Scalar Induced Gravitational Waves in the Multi-messenger Era: from fundamental physics in the early universe to PBH evolution

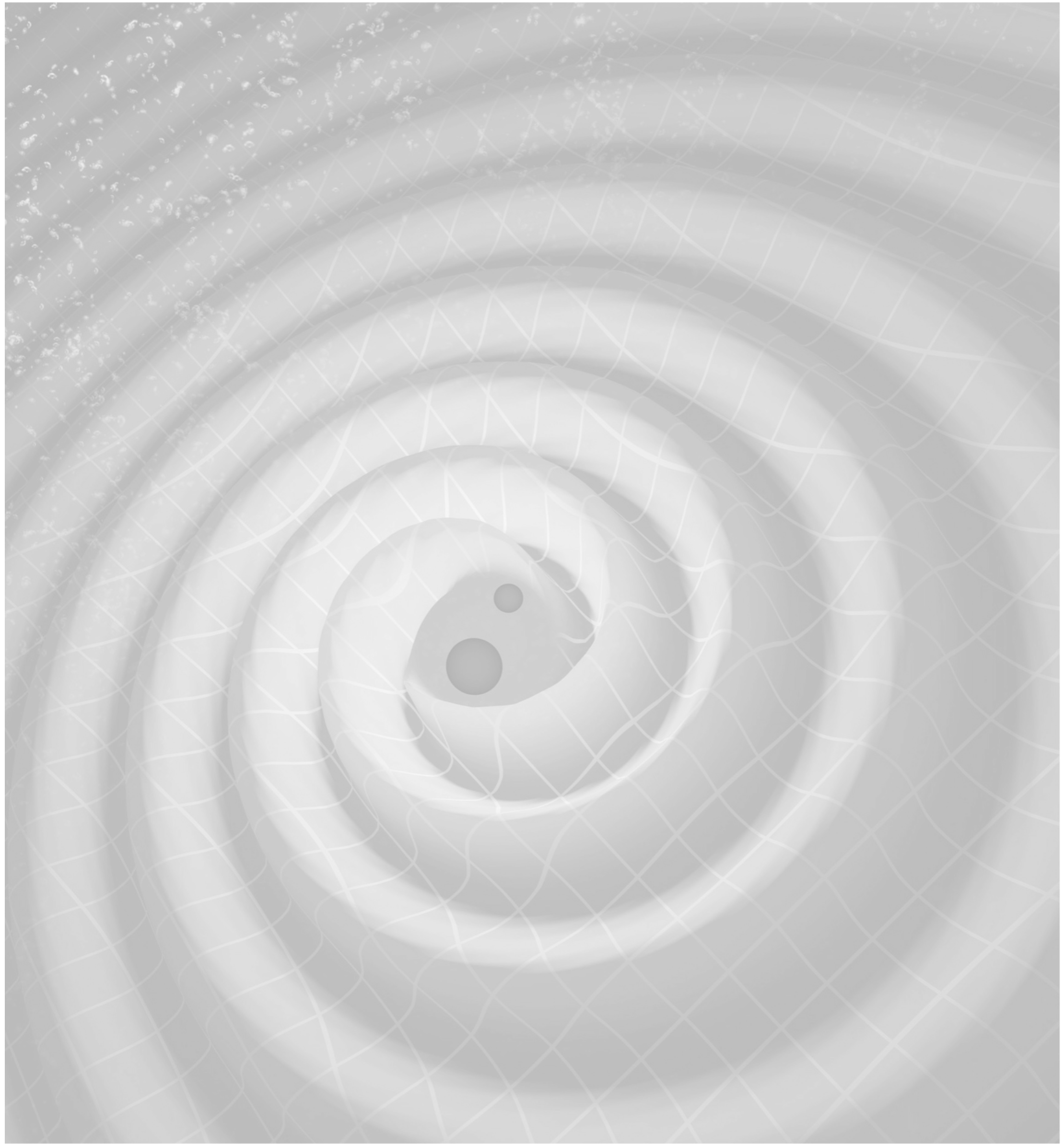
Anjali Abirami Kugarajh

Gran Sasso Science Institute

20.10.2023

Outline

- Motivation
- Second-order perturbation theory
  - Metric decomposition
  - Gauge dependence
  - Evolution equation
  - Source term for scalar-induced gravitational waves
- SIGW in modified gravity theories
  - Why modified gravity
  - *f(R)* theories
- Conclusions



