TESTS OF BINARY BLACK HOLE NATURE: CURRENT AND FUTURE PROSPECTS

ASYMMETRIC BINARIES MEET FUNDAMENTAL ASTRO-PHYSICS

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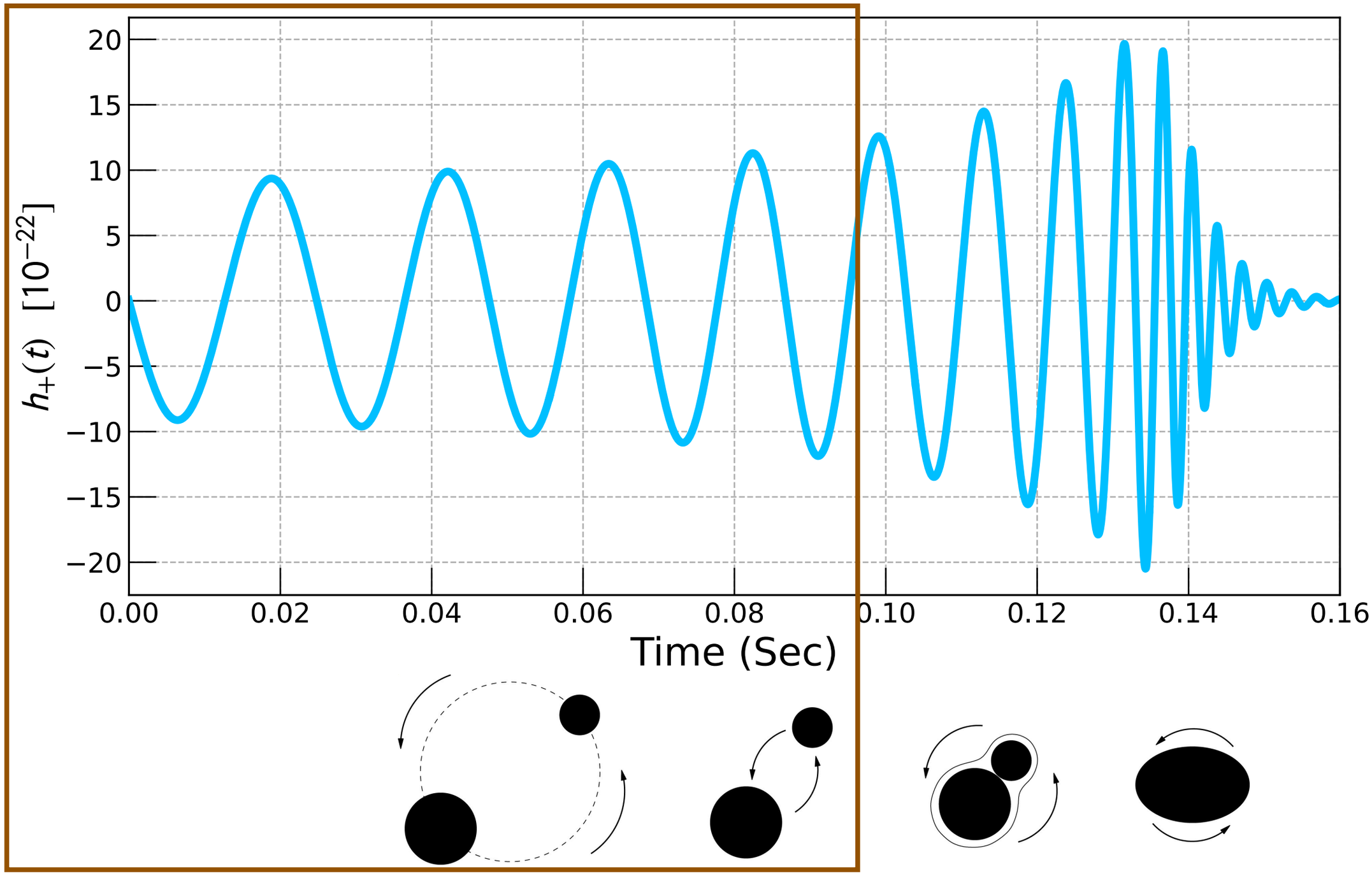
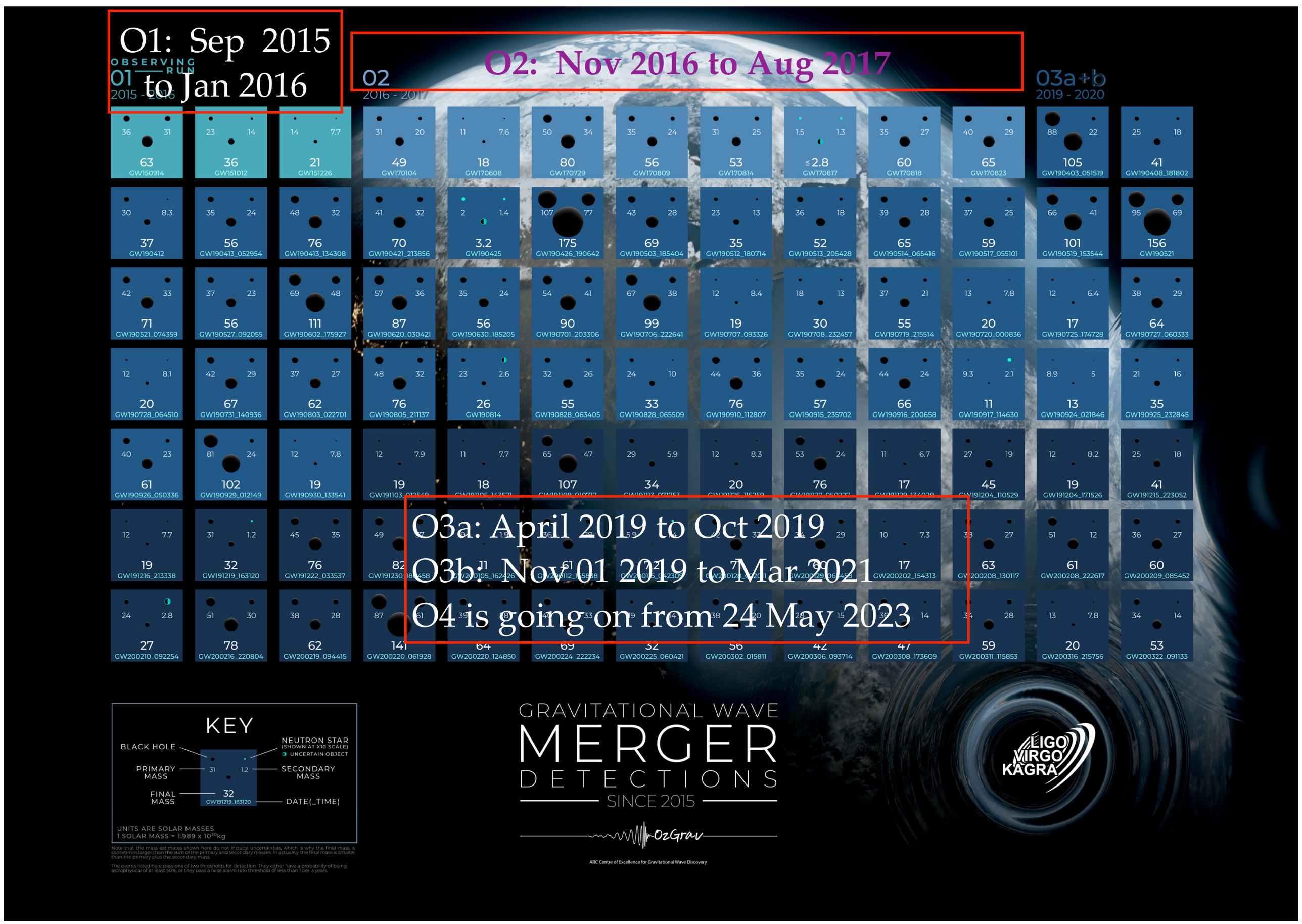
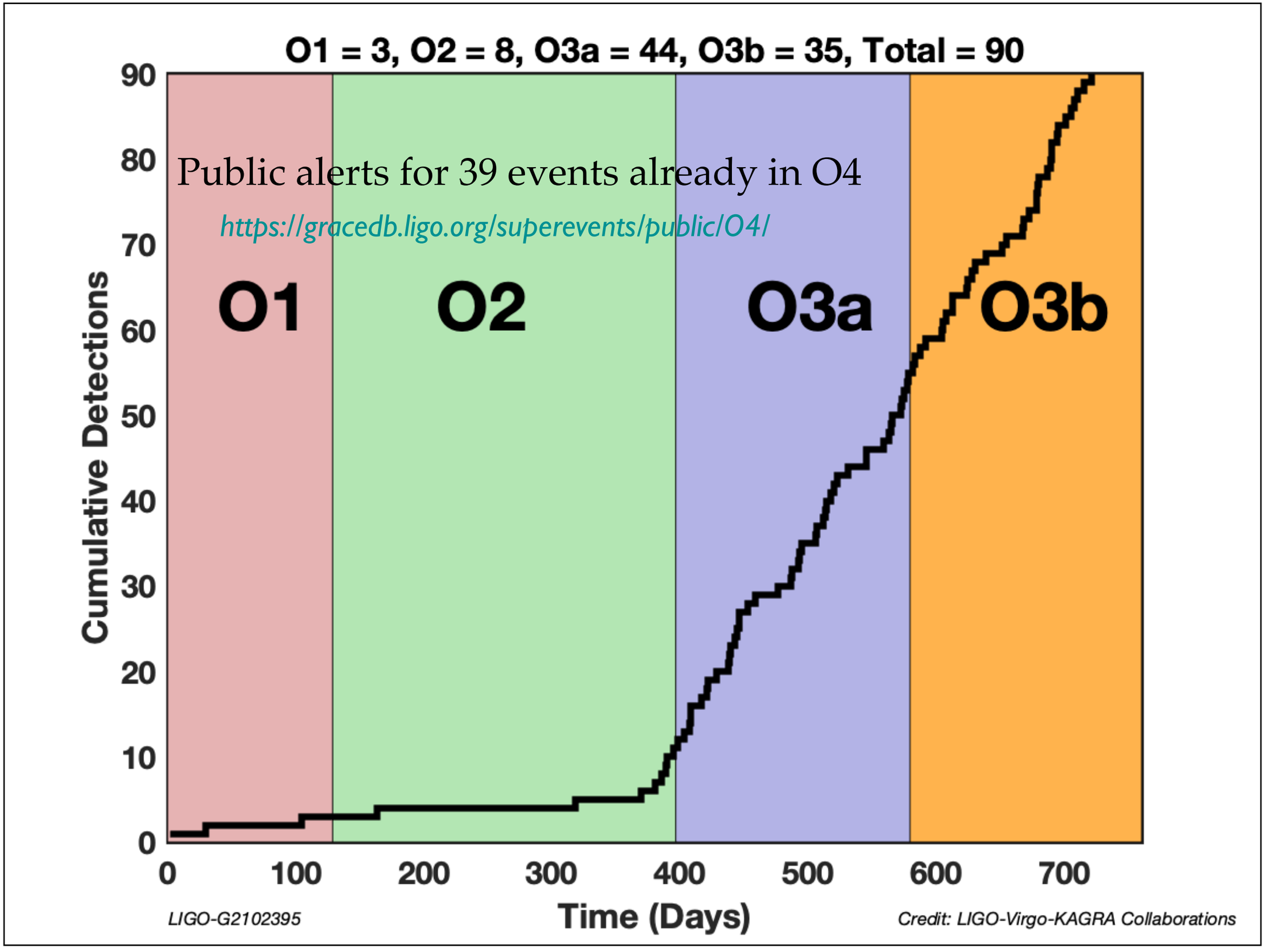
Overview

