be correlated with other measurements

Cosmological information in BBHs: dark sirens

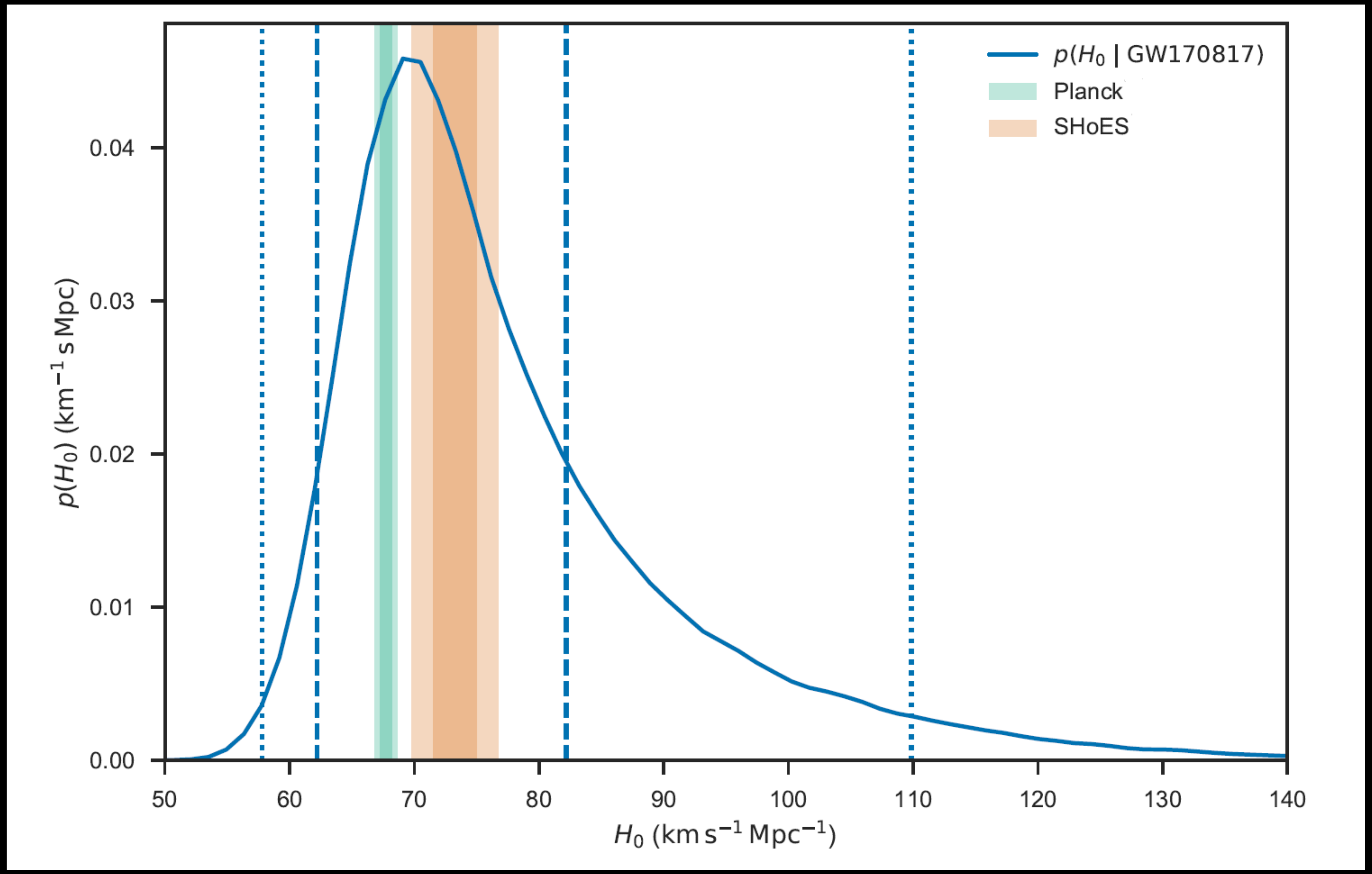
→ ***break the degeneracy:*** ***<u>independent information on z:</u>***

- Galaxy catalog ↔ matched to BBH catalog **dark sirens**

- Features in the BBH population mass spectrum encoding cosmological information **dark/spectral sirens**

- (direct EM counterpart: e.g. GW170817)

  bright sirens



LVK, Nature 551 85–88 (2017)

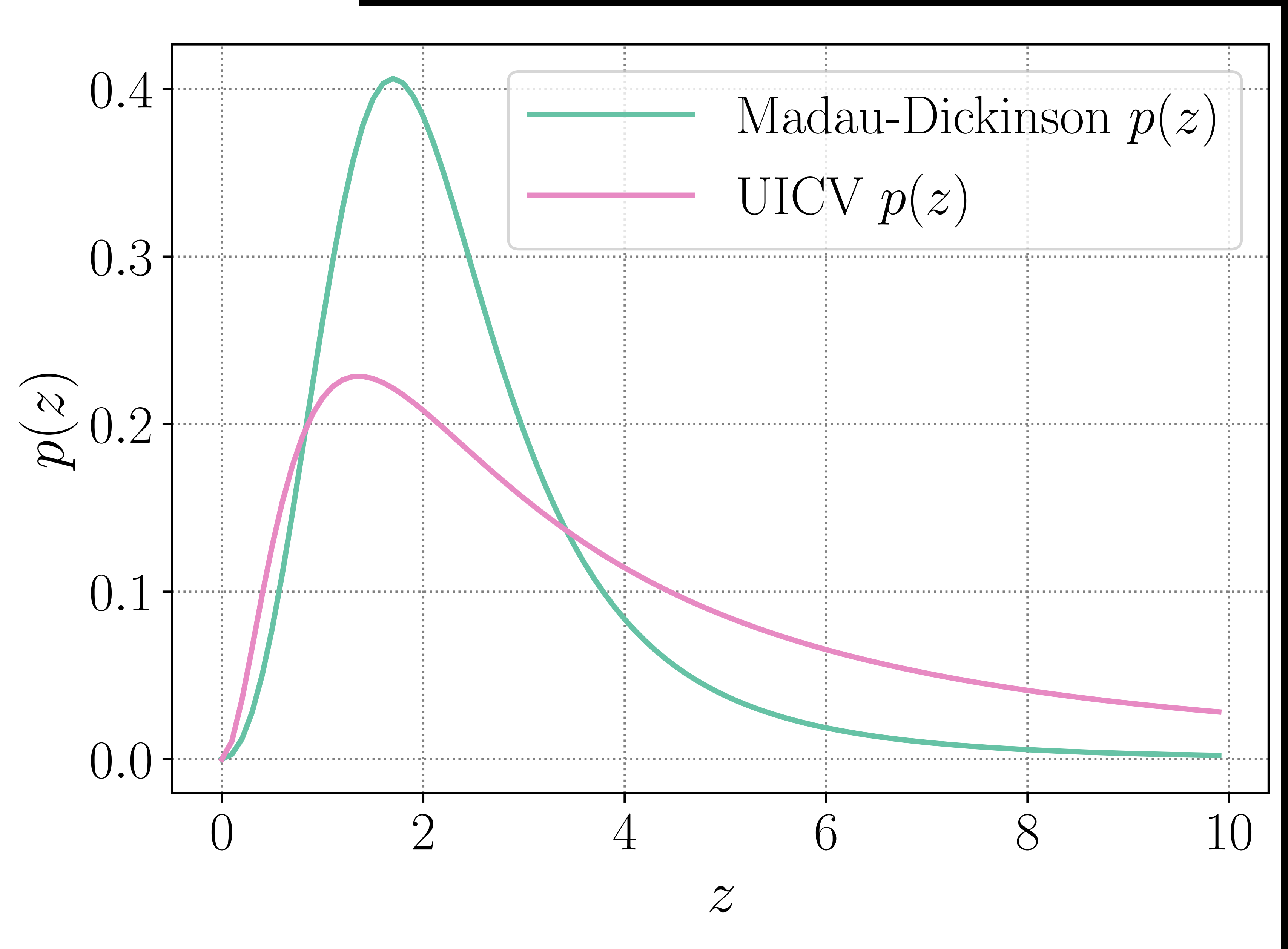
Cosmology in the BBH population : the merger rate

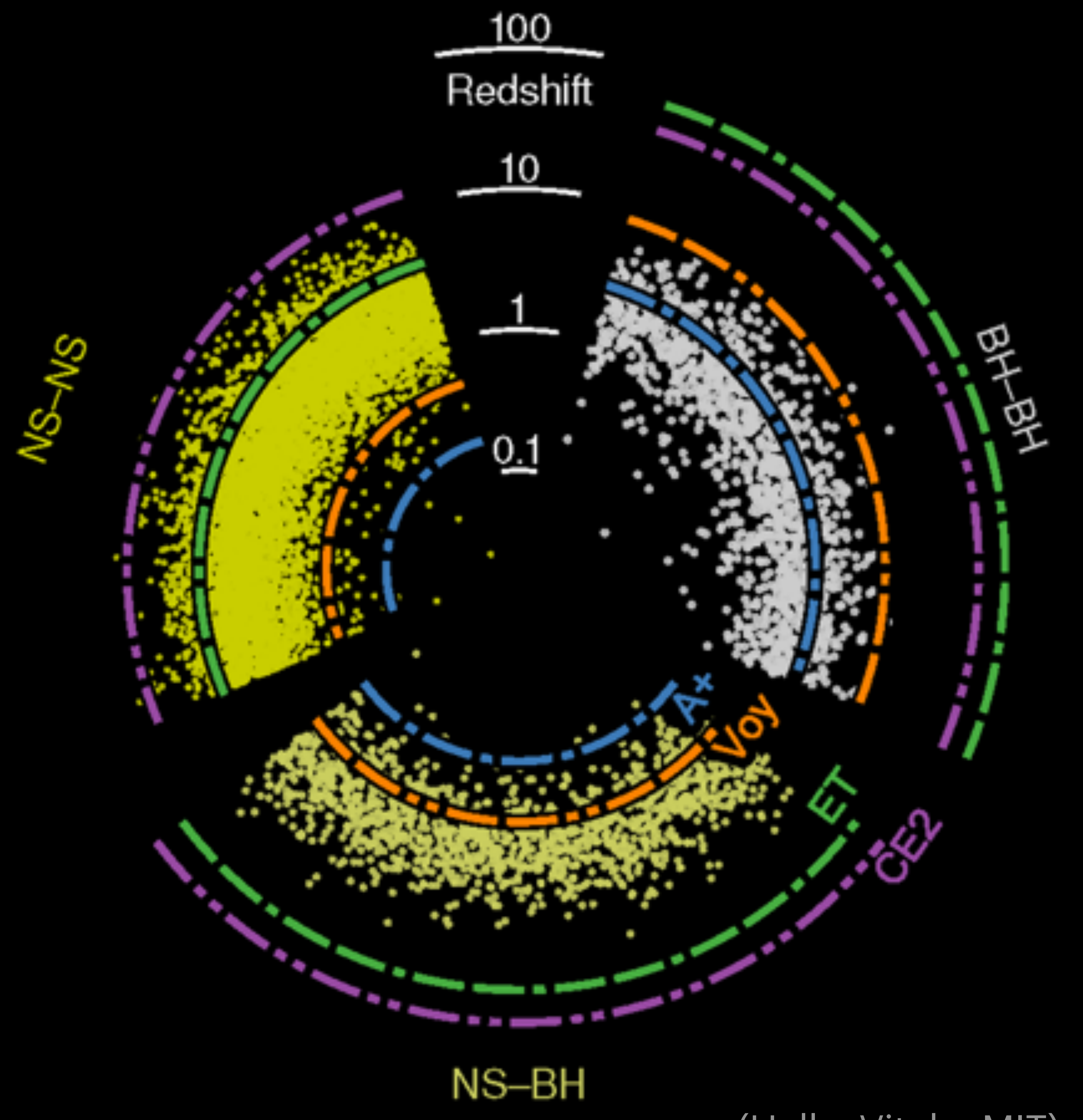
rate of mergers evolves with z !

SFR-like evolution:

$$R(z) \propto \frac{R_0\,(1+z)^{\gamma}}{1+\left(\frac{1+z}{1+z_{\rm peak}}\right)^{\kappa}}$$

$$p(z) = \frac{1}{1+z}\frac{dV_c}{dz}R(z)$$





(Hall + Vitale, MIT)

in yellow: model parameters

Dark siren cosmology: BBHs + galaxy catalogs

BBHs (probably) live in galaxies → sky locations & redshifts are correlated.

formally: use galaxy catalog as redshift prior

$$p(H_0 \,|\, d_{\rm GW}, N_{\rm obs}, \Lambda) = p(H_0)\, p(N_{\rm obs} \,|\, H_0, \Lambda) \times \prod_i^{N_{\rm obs}} p(d_{\rm GW} \,|\, \hat{D}_{\rm GW}, H_0, \Lambda)$$

localise BBH in a volume

↓

correlate with galaxies in that volume

↓

statistical z measurement

Dark siren cosmology: BBHs + galaxy catalogs

BBHs (probably) live in galaxies → sky locations & redshifts are correlated.

formally: use galaxy catalog as redshift prior in

$$p(H_0 \,|\, d_{\rm GW}, N_{\rm obs}, \Lambda) = p(H_0)\, p(N_{\rm obs} \,|\, H_0, \Lambda) \times \prod_i^{N_{\rm obs}} p(d_{\rm GW} \,|\, \hat{D}_{\rm GW}, H_0, \Lambda)$$

localise BBH in a volume

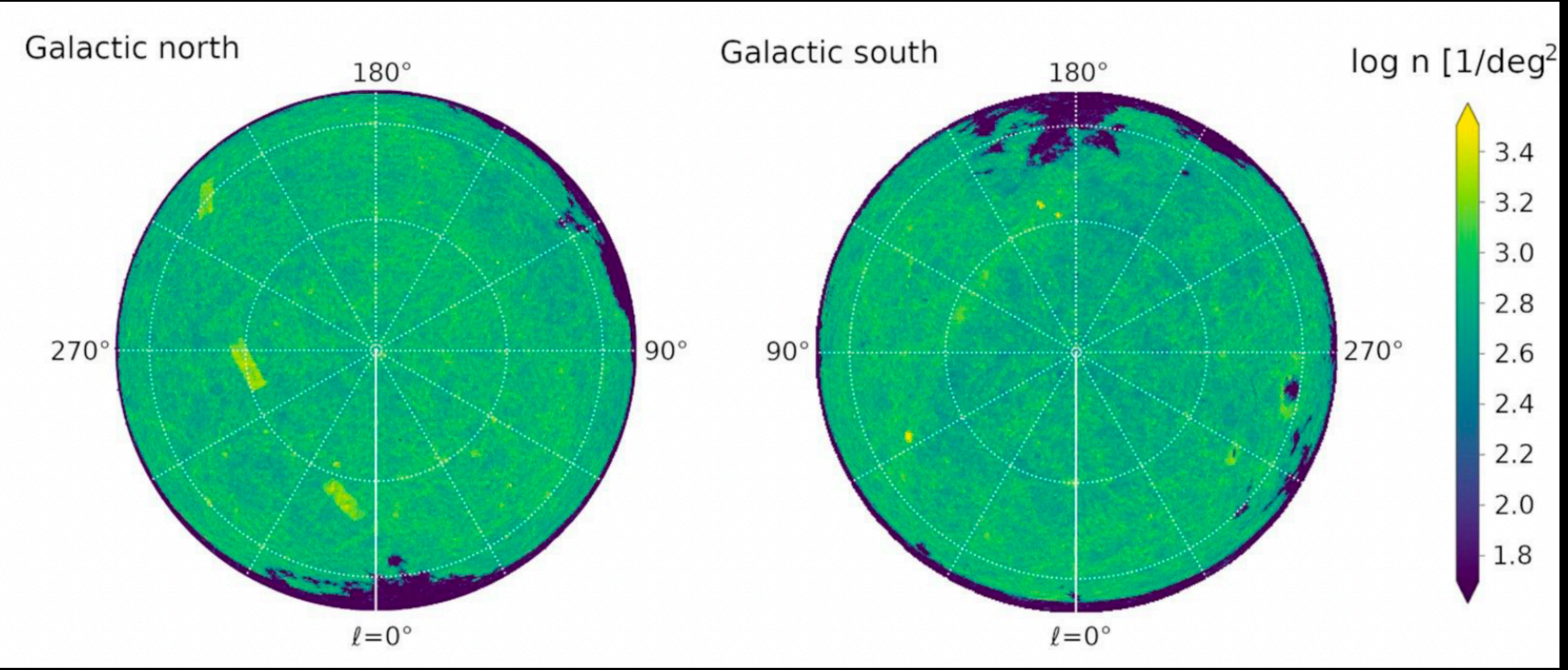
↓

correlate with galaxies in that volume

↓

statistical z measurement

... however, galaxy catalogs are incomplete:



GLADE+ : an extended galaxy catalogue for multimessenger searches with advanced gravitational-wave detectors, G. Dalya *et al.*, *MNRAS*, Volume 514, Issue 1, July 2022, Pages 1403–1411

