Glitches in rotating supersolids

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Outline

- Glitches in Neutron Stars
- Ultracold matter: supersolidity
- Glitches in rotating dipolar supersolids

Neutron Stars

Crab Pulsar

Credits: NASA (Hubble + Chandra)

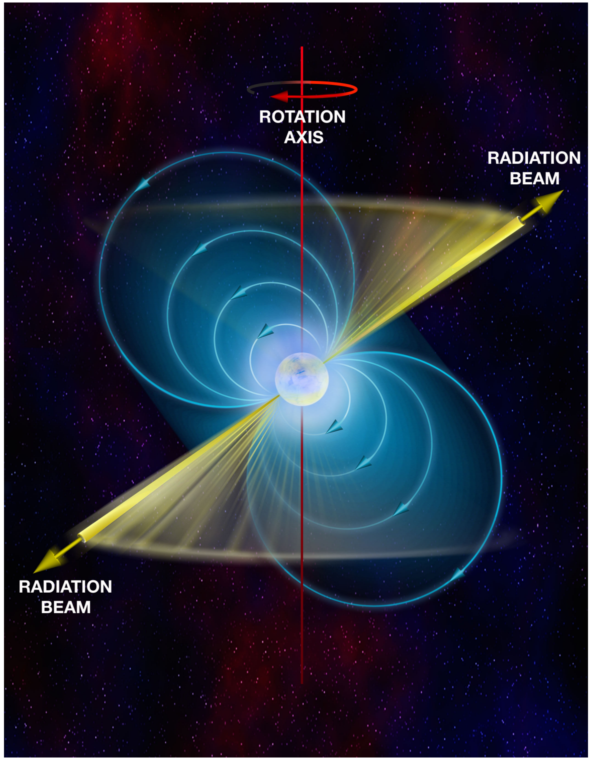
$M \sim 1 \div 2 M_{\odot}$

$R \sim 10\text{km}$

$T \sim \text{keV}$

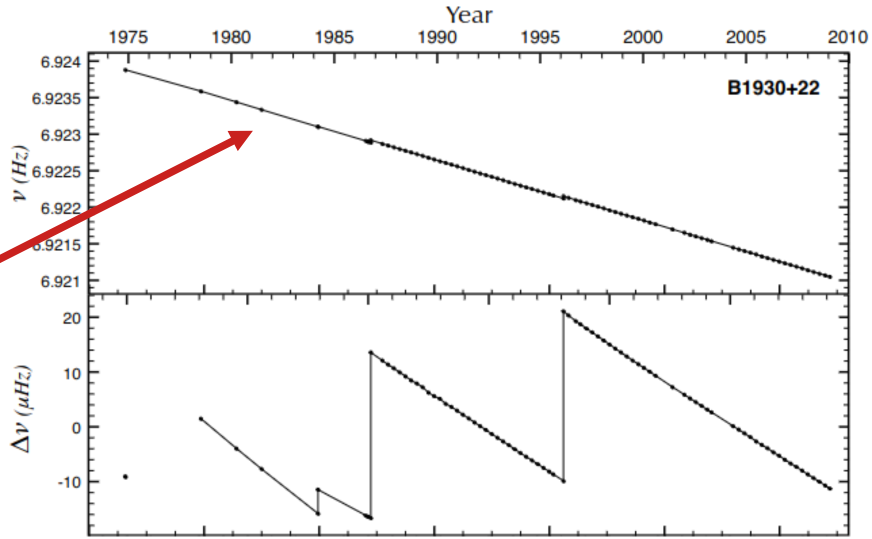
$P \simeq 33\text{ms}$

Glitches in Neutron Stars



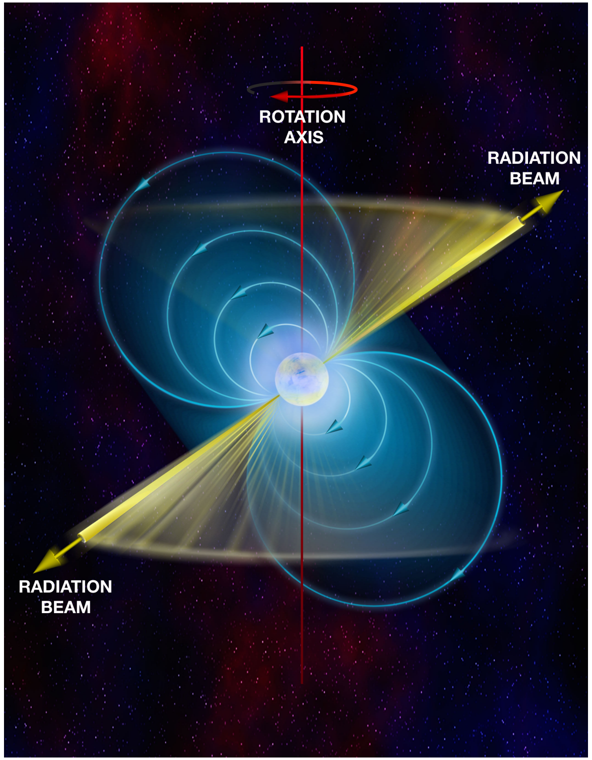
Spin-down

$$\dot{\Omega} \propto -\Omega^n$$



Espinoza *et al*, MNRAS, 414, 2, pp. 1679-1704 (2011)