- Gravitational wave transient catalog
- Tests of blackhole nature using spin-induced multipole moments
- Constraints from the gravitational wave transient catalogue
- Combining information from multiple events
- Future prospectives: more accurate models, more sensitive detectors

Time-domain gravitational waveform model of a binary black hole merger




O1: Sep 2015 to Jan 2016
O2: Nov 2016 to Aug 2017
O3a: April 2019 to Oct 2019
O3b: Nov 01 2019 to Mar 2021
O4 is going on from 24 May 2023
OBSERVING RUN 01 2015 - 2016
02 2016 - 2017
03a+b 2019 - 2020
KEY
BLACK HOLE
NEUTRON STAR (SHOWN AT X10 SCALE)
UNCERTAIN OBJECT
PRIMARY MASS
SECONDARY MASS
FINAL MASS
DATE(_TIME)
UNITS ARE SOLAR MASSES
1 SOLAR MASS = 1.989 x 10^30 kg
Note that the mass estimates shown here do not include uncertainties, which is why the final mass is sometimes larger than the sum of the primary and secondary masses. In actuality, the final mass is smaller than the primary plus the secondary mass.
The events listed here pass one of two thresholds for detection. They either have a probability of being astrophysical of at least 50%, or they pass a false alarm rate threshold of less than 1 per 3 years.
GRAVITATIONAL WAVE MERGER DETECTIONS SINCE 2015
OzGrav
ARC Centre of Excellence for Gravitational Wave Discovery
LIGO VIRGO KAGRA


O1 = 3, O2 = 8, O3a = 44, O3b = 35, Total = 90
Public alerts for 39 events already in O4
https://gracedb.ligo.org/superevents/public/O4/
O1
O2
O3a
O3b
Cumulative Detections
0
10
20
30
40
50
60
70
80
90
0
100
200
300
400
500
600
700
Time (Days)
LIGO-G2102395
Credit: LIGO-Virgo-KAGRA Collaborations

FIRST DETECTION, THIRD TRANSIENT CATALOG WITH ~100 MERGERS, AND MANY MORE!

- Provides information about source parameters- masses, spins, orientation, location etc. *[astrophysical properties]*

- Possibility of performing several tests to check the consistency of detected signal to that of binary black hole mergers in general relativity *[strong-filed tests of general relativity]*

- We can use waveform models that allow parametrised deviations from general relativity to perform such tests *[requirement of waveform models and efficient techniques]*

- Given the statistical uncertainties in various consistency tests, black hole mimicker models, such as boson stars, gravastars etc., can not be completely ruled out *[tests of black hole nature]*

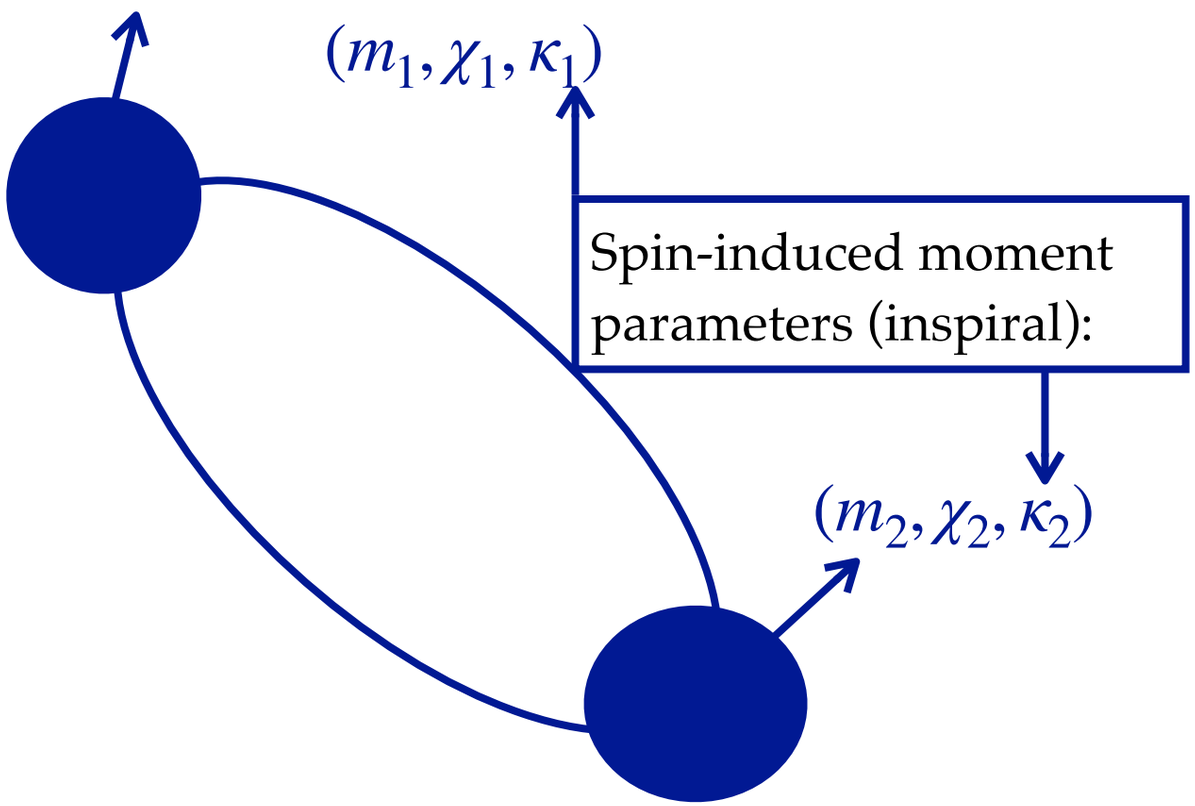
Tests of binary black hole nature using gravitational waves

- Inspiral-based tests of binary black hole nature:
  - Spin-induced multipole moments
  - Tidal deformability
  - Tidal heating
  - …
- Tests based on the post-merger signal:
  - Quasi-normal-mode ringdown analysis
  - Late ringdown echoes
  - …

[Cardoso et al., Phys. Rev. D 95, 084014 (2017); Nathan K. Johnson-McDaniel et al., Phys. Rev. D 102, 123010 (2020); Maselli et al., Phys. Rev. Lett. 120, 081101 (2018); Pani et al., Phys. Rev. D 80, 124047 (2009); Chirenti and L. Rezzolla, Classical Gravity 24, 4191 (2007); Cardoso et al., Nature Astron. 1, 586 (2017), 1709.01525]

- Deformations due to the spin of a compact object lead to **spin-induced multipole moments,** which are unique for a Kerr blackhole given mass and spin.

- **Spin-induced multipole moment coefficients** are unity for Kerr BHs, while for other objects the value is different and will vary depending up on the internal structure of the star.