Dark siren cosmology: BBHs + galaxy catalogs

Gray et al. '21 approach: pixelise the sky and keep track of incompleteness

$$p(d_{\rm GW}\,|\,\hat{D}_{\rm GW}, H_0, \Lambda) = \frac{1}{N_{\rm pix}} \sum_{\rm pix} p(d_{\rm GW}\,|\,\Omega_{\rm pix}, \hat{D}_{\rm GW}, H_0, \Lambda)$$

$$= \frac{1}{N_{\rm pix}} \sum_{\rm pix} p(d_{\rm GW}\,|\,\Omega_{\rm pix}, G, \hat{D}_{\rm GW}, H_0, \Lambda) p(G\,|\,\Omega_{\rm pix}, \hat{D}_{\rm GW}, H_0, \Lambda)$$

in the catalogue

$$+ p(d_{\rm GW}\,|\,\Omega_{\rm pix}, \bar{G}, \hat{D}_{\rm GW}, H_0, \Lambda) p(\bar{G}\,|\,\Omega_{\rm pix}, \hat{D}_{\rm GW}, H_0, \Lambda)$$

not in the catalogue

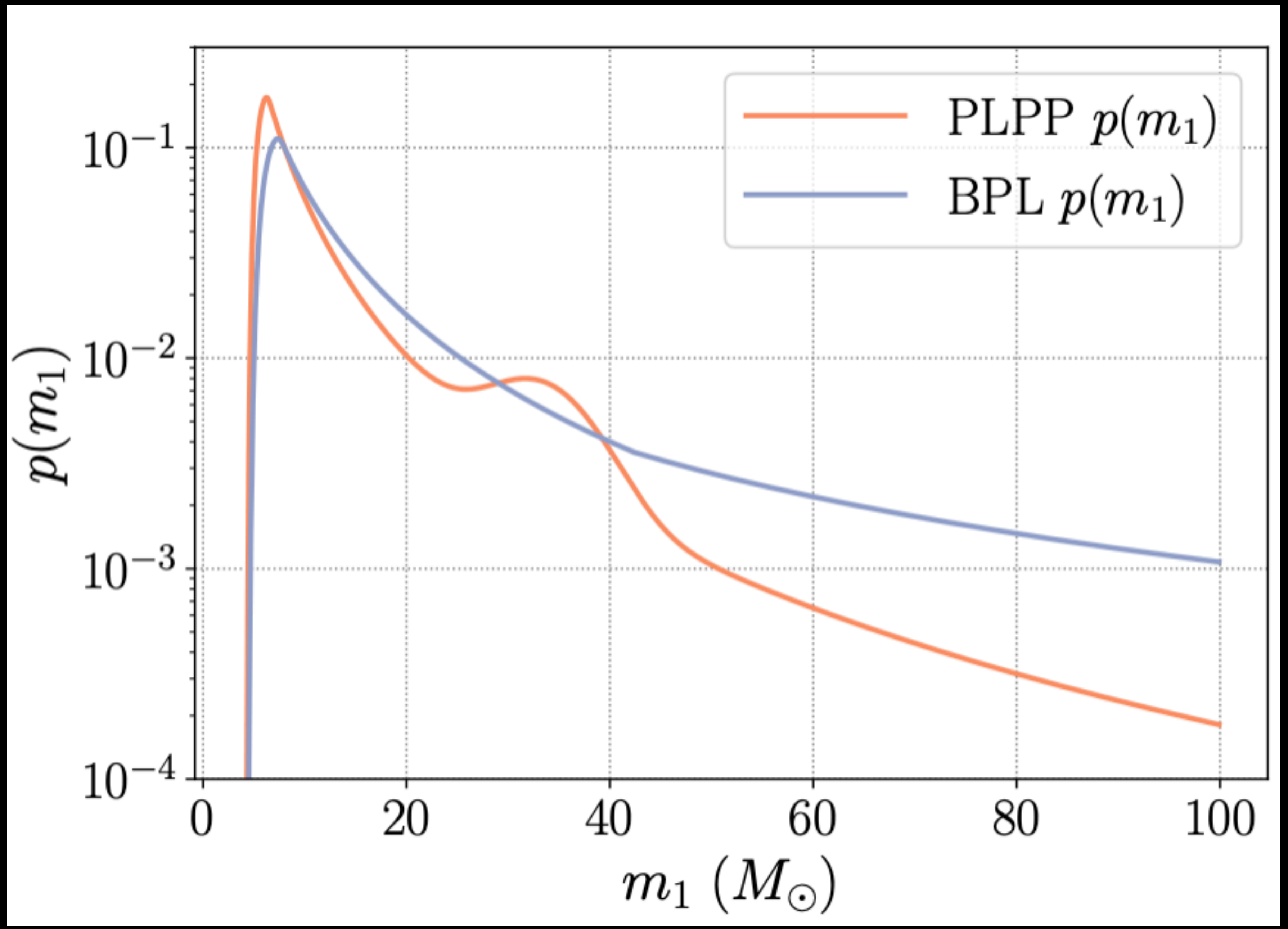
Dark siren cosmology: population models

Infer cosmology *together* with population hyper-parameters Λ:

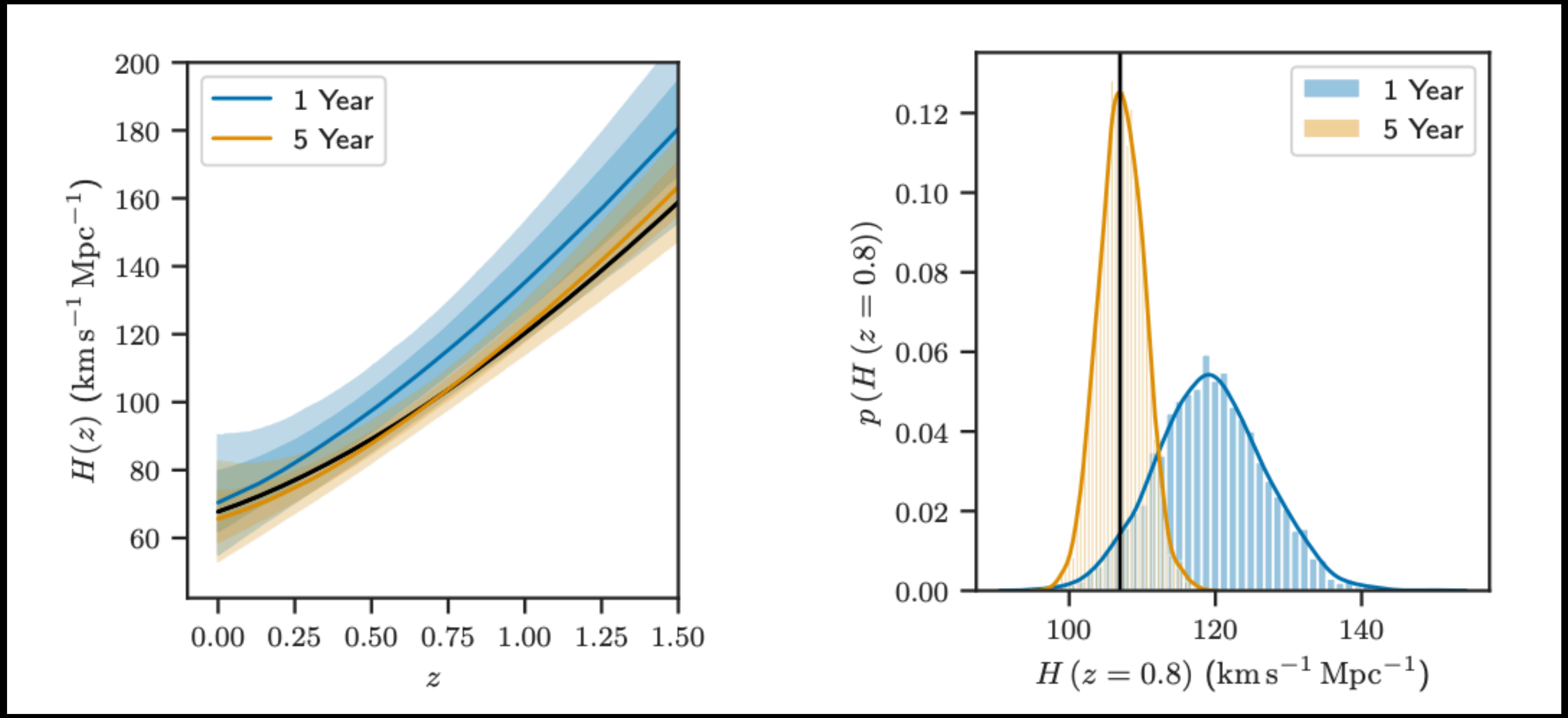
$$p_{\rm BBH}(\theta\,|\,\Lambda, H_0, \Omega_{\rm M}, w_0) \propto p(m_1, m_2\,|\,\Lambda_m)\, p(z\,|\,\Lambda_z)\, \frac{p(z\,|\,H_0, \Omega_{\rm M}, w_0)}{1+z}$$

→ many degeneracies...

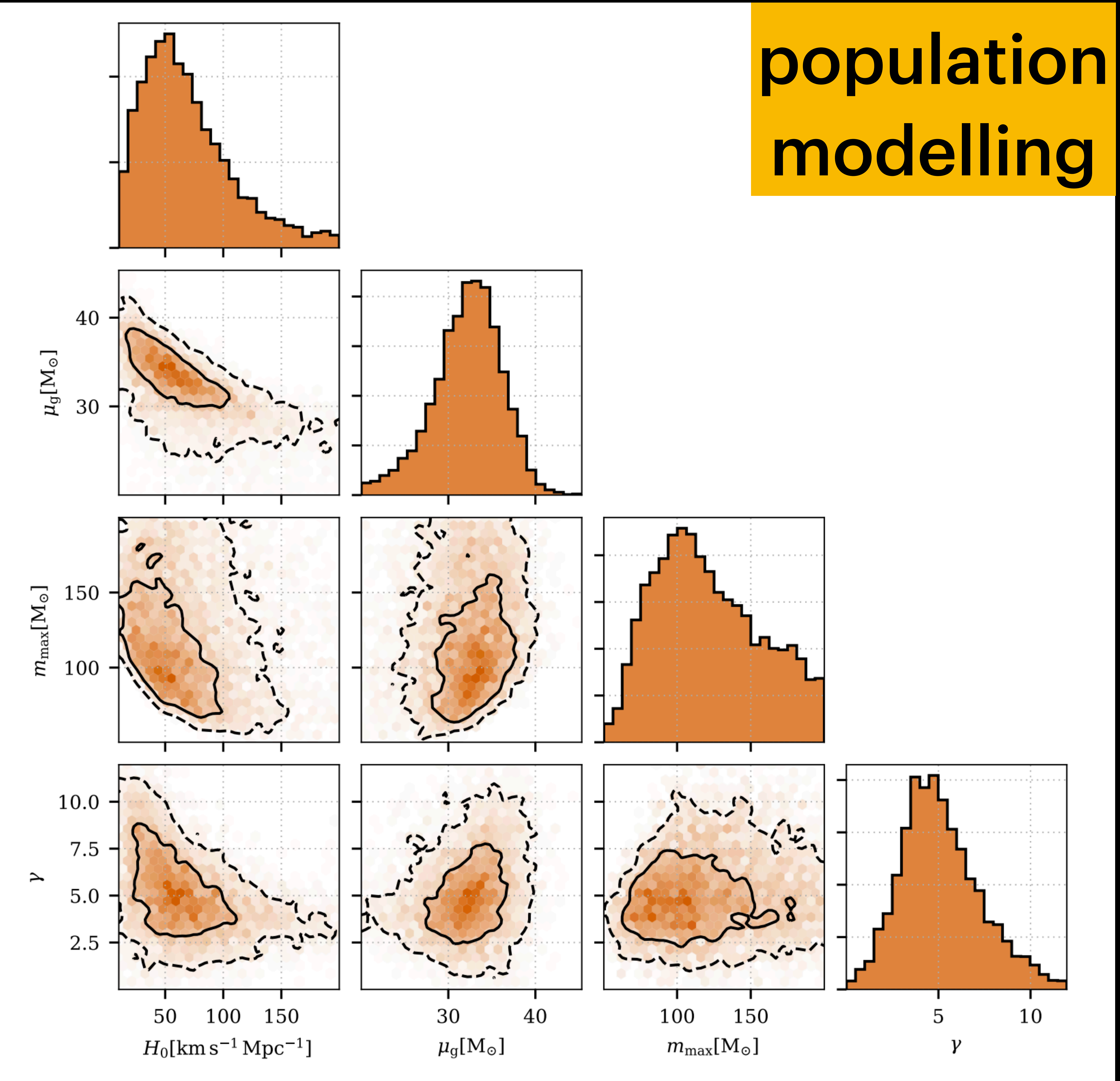
... *unless!* **we can leverage sharp features in the population distributions:**

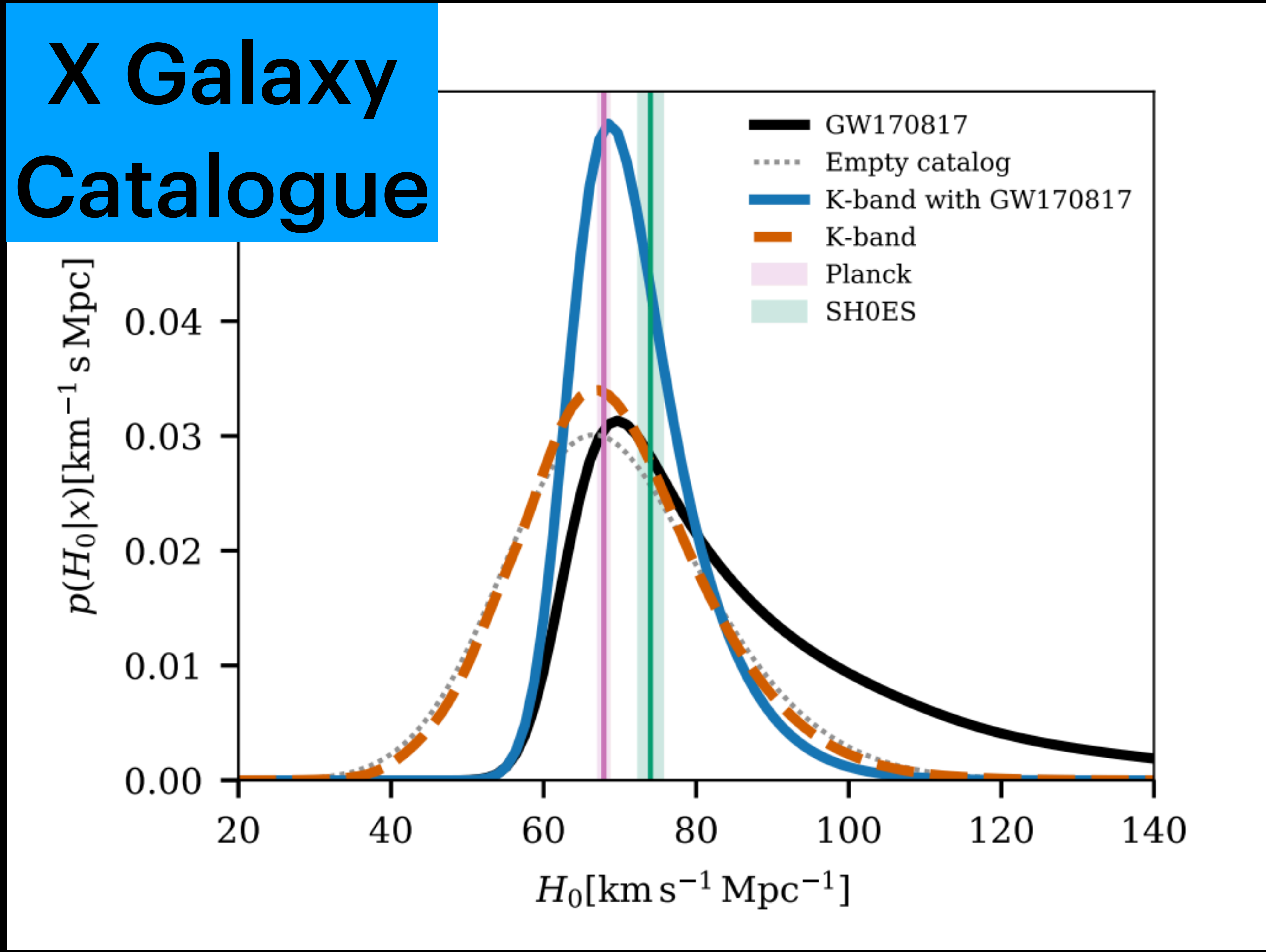


- [Farr, Fishbach, Ye, & Holz, ApJL 883 L42 (2019)]
- [Mastrogiovanni+, PRD 104 062009 (2021)]



Dark siren cosmology: LVK constraints





BOTH will become more sensitive with 3G!

which will be the most sensitive?

quality of catalogues?

systematics?