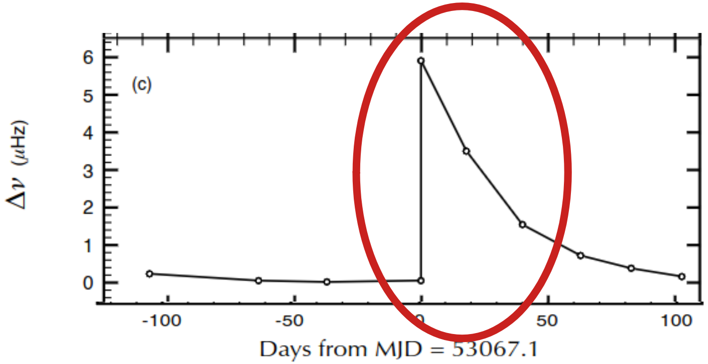
Glitches in Neutron Stars



$$\dot{L}_{\text{tot}}(t) = -N_{\text{em}}$$

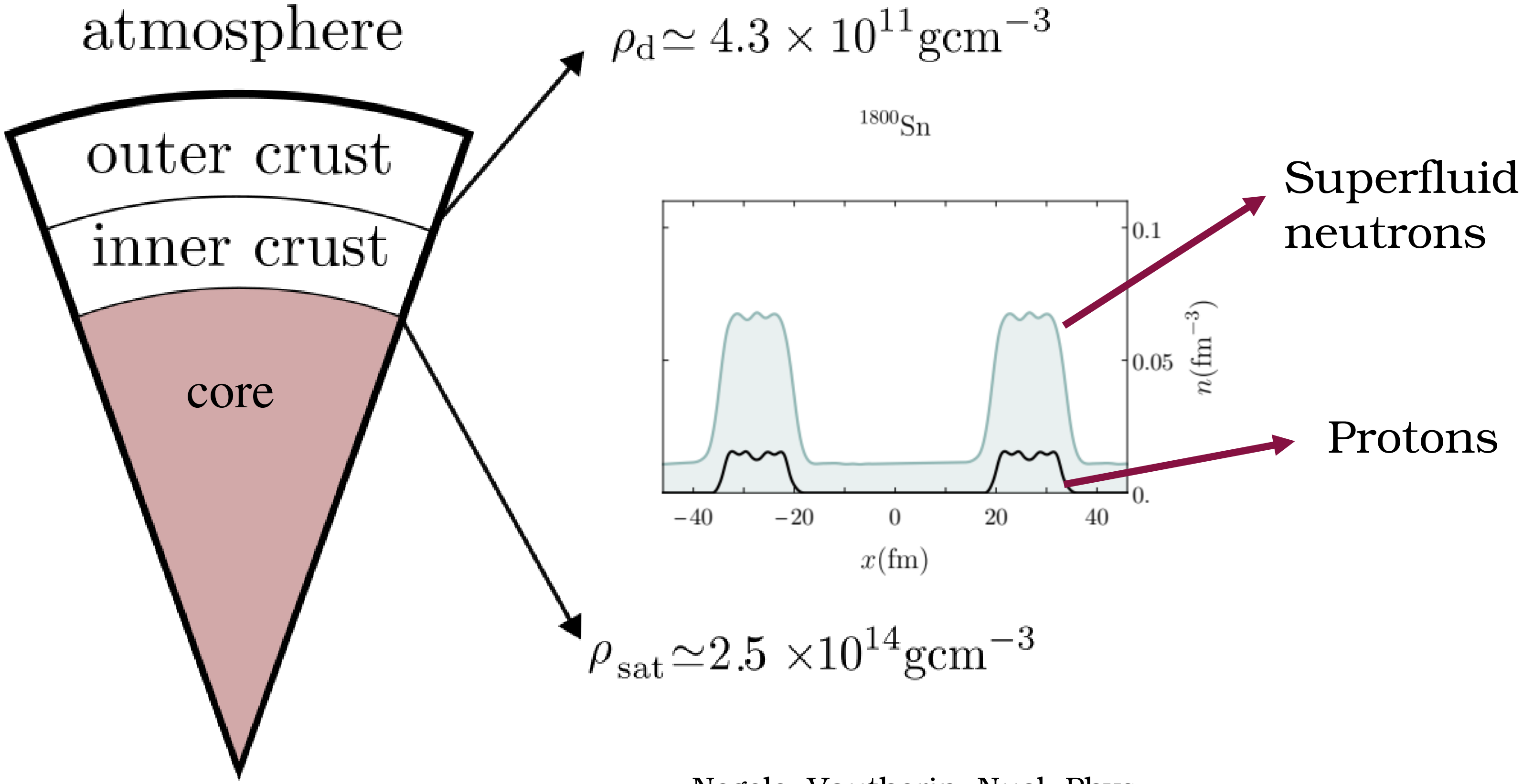
$$\frac{\Delta\Omega}{\Omega} \sim 10^{-12} \div 10^{-3}$$

Glitch event



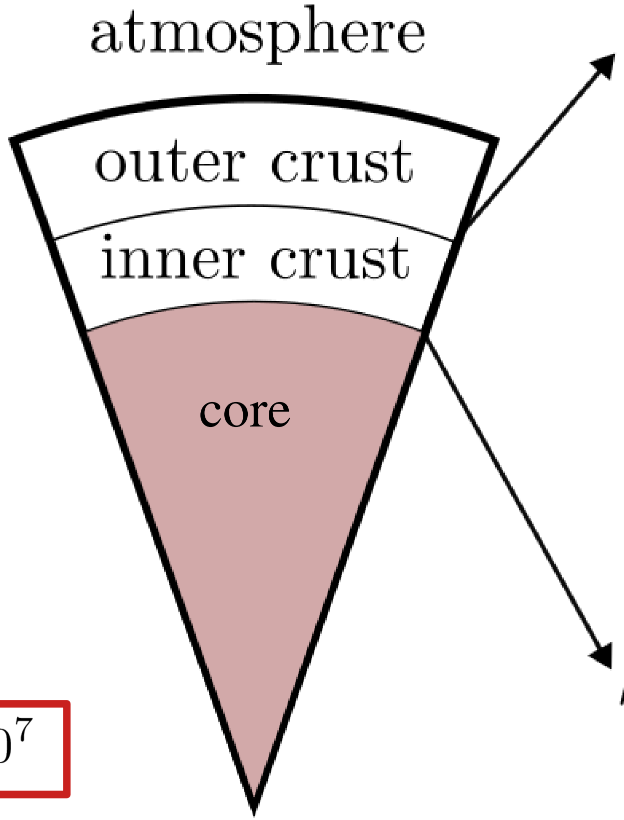
Espinoza et al, MNRAS, 414, 2, pp. 1679-1704 (2011)

Neutron Star structure

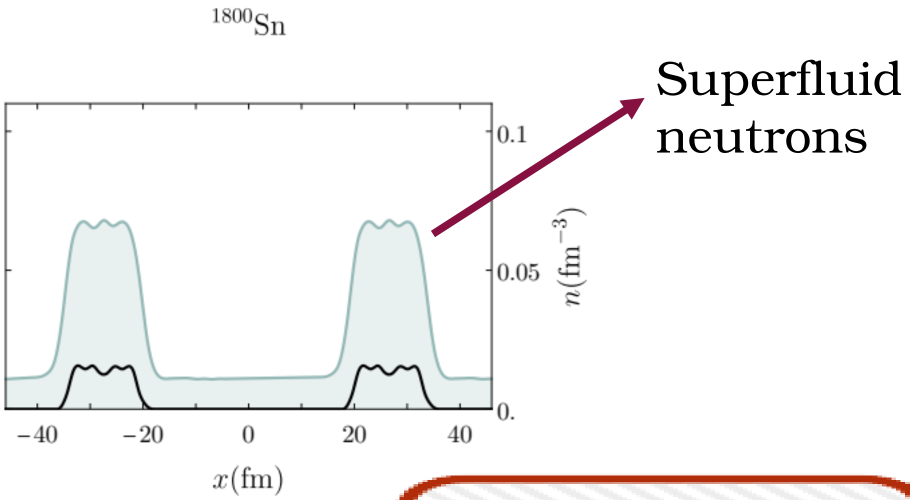


Negele, Vautherin, Nucl. Phys. A207, 298 (1973)