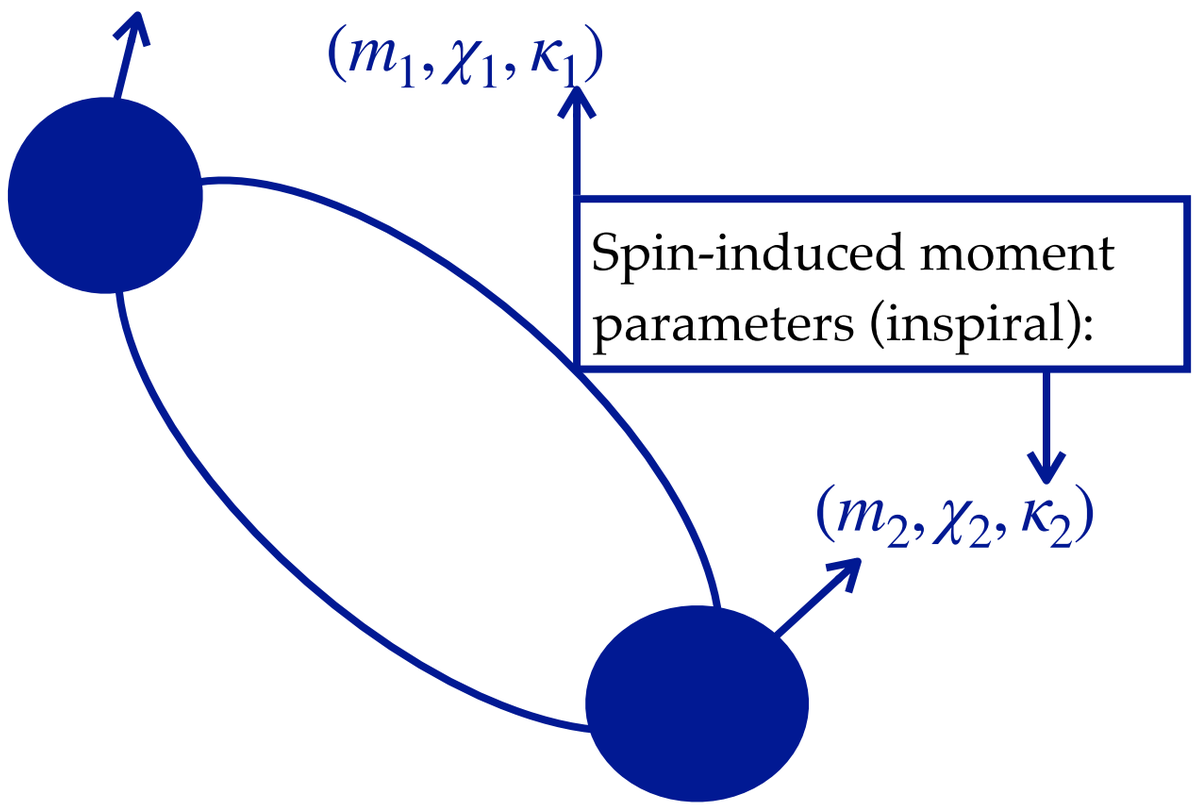
- $\kappa_{\text{Blackholes}} = 1$
- $\kappa_{\text{NeutronStars}} \sim 2 - 14$
- $\kappa_{\text{BosonStars}} \sim 10 - 150$

[R. O. Hansen et. al, 1974; B. Carter et al, 1971; Poisson 1998; Pappas 2012 (a); Pappas 2012 (b); F. D. Ryan; 1997; N. Uchikata et. al; 2016;Vaglio et al. 2023;Vaglio et al. 2022; Pacilio et al. 2020;]

Can we distinguish black holes from other compact objects using the gravitational wave measurements of **spin-induced multipole moment coefficients?**

- To perform the null-test, we need:
    - A parametrized waveform model with explicit κ dependence,
    - A parameter estimation algorithm, Fisher matrix analysis? Bayesian analysis?
- Spin-induced quadrupole moments **start to appear at 2PN in the post-Newtonian phasing formula**. Higher order contributions at **3PN and 3.5PN** are also available.

[C. K. Mishra, A Kela, Arun, G Faye Phys. Rev. D 93, 084054 (2016); Krishnendu, Arun, Mishra, Phys. Rev. Lett. 119, 091101 (2017)]



Introduce one parameters deformation of the form,
$\kappa_1 = 1 + \delta\kappa_1$ and $\kappa_2 = 1 + \delta\kappa_2$

$\delta\kappa$ vanishes for black holes and the value vary for other compact objects like neutron stars, boson stars depending upon their internal composition

Measure the symmetric combination, $\delta\kappa_s$ assuming $\delta\kappa_a = 0$

Krishnendu, Arun, Mishra, Phys. Rev. Lett. 119, 091101 (2017)

Note: One can extend this test to higher order multipoles, currently the spin-induced octupole tests are implemented

[Saini and Krishnendu, 2023]

Basics of Bayesian analysis: posteriors, Bayes factors and combining information from multiple events

- Posterior on binary parameters: $\mathrm{p}(\theta, \delta\kappa_\mathrm{s} \,|\, \mathrm{d_j}, \mathrm{H}) = \dfrac{\pi(\theta, \delta\kappa_\mathrm{s} \,|\, \mathrm{H})\, \mathrm{L}(\mathrm{d_j} \,|\, \theta, \delta\kappa_\mathrm{s}, \mathrm{H})}{\mathrm{Z_j^{nbh}}}$

- Main ingredients:

  - Likelihood $L(d_j \,|\, \theta, \delta\kappa_s, H)$, the data from j^{th} event d_j carries a signal $h_j(\theta, \delta\kappa_s)$ plus coloured Gaussian noise

  - Prior $\pi(\theta, \delta\kappa_s \,|\, H)$ on $\{\theta, \delta\kappa_s\}$

  - Evidence $\mathrm{Z_j^{nbh}} = \int \pi(\theta, \delta\kappa_\mathrm{s} \,|\, \mathrm{H}) \mathrm{L}(\mathrm{d_j} \,|\, \theta, \delta\kappa_\mathrm{s}, \mathrm{H})\, \mathrm{d}\theta\, \mathrm{d}\delta\kappa_\mathrm{s}$

  - 1-D marginalised posterior on $\delta\kappa_s$, $\mathrm{p}(\delta\kappa_\mathrm{s} \,|\, \mathrm{d_j}, \mathrm{H}) = \int \mathrm{p}(\theta, \delta\kappa_\mathrm{s} \,|\, \mathrm{d_j}, \mathrm{H})\, \mathrm{d}\theta$

Basics of Bayesian analysis: posteriors, Bayes factors and combining information from multiple events

- Joint likelihood analysis (Restricted method): Assumes universal value for $\delta\kappa_s$ irrespective of the source properties

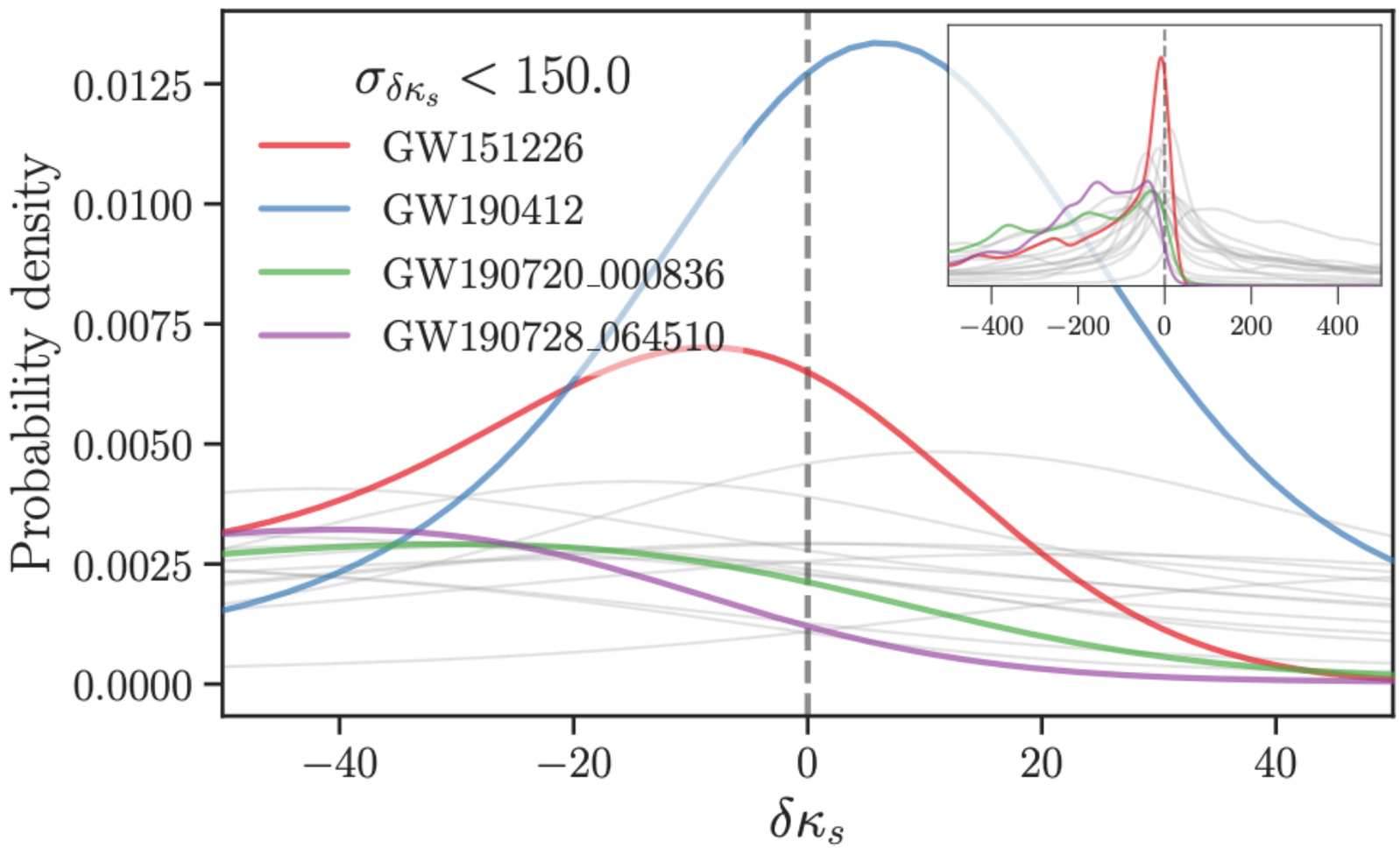
- Hierarchical method: $p(\delta\kappa_s \,|\, \alpha) = N(\mu, \sigma^2)$ from the posterior on μ and σ,
  $p(\delta\kappa_s \,|\, d) = \int p(\delta\kappa_s \,|\, \alpha)\, p(\alpha \,|\, d)\, d\alpha$