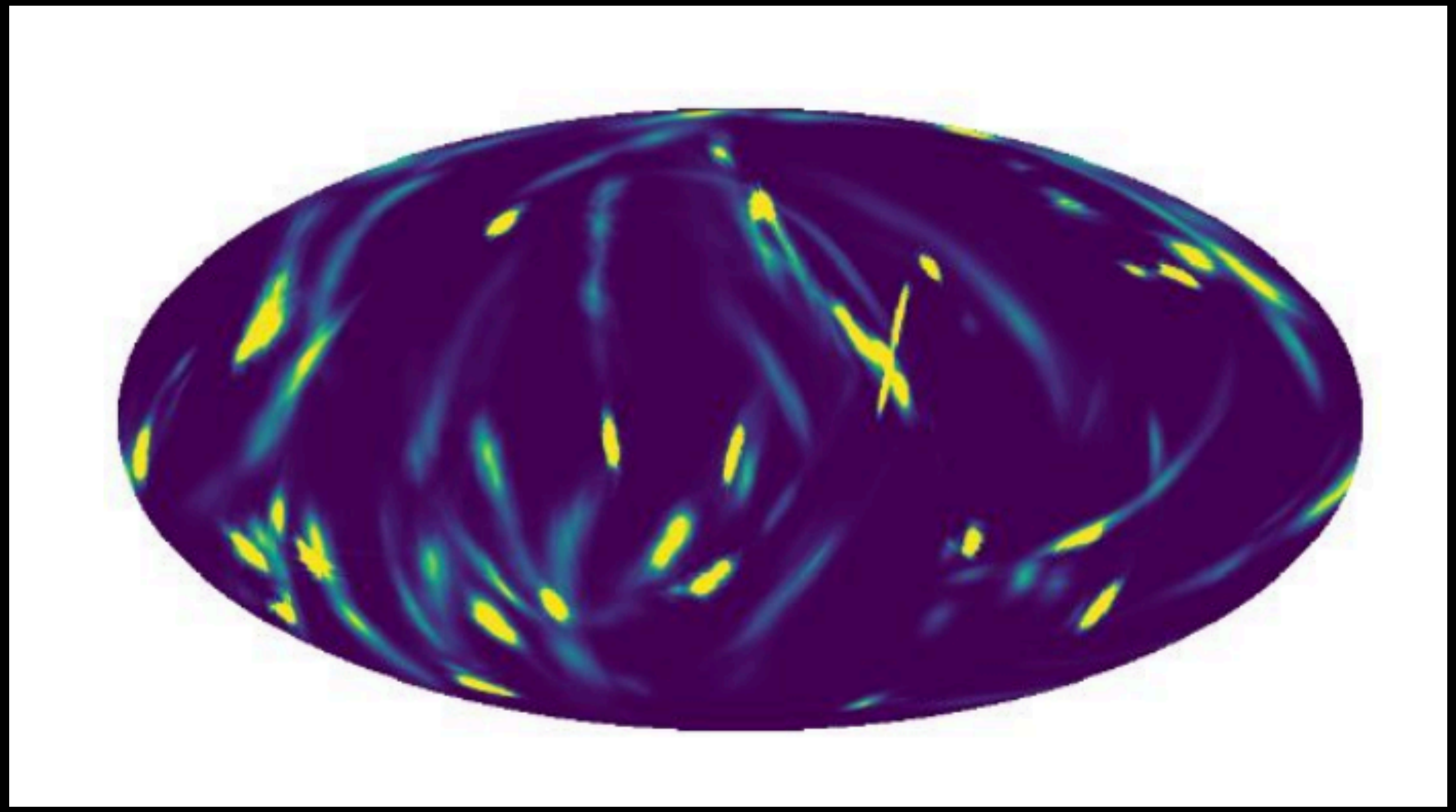
BBH/NSBH/BNS rates?

?

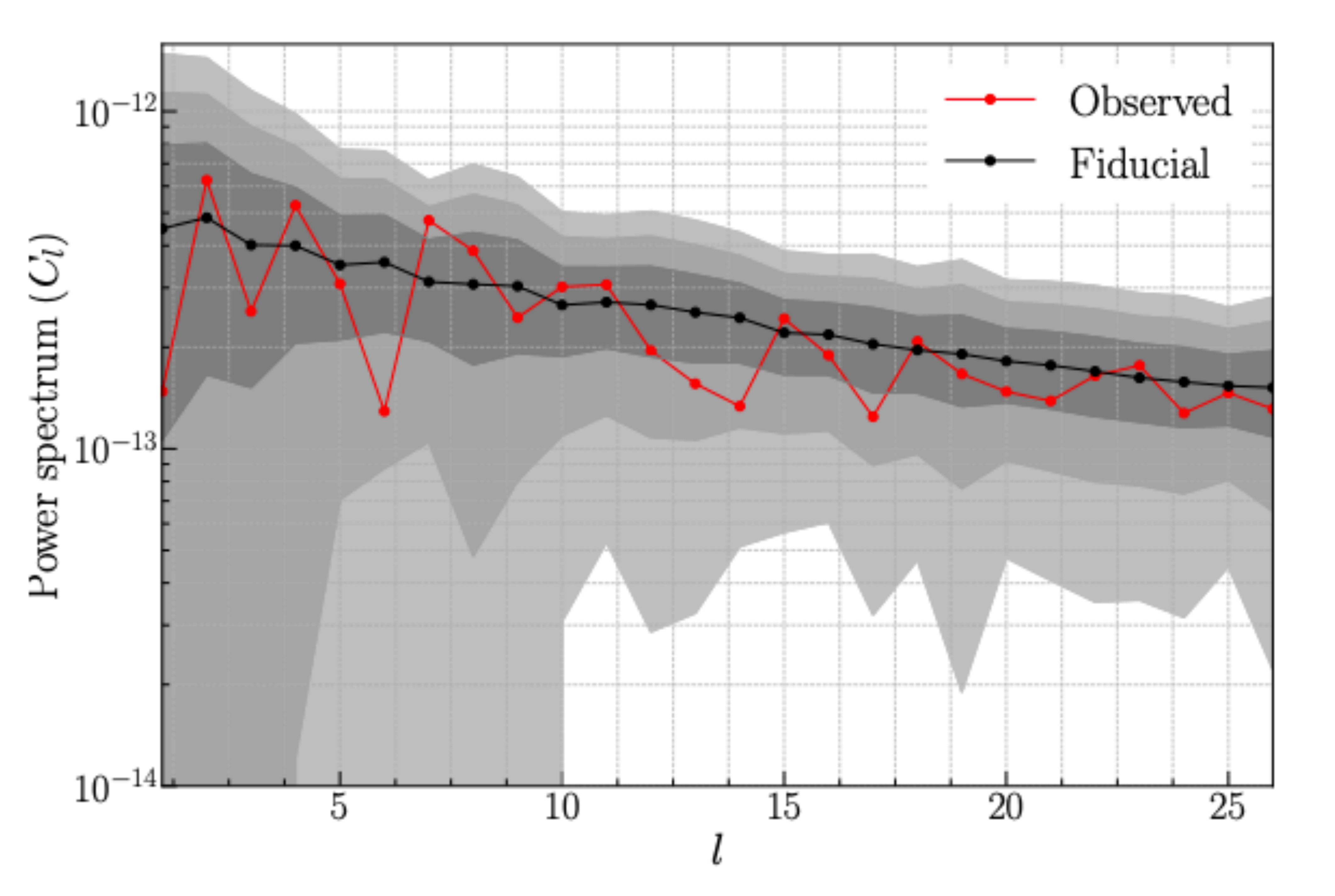
An alternative? Tracing LSS statistically

posteriors on the sky locations of the measured BBHs:

$$M^{\text{stack}}(\theta, \varphi) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\theta, \varphi) \quad \rightarrow \quad C_\ell \text{ s}$$

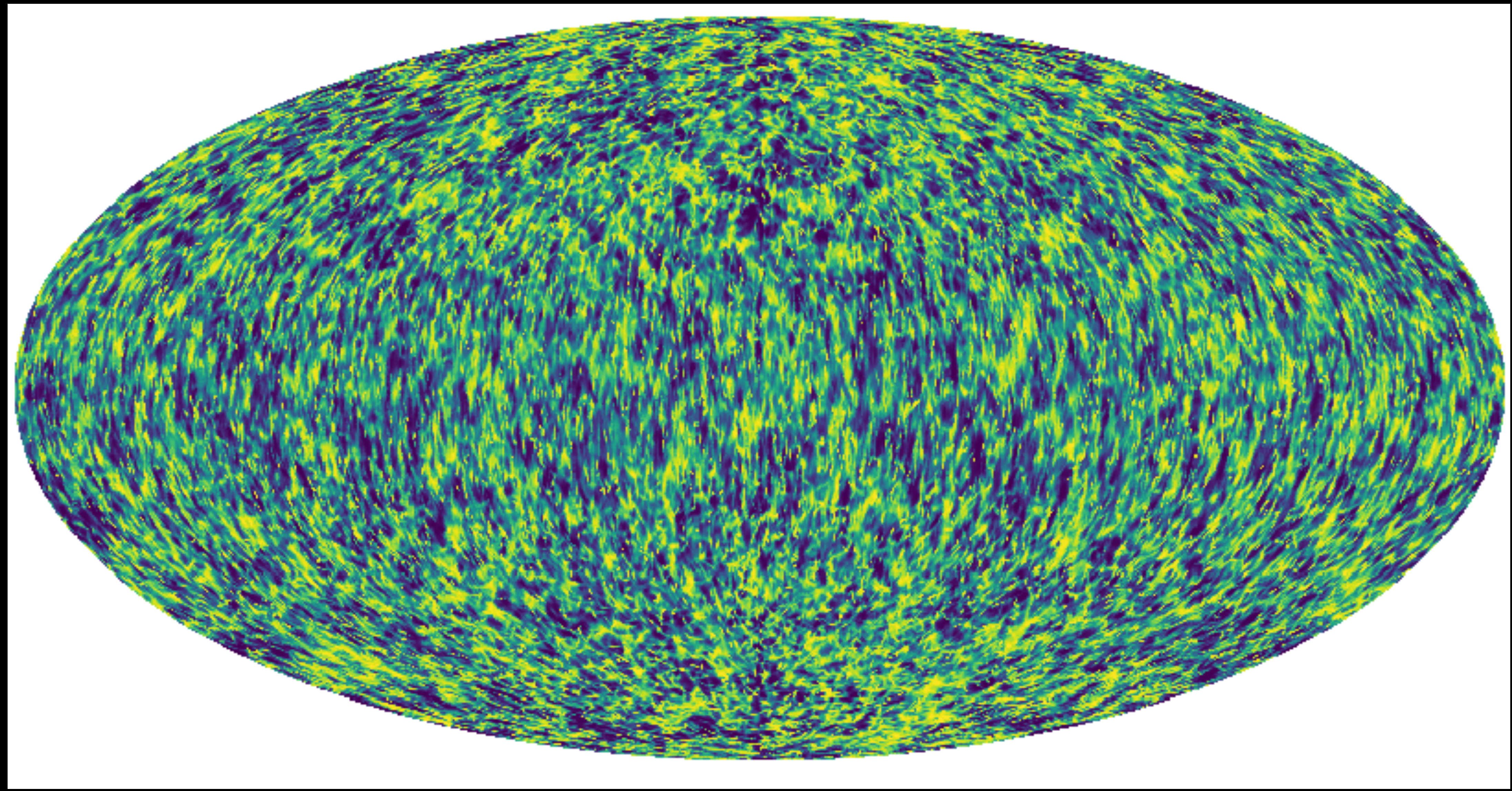


observed angular power spectrum of the BBHs:



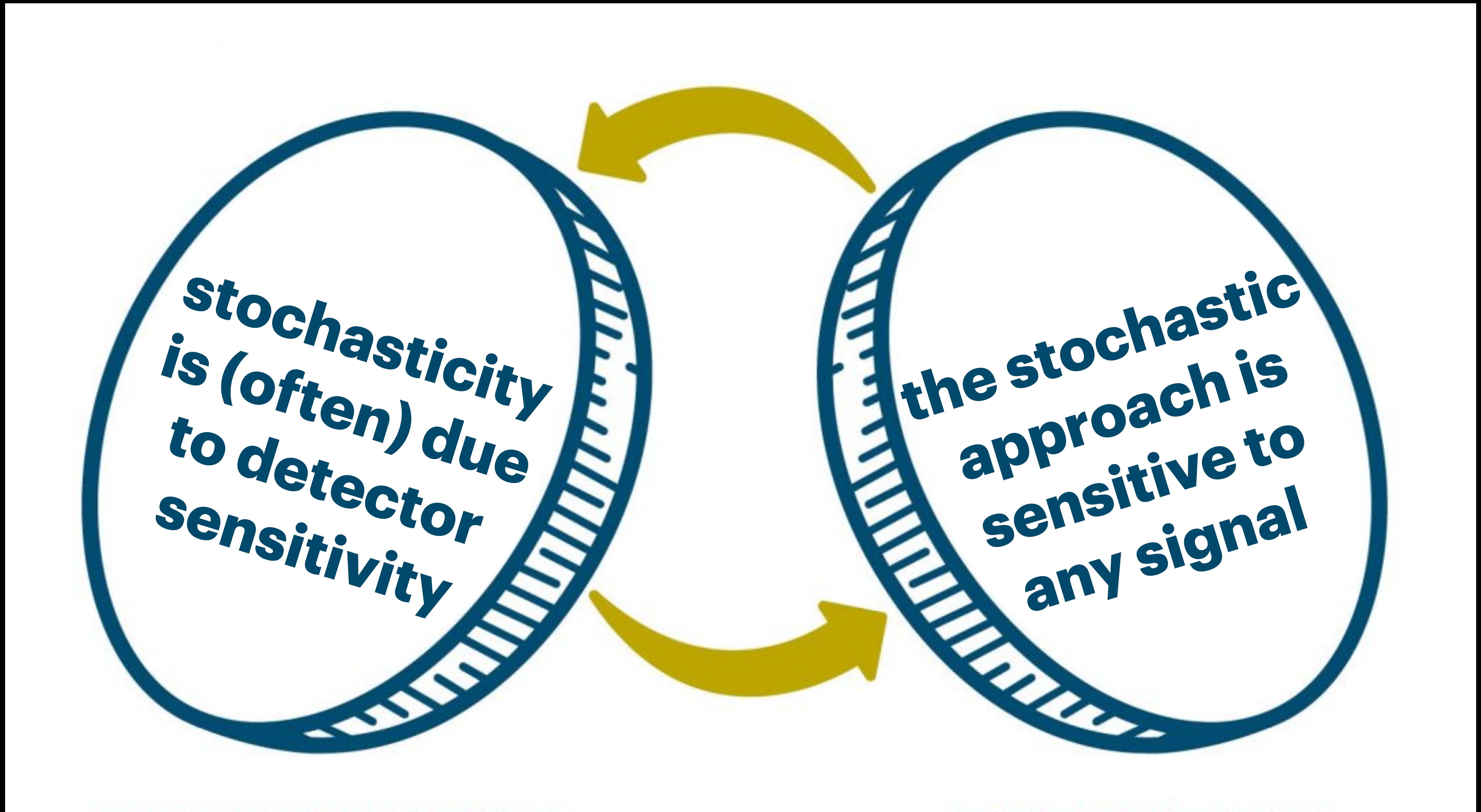
(fiducial: simulations that match observed population)

PREVIEW: 3G maps with 10^5 BBH events per year



A (stochastic?) gravitational-wave "background"

incoherent superposition of many GWs → unresolved by detectors → stochastic variables



