TÉCNICO LISBOA

Fundação para a Ciência e a Tecnologia

Extreme mass-ratio inspirals into black holes surrounded by boson clouds

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Based on: RB and Shreya Shah, arXiv:2307.16093 [gr-qc]

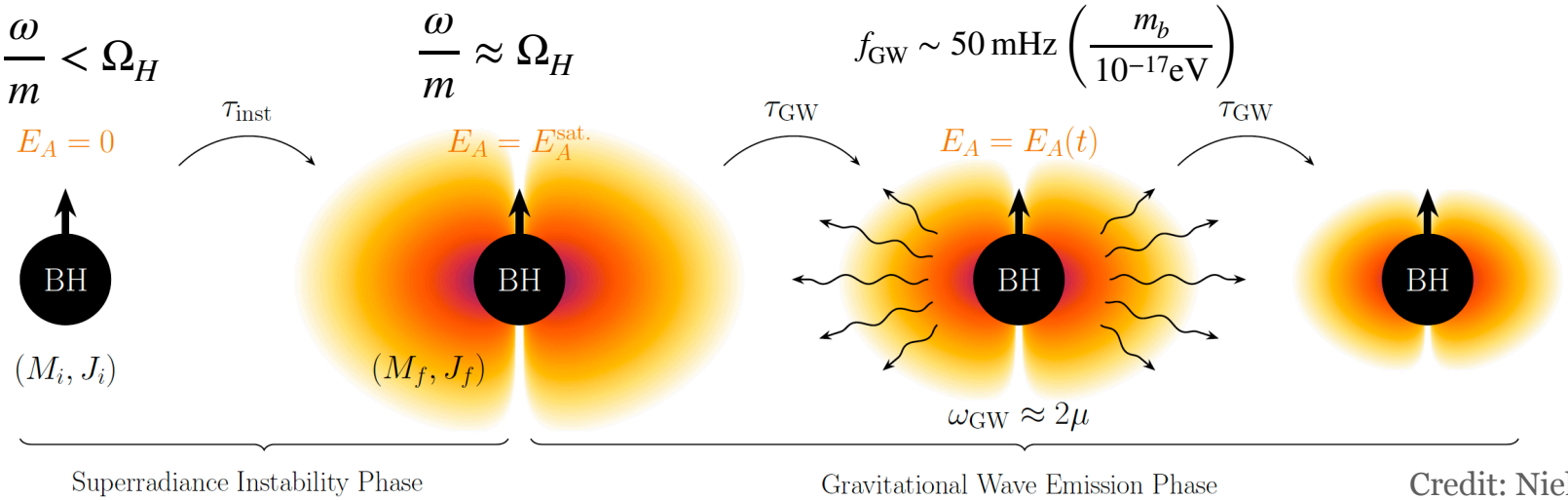
centra
center for astrophysics and gravitation

grit
gravitation in técnico

Boson clouds

Damour '76; Zouros & Eardley '79; Detweiler '80; Dolan '07; Arvanitaki+ '10, Rosa & Dolan '12; Pani+ '12; RB, Cardoso & Pani '13; Baryakthar+ '17; East '17; Cardoso+ '18; Frolov+ '18; Dolan '18; Baumann+ '19; RB, Grillo & Pani '20; Dias+ 23,...

- **Massive bosons** can form (quasi-)**bound-states** around black holes
- Around spinning black-holes, bound-states can **grow exponentially** by extracting energy and angular momentum from the black hole



Credit: Niels Siemonsen

Most efficient when:

$$2M\mu \equiv \frac{2Mm_b}{M_{\rm Pl}^2} = R_G/\lambda_C \sim \mathcal{O}(1)$$

$$M\mu \sim 0.1\left(\frac{M}{10^6 M_\odot}\right)\left(\frac{m_b c^2}{10^{-17}{\rm eV}}\right)$$

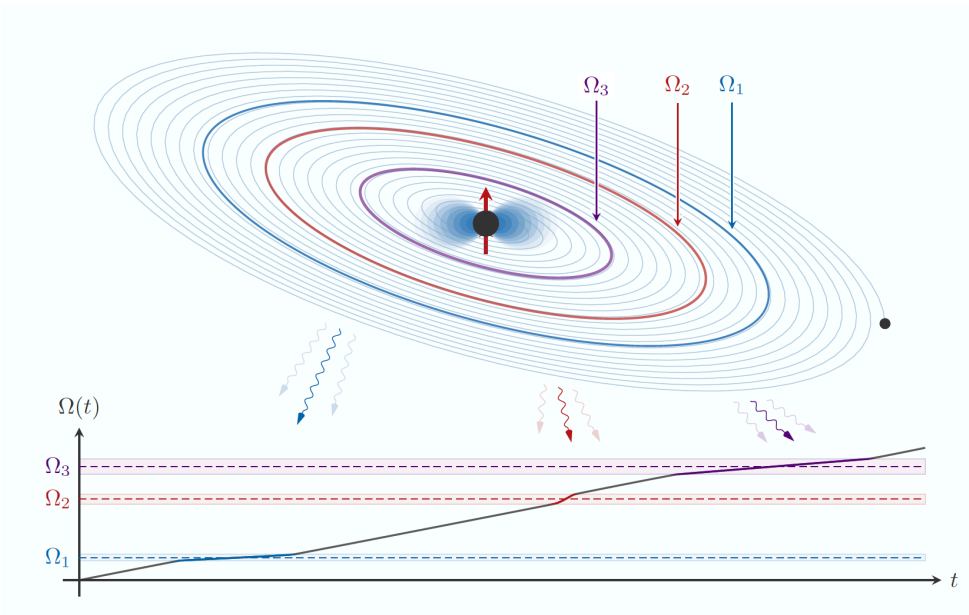
$$\tau_{\rm inst}^{\rm spin-0} \approx 10^4\,{\rm yrs}\left(\frac{M_i}{10^6\,M_\odot}\right)\left(\frac{0.1}{M_i\mu}\right)^9\left(\frac{0.9}{J_i/M_i^2}\right), \quad \tau_{\rm inst}^{\rm spin-1} \approx 1\,{\rm yr}\left(\frac{M_i}{10^6\,M_\odot}\right)\left(\frac{0.1}{M_i\mu}\right)^7\left(\frac{0.9}{J_i/M_i^2}\right)$$