Introduction

- Observations of gravitational waves (GWs) from merger of binary black holes
- GWs weakly interacting with matter → primordial gravitational waves
- Standard generation mechanism for primordial GW: quantum vacuum fluctuations
- Primordial GWs come in form of stochastic background (SGWB) → superposition of incoming GWs

Scalar-induced GWs

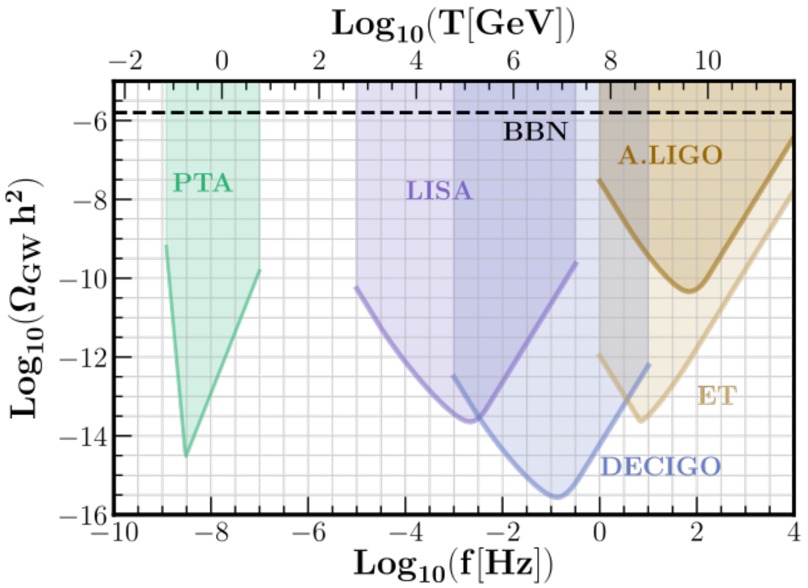
“Scalar-induced” gravitational waves (SIGW) → second order effect and coupling of scalar fluctuations

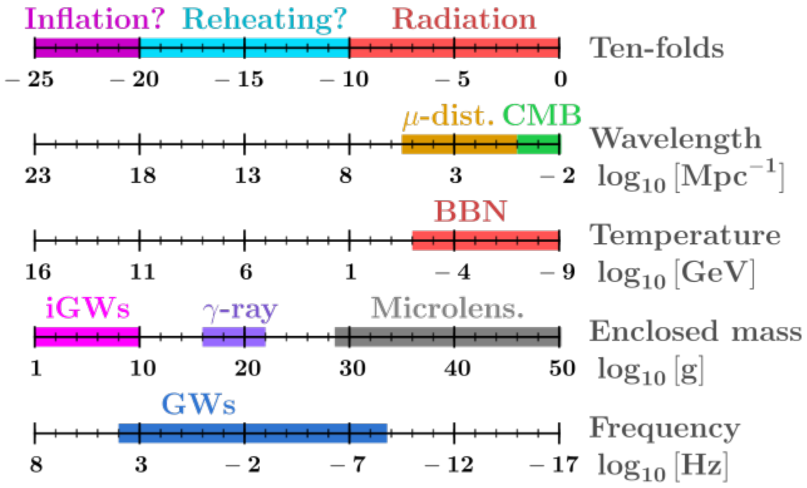
- First noticed by K. Tomita (1967) and later by Matarrese, Pantano and Saez (1993, 1994) when studying perturbations in dust dominated universe
- Studies of using SIGW to constrain primordial spectrum and spectral tilt of primordial fluctuations
- Studying characteristic signatures in the SIGW spectrum can help in understanding early universe era

Motivation

Number 1. Detection

Characteristic frequencies fall in the band of current and future GW interferometers





arXiv:2109.01398 [gr-qc]

Left plot show power law integrated sensitivity curve.
Right plot shows typical quantities associated to cosmological horizon at particular time and ranges covered by cosmological probes.