and... less model dependent! because we can use a sufficient statistic

Gravitational-Wave Background Sources

fractional GW energy density $\Omega_{GW}(f)$

Primordial

GWs from inflation

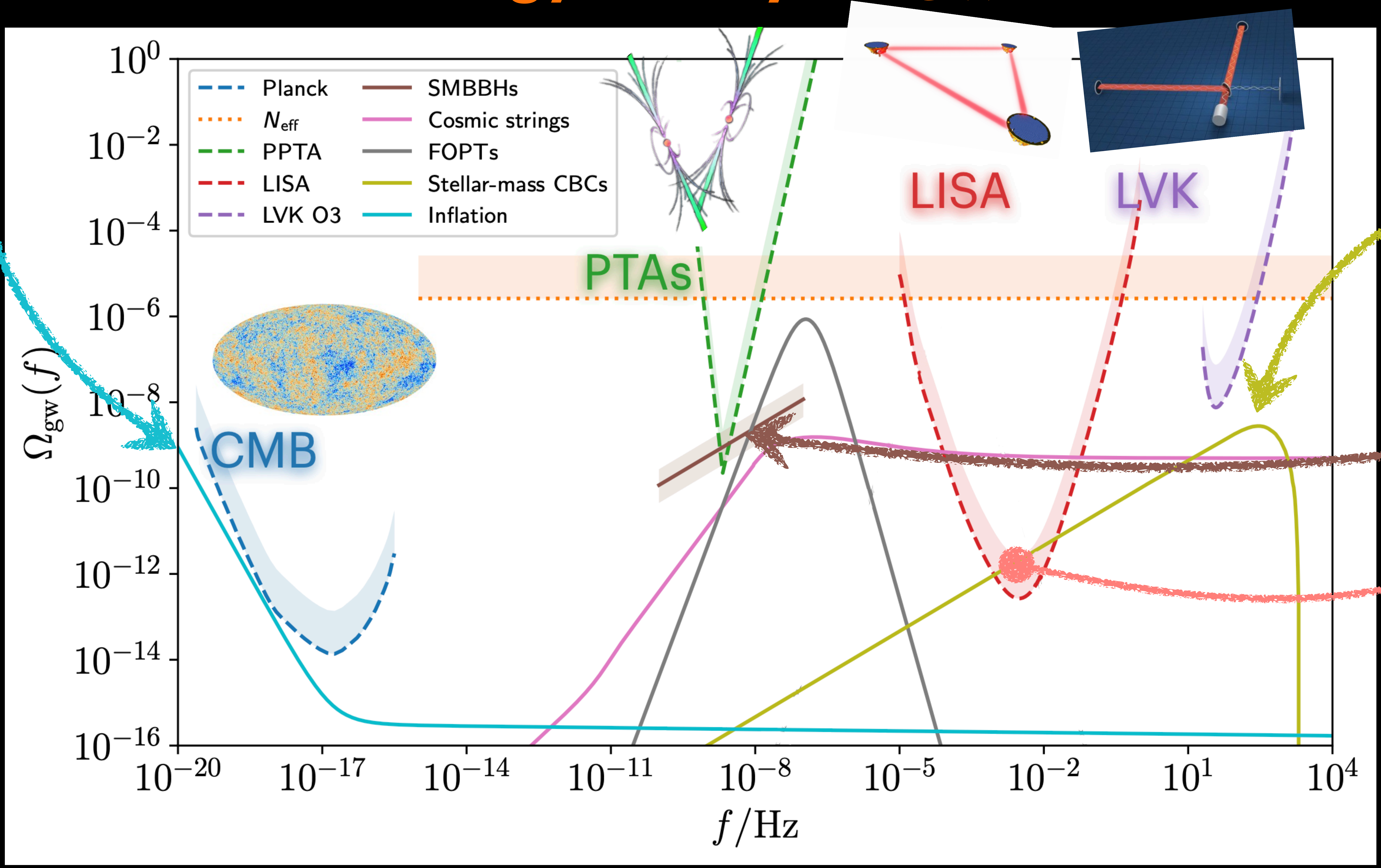
first order phase transitions

cosmic strings

primordial black holes

Beyond GR

TVS polarisations



Astrophysical

stellar mass compact binary coalescences

asymmetric rotating compact objects

supermassive black hole binary inspirals

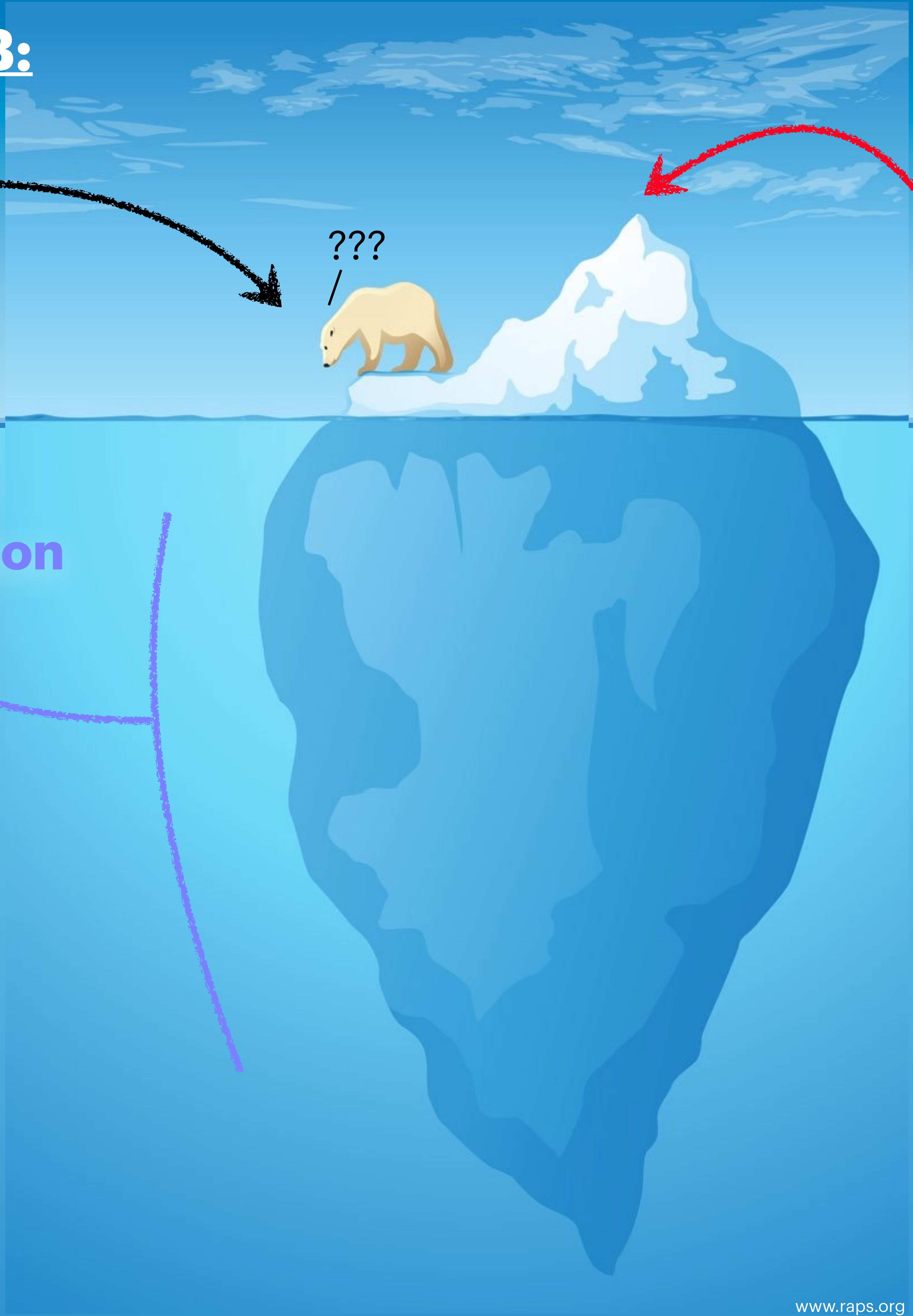
core-collapse supernovas

binary white dwarfs

From [AIR et al.]

The compact binary GWB:

US



resolved binary black hole and neutron star events

noise threshold

noise threshold

"stochastic" background of binary black holes and neutron stars

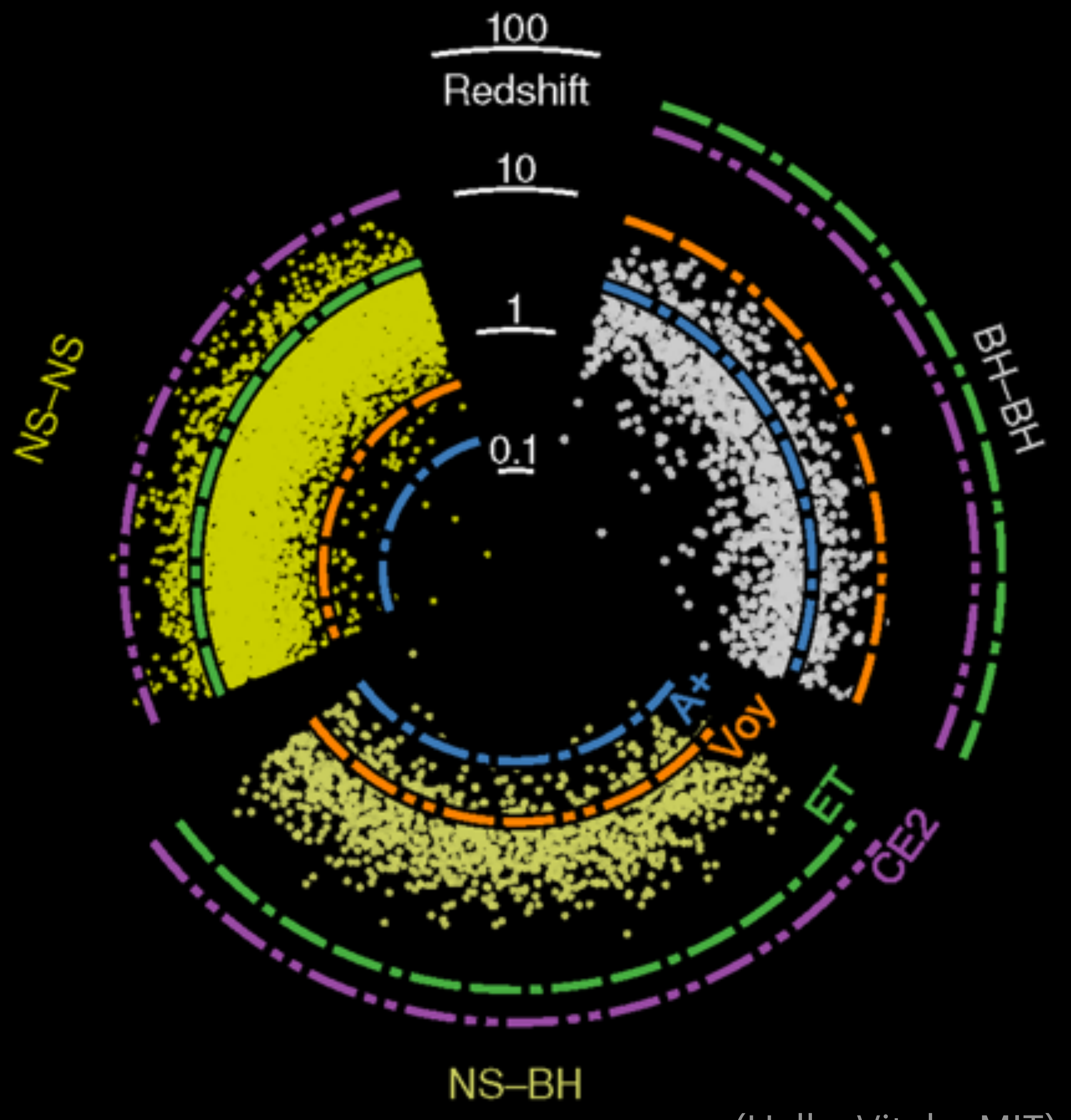
Ω_{GW} : the GW history of the Universe

from Phinney '01 :

number of events in unit comoving volume

$$\Omega_{GW}(f) \propto f \int_0^\infty dz \, \frac{R(z)}{H(z)(1+z)} \, f_s \, \frac{dE_{GW}}{df_s}$$

(redshifted) energy radiated per event per source-frame frequency

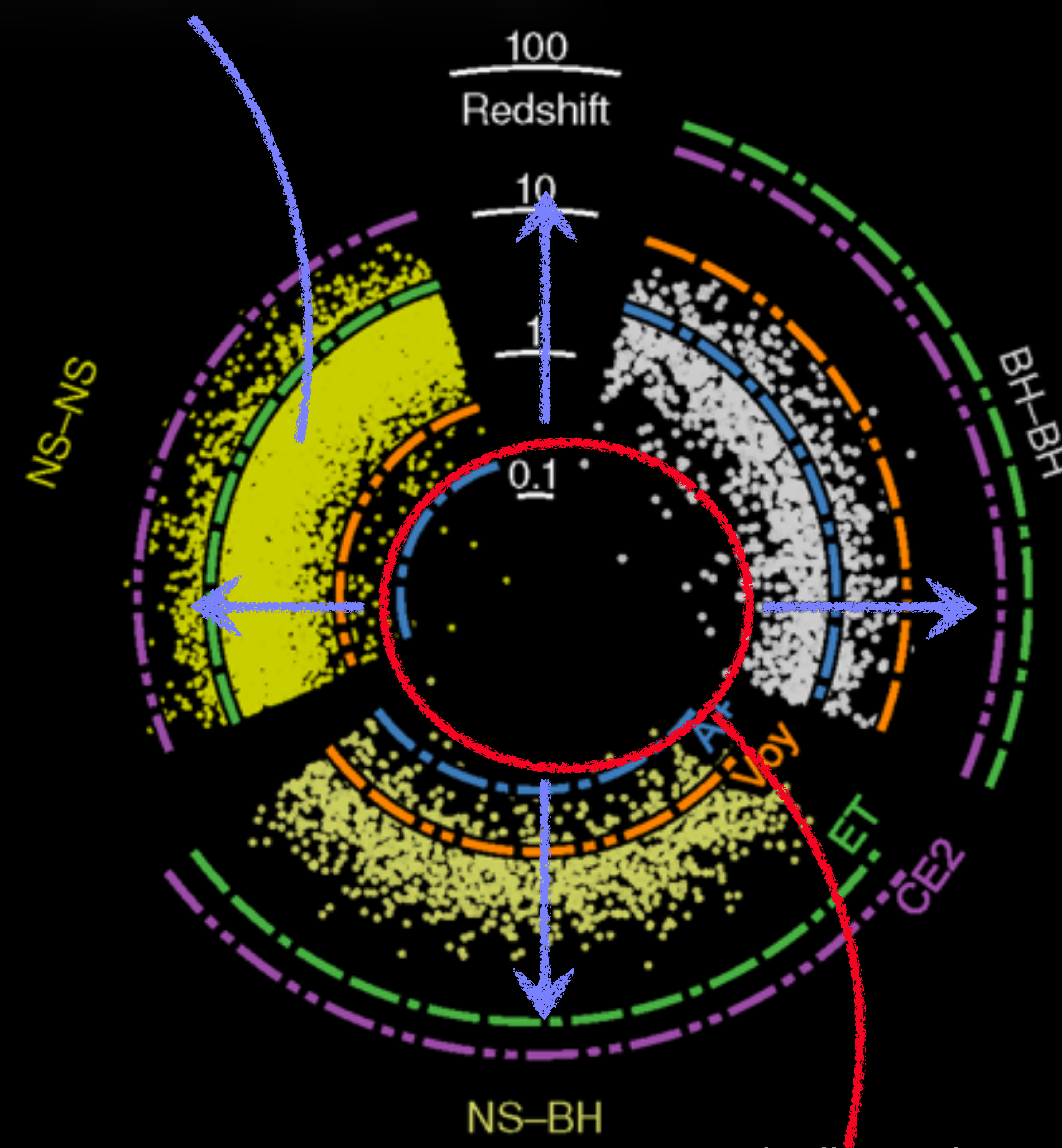


(Hall + Vitale, MIT)

Ω_{GW} : the GW history of the Universe

$$\Omega_{GW}(f) \propto f \int_0^\infty dz \frac{R(z)}{H(z)(1+z)} f_s \frac{dE_{GW}}{df_s}$$

target for stochastic searches



(Hall + Vitale, MIT)

matched-filter search horizon

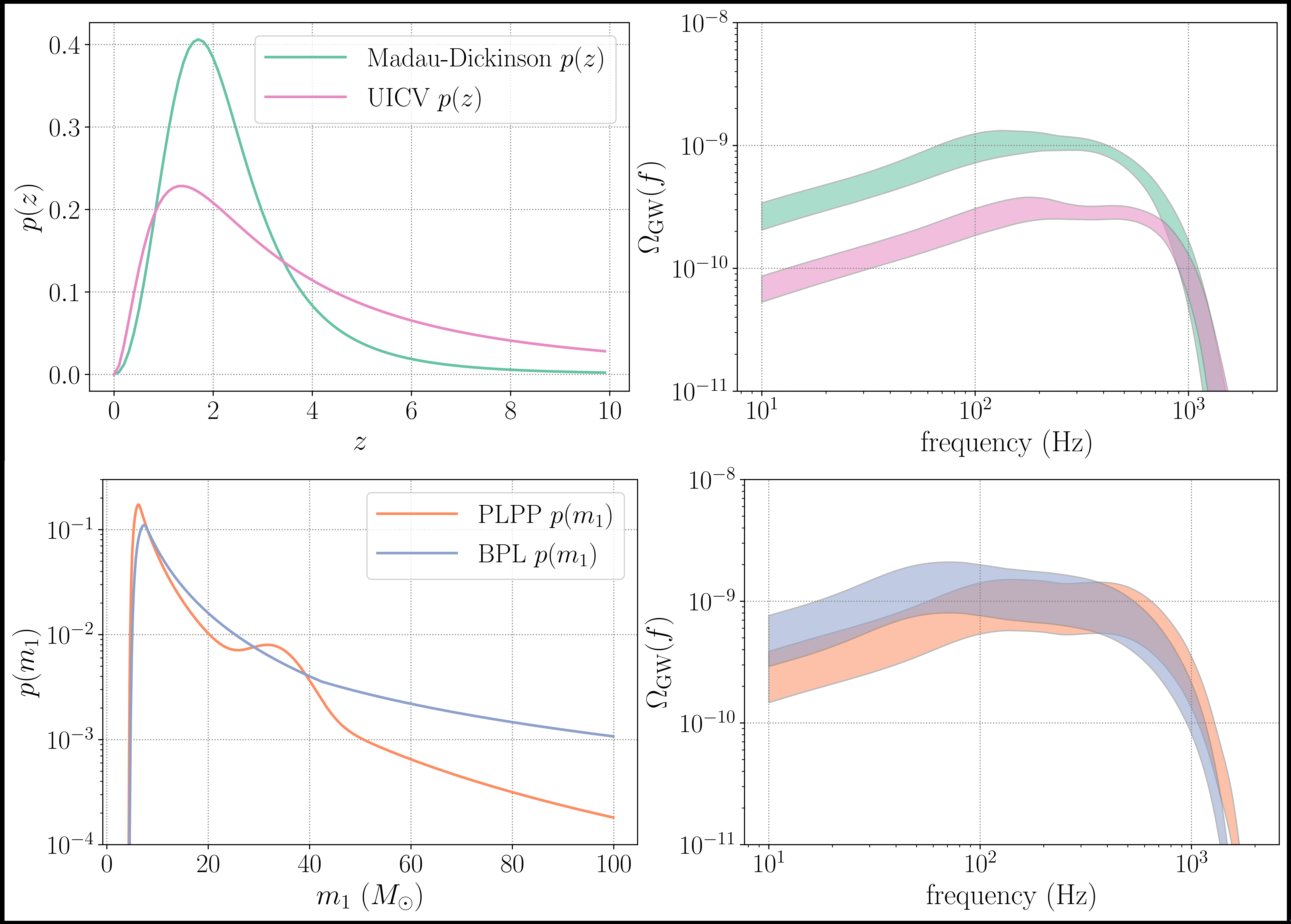
Access to:

- GWB power spectrum shape → GW sources
  Mandic et al. '12, LVK O2 ('19), O3 ('21), …
- Merger rate amplitude and evolution
  Callister et al. '20, LVK O3 ('21), …
- Mass spectrum information and evolution
  Bavera et al. '21, L. A. C. van Son et al. '22, …
- Spatial distribution of sources (anisotropic)
  Cusin et al. '18, Jenkins et al. '18, LVK Anisotropic O1 ('17), LVK O2 ('19), O3 ('21), …

Population hyper-prior differences

popstock

new open-source codebase to calculate your Ω_{GW} just released!