$$\tau_{\rm GW}^{\rm spin-0} \approx 10^{10}\,{\rm yrs}\left(\frac{M_i}{10^6\,M_\odot}\right)\left(\frac{0.1}{M_i\mu}\right)^{15}\left(\frac{0.5}{\Delta(J/M^2)}\right), \quad \tau_{\rm GW}^{\rm spin-1} \approx 10^3\,{\rm yrs}\left(\frac{M_i}{10^6\,M_\odot}\right)\left(\frac{0.1}{M_i\mu}\right)^{11}\left(\frac{0.5}{\Delta(J/M^2)}\right)$$

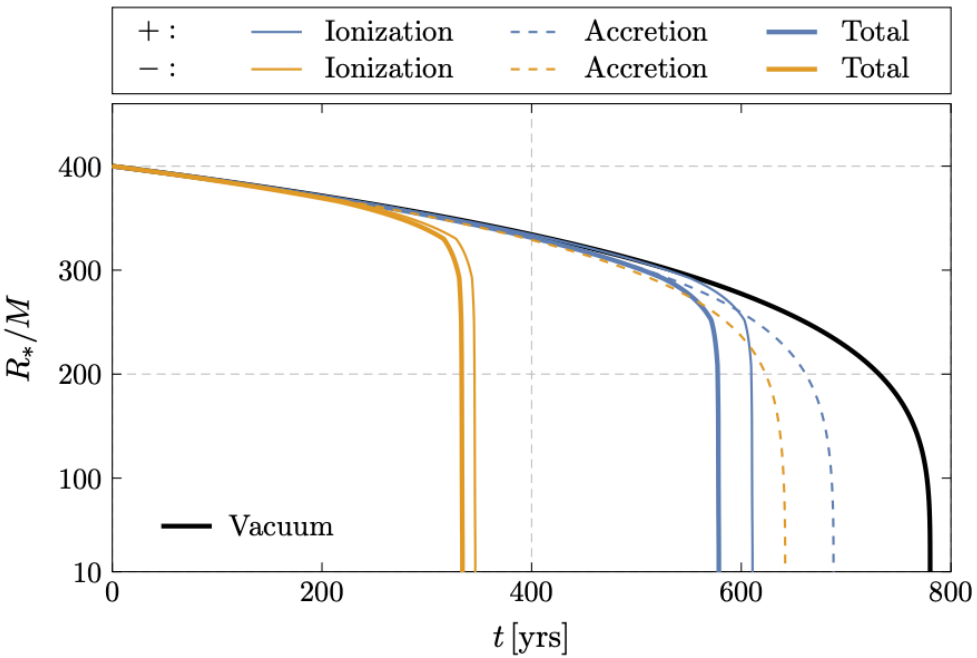
Probing boson clouds with EMRIs

Baumann+'18, '19, '21; Hannuksela+ '19; Tomaselli+'23...

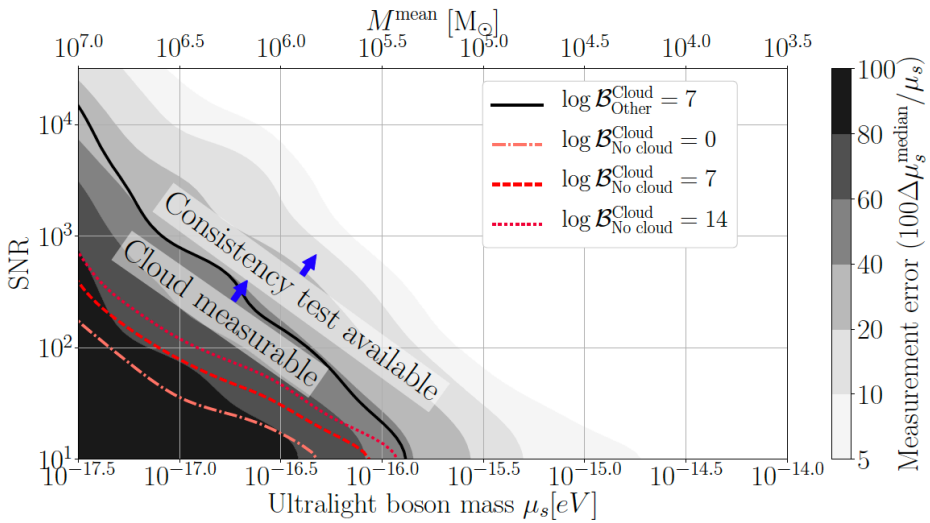
- Several effects induced by the presence of a boson cloud mostly studied within **Newtonian approximations**:
  - **Floating/Sicking orbits** at specific orbital frequencies due to excitation of **resonances**
  - Different orbital evolution due to **dynamical friction** ("ionization"), **accretion** and **self-gravity** of the cloud