- Mixture likelihood approach: Given the population of events, what fraction of non-BBHs are present? $L(d_j \,|\, \theta, \delta\kappa_s, f_{nbh}) = (1 - f_{nbh}) L(d_j \,|\, \theta) + f_{nbh} L(d_j \,|\, \theta, \delta\kappa_s)$

[Isi, Chatziioannou, Farr, PRL 123, 121101(2019); M Saleem, Krishnendu, Abhirup Ghosh, Anuradha Gupta, W Del Pozzo, Archisman Ghosh, K G Arun, PRD 105, 104066 (2022)]

SPIN-INDUCED QUADRUPOLE ESTIMATES FROM GWTC-2

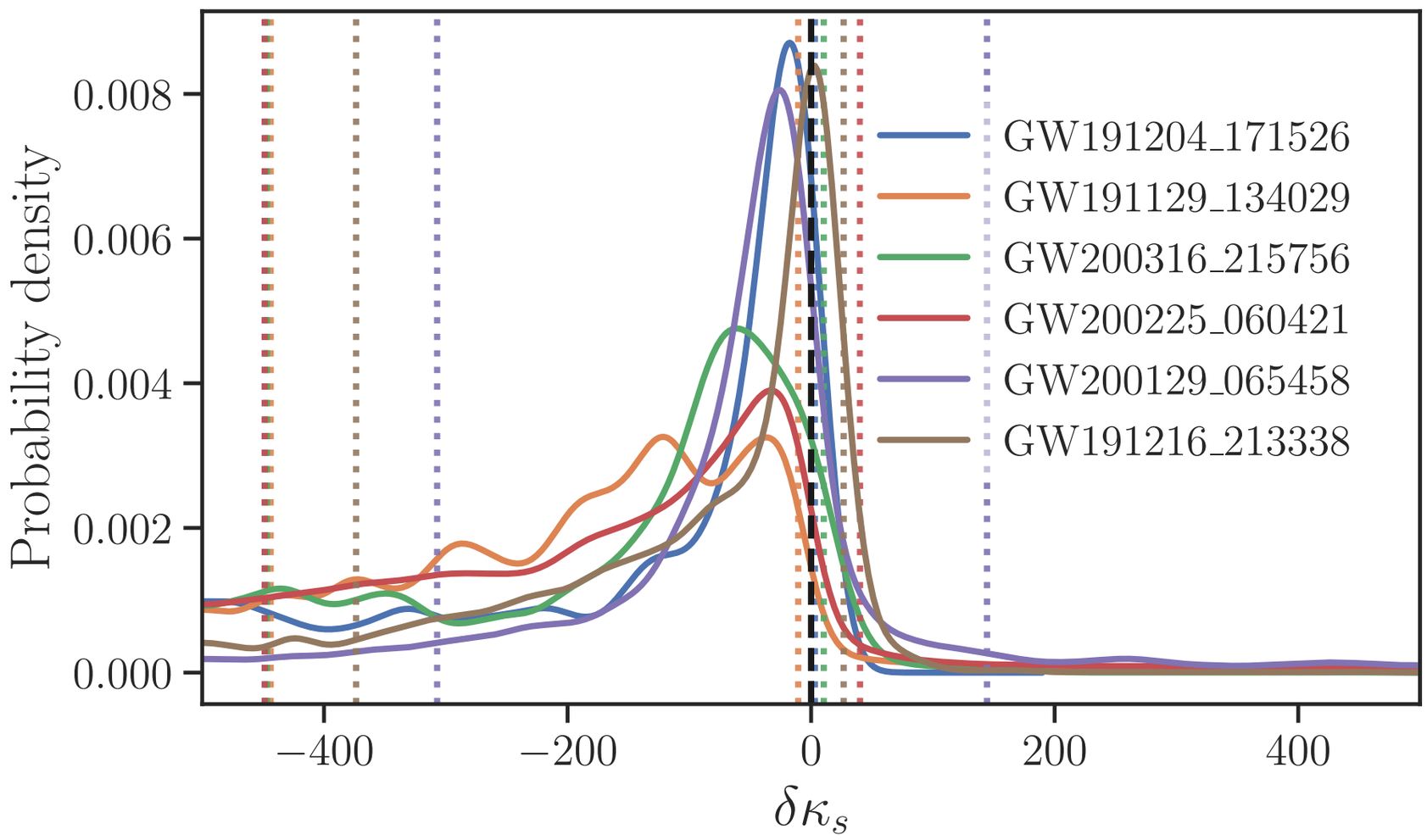
LVC; Phys. Rev. D 103, 122002 (2021)



- IMRPhenomPv2
- Inspiral-dominated events

Spin-induced quadrupole estimates from GWTC-3

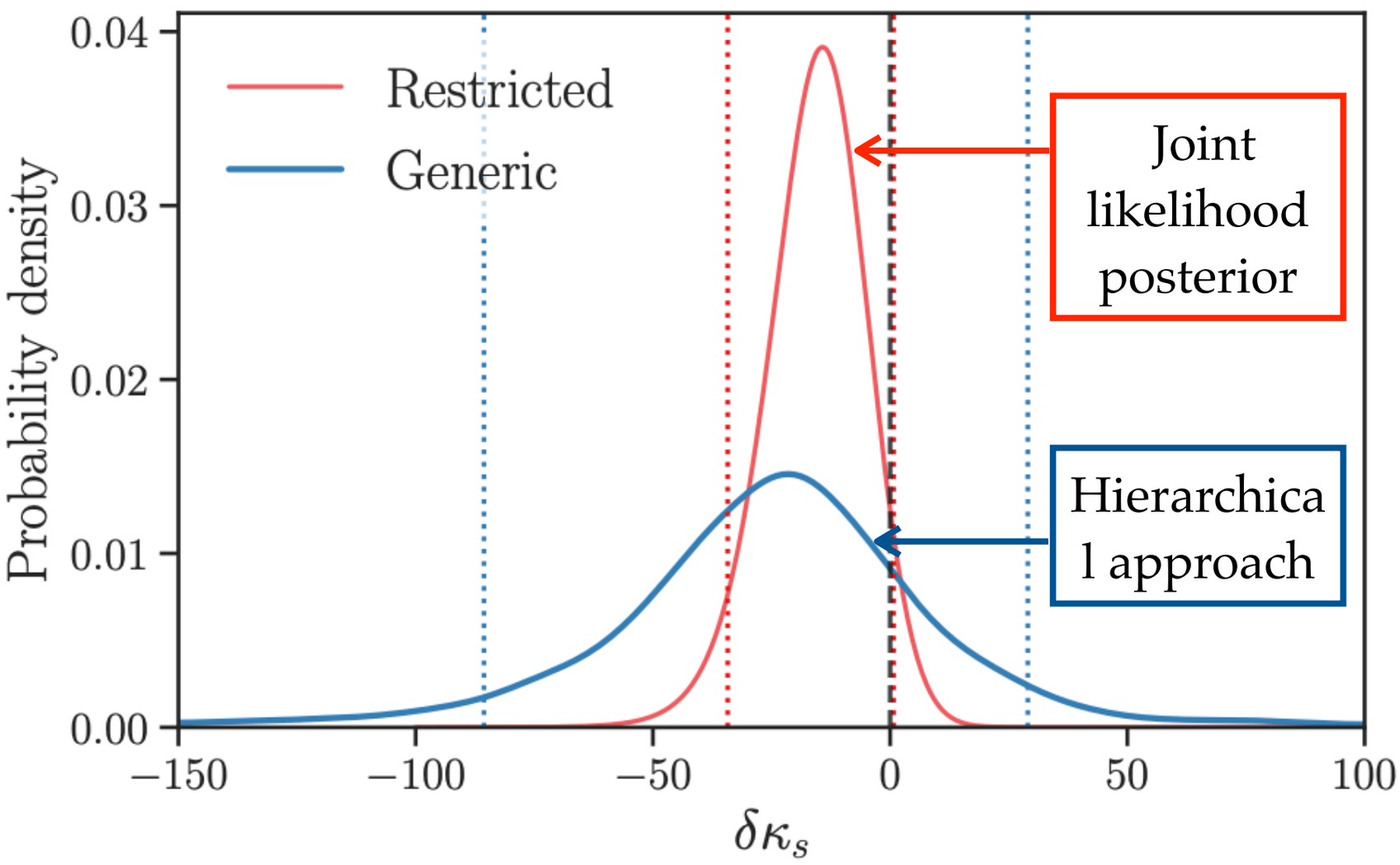
LVK, 2021



- IMRPhenomPv2
- Inspiral-dominated events with non-zero spin effects

Estimates from combining information from multiple events

LVK, 2021



- Even though the individual events are poorly constrained the combined result is promising

1. With 2G detectors, single event constraints are uninformative, need population inference to look for non-BH signature in the data

2. Before we make that conclusion: the spins, mass ratio and the signal-to-noise ratio are crucial

3. Waveform model with higher harmonics and full precession can lead to better constraints

4. Future detectors

Population inference of spin-induced quadrupole moments as a probe for BH mimickers

- Mass distributions: Model-C in GWTC-2
- Spin distribution: Default model in GWTC-2
- Three different distributions for $\delta\kappa_s$ to create non-BBH signals
  - Uniform [-40, 40]
  - Gaussian with $\mu = 25, \sigma = 5$
  - Gaussian with $\mu = -25, \sigma = 5$