- shading: uncertainty on PLPP mass model
- shading: uncertainty on local merger rate

samples from [LVK '23 PRX 13 1 011048]

Inferring the merger rate evolution

Joint Analysis

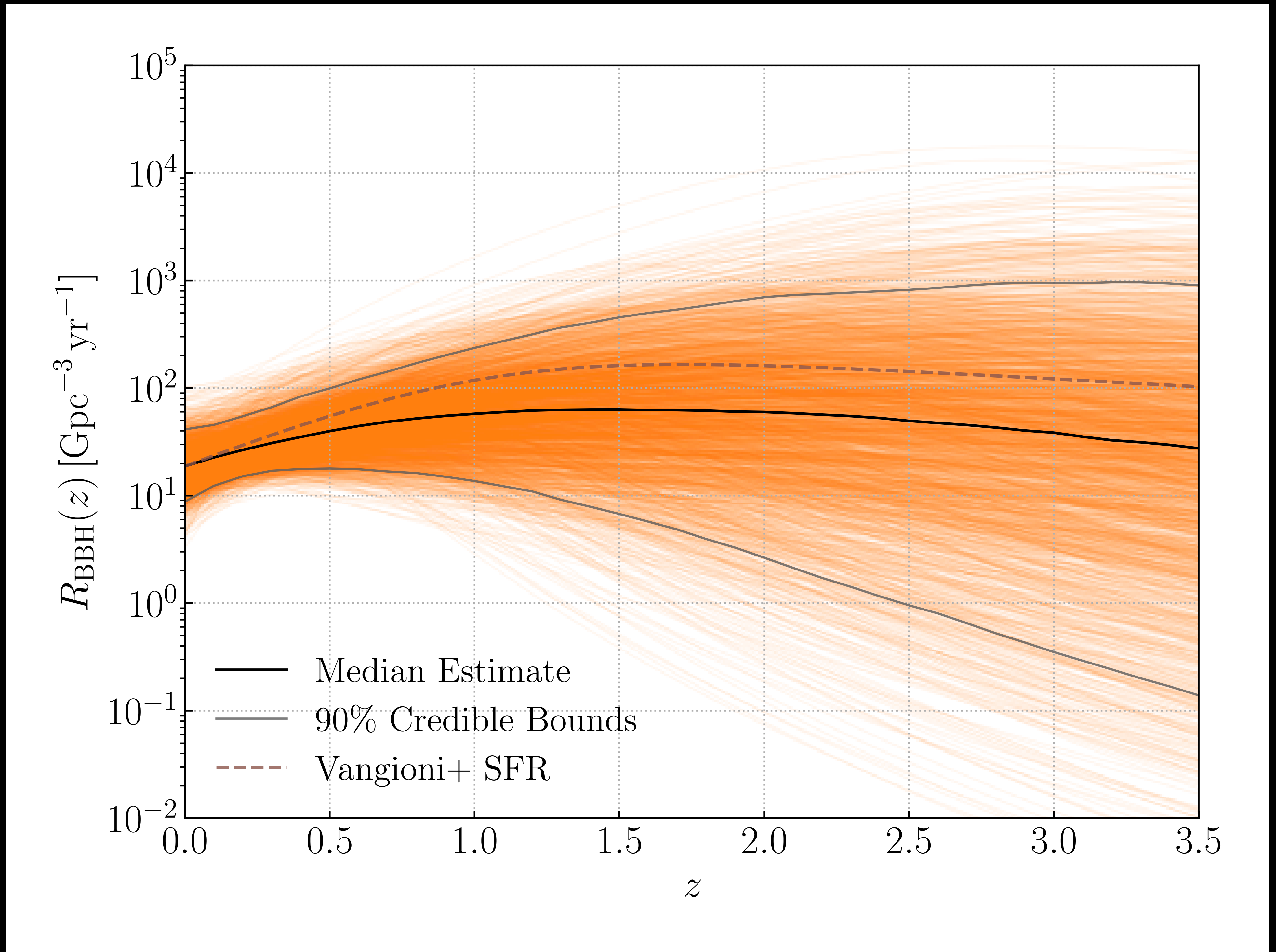
combine resolved sources with the GWB to constrain *R(z)*

$$\Omega_{GW}(f) \propto f \int_0^\infty dz \, \frac{R(z)}{H(z)(1+z)} \, f_s \frac{dE_{GW}}{df_s}$$

Phinney '01

$$R(z) = \mathscr{C}(\alpha, \beta, z_p) \frac{R_0 \, (1+z)^\alpha}{1 + \left(\frac{1+z}{1+z_p}\right)^{\alpha+\beta}}$$

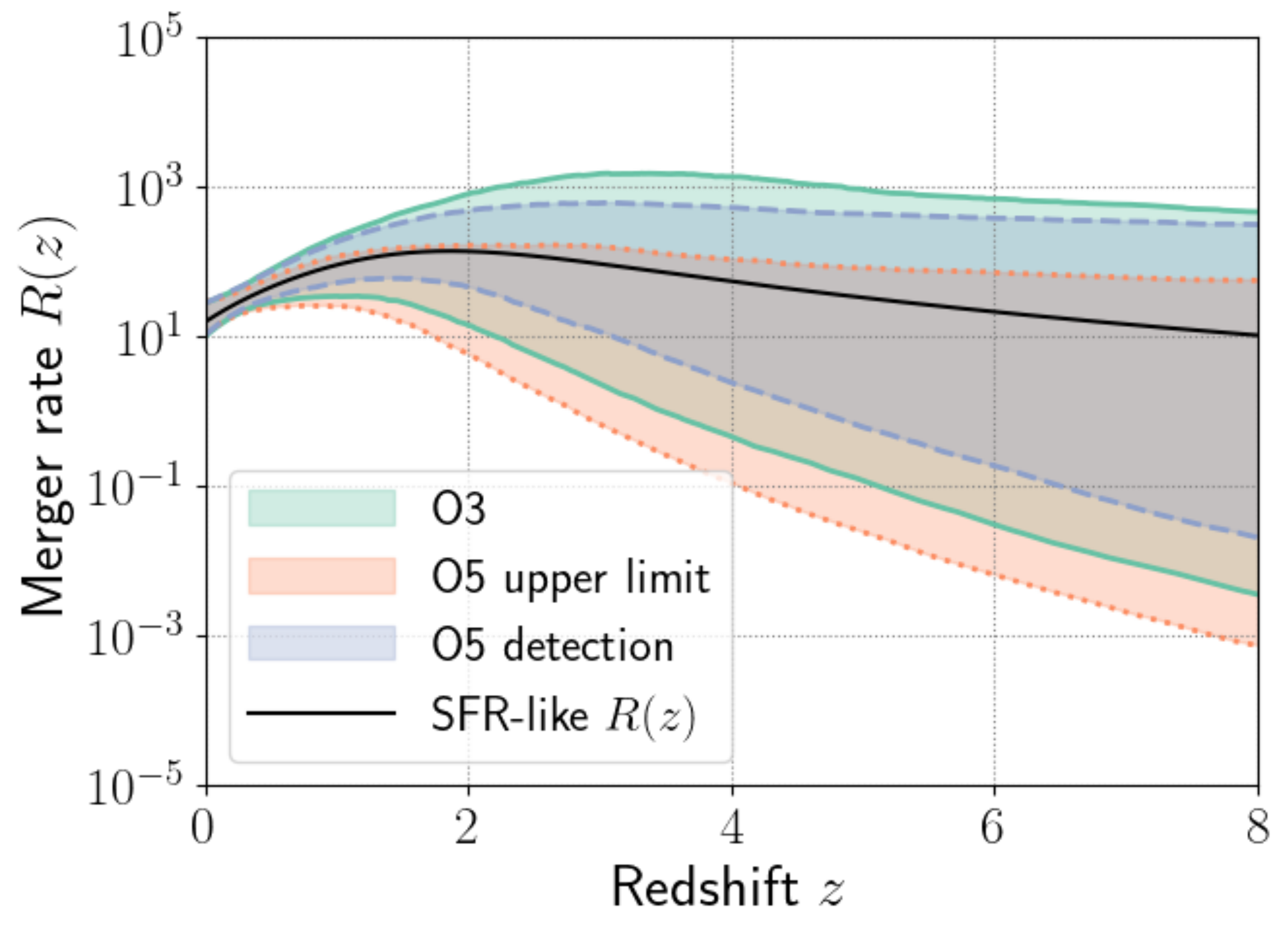
⟷ model for star formation rate

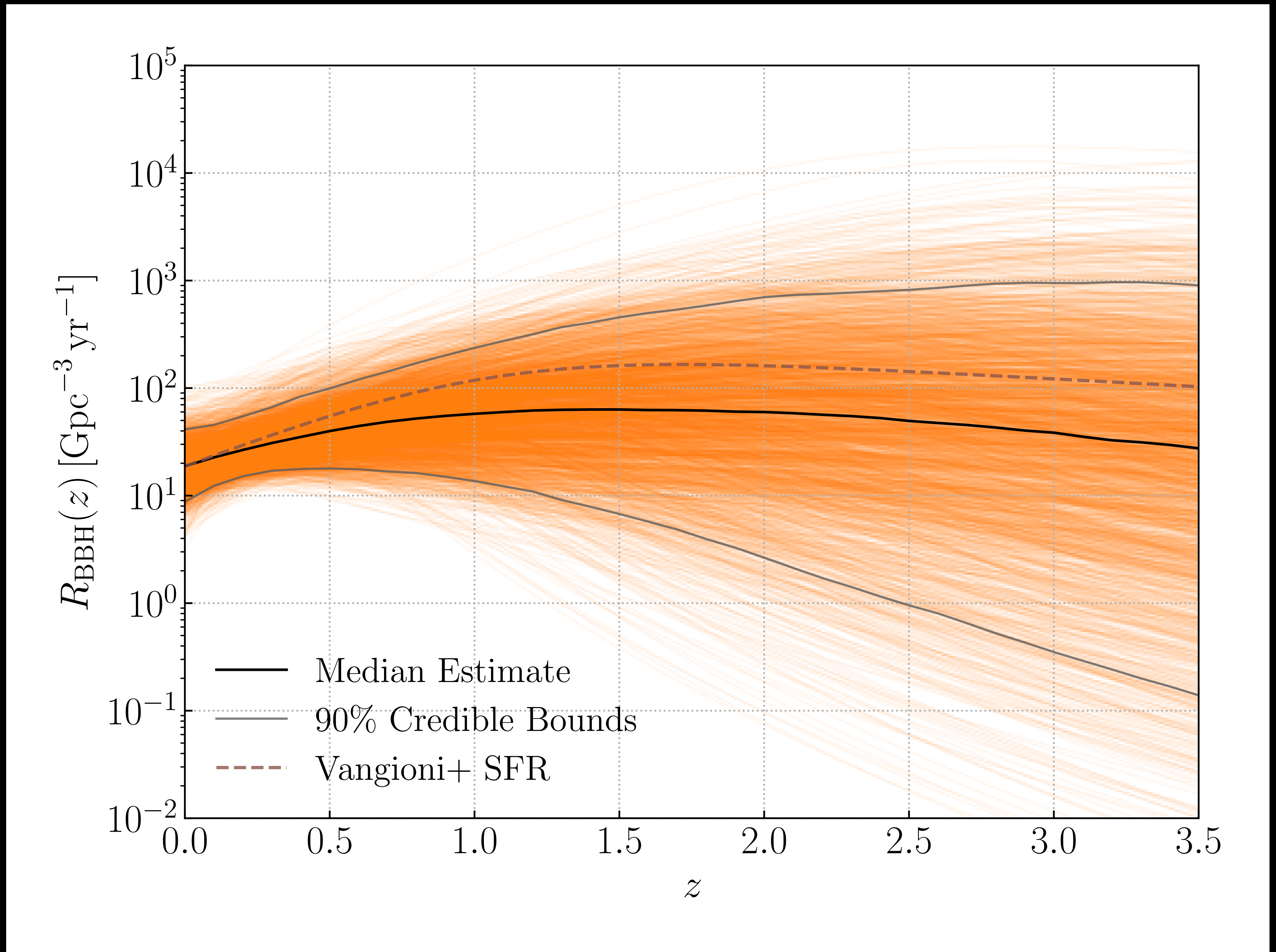


from LVK '21 for BBHs

Callister et al. '16, Callister et al. '20

Inferring the merger rate evolution



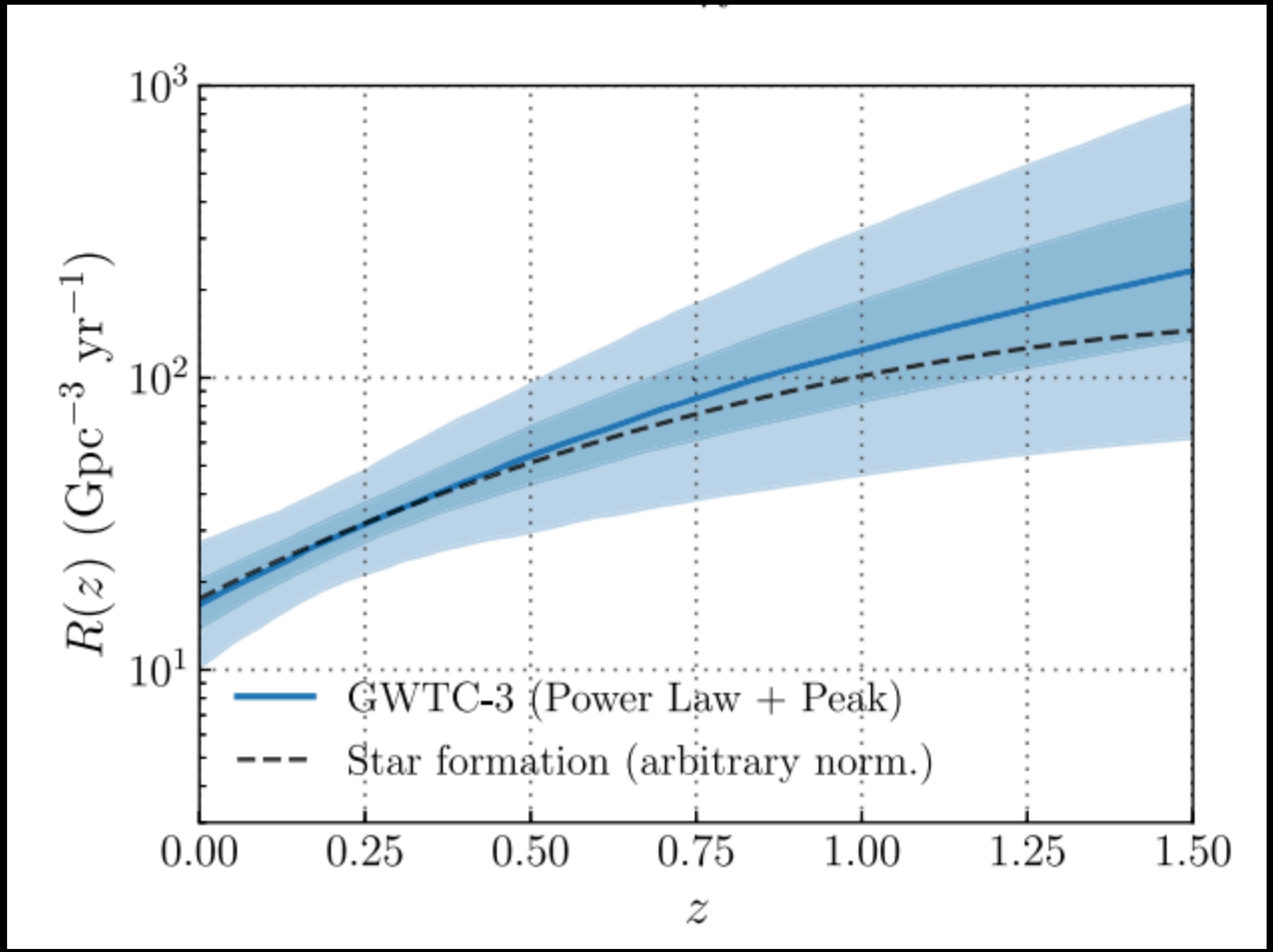


from LVK '21 for BBHs

O5 data run will be pivotal for binary merger rate inference, with or without a detection.