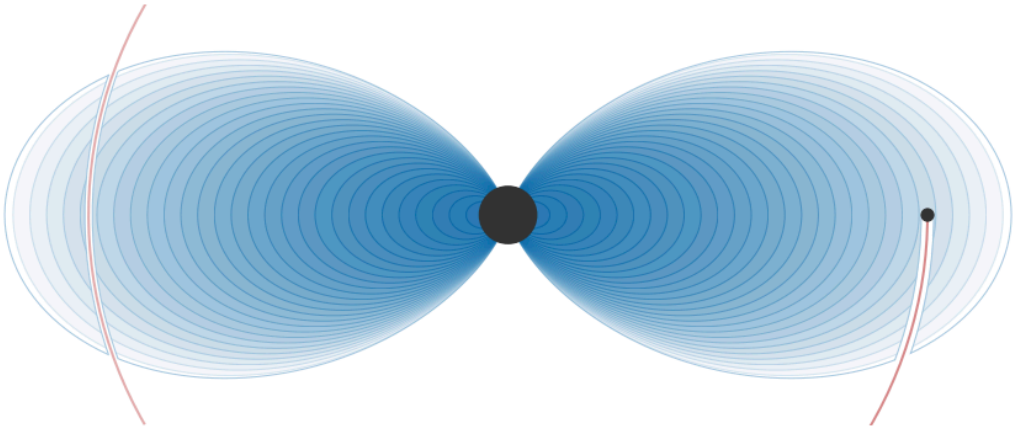
From: Baumann *et al*, PRD101, 083019 (2020)



From: Baumann *et al*, PRD105, 115036 (2022)



From: Hannuksela+, Nature Astronomy 3, 447 (2019)



From: Baumann *et al*, PRD105, 115036 (2022)

Goal:

Study EMRIs in boson clouds within a **fully relativistic** setup (i.e. using black-hole perturbation theory)

Main challenges:

- Superradiant clouds are not spherically symmetric, i.e. cannot simply use tools developed in Cardoso+ arXiv:2210.01133
- Exact stationary solutions describing spinning BHs+(complex) boson clouds exist, but can only be constructed numerically [Herdeiro&Radu '14]. One could perturb such solutions, but life's too short…

EMRIs in boson clouds: perturbative scheme

$$G_{\mu\nu} = 8\pi(T^{\Phi}_{\mu\nu} + T^{p}_{\mu\nu}), \qquad \Box\Phi - \mu^2\Phi = 0$$

$$T^{\Phi}_{\mu\nu}[\Phi, \Phi^*] = 2\partial_{(\mu}\Phi\partial_{\nu)}\Phi^* - g_{\mu\nu}\left(\partial_\alpha\Phi\partial^\alpha\Phi^* + \mu^2\Phi^*\Phi\right), \qquad T^{p,\mu\nu} = m_p\int u^\mu_p u^\nu_p \frac{\delta^{(4)}\left(x^\mu - x^\mu_p(\tau)\right)}{\sqrt{-g}} d\tau$$

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- Expand fields as: $g_{\mu\nu} = g^{(0)}_{\mu\nu} + \epsilon^2 g^{(2)}_{\mu\nu} + q h_{\mu\nu} + \ldots \,, \qquad \Phi = \epsilon(\Phi^{(1)} + q\Phi^{(q)}) + \ldots$

$$q \equiv m_p/M \ll 1 \,, \quad \epsilon \ll 1$$

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$$q \equiv m_p/M \ll 1 \,, \quad \epsilon \ll 1$$

$\mathcal{O}(q^0, \epsilon^0)$: $G_{\mu\nu}[g^{(0)}] = 0$

Background black hole solution

We have analytical solutions

$\mathcal{O}(q^0, \epsilon^1)$: $\left(\Box^{(0)} - \mu^2\right)\Phi^{(1)} = 0$

Klein-Gordon eq. on a black hole background: obtain solution for the "scalar cloud" profile.