Motivation

Number 1. Detection

Characteristic frequencies fall in the band of current and future GW interferometers

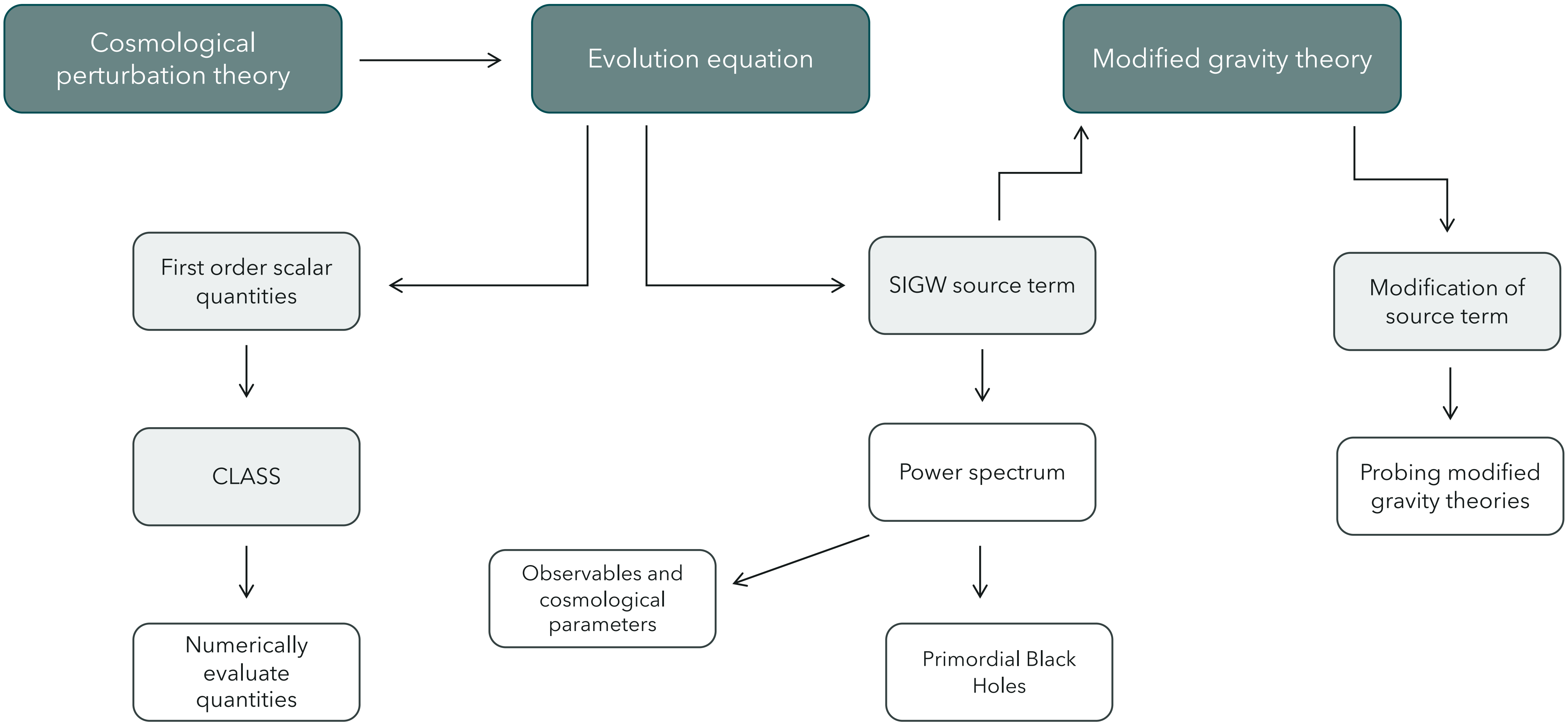
Pulsar Timing Arrays (PTA) observation providing strong evidence for a SGWB
→ compatible with "scalar-induced" background