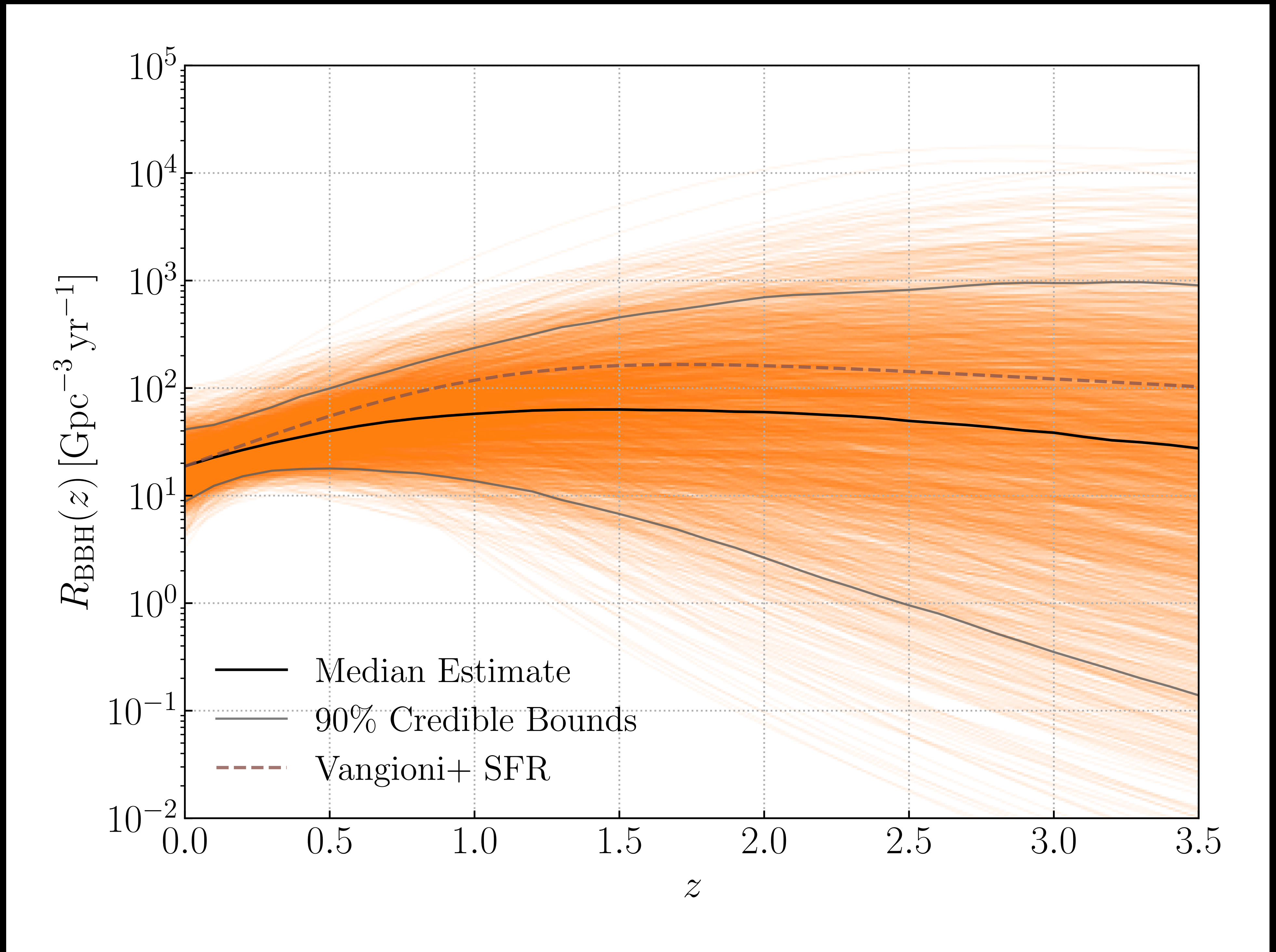
Callister et al. '16, Callister et al. '20

Inferring the merger rate evolution



from LVK '23 population paper

VS.



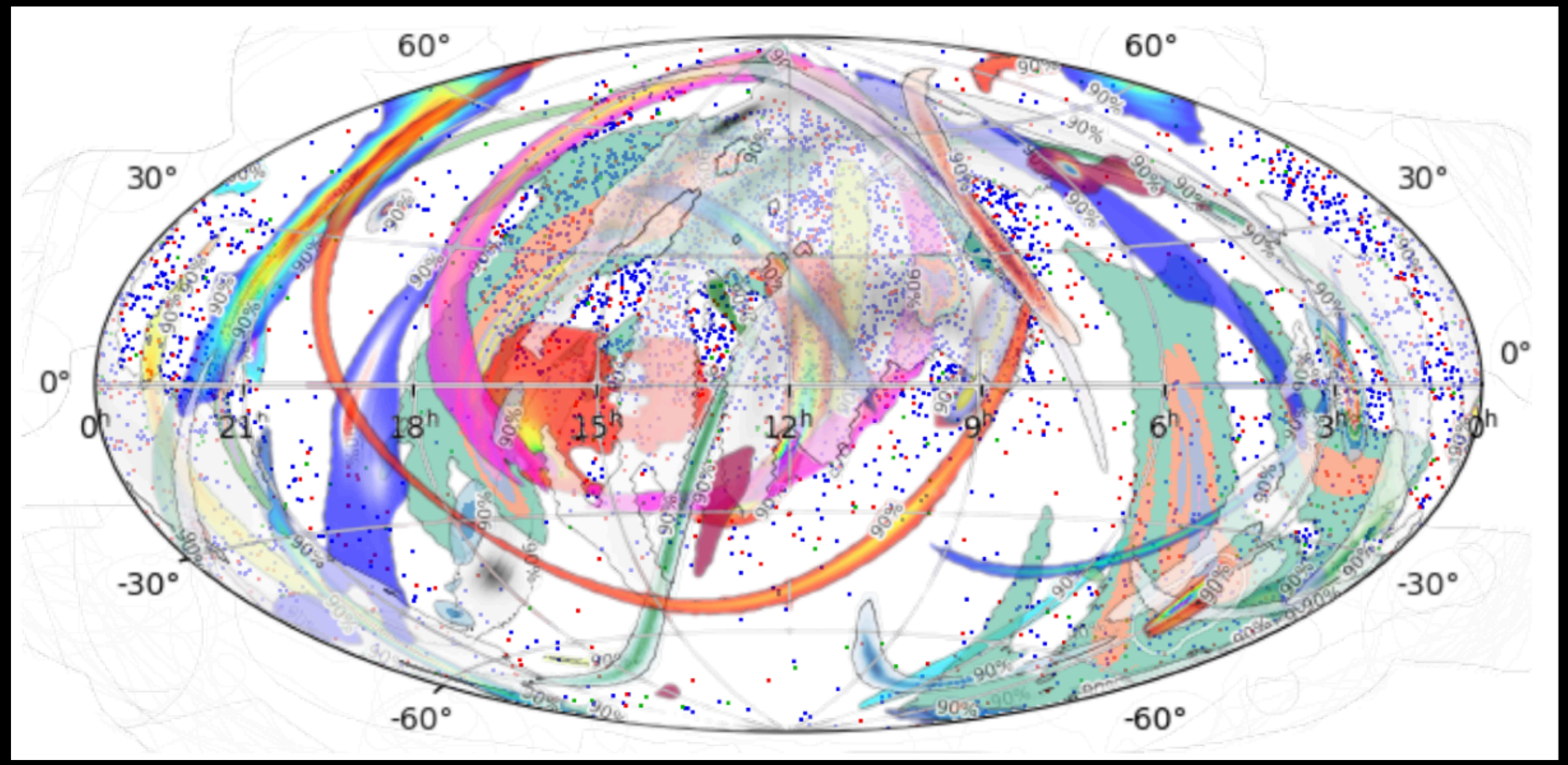
from LVK '21 for BBHs

... what about combining spectral sirens with the stochastic background?

Callister et al. '16, Callister et al. '20

Ω_{GW} : the GW history of the Universe

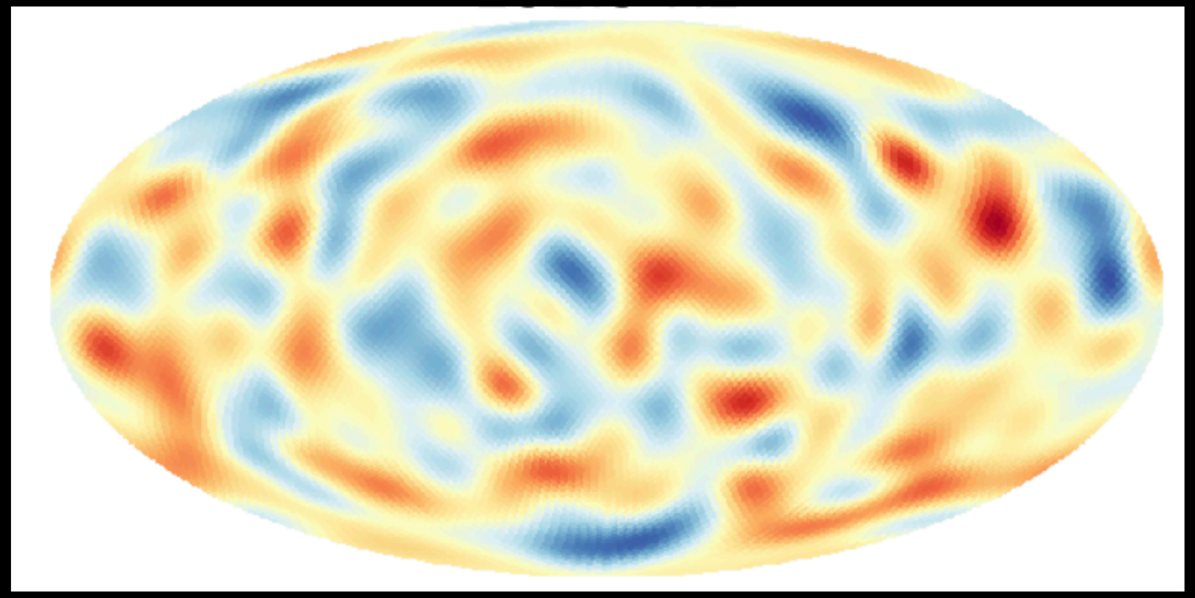
$$\Omega_{GW}(f) \propto f \int_0^\infty dz \, \frac{R(z)}{H(z)(1+z)} \, f_s \frac{dE_{GW}}{df_s}$$



Veronesi et al. '23

AGN

GWB



AIR & Contaldi '19

Access to:

- GWB power spectrum shape

  → *characterise the power spectrum, separate BH/NS*
- Merger rate amplitude and evolution

  → *binary progenitors: time-to-merger delay, metallicity dependence, formation channels*
- Mass spectrum information and evolution

  → *binary progenitors: high redshift mass distribution, SN pair instability, ...*
- Spatial distribution of sources (anisotropic)

  → *cross-correlation with LSS; tracer of BBH accretors (AGNs) and BNS counterparts (GRBs)*

Stochastic sources: anisotropic spectra

$$\Omega_{\rm GW}(f,\hat{n}) := \frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\ln f\, d\hat{n}} \longrightarrow \Omega_{\rm GW}(f,\hat{n}) \approx E(f)\,\Omega_{\rm GW}(\hat{n})$$

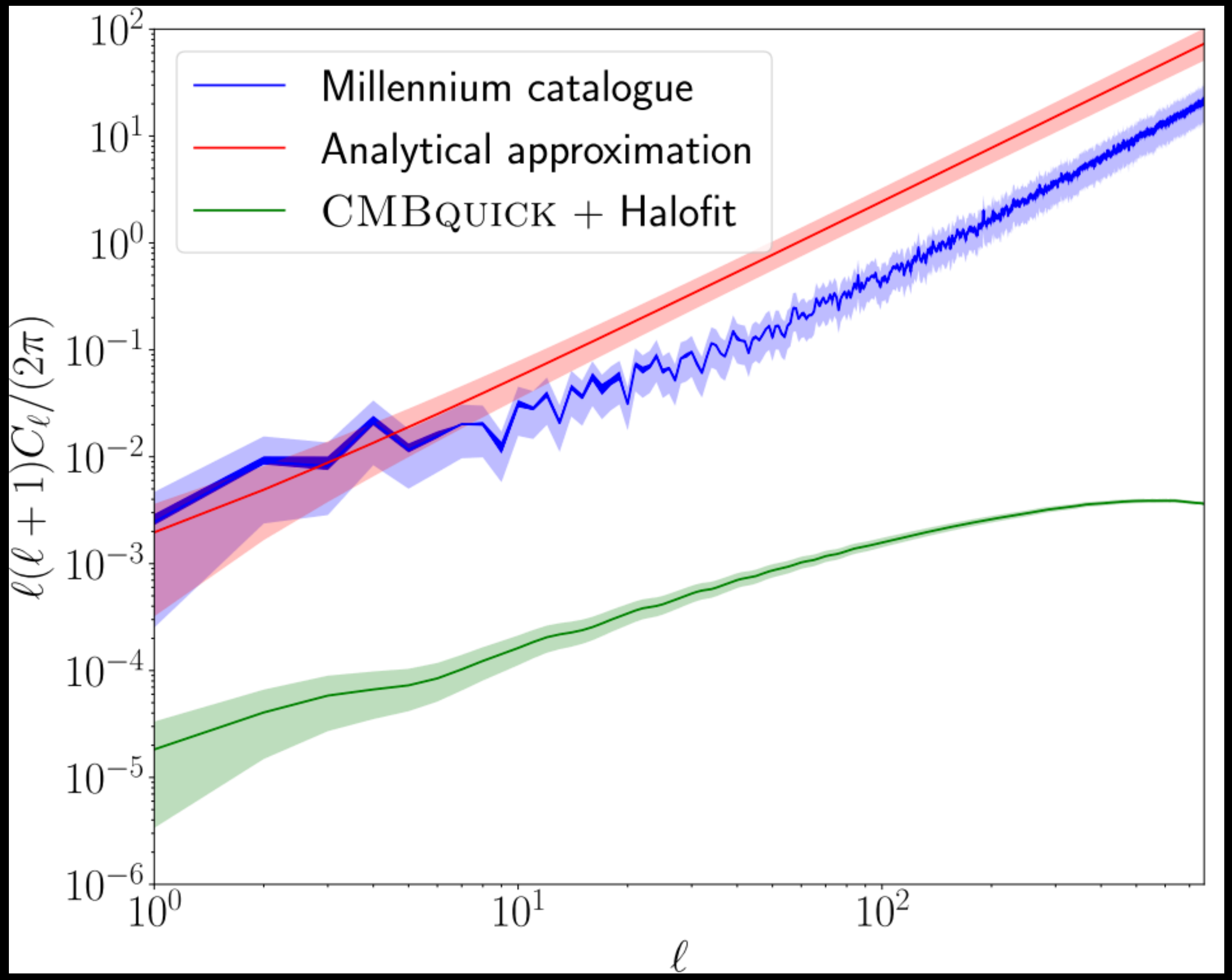
spectral shape — power on the sky

$$\Omega_{\rm GW}(\hat{n}) = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{+\ell} \Omega^{\rm GW}_{\ell m}\, Y_{\ell m}(\hat{n})$$

spherical harmonic expansion

assuming Gaussianity

$$C^{\Omega}_{\ell} = \frac{1}{2\ell+1}\sum_{m=-\ell}^{+\ell}\left\langle \Omega^{\rm GW}_{\ell m}\,\Omega^{\rm GW}_{\ell m}\right\rangle$$



Expected CBC GWB angular power spectrum from Cusin et al. & Jenkins et al.

Mapping Ω, estimating the angular spectrum

min χ^2 ("frequentist")

$$\Omega_{\rm GW}(\hat{n}) = \mathscr{F}(\hat{n}, \hat{n}')^{-1} z(\hat{n}')$$

LVK anisotropic, AIR & Contaldi '18, '19, '20, +

issue: unresolved modes lead to ill-conditioned Fisher matrix.

regularisation efforts include Agarwal et al. '21, Xiao & AIR in prep., +

Bayesian

$$\mathscr{L}(d \,|\, I(\hat{n})) = \frac{1}{(2\pi)^{\mathscr{N}/2} |C|^{1/2}} e^{-\frac{1}{2} d^{\dagger} C^{-1} d}$$

Banagiri et al. '21

issue: too many parameters; usually uses spherical harmonics and reduces to few modes.