We know how to solve it

EMRIs in boson clouds: perturbative scheme

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$$q \equiv m_p/M \ll 1 \,, \quad \epsilon \ll 1$$

$\mathcal{O}(q^1, \epsilon^0)$: $\delta G^{(0)}_{\mu\nu}[h] = 8\pi T^{p}_{\mu\nu}[g^{(0)}]$

$\mathcal{O}(q^1, \epsilon^1)$: $\left(\Box^{(0)} - \mu^2\right)\Phi^{(q)} = S^{\Phi}[h, \Phi^{(1)}]$

EMRIs in a "vacuum" black hole background

We know how to solve it

Scalar perturbations sourced by background "cloud"+ metric perturbations

Can be solved using standard techniques

Solving the perturbation equations

- **Proof-of-principle example:** assume $g_{\mu\nu}^{(0)}$ given by a Schwarzschild black hole and assume point-particle in equatorial, circular orbits.
- **Note:** in a Schwarzschild black hole, scalar cloud always decays, unlike in a Kerr spacetime where true bound-states can exist. However, when $M\mu \ll 1$, decay timescale is very large, so I will neglect this decay.

Solving the perturbation equations

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A quick recipe to solve pert. equations up to $\mathcal{O}(q^1, \epsilon^1)$:

- **Step 1:** get scalar cloud profile:

$$\mathcal{O}(q^0, \epsilon^1): \quad \Box^{(0)}\Phi^{(1)} - \mu^2\Phi^{(1)} = 0\,, \qquad \Phi^{(1)}(t, r, \theta, \phi) = R_{n_i\ell_i}(r)Y_{\ell_i m_i}(\theta, \phi)e^{-i\omega t}$$

Solving the perturbation equations

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- **Step 2:** get metric perturbations (notice different indices for angular numbers):

$$\mathcal{O}(q^1, \epsilon^0): \quad \delta G_{\mu\nu}[g^{(0)}, h] = 8\pi T^p_{\mu\nu}[g^{(0)}]$$

$$h_{\mu\nu}(t, r, \theta, \phi) = \Re \sum_{l,m} \int e^{-i\sigma t} \left[h^{\text{axial},lm}_{\mu\nu}(\sigma, r, \theta, \phi) + h^{\text{polar},lm}_{\mu\nu}(\sigma, r, \theta, \phi) \right] d\sigma$$

For circular orbits: $\quad h^{\text{axial/polar},lm}_{\mu\nu} \sim \delta(\sigma - m\Omega_p)\,, \quad \Omega_p = \pm\sqrt{M/r_p^3}$

Solving the perturbation equations

- **Step 3:** solve eq. for the perturbations of the cloud (notice different indices for angular numbers):

$$\mathcal{O}(q^1, \epsilon^1): \left(\Box^{(0)} - \mu^2\right)\Phi^{(q)} = S^{\Phi}[h, \Phi^{(1)}]$$

$$\Phi^{(q)} = \frac{1}{2r}\sum_{\ell_j, m_j}\int d\sigma \left[Z_+^{\ell_j m_j}(r) Y_{\ell_j m_j}(\theta, \phi) e^{-i\sigma t} + (Z_-^{\ell_j m_j}(r))^* Y^*_{\ell_j m_j}(\theta, \phi) e^{i\sigma t}\right] e^{-i\omega t}$$

Note: equations imply $Z_-^{\ell_j, m_j}(\sigma; r)^* = (-1)^{m_j} Z_+^{\ell_j, -m_j}(-\sigma; r)$, so I'll focus on Z_+ in what follows

Solving the perturbation equations

- **Step 3:** solve eq. for the perturbations of the cloud (notice different indices for angular numbers):

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$$\downarrow$$

$$\sum_{\ell_j, m_j} Y_{\ell_j m_j}\left[\frac{d^2}{dr_*^2} + (\omega+\sigma)^2 - V_j(r)\right] Z_+^{\ell_j m_j}(r) = \sum_{l,m} S^+_{lm;\ell_i m_i}(r,\theta,\phi)$$

$$S^+_{lm}(r,\theta,\phi) = P_{lm}(r) Y_{lm} Y_{\ell_i m_i} + \hat{P}_{lm}(r)\left(Y^{lm}_{,\theta} Y^{\ell_i m_i}_{,\theta} + \frac{Y^{lm}_{,\phi} Y^{\ell_i m_i}_{,\phi}}{\sin^2\theta}\right) + A_{lm}(r)\frac{Y^{lm}_{,\theta} Y^{\ell_i m_i}_{,\phi} - Y^{lm}_{,\phi} Y^{\ell_i m_i}_{,\theta}}{\sin\theta}$$

Depends on couplings between **polar** metric perturbations and cloud's profile

Depends on couplings between **axial** metric perturbations and cloud's profile

Solving the perturbation equations

$$\sum_{\ell_j,m_j} Y_{\ell_j m_j}\left[\frac{d^2}{dr_*^2}+(\omega+\sigma)^2-V_j(r)\right]Z_+^{\ell_j m_j}(r)=\sum_{l,m} S^+_{lm;\ell_i m_i}(r,\theta,\phi)$$

project onto spherical harmonics

$$\left[\frac{d^2}{dr_*^2}+(\omega+\sigma)^2-V_j\right]Z_+^{\ell_j m_j}(r)=(\tilde{S}^+)^{\ell_j,\ell_i}_{m_j,m_i}(r)$$