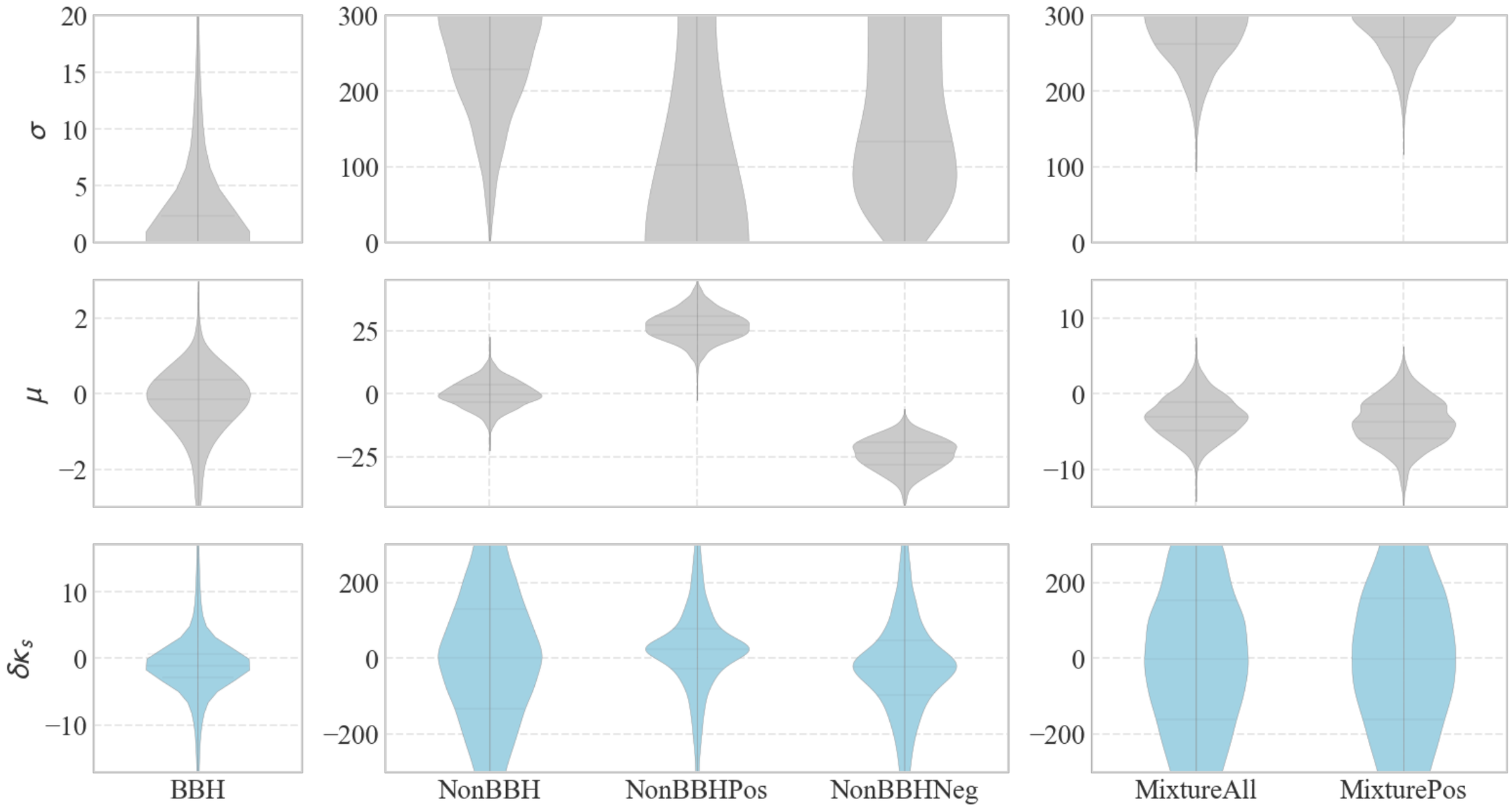
Population models created from the injected binary signals

Model	f_{nbh}	BBH	Uniform	GausPos	GausNeg	N_{tot}
BBH	0.0	50	0	0	0	50
NonBBH	1.0	0	20	15	15	50
NonBBHPos	1.0	0	10	15	0	25
NonBBHNeg	1.0	0	10	0	15	25
MixtureAll	0.5	30	10	10	10	60
MixturePos	0.5	20	10	10	0	40

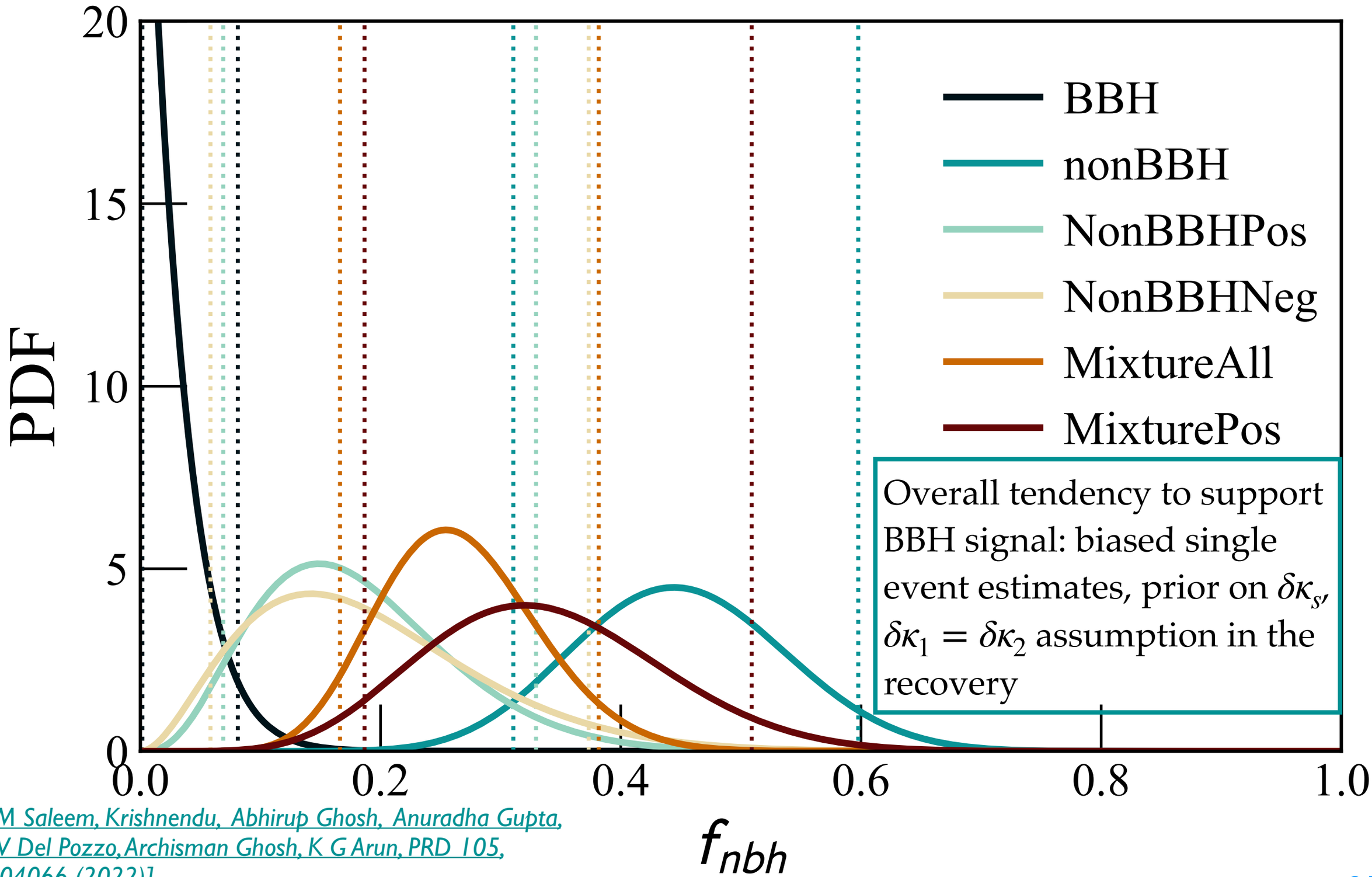
Employing IMRPhenomPv2 waveform model and parameter estimation is performed using LALInference

[M Saleem, Krishnendu, Abhirup Ghosh, Anuradha Gupta, W Del Pozzo, Archisman Ghosh, K G Arun, PRD 105, 104066 (2022)]

Population inference of $\delta\kappa_s$ from simulated binaries using hierarchical inference