Ultimate goals: have a map/angular spectrum, and/or cross-correlate with LSS:

Yang et al. '20 , Alonso et al. '20

Mapping Ω, estimating the angular spectrum

Maximum Likelihood

$$\Omega_{\rm GW}(\hat{n}) = \mathscr{F}(\hat{n},\hat{n}')^{-1} z(\hat{n}')$$

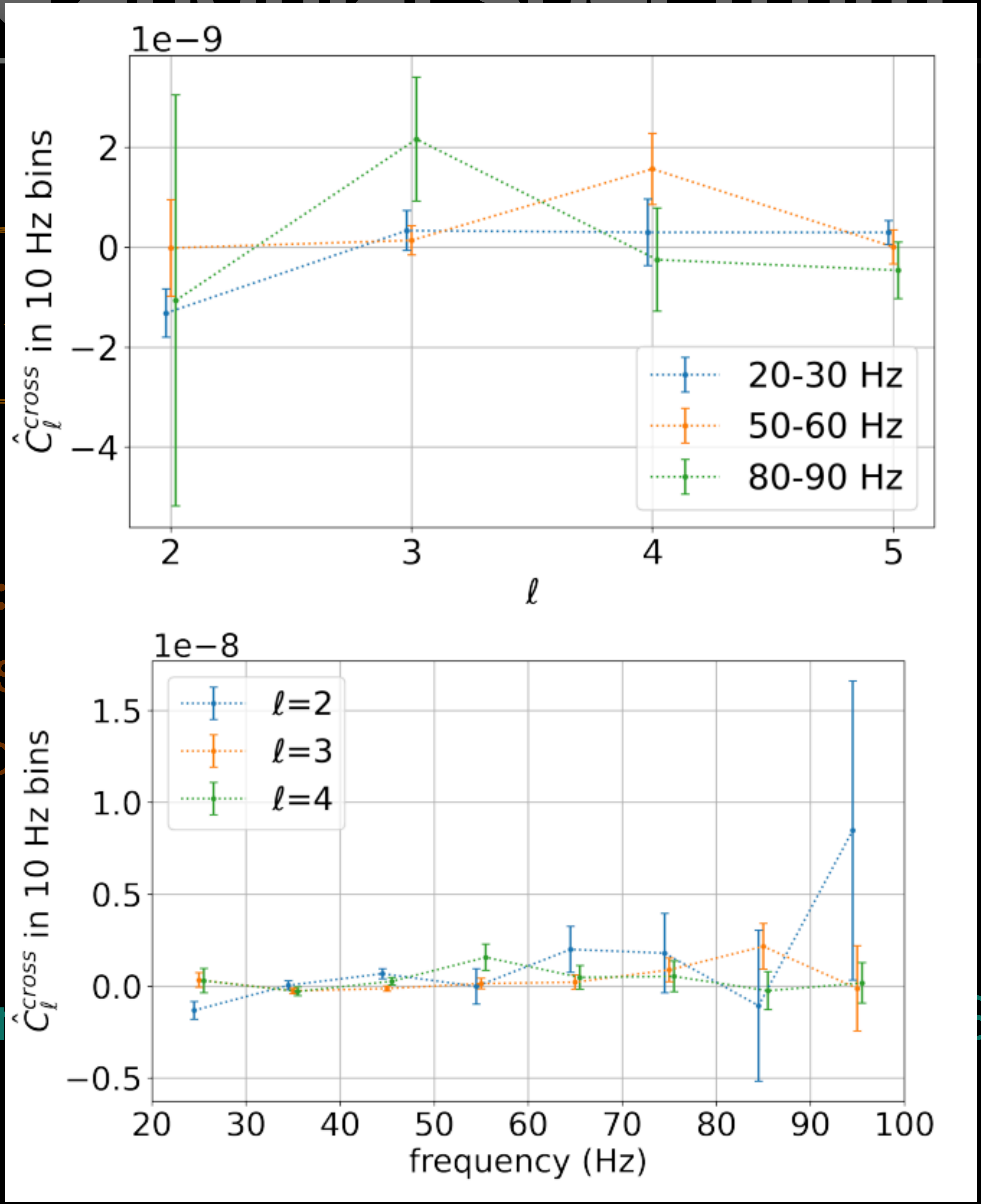
$\mathscr{L}(d|$

LVK anisotropic, AIR & Contaldi '18, '19, '20, +

issue: unresolved modes lead to ill-conditioned Fisher matrix.

issue:
uses s
reduc

regularisation efforts include Agarwal et al. '21, Xiao & AIR in prep., +



Ultimate goals: have a map/angular spectru... S:

Yang et al. '23, Alonso et al. '20

LVK results: Pixel & Spherical Harmonic searches

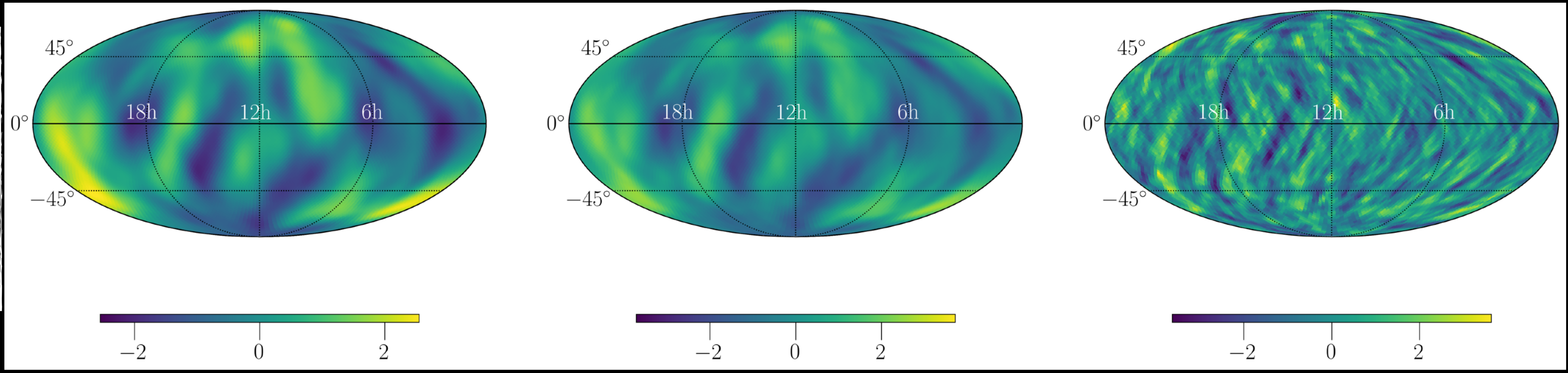
$\alpha = 0$ $\alpha = 2/3$ $\alpha = 3$

ASSUME NO CORRELATED POWER BETWEEN PIXELS

↓

only use diagonal of $\mathscr{F}$

3.5 — 3.8 improvement w.r.t. O2

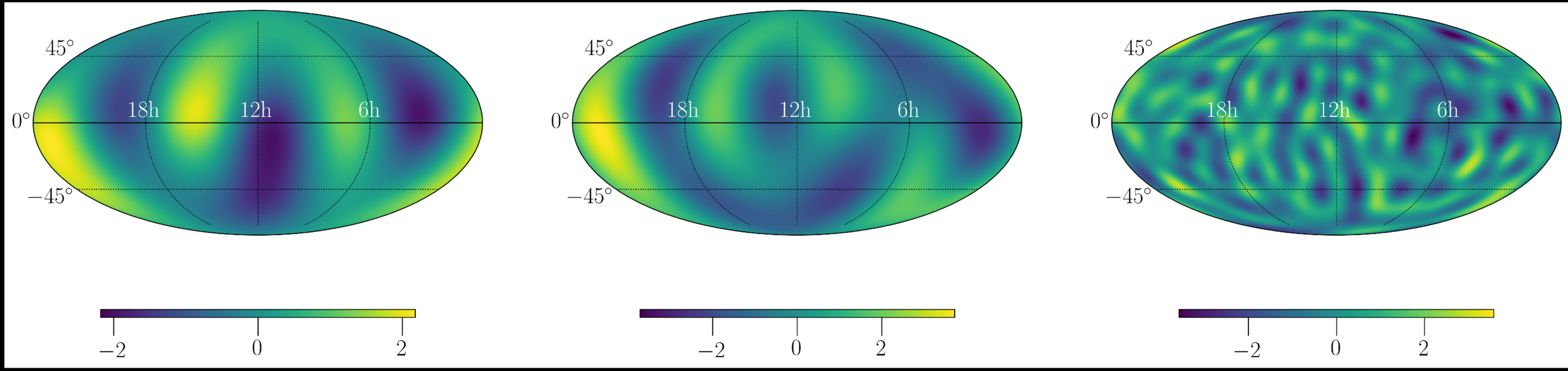


EXTRA CHALLENGE: INVERSION OF FULL $\mathscr{F}$

↓

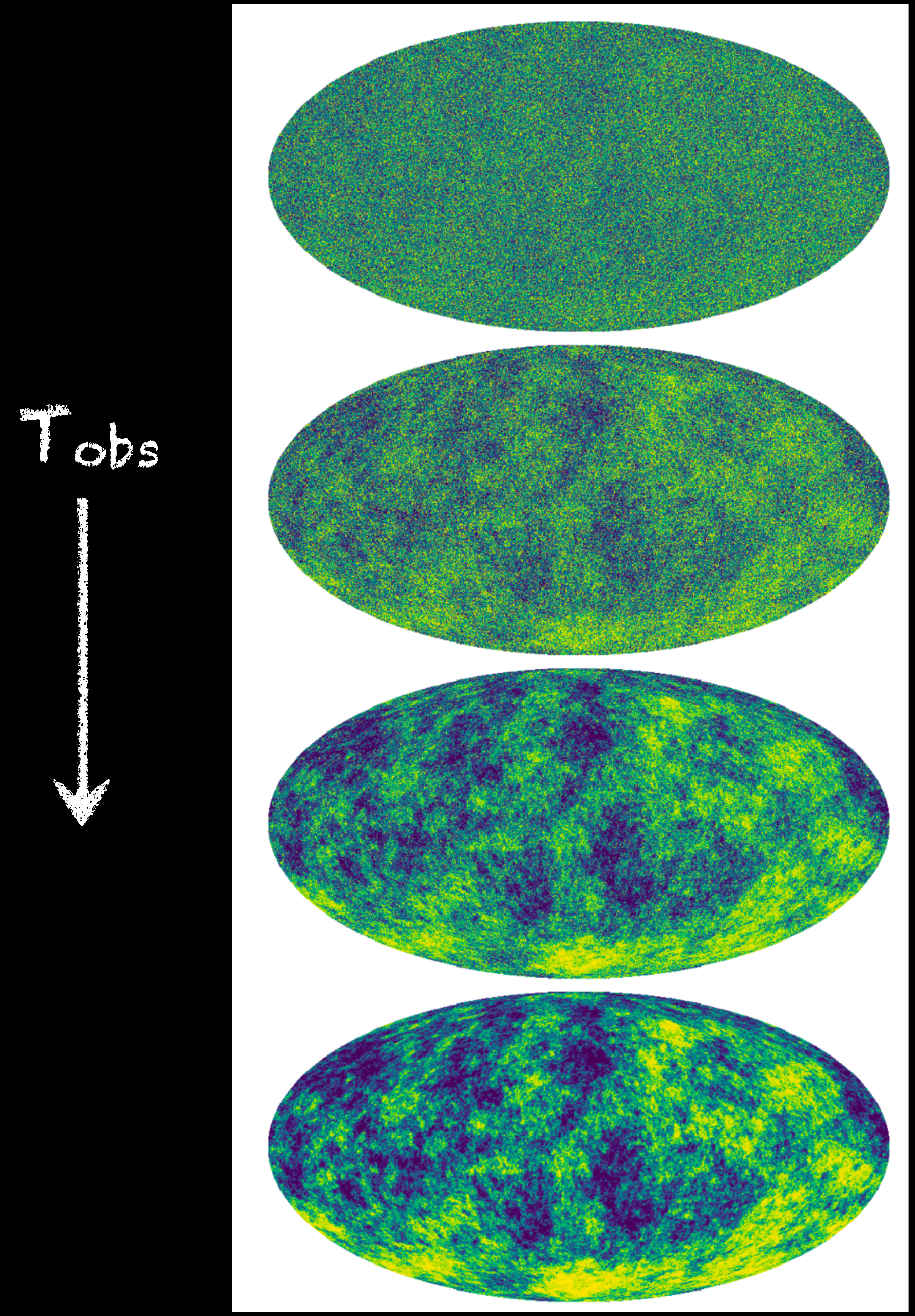
use SVD and Virgo

2.8 — 3.2 improvement w.r.t. O2



from ArXiv 2103.08520

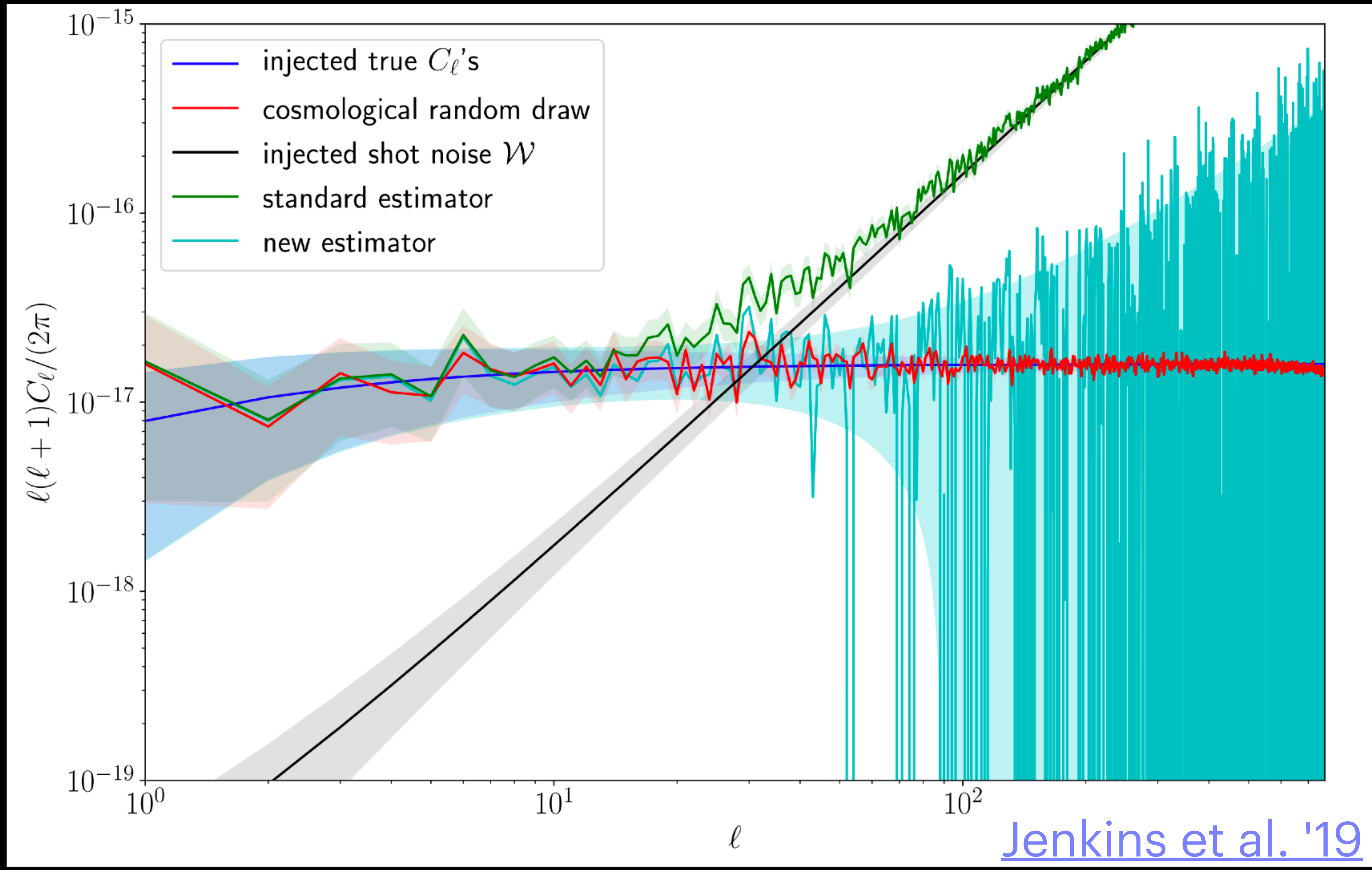
Spatial shot noise: de-biasing approach



How to avoid biasing the $\mathscr{C}_\ell$ measurement:

★cross-correlate over multiple time chunks

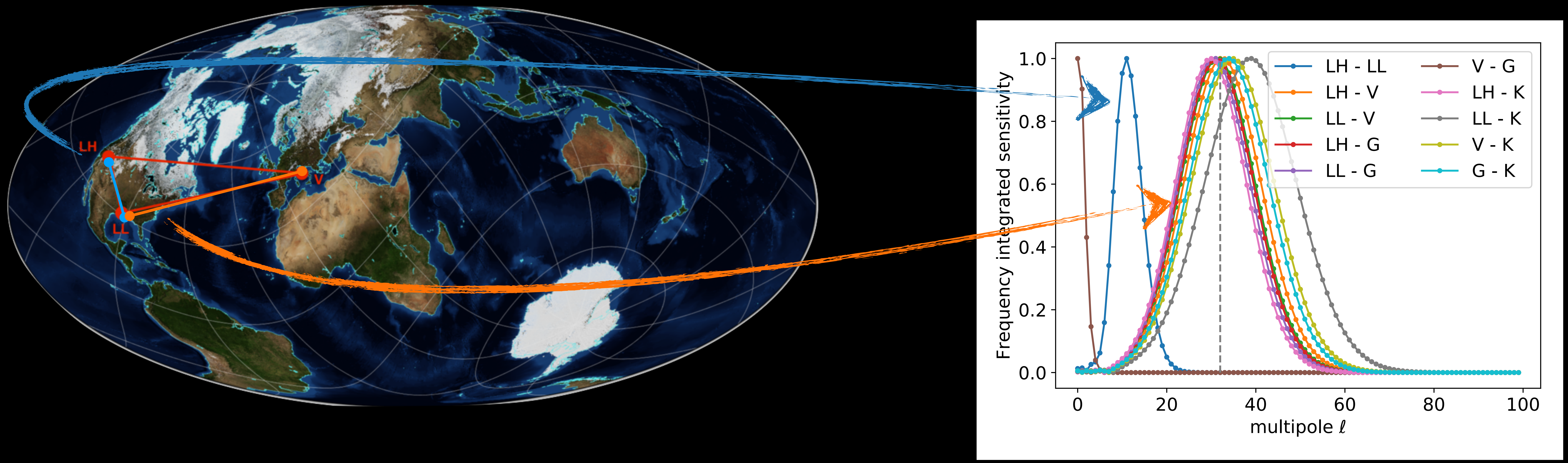
Jenkins et al. '19, Kouvatsos et al. (**AIR**) '23



★cross-correlate with LSS and/or other maps

Alonso et al. '20, Yang et al. '20, ...