- Task reduces to computing following integrals:

$$\int d\Omega\, Y^*_{\ell_j m_j} Y_{lm} Y_{\ell_i m_i}\,, \qquad \int d\Omega\, Y^*_{\ell_j m_j}\mathbf{Y}^{lm}_a \mathbf{Y}^{\ell_i m_i}_b \gamma^{ab}\,, \qquad \int d\Omega\, Y^*_{\ell_j m_j}\mathbf{X}^{lm}_a \mathbf{Y}^{\ell_i m_i}_b \gamma^{ab}$$

$$\mathbf{Y}^{lm}_a(\theta,\phi)=\left(Y^{lm}_{,\theta}\,,\,Y^{lm}_{,\phi}\right) \qquad \mathbf{X}^{lm}_a(\theta,\phi)=\left(-\frac{Y^{lm}_{,\phi}}{\sin\theta}\,,\,\sin\theta Y^{lm}_{,\theta}\right) \qquad \gamma^{ab}=\mathrm{diag}(1,1/\sin^2\theta)$$

- All these integrals can be computed explicitly and written in terms of Wigner 3-j symbols

Solving the perturbation equations

$$\sum_{\ell_j, m_j} Y_{\ell_j m_j} \left[\frac{d^2}{dr_*^2} + (\omega + \sigma)^2 - V_j(r) \right] Z_+^{\ell_j m_j}(r) = \sum_{l,m} S^+_{lm;\ell_i m_i}(r, \theta, \phi)$$

project onto spherical harmonics

$$\left[\frac{d^2}{dr_*^2} + (\omega + \sigma)^2 - V_j \right] Z_+^{\ell_j m_j}(r) = (\tilde{S}^+)^{\ell_j, \ell_i}_{m_j, m_i}(r)$$

Selection rules for cloud with given $\{\ell_i, m_i\}$:

- $\tilde{S}^+$ only depends on metric perturbations with magnetic angular number $m = m_j - m_i$, and orbital number l that satisfies $|\ell_j - \ell_i| \leq l \leq \ell_j + \ell_i$
- $\tilde{S}^+$ only depends on polar metric perturbations that satisfy $\ell_j + \ell_i + l = 2p$ with $p \in \mathbb{N}$, e.g. $\ell_i = 0 \implies l = \ell_j$; $\ell_i = 1 \implies l = \ell_j \pm 1$
- $\tilde{S}^+$ only depends on axial metric perturbations that satisfy $\ell_j + \ell_i + l = 2p + 1$, e.g. $\ell_i = 1 \implies l = \ell_j$ (**note:** no coupling to axial perturbations if $\ell_i = 0$)

Power lost by a point particle in a scalar cloud

- Assume adiabatic evolution:

rate of change of the point particle 's orbital energy

GW fluxes

scalar radiation fluxes

$$\dot{E}_{\rm orb} + \dot{M}_b = -\dot{E}^{g,\infty} - \dot{E}^{g,H} - \dot{E}^{\Phi,\infty} - \dot{E}^{\Phi,H}$$

scalar cloud mass loss

$$\downarrow$$

$$\dot{E}_{\rm orb} = -\dot{E}^{g,\infty} - \dot{E}^{g,H} - \dot{E}^{s,\infty} - \dot{E}^{s,H}, \qquad \dot{E}^{s} = \dot{E}^{\Phi} + \dot{M}_b$$

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$$\dot{E}_{\rm orb} + \dot{M}_b = -\dot{E}^{g,\infty} - \dot{E}^{g,H} - \dot{E}^{\Phi,\infty} - \dot{E}^{\Phi,H}$$

scalar cloud mass loss

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$$\dot{E}_{\rm orb} = -\dot{E}^{g,\infty} - \dot{E}^{g,H} - \dot{E}^{s,\infty} - \dot{E}^{s,H}\,, \qquad \dot{E}^{s} = \dot{E}^{\Phi} + \dot{M}_b$$

- For a complex scalar field: $M_b = \omega Q \rightarrow \dot{M}_b = \omega \dot{Q}$ (Q, Noether charge)

$$\downarrow \quad \dot{E}^{s} = \sum_{\ell_j, m_j} \dot{E}^{s}_{\ell_j m_j}$$

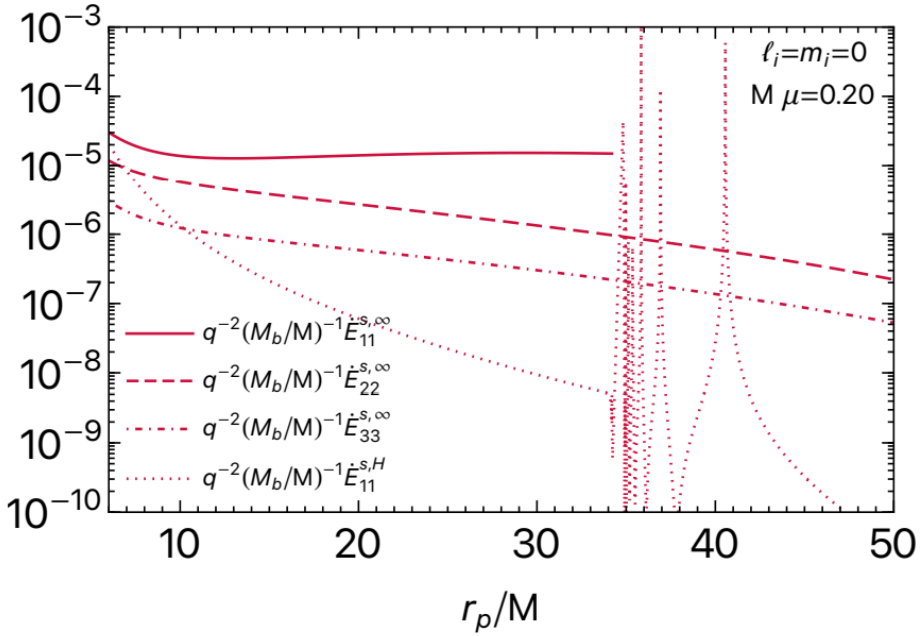
$$2\dot{E}^{s,\infty}_{\ell_j m_j} = \Omega_+ s_+ \sqrt{\left(\Omega_+ + \omega\right)^2 - \mu^2}\, \left|\tilde{Z}^{\ell_j m_j}_+(\Omega_+)\right|^2 - \Omega_- s_- \sqrt{(\Omega_- - \omega)^2 - \mu^2}\, \left|\tilde{Z}^{\ell_j m_j}_-(\Omega_-)\right|^2$$