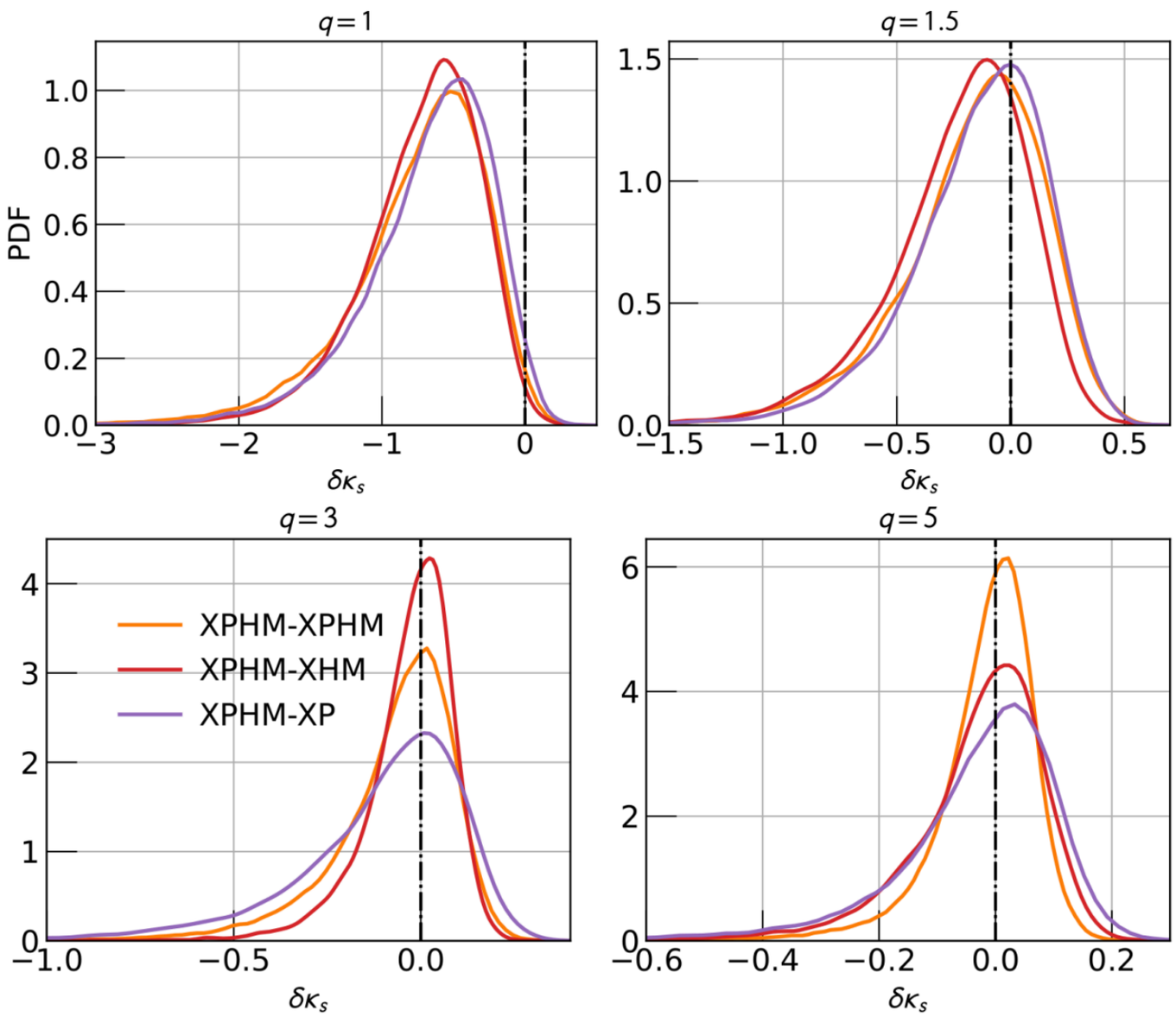
[M Saleem, Krishnendu, Abhirup Ghosh, Anuradha Gupta, W Del Pozzo, Archisman Ghosh, K G Arun, PRD 105, 104066 (2022)]



[M Saleem, Krishnendu, Abhirup Ghosh, Anuradha Gupta, W Del Pozzo, Archisman Ghosh, K G Arun, PRD 105, 104066 (2022)]

Effect of higher harmonics and constraints from a fully precessing model

[Divyajyoti NLN, Krishnendu, C K Mishra and C K Mishra, in preparation]

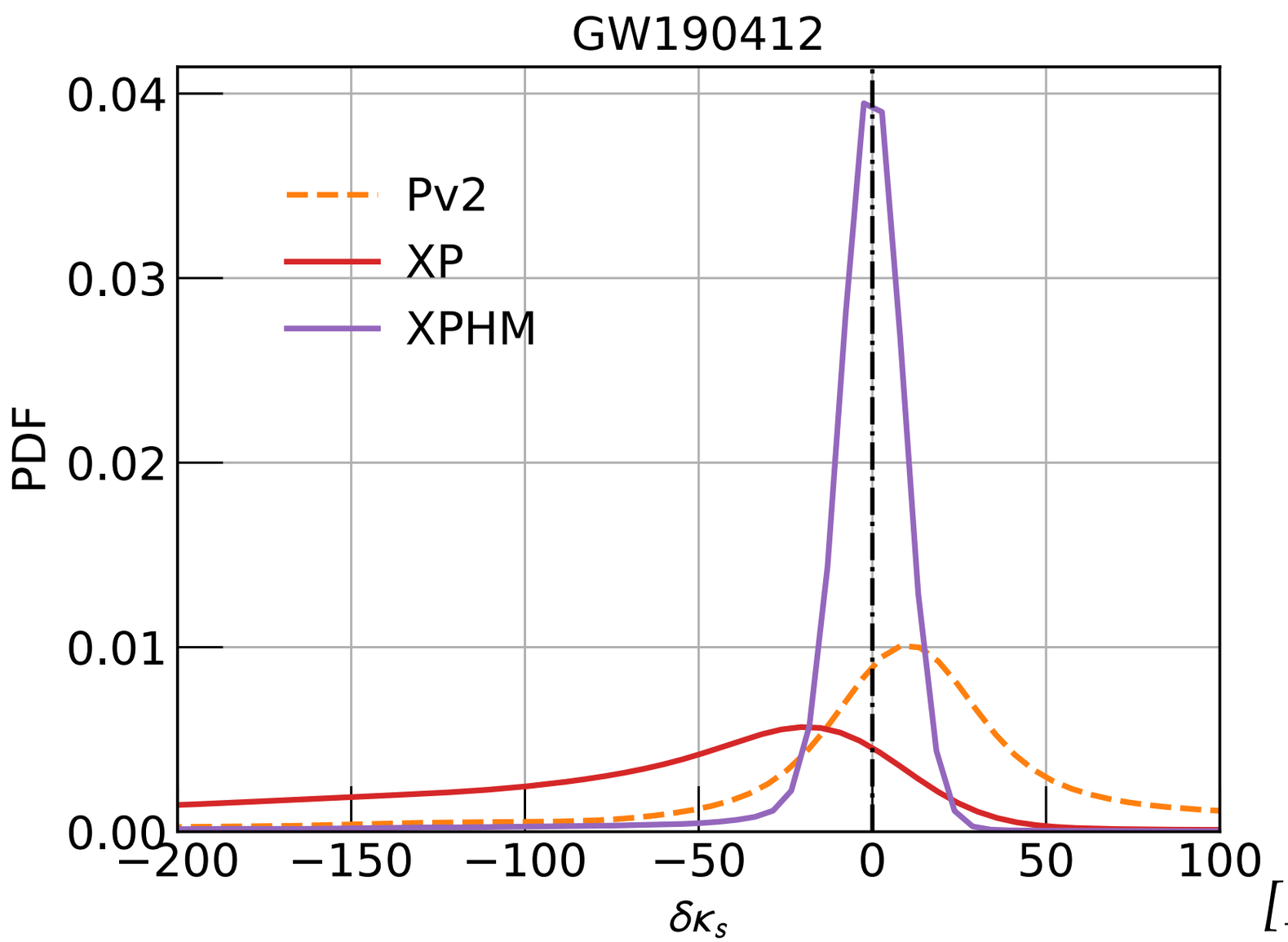


- Fully precessing models from IMRPhenomXP family, comparing the results to the higher mode version, non-precessing version and dominant model version for simulation

[Preliminary]

Effect of higher harmonics and constraints from a fully precessing model

[Divyajyoti NLN, Krishnendu, C K Mishra and C K Mishra, in preparation]



[Preliminary]

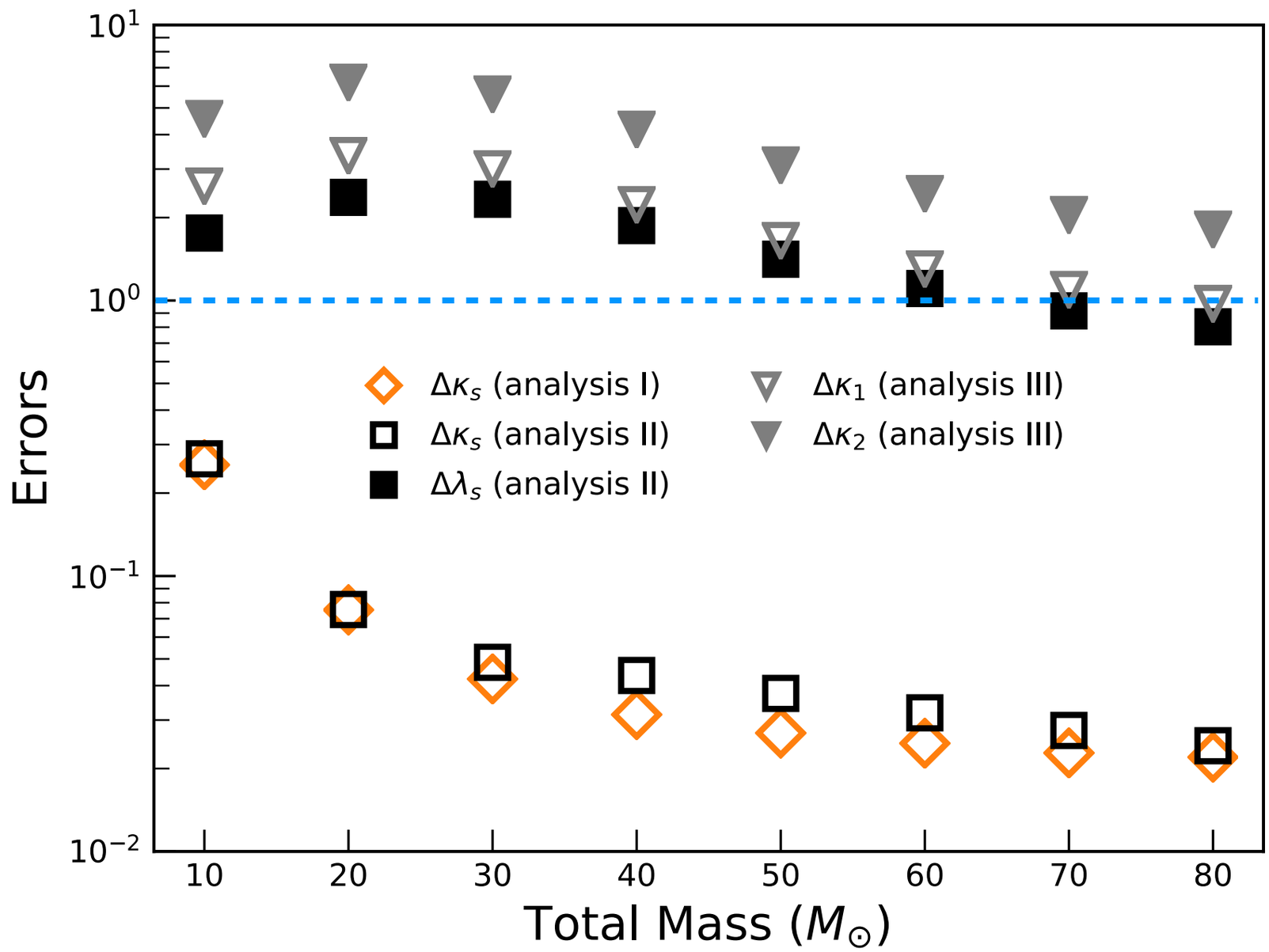
- Selection bias studies: Efforts to include the exotic compact object signatures in the initial template bank generation

- Efforts to simultaneously measure the tidal-deformability parameter and spin-induced quadrupole moment parameters

[Chia, Edwards, Wadekar, Zimmerman, Olsen, Roulet, arXiv:2306.00050, Tejaswi Venumadhav, Barak Zackay, Matias Zaldarriaga; Vaglio et al. 2023; Vaglio et al. 2022; Pacilio et al. 2020; Narikawa, Uchikata, Tanaka, Phys. Rev. D 104, 084056 (2021)]

WHAT DO WE KNOW ABOUT THE CONSTRAINTS FROM FUTURE DETECTORS?