Neutron Star glitches



$\rho_{\rm d} \simeq 4.3 \times 10^{11} {\rm gcm}^{-3}$



$\rho_{\rm sat} \simeq 2.5 \times 10^{14} {\rm gcm}^{-3}$

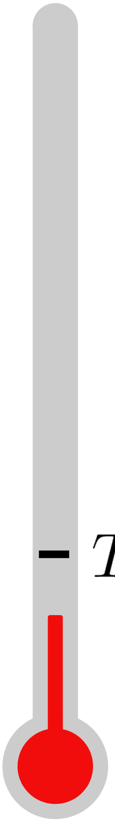
$N_{\rm vort,glitch} \sim 10^7$

Vortices unpinning creates glitches

Warszawski, Melatos, MNRAS, 415,1611 2011

Ultracold bosons

$T(\mathrm{K})$



Thermal gas

$T_c \sim \mu\mathrm{K}$

Bose Einstein Condensate

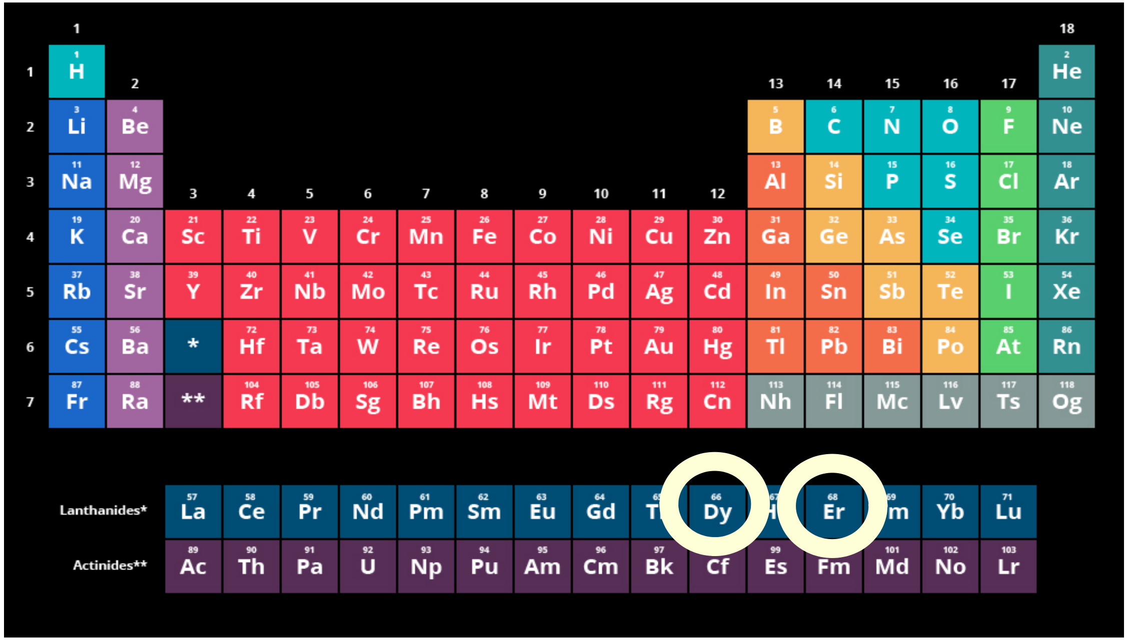
Dilute gas with short-range isotropic interaction

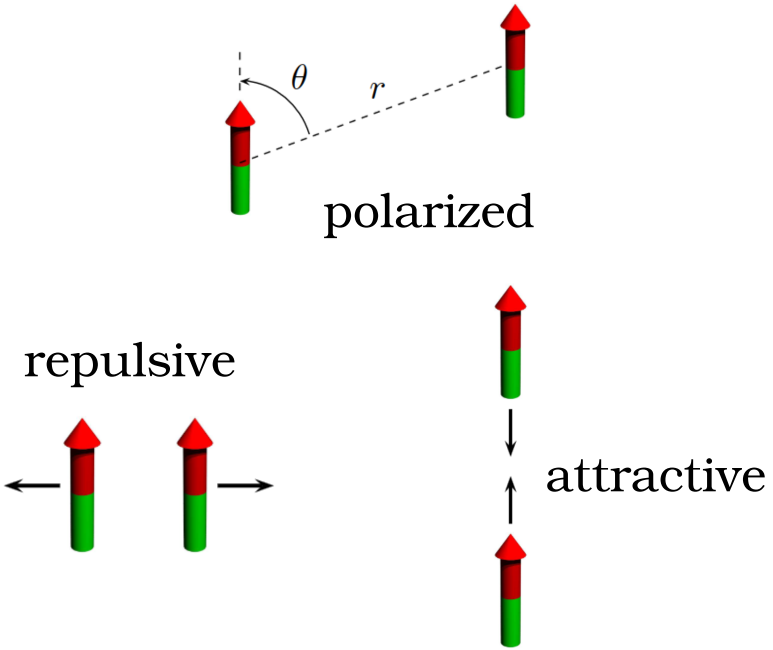
$$U_c(\boldsymbol{r}) = \frac{4\pi\hbar^2 a_s}{m}\delta(\boldsymbol{r})$$

Mean Field theory

Gross- Pitaevskii Equation

Ultracold dipolar atoms



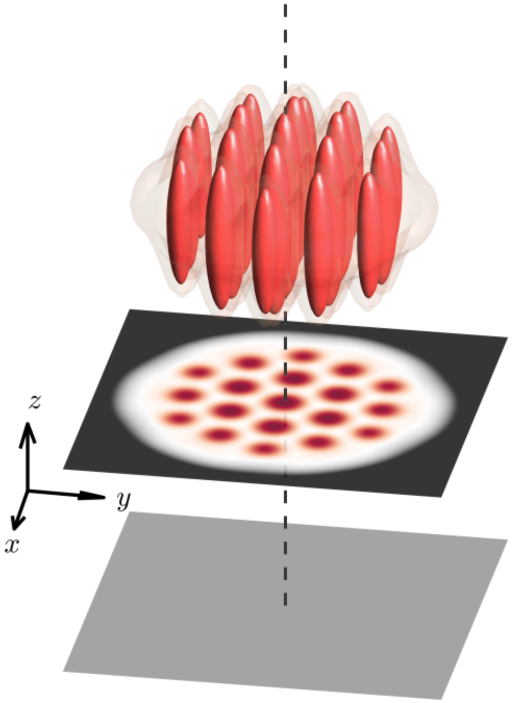


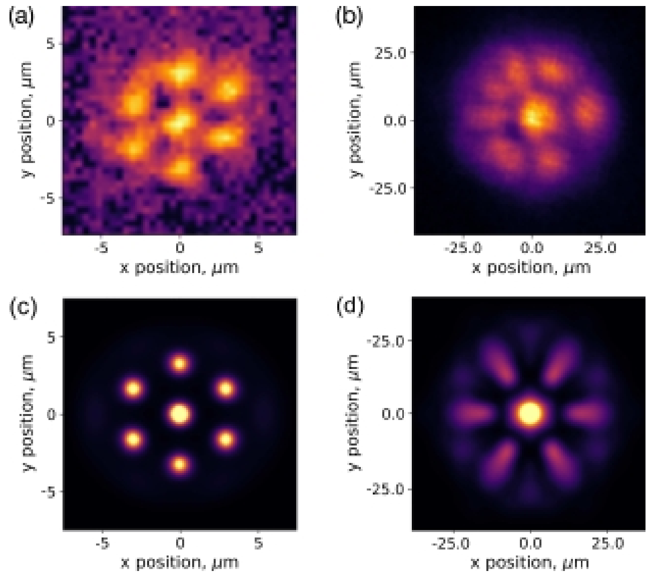
Long-range interaction $U_{\mathrm{dd}}(\boldsymbol{r}) = \dfrac{3\hbar^2 a_{\mathrm{dd}}}{m}\dfrac{1-3\cos^2\theta}{r^3}$ $\qquad a_{\mathrm{dd}} = \dfrac{\mu_0 \mu^2 m}{12\pi\hbar^2}$

Supersolidity

$$\epsilon_{\rm dd} = \frac{a_{\rm dd}}{a_s}$$

Observations of supersolid phase in MIT, Pisa/LENS, Stuttgart

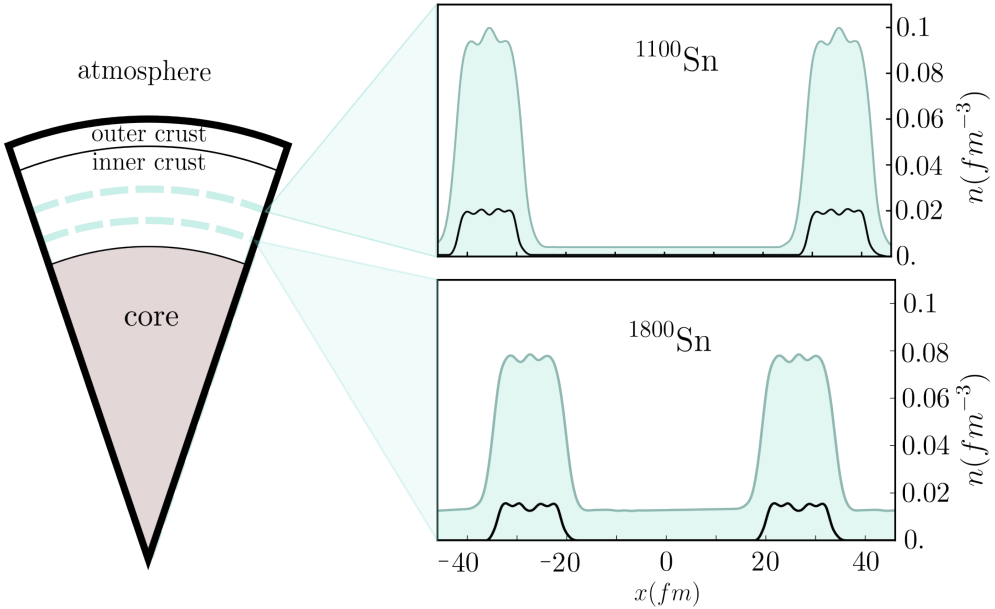




Bland *et al* PRL 128, 195302 (2022)
[Dy experiment (top) *vs* numerical (bottom)]