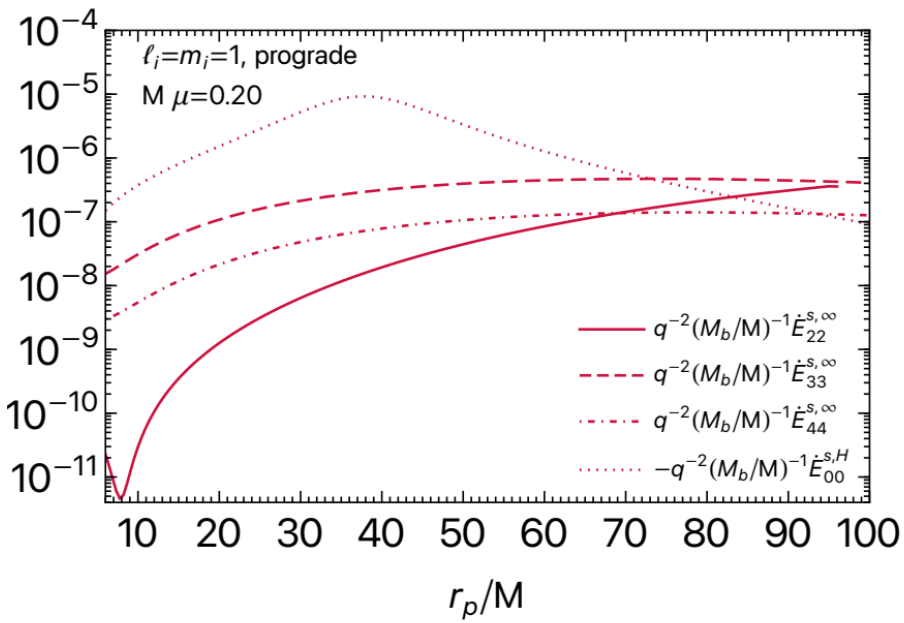
$$2\dot{E}^{s,H}_{\ell_j m_j} = \Omega_+ \left(\omega + \Omega_+\right) \left|\tilde{Z}^{\ell_j m_j}_+(\Omega_+)\right|^2 - \Omega_- \left(\omega - \Omega_-\right) \left|\tilde{Z}^{\ell_j m_j}_-(\Omega_-)\right|^2$$

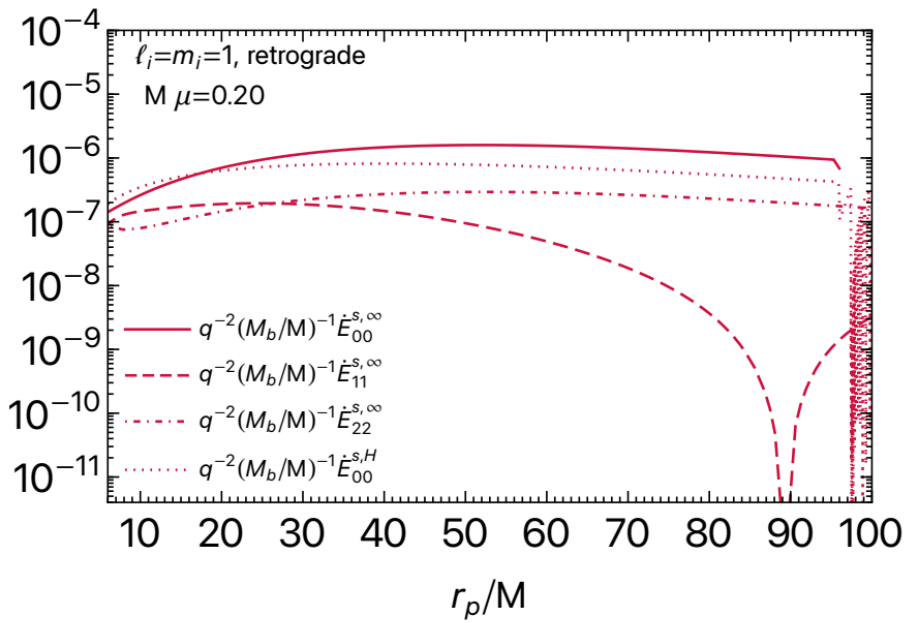
$$\Omega^{m_j}_{\pm} = \left(m_j \mp m_i\right) \Omega_p\,, \quad s_{\pm} = \mathrm{sgn}(\omega \pm \Omega_{\pm})$$