Motivation

Number 2. Primordial Black Hole

- Compact binaries composed of PBHs can be potentially observed
- Study of primordial black holes (PBHs) and relevance to scalar fluctuations
- SIGW can provide constrains on PBH abundance
- Large primordial fluctuations may collapse to form primordial black holes
- If PBHs are detected, an induced GW counterpart could also be seen

Structure of project



Relativistic cosmological perturbation theory

- Small inhomogeneous perturbations, i.e. deviations, on homogenous and isotropic FRW background
- Decomposition of metric tensor into unperturbed background and perturbed parts

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + \delta^{(1)} g_{\mu\nu} + \frac{1}{2}\delta^{(2)} g_{\mu\nu}, \tag{1}$$

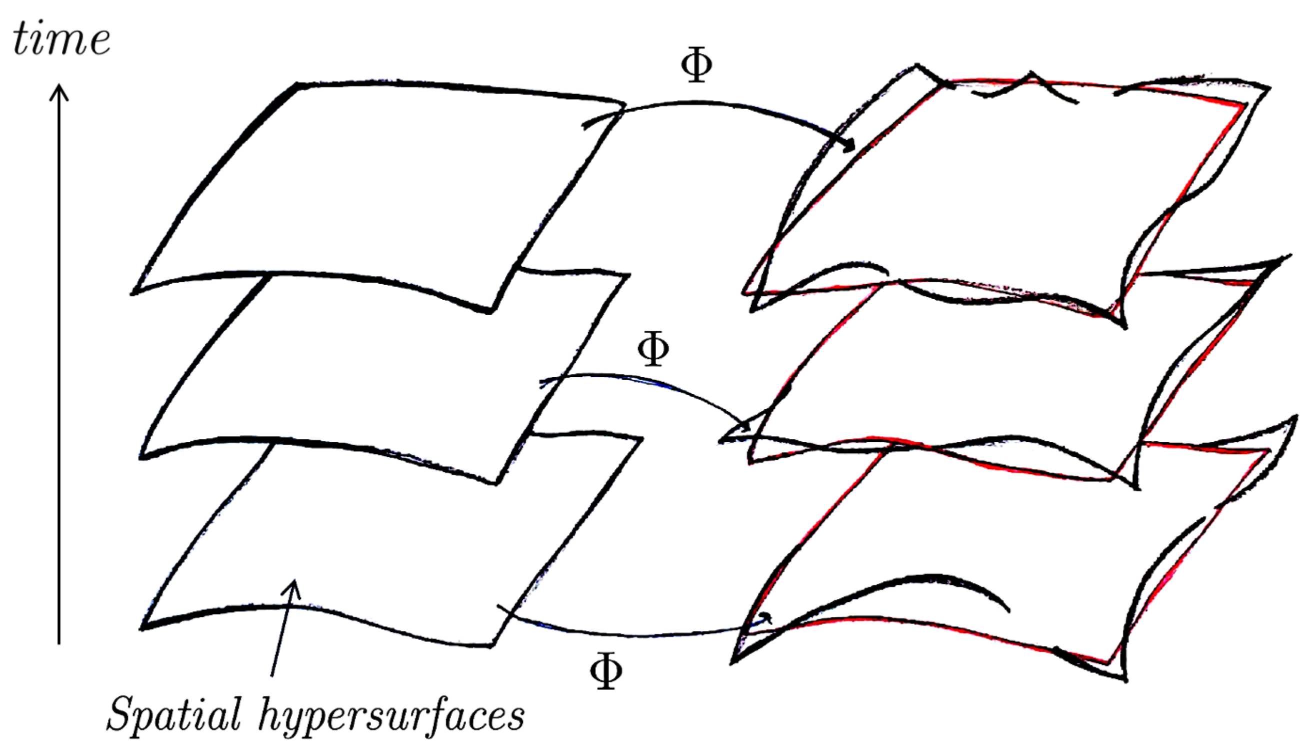
Metric decomposition

- Metric includes scalar, vector and tensor parts related to observable quantities
  - Scalar perturbation → curvature fluctuations
  - Tensor perturbation → gravitational waves

$$g_{00} = -a^2(\eta)\left(1 + 2\sum_{r=1}^{\infty}\frac{1}{r!}\phi^{(r)}\right), \tag{2}$$

$$g_{0i} = a^2(\eta)\sum_{r=1}^{\infty}\frac{1}{r!}\hat{w}_i^{(r)}, \tag{3}$$

$$g_{ij} = a^2(\eta)\left\{\left[1 - 2\left(\sum_{r=1}^{\infty}\frac{1}{r!}\psi^{(r)}\right)\right]\delta_{ij} + \sum_{r=1}^{\infty}\frac{1}{r!}\hat{\chi}_{ij}^{(r)}\right\}, \tag{4}$$