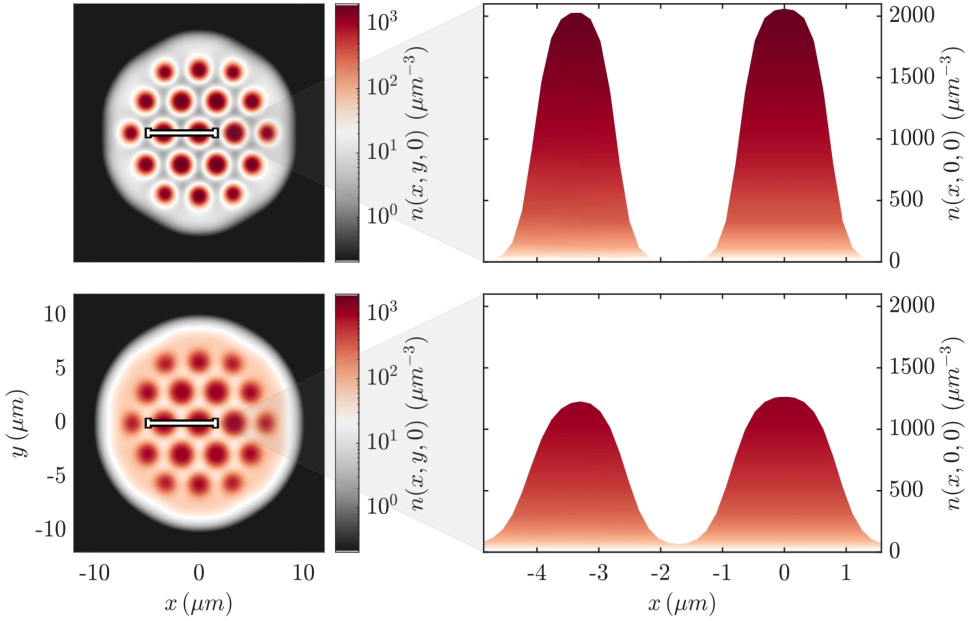
Neutron Stars *vs* Supersolids

$n_b \simeq 5.7 \times 10^{-3} \mathrm{fm}^{-3}$



$n_b \simeq 2.2 \times 10^{-2} \mathrm{fm}^{-3}$

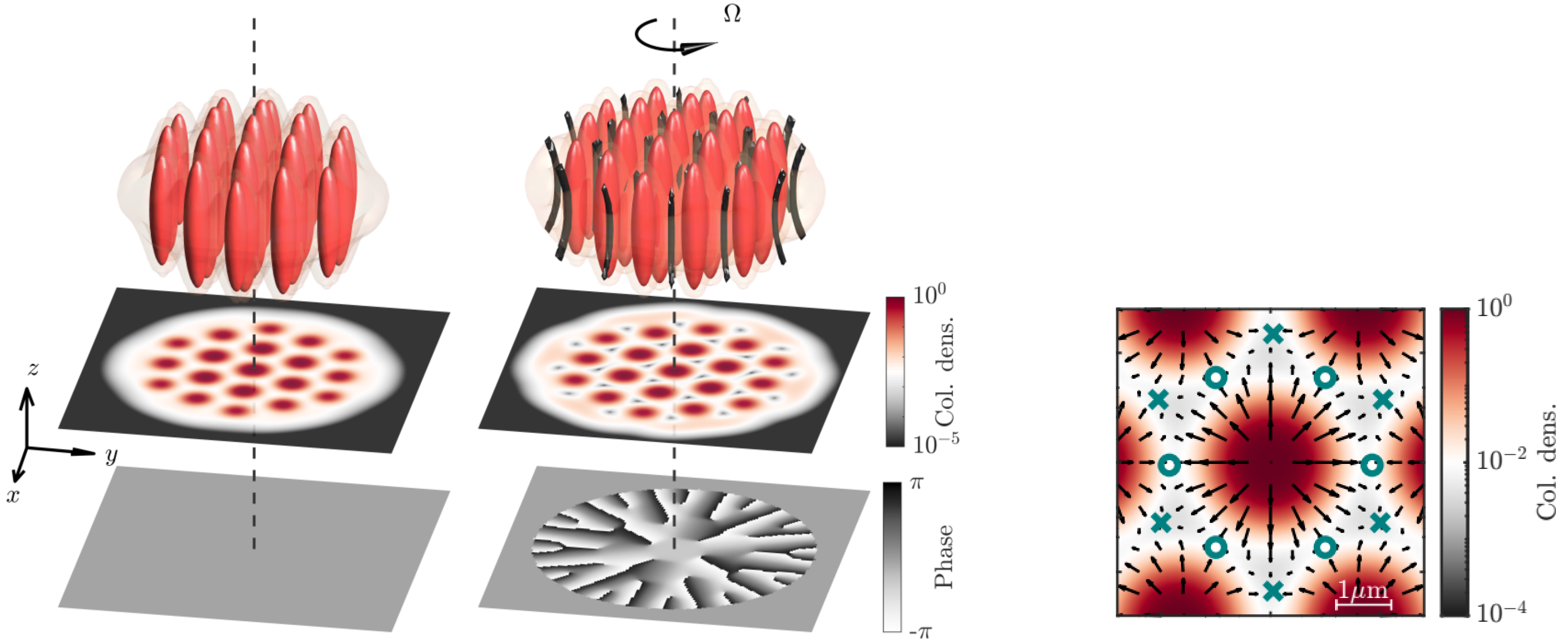
$a_s = 88a_0$



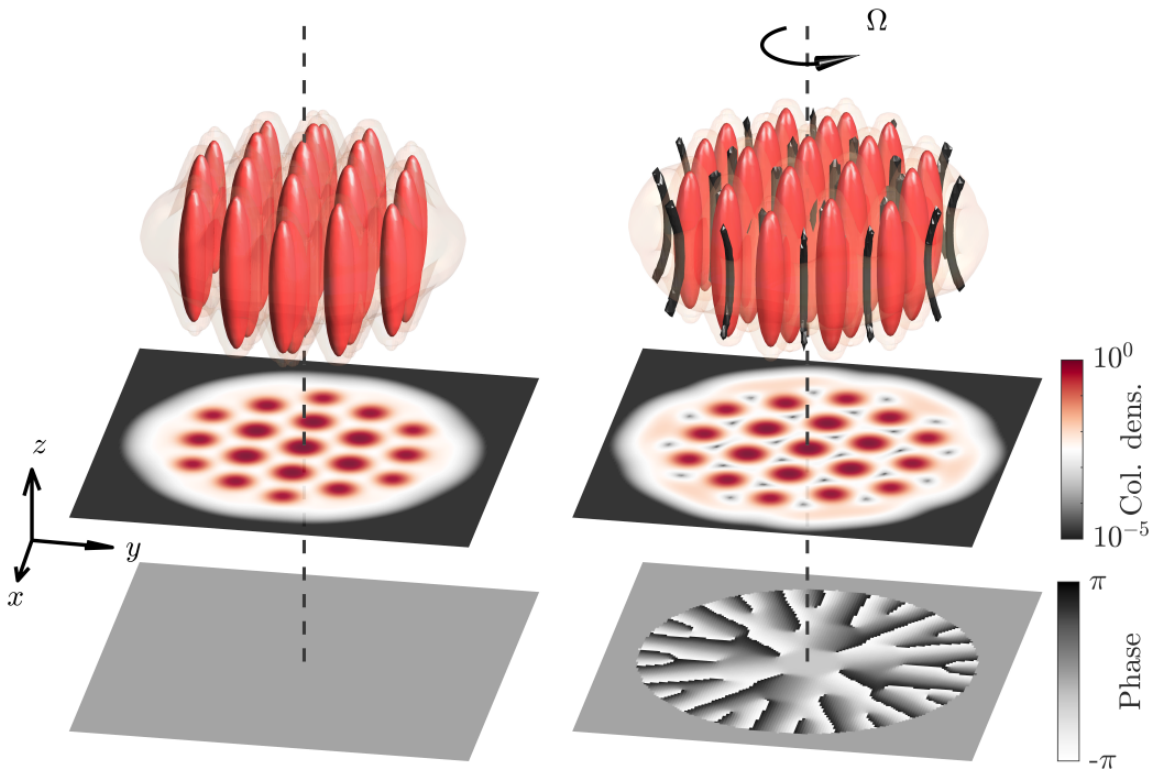
$a_s = 93a_0$

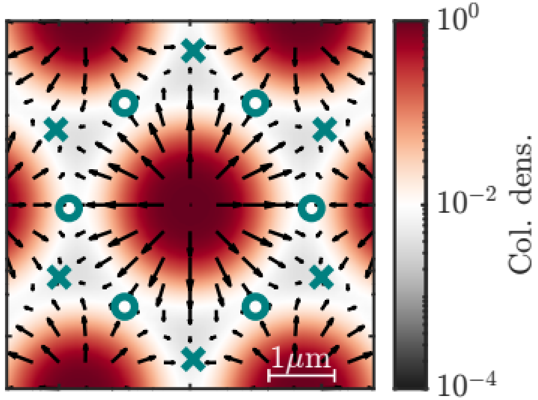
Poli, Bland, White, Mark, Ferlaino, ST, Mannarelli (arXiv:2306.09698)

Rotating a supersolid



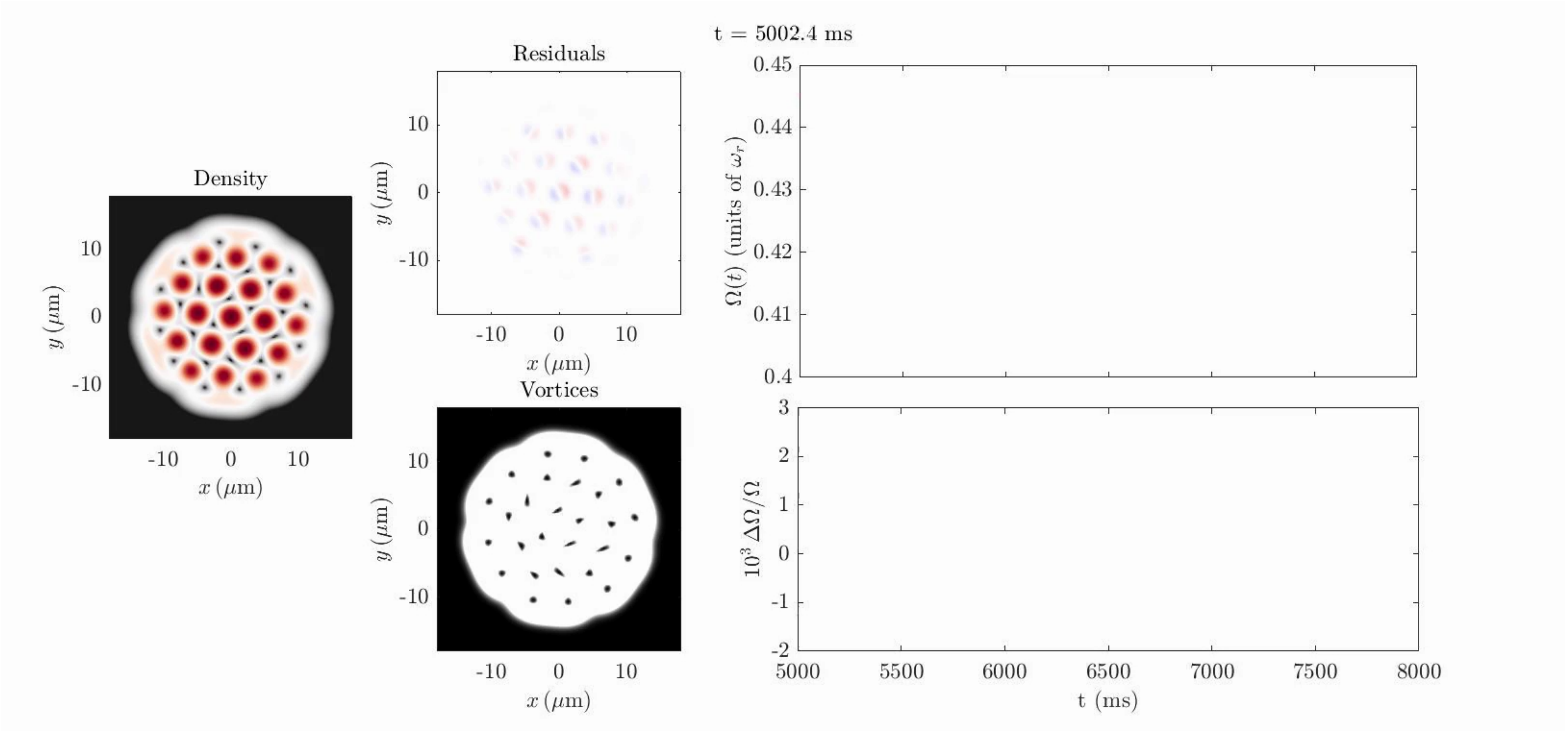
Rotating a supersolid





$$I_s(t)\dot{\Omega}(t) = -N_{\rm em} - \dot{I}_s(t)\Omega(t) - \dot{L}_{\rm vort}(t)$$

Glitches in supersolids



Parameters $a_s = 91a_0$, $\gamma = 0.05$, $N_{\mathrm{em}} = 4.3 \times 10^{-35} \mathrm{kg\ m^2/s^2}$ $\boldsymbol{\omega}_{\mathrm{trap}} = 2\pi \times (50, 130)\mathrm{Hz}$ $\mathrm{N} = 3 \times 10^5$

Summary

- Numerical observation of supersolid glitches
- Simulations of inner crust vortex interstitial pinning
- Observation of crystal oscillations and vortices percolation

✱ Scalability of pulse shape with number of vortices involved

✱ Investigate different spin-downs to mimic $\dot{\Omega} \propto -\Omega^n$

✱ Extension to nuclear pinning

Thank you for the attention!