Power lost due to scalar cloud

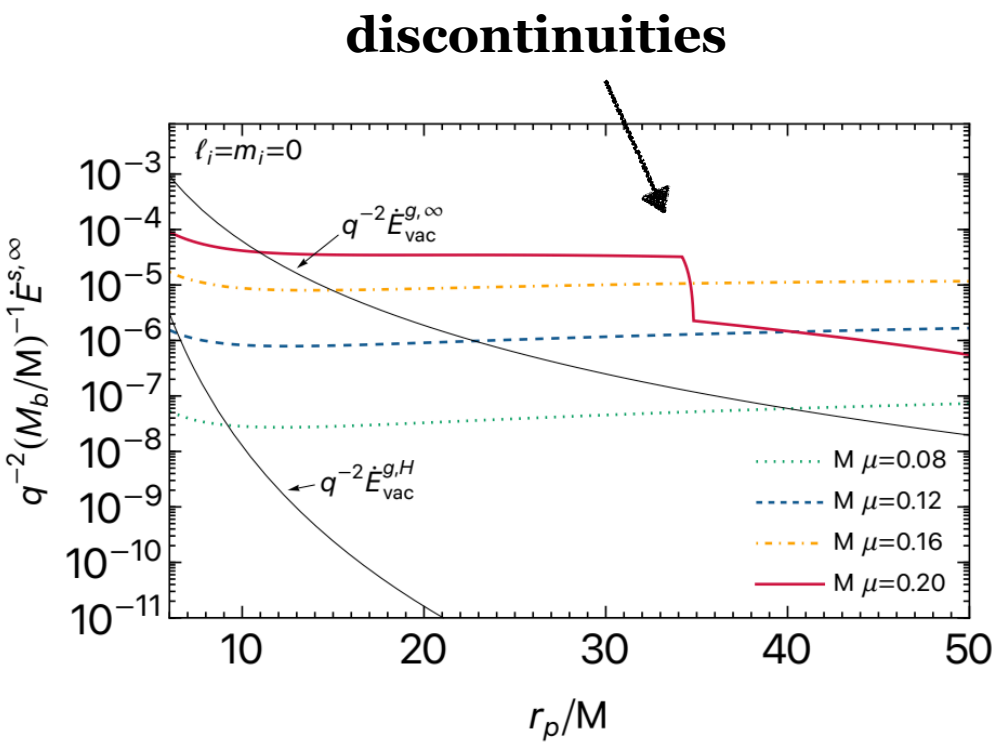
- Multipoles ℓ_j, m_j that contribute the most depend on the cloud configuration as well as direction of the orbit for $m_i \neq 0$