Gauge dependence

Gauge → mapping between background and perturbed space

Gauge chosen to best suit the model and problem → helps simplify

Goal 1: derive Einstein tensor and source term of SIGW in generic gauge

Evolution equation

- General expression:

$$G^{\mu}_{\nu} = \kappa^2 T^{\mu}_{\nu} \tag{5}$$

$$G^{\lambda}_{\mu} = g^{\nu\lambda}\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) \tag{6}$$

$$T^{\mu}_{\nu} = (\rho + p)u^{\mu}u_{\nu} + p\delta^{\mu}_{\nu} + \pi^{\mu}_{\nu}. \tag{7}$$

First order quantities

- Using first order spatial component and imposing a gauge known as Newtonian gauge.

$$\delta^{(1)} G^i_j = \kappa^2 T^{(1)i}_j. \tag{8}$$

$$-a^{-2}\left(\partial^i \partial_j - \frac{1}{3}\nabla^2 \delta^i_j\right)\left(\phi^{(1)} - \psi^{(1)}\right) = \kappa^2 a^{-2} \pi^{(1)i}_j \tag{9}$$

- In absence of anisotropic stress, we see that $\phi = \psi$
- Solving following equation to find the first order quantities:

$$\phi^{(1)\prime\prime} + 3\mathcal{H}\left(1 + c_s^2\right)\phi^{(1)\prime} + \left(2\mathcal{H}' + \left(1 + 3c_s^2\right)\mathcal{H}^2 - c_s^2 \nabla^2\right)\phi^{(1)} = 0 \tag{10}$$

Second-order evolution equation

$$\delta^{(2)} G^i_j = \kappa^2 T^{(2)i}_j . \tag{11}$$

$$\chi^{i(2)''}_j + 2\frac{a'}{a}\chi^{i(2)'}_j - \nabla^2 \chi^{i(2)}_j = -4S^i_j , \tag{12}$$