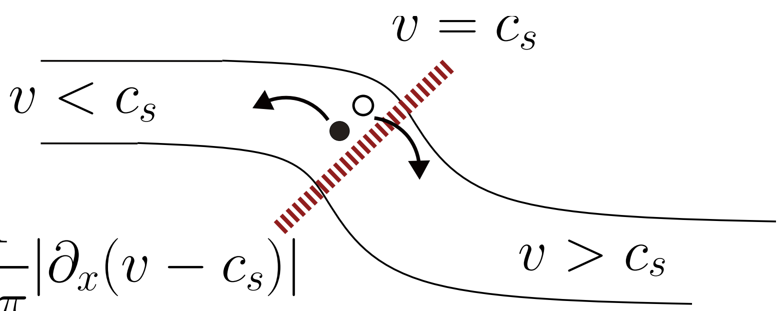
Backup slides

Dissipation at the acoustic horizon

Acoustic gravity analogs

Inviscid, barotropic irrotational flow at T=0

Scale separation: background + fluctuations



$$T_H = \frac{1}{2\pi} |\partial_x (v - c_s)|$$

Unruh PRL 1970
Barcelo *et al* , Liv.Rev. In Rel.,

Covariant kinetic theory for phonons: $f(x,p)$ distribution function

$$T^{\mu\nu}_{\rm ph} = \int p^\mu p^\nu f(x,p) d\mathcal{P}$$

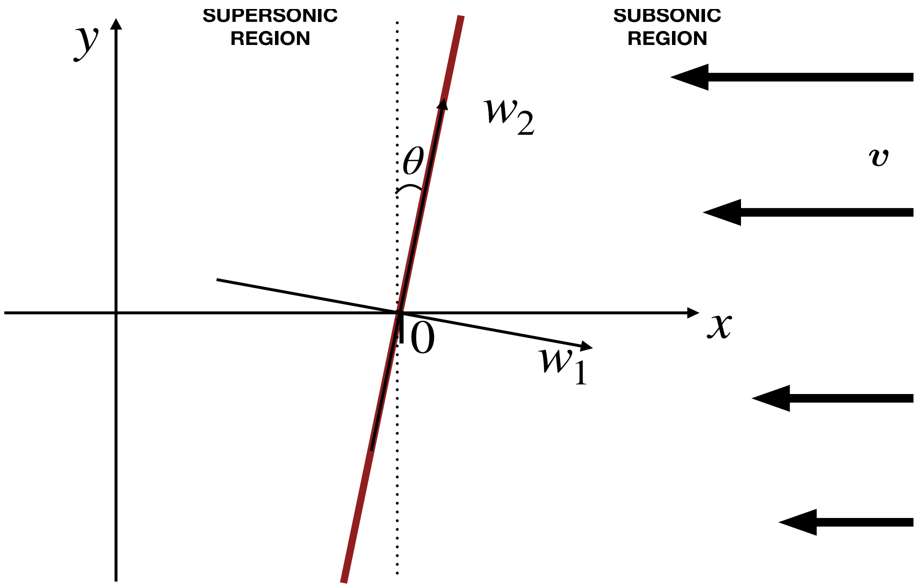
$$s^\alpha_{\rm ph} = -\int p^\alpha [f \ln f - (1+f)\ln(1+f)] d\mathcal{P}$$

measure in momentum space

Mannarelli, Grasso, ST, Chiofalo PRD103, 076001 (2021)

$$v_x \simeq c_s - 2\pi T_H x + ky$$

$$\sigma'_{ij} = \eta(\partial_i v_j + \partial_j v_i) + \zeta \delta_{ix}\delta_{jx} \nabla \cdot \mathbf{v}$$

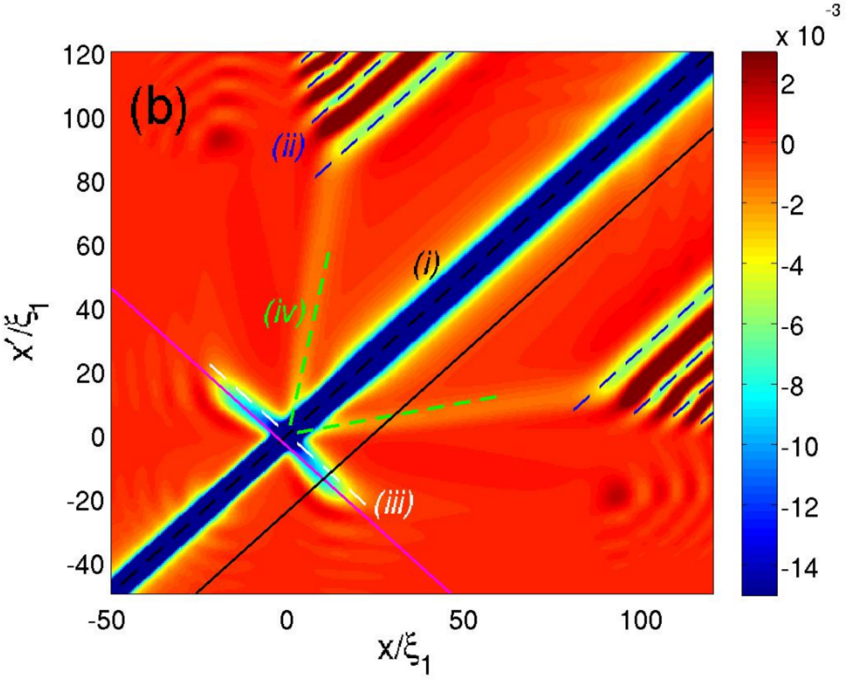


$$T_{ij}^{\mathrm{ph}} = \sigma'_{ij} \qquad \text{yields} \qquad \frac{\zeta}{s_{\mathrm{ph}}} = \frac{\eta}{s_{\mathrm{ph}}} = \frac{1}{4\pi}$$

Chiofalo, Grasso, Mannarelli, ST, arxiv: 2202.13790 [gr-qc]

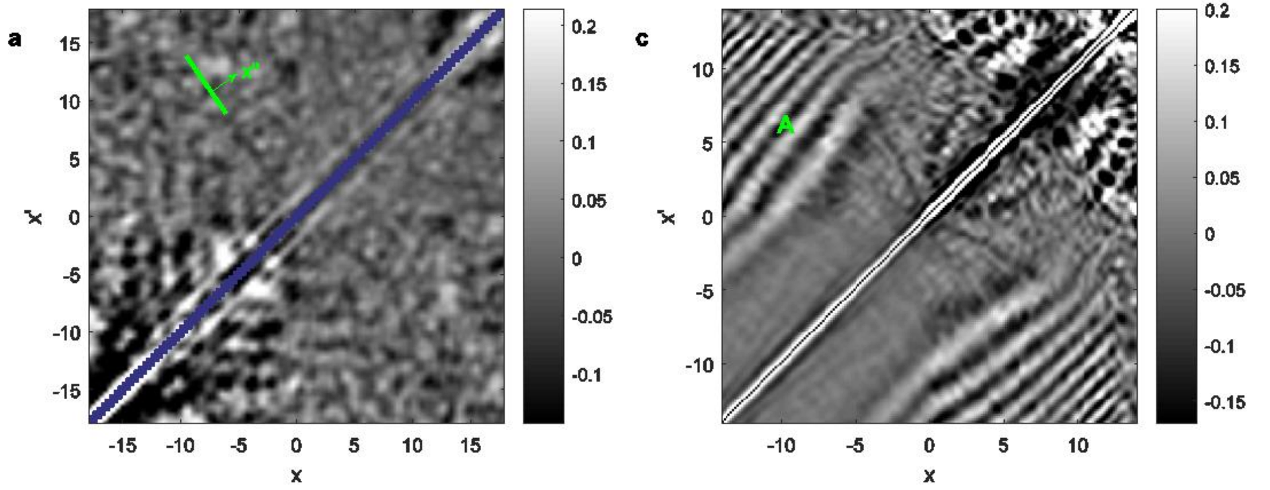
Correlation functions

Hawking-like emission is encoded in the density-density correlation function



Carusotto *et al* NJP 10 103001 (2008)