Angular resolution: LVK and beyond

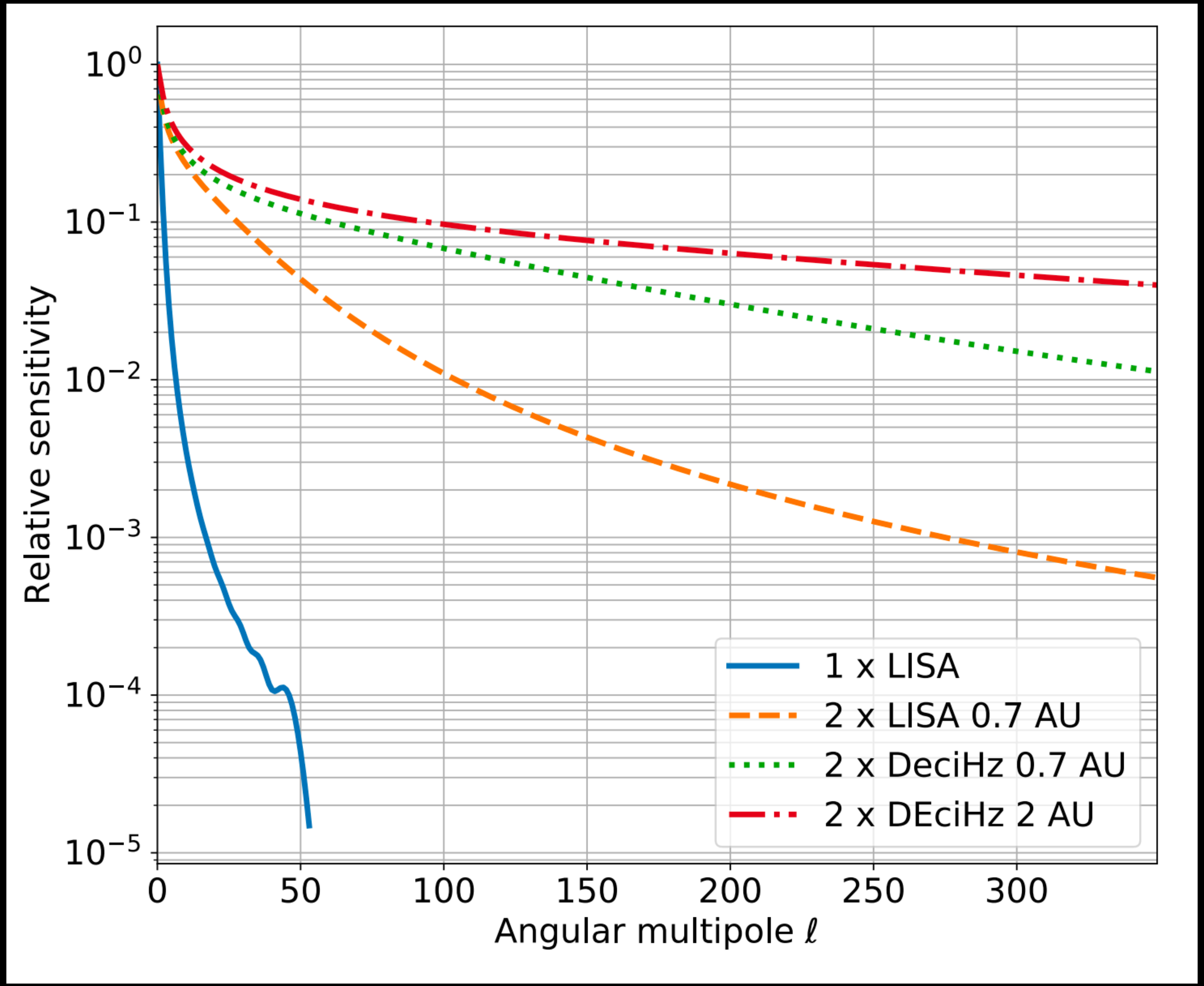


baseline: distance between simultaneous measurements ~ aperture

(similar to radio interferometry)

... need to go to space for high resolution

Angular resolution: LISA and beyond



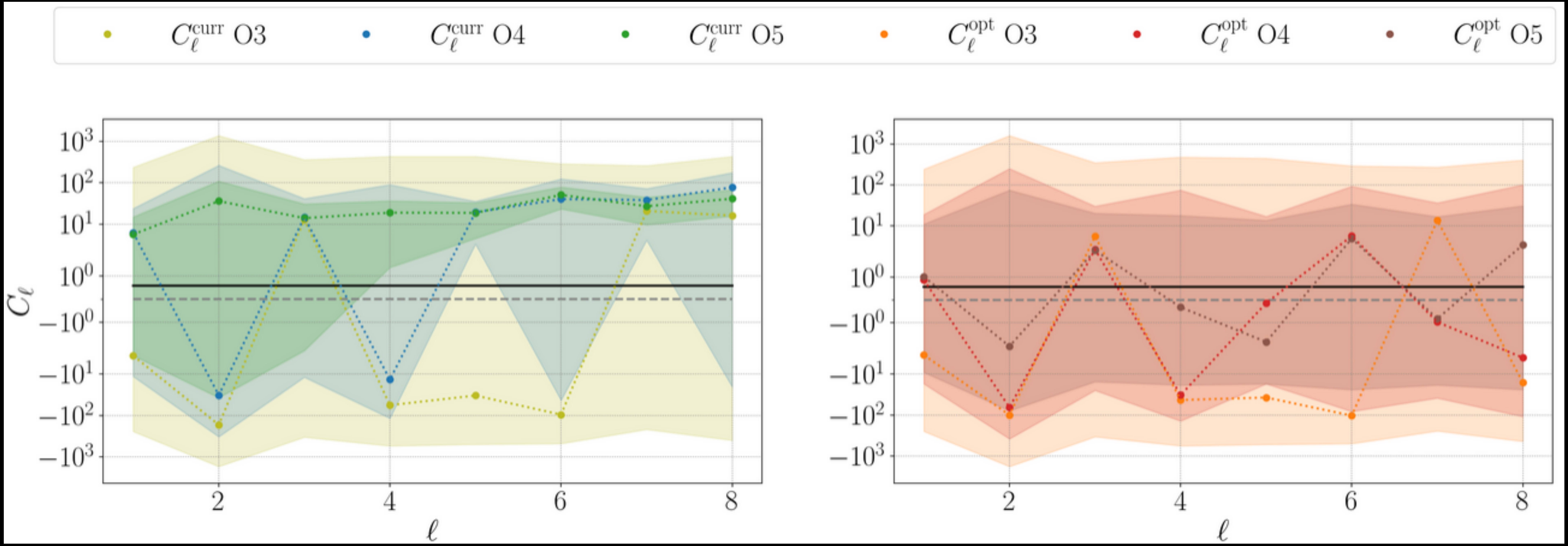
ESA Voyage 2050
White Paper

only hope : MORE DETECTORS IN SPACE

Spatial shot noise: de-biasing approach

★ cross-correlate over multiple time chunks, i and j, ignoring auto-correlations which are shot-noise dominated:

$$\hat{C}_\ell^{\rm curr} = \frac{1}{2\ell+1}\frac{1}{n(n-1)}\sum_m \sum_i \hat{\Omega}^i_{\ell m}\hat{\Omega}^{i*}_{\ell m} \longrightarrow \hat{C}_\ell^{\rm opt} = \frac{1}{2\ell+1}\frac{1}{n(n-1)}\sum_m \sum_{i\neq j} \hat{\Omega}^i_{\ell m}\hat{\Omega}^{j*}_{\ell m}$$



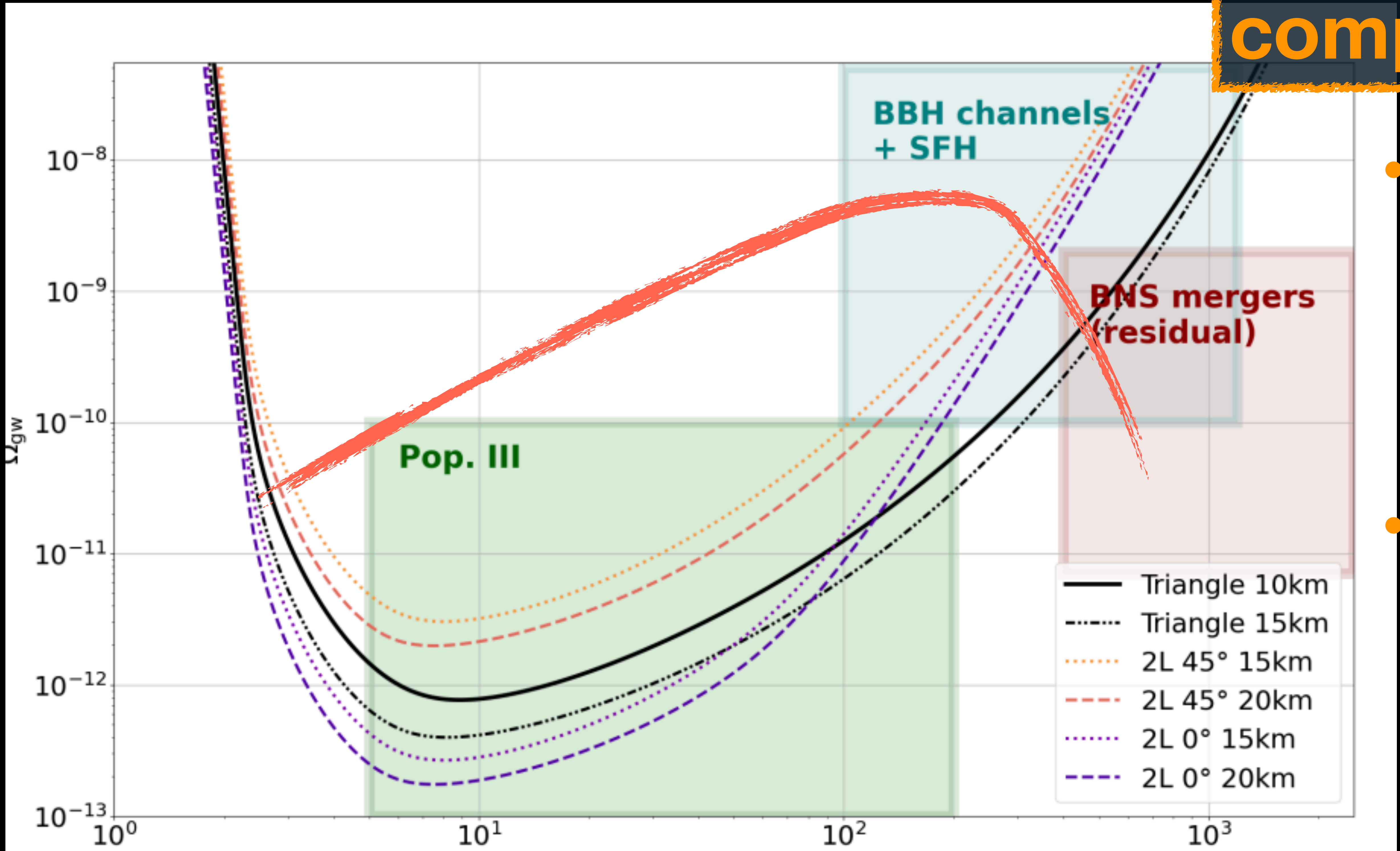
— already relevant in O4; crucial for O5 searches! —

3G: no more “backgrounds”?

Extend the depth of ground surveys up to $z \approx 10 \longrightarrow$ resolve all BBHs!

“new” stochastic signals within reach:

- BNS/BHNS background
- pop III / high z BBH background
- cosmological backgrounds …



Branchesi et al. '23

CHALLENGE: component separation

- subtraction of high SNR signals
  - Regimbau et al. '17
  - Cutler & Harms '06
- simultaneous estimation of multiple signals
  - Biscoveanu et al. ‘20
  - Martinovic et al. ‘20

The future: LISA

galactic binaries

[Edlund et al. '05](#)

Cornish & Robson '17

Finch et al. '23

...

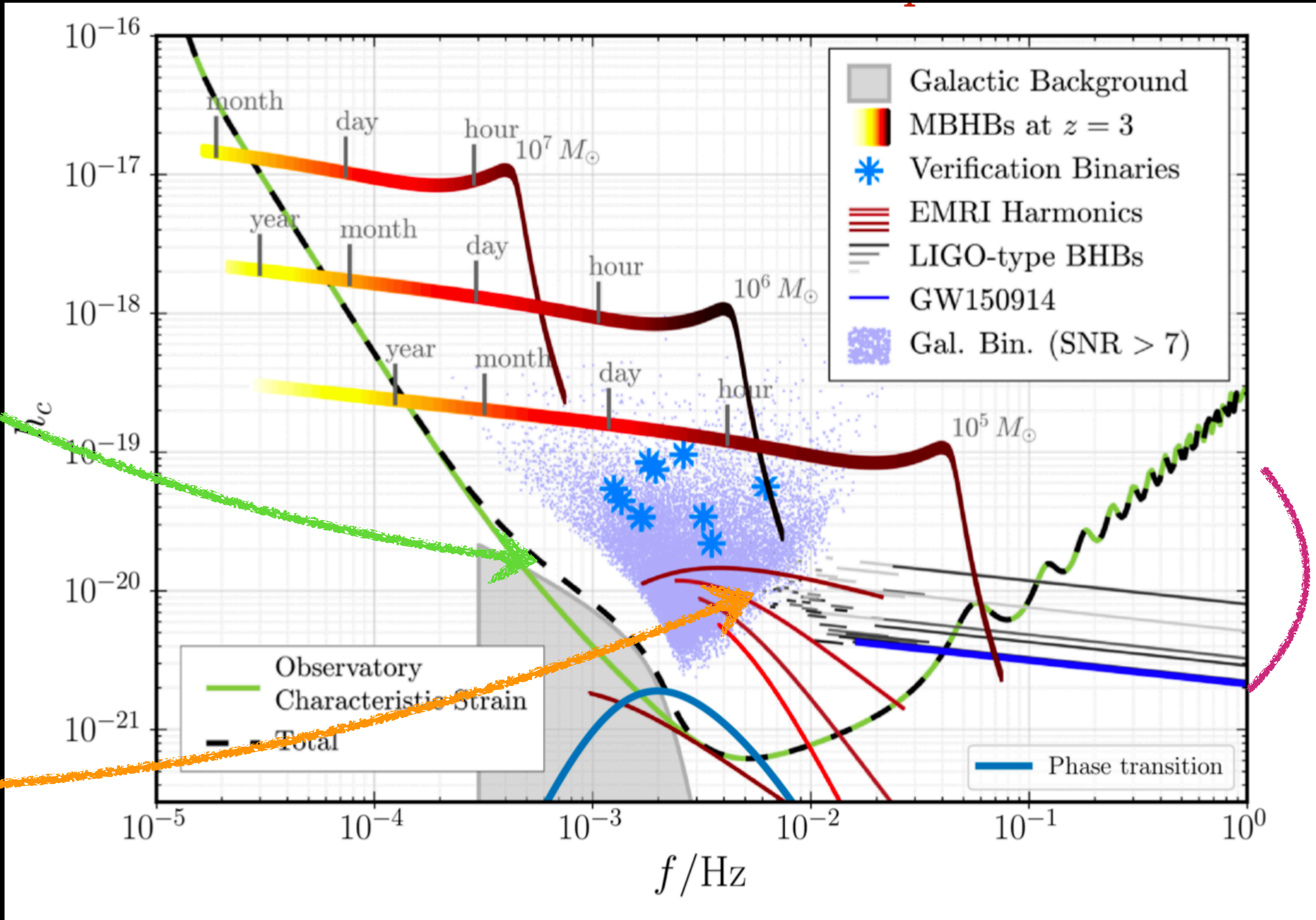
EMRIs

Babak et al. '17

Oliver et al. '23

Pozzoli et al. '23

...



... but, LISA won't really see high z

BBH background

Chen et al. '19

Babak et al. '23

Muratore et al. '23

...

some thoughts...

- lots of (independent?) cosmology can be potentially done with BBHs
- next generation detectors will open up many new avenues: both <u>cross-correlating</u> information and as <u>stand-alone</u> searches
- people are working on methods NOW
- if ET ends up being 2 detectors/we have 2CEs/..., maps will become more interesting

Thanks!

Data analysis: the cross-correlation statistic

GW detectors collect timestream data :

$$d(t) = s(t) + n(t)$$

If noise is uncorrelated between detectors, search for GWB with

cross correlation: $C_{12}(f) = \tilde{d}_1(f)\,\tilde{d}_2^{\star}(f) \approx \tilde{s}_1(f)\,\tilde{s}_2^{\star}(f)$

$$\langle C_{12}(f)\rangle \propto T_{\rm obs}\,\Gamma_{12}(f)\,f^{-3}\,\hat{\Omega}_{\rm GW}(f)$$

eg: $\Omega_{\rm GW}(f) = \Omega_{\rm ref}\left(\frac{f}{f_{\rm ref}}\right)^{\alpha}$

parameter estimation

$$\log\mathscr{L}(\hat{\Omega}_{\rm GW}(f)\,|\,\Theta) \propto \frac{1}{2}\sum_{f,\tau}\left(\frac{\hat{\Omega}_{\rm GW}(f) - \Omega_{\rm GW}(f\,|\,\Theta)}{\sigma_{\Omega}^2(f)}\right)^2$$