Source term for SIGW

$$
\begin{aligned}
S^i_j = {} & \partial^i\phi^{(1)}\partial_j\phi^{(1)} + 2\phi^{(1)}\partial^i\partial_j\phi^{(1)} - 2\psi^{(1)}\partial^i\partial_j\phi^{(1)} - \partial_j\phi^{(1)}\partial^i\psi^{(1)} - \partial^i\phi^{(1)}\partial_j\psi^{(1)} \\
& + 3\partial^i\psi^{(1)}\partial_j\psi^{(1)} + 4\psi^{(1)}\partial^i\partial_j\psi^{(1)} \\
& - \frac{4}{3\mathcal{H}^2(1+w)}\left[\partial^i\left(\psi^{(1)\prime} + \mathcal{H}\phi^{(1)}\right)\partial_j\left(\psi^{(1)\prime} + \mathcal{H}\phi^{(1)}\right)\right] \\
& + 4\psi^{(1)}\delta^{ik}\left[\left(\partial_j\partial_k - \frac{1}{3}\nabla^2\delta_{jk}\right)\left(\phi^{(1)} - \psi^{(1)}\right)\right] .
\end{aligned}
\tag{13}
$$

Goal 1 ✅

Modified gravity theories

Due to issues in standard cosmological model, alternative theories have been considered

Standard cosmological model

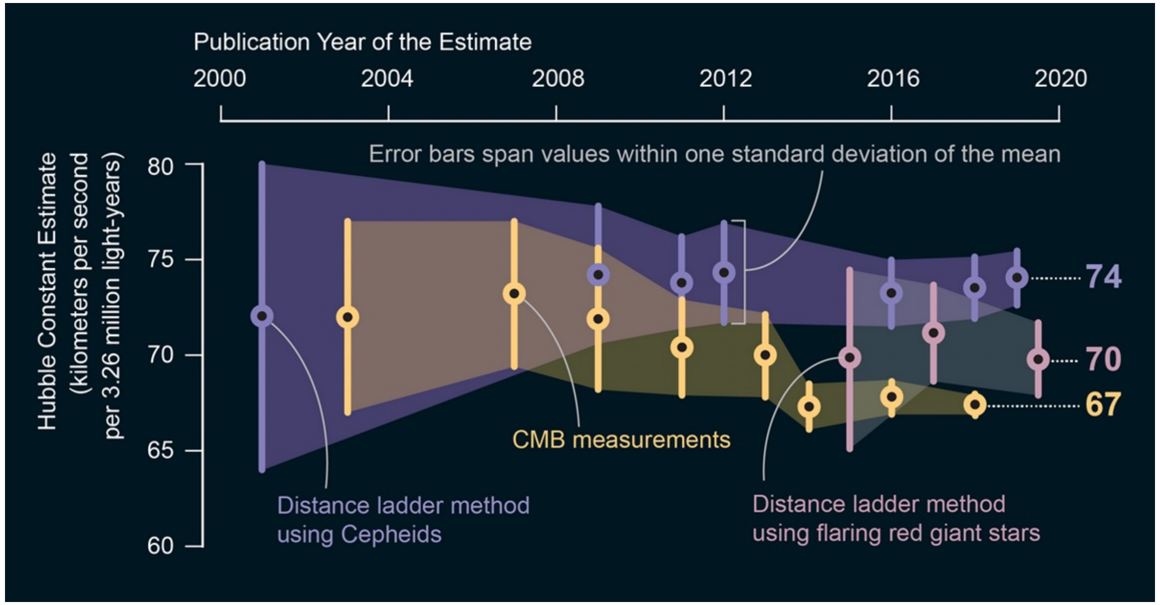
- What are the issues?
  - Cosmological constant problem → calculated value for Λ is **120 order of magnitude larger** than observed value

Standard cosmological model

- What are the issues?
  - Cosmological constant problem
  - How to describe primordial universe → understanding of inflation