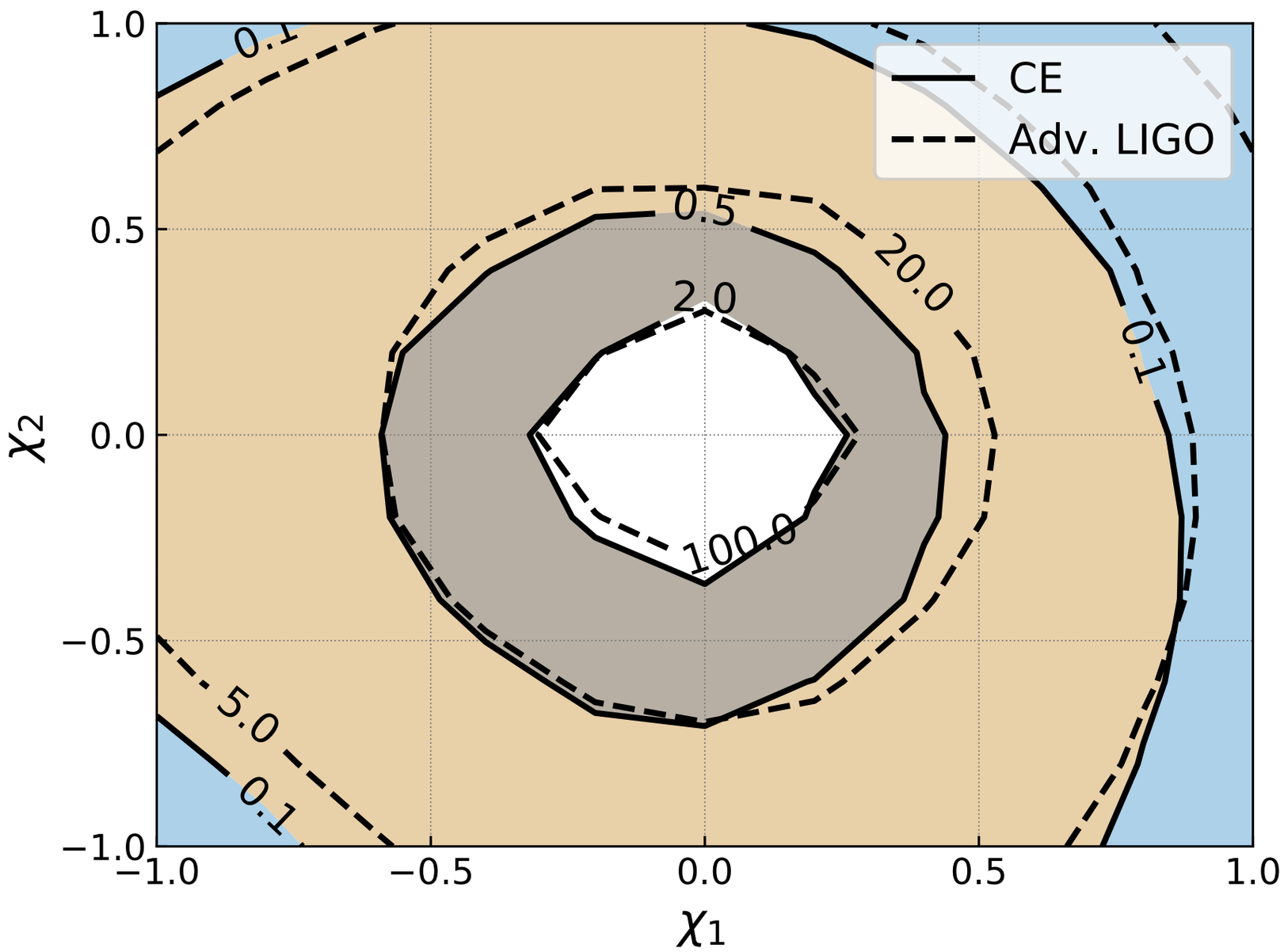
Possibilities with 3G detectors: simultaneous measurement of two parameters



- $1-\sigma$ errors using Fisher information matrix analysis
- Nearly equal mass binary with spins: (0.9,0.8) and located at 400Mpc
- Regions with less than 10 % accuracy

[Krishnendu, C K Mishra, Arun., PRD D 99, 064008 (2019)]

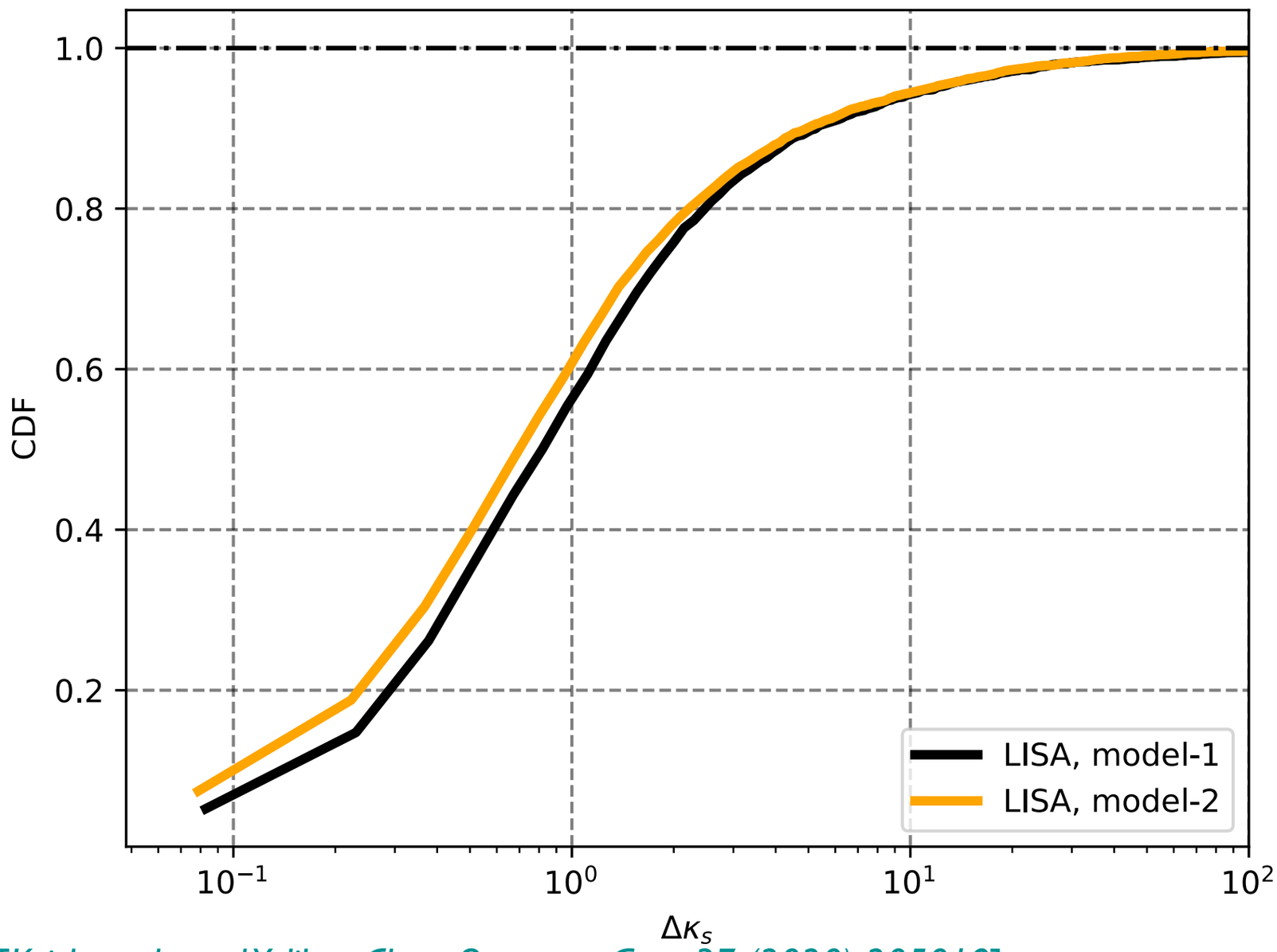
Possibilities with 3G detectors:Improvement compared to 2G detectors



- $1-\sigma$ errors using Fisher information matrix analysis
- Nearly equal mass binary with total mass $30M_\odot$ and located at 400Mpc
- Order of $\sim 40-50$ factor improvement

[Krishnendu, C K Mishra, Arun., PRD D 99, 064008 (2019)]