Experimental observation of Hawking-like emission (left) vs numerics (right)



Steinhauer, Nature 12, 959–965 (2016)

A new approach

Effective Field Theory for inhomongeneous flows based on perturation theory

Scale separation $\rho = \rho_0 + \tilde{\rho}\,,\ \theta = \theta_0 + \tilde{\theta}$ $\longrightarrow$ $\Psi(x) = (\tilde{\rho}(x), \tilde{\theta}(x))^t$

Partition function $$Z[J] = \int \mathcal{D}\Psi \exp\left(i\mathcal{S}[\Psi] + i\int d^d x J^t(x)\Psi(x)\right),$$

$$\mathcal{S}[\Psi] = \int d^d x \int d^d x' \frac{1}{2}\Psi^t(x) D^{-1}(x,x')\Psi(x')$$

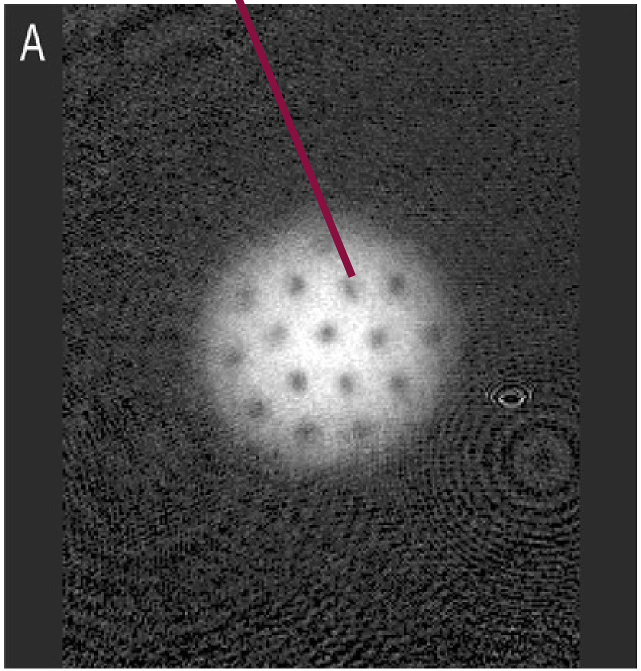
$$D^{-1}(x,x') = \delta^d(x-x')\begin{pmatrix} -\Box - \tilde{m}^2 & V^\mu \partial_\mu \\ -V^\mu \partial_\mu & -B^2\Box \end{pmatrix}$$

Biondi, Chiofalo, Liberati, Mannarelli, ST *in preparation*

Glitches - Backup slides

Angular momentum in BECs

Vortex core



BEC experiment

Vortices arrange in triangular Abrikosov lattice

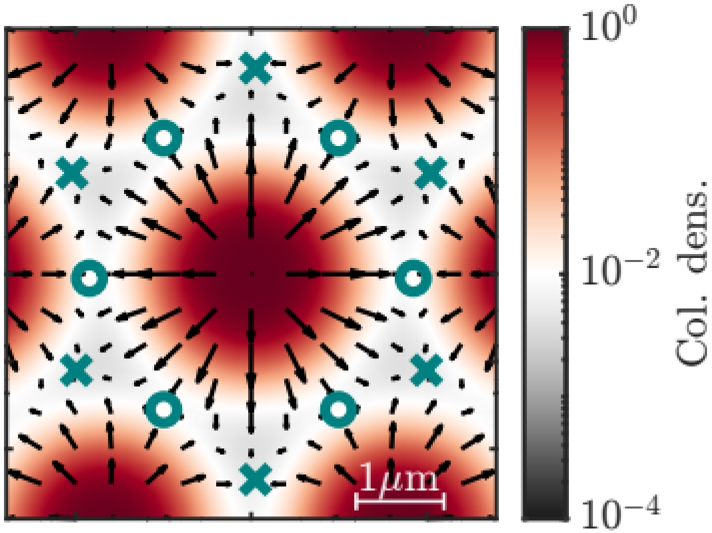
$$L_z = N_{\mathrm{vort}} \ell_z$$

No inertia

Abo-Shaeer *et al*, Science 2001 Vol 292, 5516, pp. 476-479 DOI: 10.1126/science.1060182

Angular momentum in supersolids

$a_s = 90a_0\,, \quad \boldsymbol{\omega}_{\text{trap}} = 2\pi \times (50, 130)\text{Hz}$



X = stable
O= metastable

$$I_{\text{rigid}}(t) = \langle x^2 + y^2 \rangle_{\Psi(t)}$$

$$I_{\text{s}}(t) = \alpha I_{\text{rigid}}(t)$$

related to the "amount" superfluidity

$$L_s(t) = I_s(t)\Omega(t)$$

Legget PRL, 25, 1543 (1970)

Educated ansatz

$$L_{\text{tot}}(t) = L_s(t) + L_{\text{vort}}(t)$$

Evolution

Given $\dot{L}_{\mathrm{tot}}(t) = -N_{\mathrm{em}}$

$$I_s(t)\dot{\Omega}(t) = -N_{\mathrm{em}} - \dot{I}_s(t)\Omega(t) - \dot{L}_{\mathrm{vort}}(t)$$

feedback

$$i\hbar\partial_t\Psi = (1 - i\gamma)[\mathcal{L}[\Psi; a_s, a_{\mathrm{dd}}, \boldsymbol{\omega}_{\mathrm{trap}}] - \Omega(t)\hat{L}_z]\Psi$$

eGPE

Evolution

Given $\dot{L}_{\text{tot}}(t) = -N_{\text{em}}$

$$I_s(t)\dot{\Omega}(t) = -N_{\text{em}} - \dot{I}_s(t)\Omega(t) - \dot{L}_{\text{vort}}(t)$$

feedback

$$i\hbar\partial_t\Psi = (1 - i\gamma)[\mathcal{L}[\Psi; a_s, a_{\text{dd}}, \boldsymbol{\omega}_{\text{trap}}] - \Omega(t)\hat{L}_z]\Psi$$

eGPE

Evolution

Given $$\dot{L}_{\rm tot}(t) = -N_{\rm em}$$

$$I_s(t)\dot{\Omega}(t) = -N_{\rm em} - \dot{I}_s(t)\Omega(t) - \dot{L}_{\rm vort}(t)$$

Dissipation

$$i\hbar\partial_t\Psi = (1 - i\gamma)[\mathcal{L}[\Psi; a_s, a_{\rm dd}, \boldsymbol{\omega}_{\rm trap}] - \Omega(t)\hat{L}_z]\Psi$$

$$\mathcal{L}[\Psi; a_s, a_{\rm dd}, \boldsymbol{\omega}] = -\frac{\hbar^2\nabla^2}{2m} + \frac{1}{2}m\left[\omega_r^2(x^2+y^2) + \omega_z^2 z^2\right] + \int d^3\mathbf{r}'\left[U_c(\mathbf{r}-\mathbf{r}') + U_{\rm dd}(\mathbf{r}-\mathbf{r}')\right]\left|\Psi(\mathbf{r}',t)\right|^2 + \gamma_{\rm QF}|\Psi(\mathbf{r},t)|^3 - \mu\,,$$