Standard cosmological model

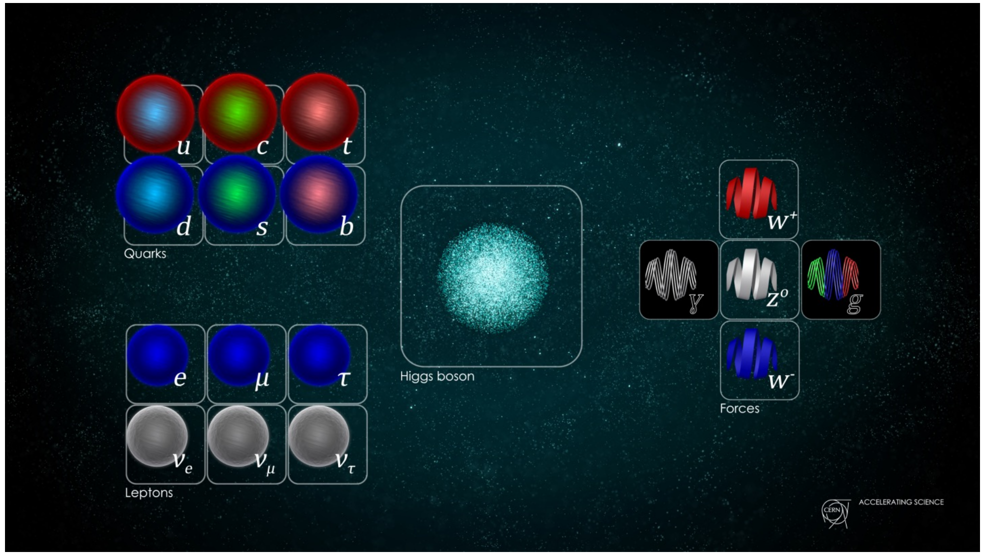
- What are the issues?
  - Cosmological constant problem
  - How to describe primordial universe
  - Tensions within data, for example Hubble tension → tensions between data sets of direct and indirect measurement

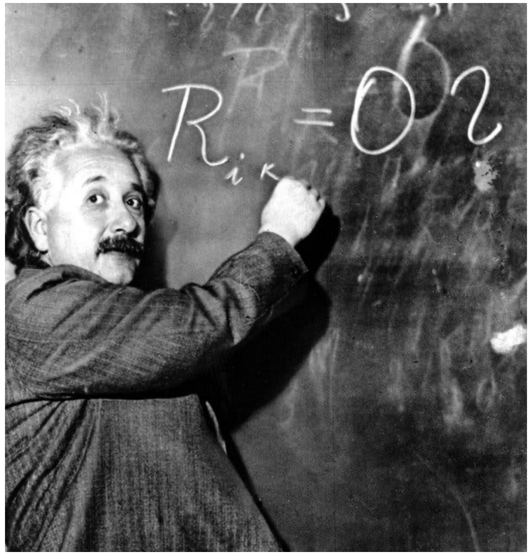


Panek (2020)

Modified gravity theories

- What are the issues?
  - Cosmological constant problem
  - How to describe primordial universe
  - Tensions within data, e.g. Hubble tension
  - General relativity does not mathematically connect with nature's other three forces → lack in compatibility of quantum field theory/quantum mechanics and general relativity





Modified gravity theories

- Due to issues in standard cosmological model, alternative theories have been considered
- What are the issues?
  - Cosmological constant problem
  - How to describe primordial universe
  - Tensions within data, e.g. Hubble tension
  - Lack of Quantum Gravity Theory
- Need to modify something: ***universe content*** or ***theory of gravity***

SIGW in modified gravity theories

- In standard GR, contribution from anisotropic stress is small → anisotropic stress negligible
- In the source term for SIGW, coupling between scalar perturbation and anisotropic stress
- In certain MG models, a correction occurring from anisotropic stress
  - Non-standard relation between scalar potentials, i.e. $\phi^{(1)} - \psi^{(1)} \neq 0.$
  - Due to modification in geometric part of Einstein equation

Goal 2: *study relation between scalar potentials and anisotropic stress contribution on source term of SIGW*

$f(R)$ theory

- $f(R)$ gravity, Lagrangian density f is a function of Ricci scalar
- Modified evolution equation:

$$F(R)G^{\lambda}_{\mu} = g^{\lambda\nu} g_{\mu\nu} \frac{(f(R) - RF(R))}{2} + g^{\lambda\nu} \nabla_{\mu} \nabla_{\nu} F(R) - g^{\lambda\nu} g_{\mu\nu} \Box F(R) + \kappa^2 T^{\lambda}_{\mu} \tag{14}$$