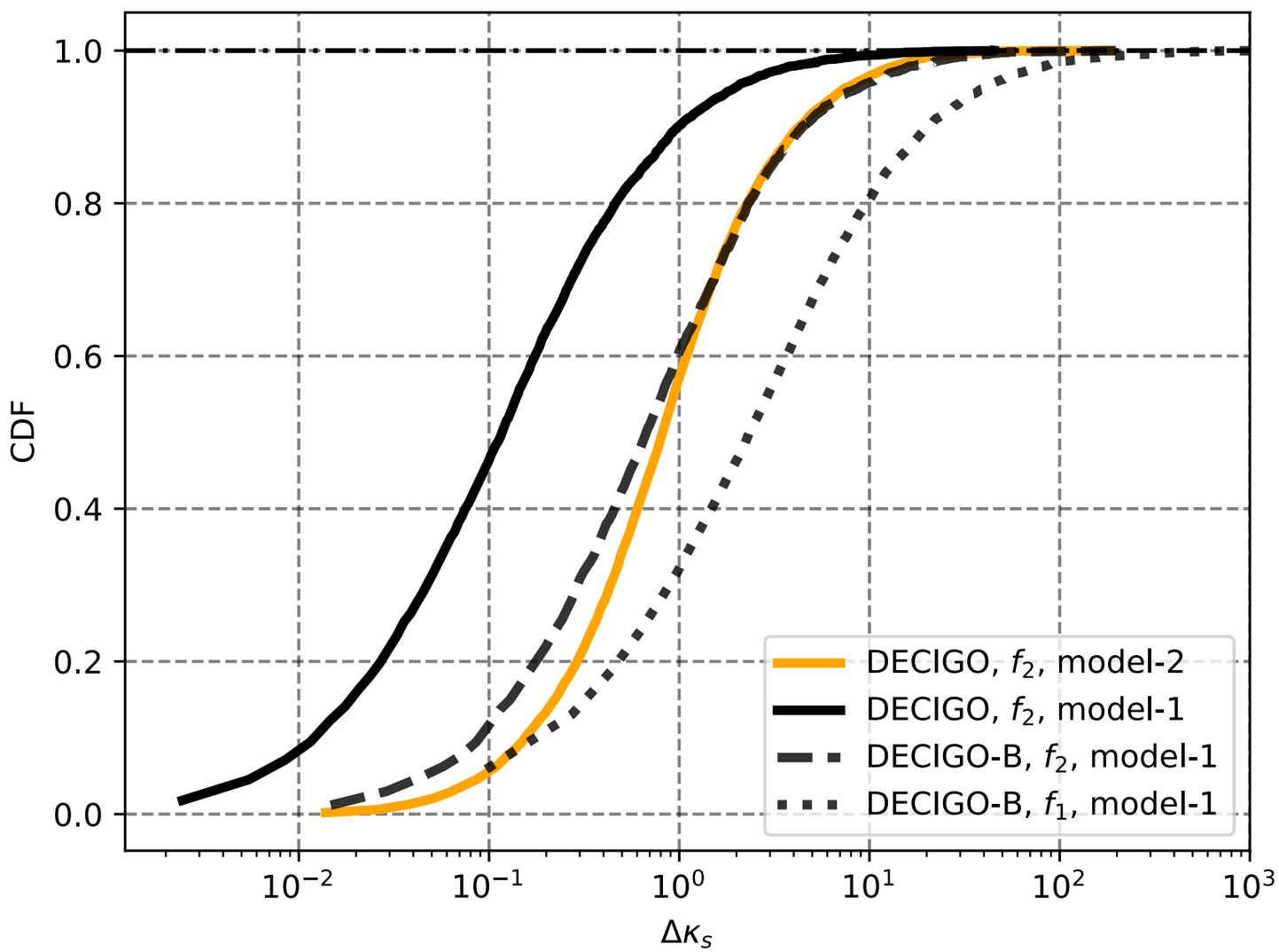
Super-massive and Intermediate mass binary black holes



- $1-\sigma$ errors using Fisher information matrix analysis
- Model 1 and 2 are astrophysical population models proposed for super massive binary BHs
- Around 60% of the sources provide $\Delta\kappa_s \leq 1$

[Krishnendu and Yelikar, Class. Quantum Grav. 37 (2020) 205019]

Super-massive and Intermediate mass binary black holes



- Model 1 and 2 are astrophysical population models for intermediate mass binary BHs
- Around 85% of the sources provide $\Delta\kappa_s \leq 1$ for DECIGO with lower cut-off frequency 0.001Hz

[Krishnendu and Yelikar, Class. Quantum Grav. 37 (2020) 205019]

- Bayesian implementation
- More accurate waveform models

SUMMARY

- The gravitational-wave astronomy community is awaiting for its forth transient catalog.
- Tests to distinguish binaries composed of black holes from other objects including exotic compact objects is one of the fundamental physics questions.
- Applicability of spin-induced quadrupole moment based tests on inspiral dominated signals with non-vanishing spin effects are promising.
- Increased detector sensitivity and improved waveform models are crucial.
- *Complementary analyses: tidal deformability measurements and tidal heating measurements*

Thank you!

- Challenges with asymmetric binaries: waveform model, data analysis technique…
-

Post-Newtonian waveform with spin-induced quadrupole moment terms (for slide 7)

$$V_k = (2\,\pi\,M\,\frac{f_{gw}}{k})^{\frac{1}{3}}$$

$$\tilde{h}(f) = \frac{M^2}{D_L}\sqrt{\frac{5\,\pi\,\nu}{48}}\sum_{n=0}^{4}\sum_{k=1}^{6} V_k^{n-\frac{7}{2}}\mathcal{C}_k^{(n)}\mathrm{Exp}[i\,\psi(f)]$$

$$M = m_1 + m_2$$

$$\nu = \frac{m_1\,m_2}{(m_1+m_2)^2}$$

$$\mathcal{C}_2^{(0)} = \frac{1}{2}[-(1+cos_\iota^2)F_+ - 2\,i\,cos_\iota F_\times]$$

- Spin-Induced quadrupole moment terms at **2 PN, 3 PN, 3.5 PN** terms are available in the literature. In addition to that, **2 PN amplitude corrections** to the spin-induced coefficients are also been calculated. [C. K. Mishra, A. Kela, K. G. Arun]

- The leading order **(3.5PN) spin-induced octupole moment coefficient** in the phasing *also calculated*

- **Power Law + Peak** (8 parameters; Appendix B.2). Similar to the Truncated model, but with two modifications. At low masses we implement a smoothing function to avoid a hard cut-off. At high masses, we include a Gaussian peak, initially introduced to model a pile-up from pulsational pair-instability supernovae (Talbot & Thrane 2018). This model is better able to accommodate the high-mass events that pose a challenge for the Truncated model. In Abbott et al. (2019a), it is referred to as "Model C."

2. For each binary, the magnitudes of the two component-spins are drawn according to the *Default Model* as named in [137]. In this model, the spin magnitudes a_i $(i = 1, 2)$ are drawn from a beta distribution

$$p(a_i|\alpha_a, \beta_a) \propto a_i^{\alpha_a - 1}(1 - a_i^{\beta_a - 1}), \tag{19}$$

where α_a and β_a are shape parameters. We choose $\alpha_a = 2.75$ and $\beta_a = 6.00$ to make sure that we do not have sources with $a_i \sim 0$, as non-spinning compact objects do not carry imprints of spin-induced multipole moments.

3. The spin orientations are randomly drawn from a mixture of *isotropic* and *aligned-to-orbital-angular-momentum* orientations. In other words, the populations include binaries with precessing spins and binaries with non-precessing spins.

Post-Newtonian waveform with spin-induced quadrupole moment terms (for slide 7)

$$\tilde{h}(f) = \frac{M^2}{D_L}\sqrt{\frac{5\,\pi\,\nu}{48}}\sum_{n=0}^{4}\sum_{k=1}^{6} V_k^{n-\frac{7}{2}}\mathcal{C}_k^{(n)}\mathrm{Exp}[i\,\psi(f)]$$

$$\psi(f) = \frac{3}{128\,\eta\,v^5}\sum_{\alpha=0}^{N}\Psi_\alpha\,v^\alpha$$

$$\begin{aligned}\Psi_{\alpha=4} = &-\frac{5}{8}(\boldsymbol{\chi}_{\mathrm{s}}\cdot\hat{\boldsymbol{L}}_{\mathrm{N}})^2\left[1+156\,\eta+80\,\delta\,\kappa_a+80(1-2\,\eta)\kappa_s\right]\\ &+(\boldsymbol{\chi}_{\mathrm{a}}\cdot\hat{\boldsymbol{L}}_{\mathrm{N}})^2\left[-\frac{5}{8}-50\,\delta\,\kappa_a-50\kappa_s+100\,\eta\,(1+\kappa_s)\right]\\ &+\frac{5}{4}(\boldsymbol{\chi}_{\mathrm{a}}\cdot\hat{\boldsymbol{L}}_{\mathrm{N}})(\boldsymbol{\chi}_{\mathrm{s}}\cdot\hat{\boldsymbol{L}}_{\mathrm{N}})\left[\delta+80\,(1-2\,\eta)\,\kappa_a+80\,\delta\,\kappa_s\right]\end{aligned}$$

Twin **LIGO** observatories. Situated in the United States, Hanford and Livingston. 4km arm length. Several science runs during 2002-2010. **The first detection was in 2015 during the first observing run, GW150914**

Virgo 3km arm length detector, in Italy. Joined the LIGO network on

GEO600, 600 m arm length. Located in Germany

Japanese detector, **KAGRA** (Kamioka Gravitational Wave Detector). Large-scale cryogenic detector (3km)

- More info: waveform model: 40, 20, 0.6, 0.5 Pv2, backward fast FFT, time axis is set to zero such that the instantaneous frequency is 40Hz