Numerical evolution

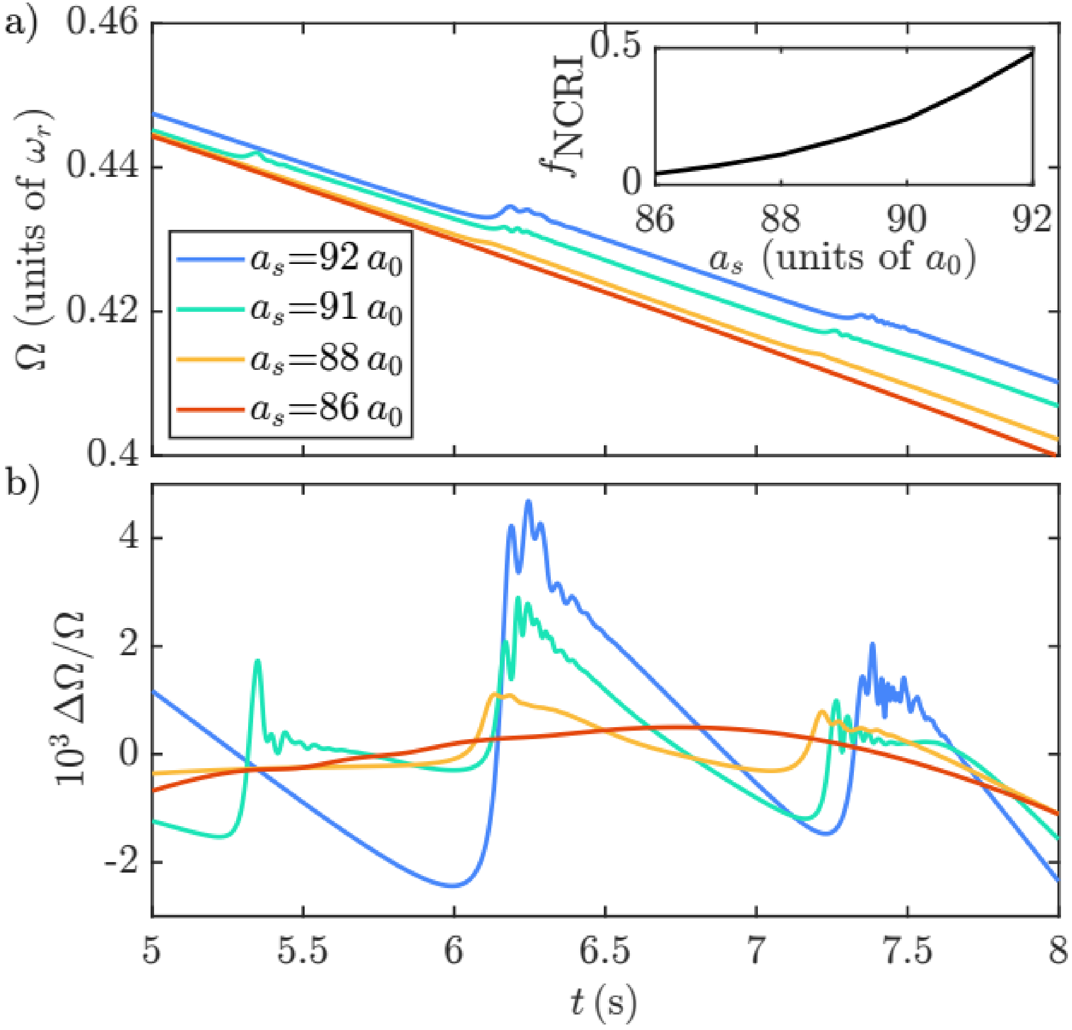
1) Imaginary time evolution to extract the ground state wavefunction in the rotating frame for fixed a_s , $a_{\rm dd}$ and Ω

2) Extract the value of $f_{\rm NCRI} = \lim_{\Omega \to 0} \frac{\langle \hat{L}_z \rangle_\Psi}{\Omega}$

3) Given an initial value of angular velocity, start the numerical evolution with eGPE

4) Use the update rule of the feedback- mechanism to get the new angular velocity

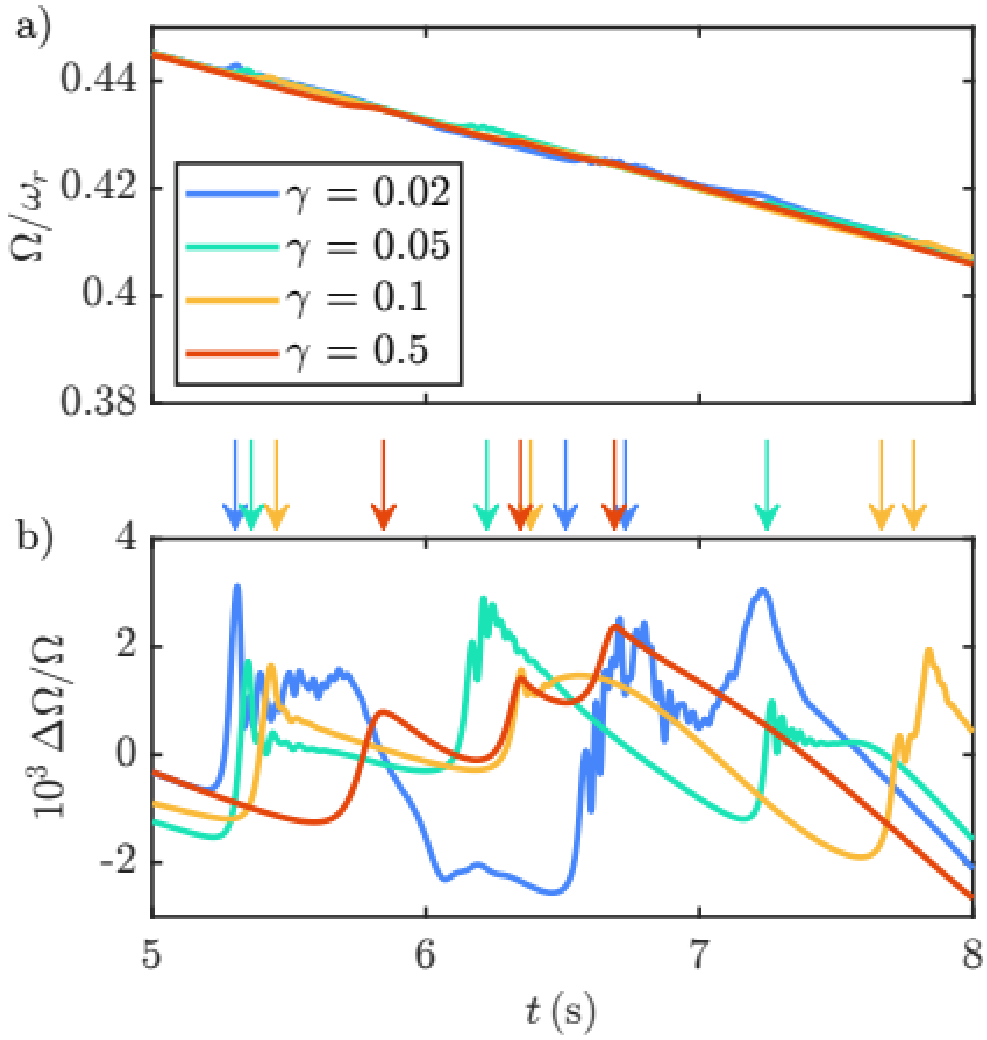
5) Insert the new value in the eGPE, continue the numerical evolution



$$I_{\mathrm{s}}(t) = \alpha I_{\mathrm{rigid}}(t)$$

$$\alpha = 1 - f_{\mathrm{NCRI}}$$

- Parameters $N_{\mathrm{em}} = 4.3 \times 10^{-35}\mathrm{kg\ m^2/s^2}\,, \quad \gamma = 0.05\,, \quad \Omega_{\mathrm{init}} = 0.5\omega_r\,.$



Different values of γ mimic different coupling with the outer crust

Parameters $N_{\rm em} = 4.3 \times 10^{-35}\text{kg m}^2/\text{s}^2\,,\ a_s = 91a_0\,,\quad \Omega_{\rm init} = 0.5\omega_r\,.$