Model considered: $$f(R) = R + \alpha R^2 \tag{15}$$

First-order evolution equation

$$
\begin{aligned}
&- \left(1 + 12\alpha a^{-2}\left[\mathcal{H}' + \mathcal{H}^2\right]\right)\left(\partial^i\partial_j - \frac{1}{3}\nabla^2\delta^i_j\right)\left(\phi^{(1)} - \psi^{(1)}\right) \\
&= 2\alpha a^{-2}\left(\partial^i\partial_j - \frac{1}{3}\nabla^2\delta^i_j\right)\left(-6\psi^{(1)\prime\prime} - 6\mathcal{H}\phi^{(1)\prime} - 18\mathcal{H}\psi^{(1)\prime} - 12\left[\mathcal{H}' + \mathcal{H}^2\right]\phi^{(1)} - 2\nabla^2\phi^{(1)} + 4\nabla^2\psi^{(1)}\right) \\
&+ \kappa^2\pi^{(1)i}_j \qquad (16)
\end{aligned}
$$

Second-order evolution equation

$$\frac{1}{4}\left(1+12a^{-2}\alpha\left[\mathcal{H}'+\mathcal{H}^2\right]\right)\left(\chi_j^{i(2)\prime\prime}+2\mathcal{H}\chi_j^{i(2)\prime}-\nabla^2\chi_j^{i(2)}\right)+3\alpha a^{-2}\left[\mathcal{H}''-2\mathcal{H}^3\right]\left(\chi_j^{(2)i}\right)'=S^i_{mg,j} \quad (17)$$

Future work

CLASS and HiCLASS

- Boltzmann solver code Cosmic Linear Anisotropy Solving System (CLASS)
- Code simulates linear perturbation evolution
- Computes observables for cosmological probes.
- "HiCLASS" implements modified gravity theory, specifically Horndeski theory, in the CLASS code.
- Numerically evaluate the basic quantities for scalar perturbations

Goal 3: Studying the power spectrum accounting for modification

Future work

Primordial Black Holes

- To understand the relation between the PBH formation and induced GWs, the peaks of induced GWs spectra is studied
- Determining abundance and mass fraction of PBHs

Future – generation detectors

- Measuring the effect of gravitational slip
- A way to probe modified gravity theories

Conferences and workshops

2021/22

- CORFU2022: 22th Hellenic School and Workshops on Elementary Particle Physics and Gravity – Workshop on Tensions in Cosmology: 7-12 September 2022 – Corfu, Greece

2022/23

- Complexity of the Cosmos: 5-7 October 2022 – L'Aquila, Italy
- Messengers of the Early Universe:Gravitational Waves and Primordial Black Holes: 12-14 December 2022 – Padova, Italy
- Progress on Old and New Themes in cosmology (PONT) 2023 : 1-5 May 2023 – Avignon, France
- New Horizons in Primordial Black Hole physics (NEHOP): 19-21 June 2023 – Naples, Italy

Attended schools

2021/22

- V Institute of Space Sciences Summer School: 4-15 July 2022 – Barcelona, Spain

2022/23

- Kavli-Villum Summer School on Gravitational Waves: 25-30 September 2023 – Corfu, Greece

Other activities

- Working from Padova: 9-15 July 2023 – Padova, Italy. During this one week period, I was able to work directly with my supervisors, Professor Sabino Matarrese and Dr. Angelo Ricciardone, in order to discuss and make progress in the calculations that I have been doing.
- Asiago Dedicated Cosmology Meeting: 4-6 October 2023 – Asiago, Italy. Planned by the cosmology group in Padova, I was invited to attend their annual dedicated group meeting at the Asiago observatory.

Memberships and collaborations

- Member of the Einstein Telescope (ET) Collaboration.