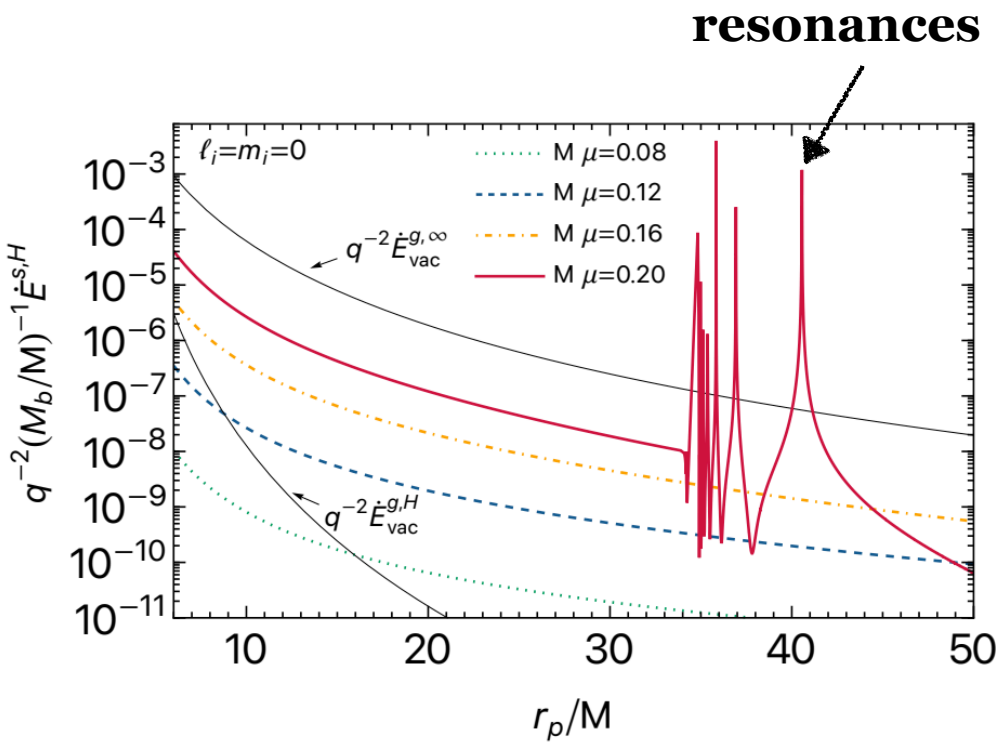






Power lost by a point particle in a **spherical** cloud

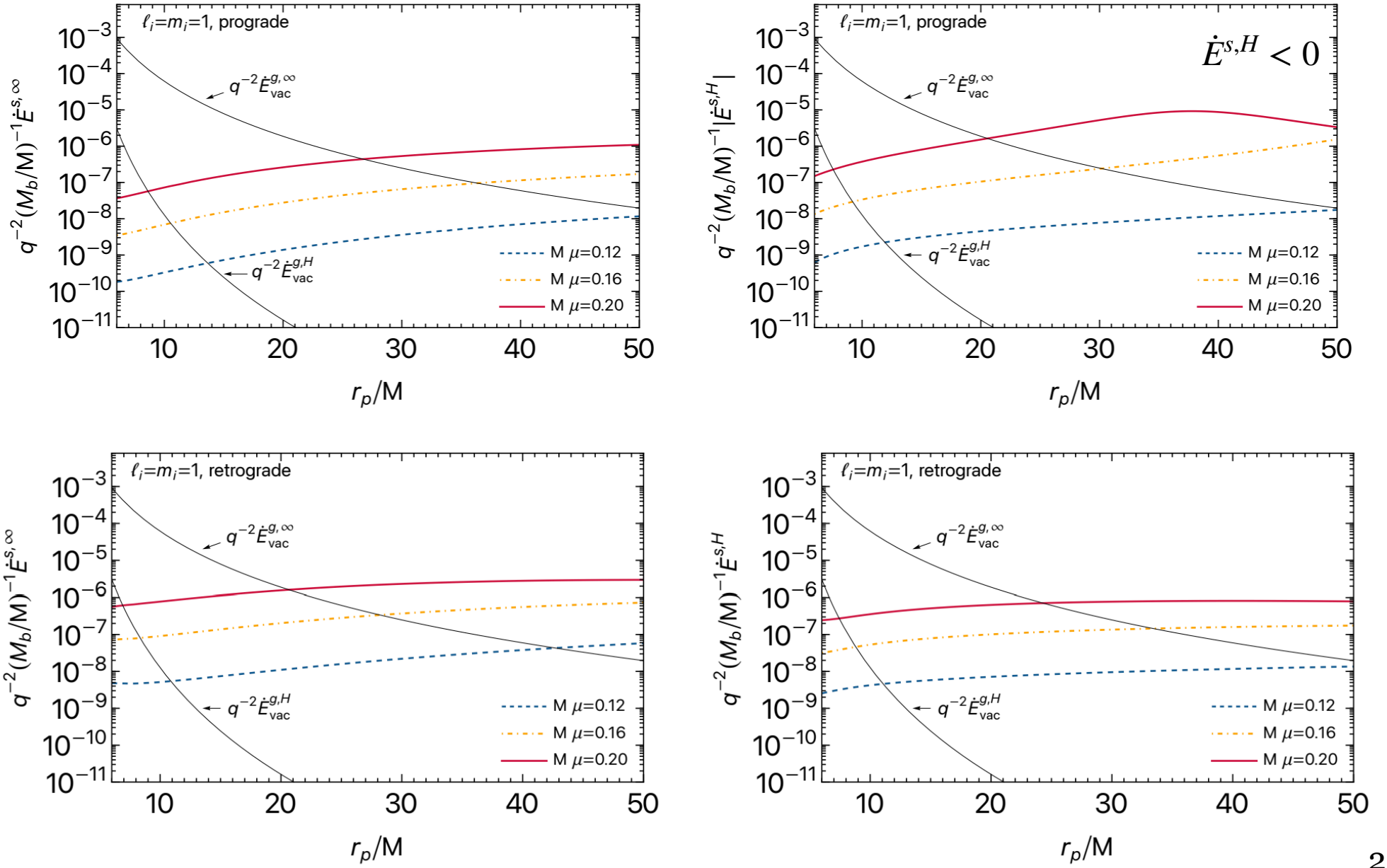




- **Resonances** when $m\Omega_p^{n_j} = \Re(\omega_{n_j \ell_j m_j}) - \Re(\omega_{n_i \ell_i m_i})$ [Baumann+'18, '19]
- When $(m\Omega_p + \omega)^2 - \mu^2 < 0$, modes cannot propagate to infinity: **discontinuities** in power lost by particle. [Baumann+'21; Tomaselli+'23]
- Power lost due to scalar cloud can **dominate** over GW emission, especially at large orbital separations

Power lost by a point particle in a **dipolar** cloud

For $\ell_i = m_i = 1$, results depend on **direction of the orbit**.



Conclusions

Presence of **boson clouds** around massive black holes could leave imprint in I/EMRI waveforms.

We build a **fully relativistic, perturbative framework** to study I/EMRIs in the presence of boson clouds. Proof-of-principle results promising and **confirm presence of striking signatures** associated with the presence of the cloud.

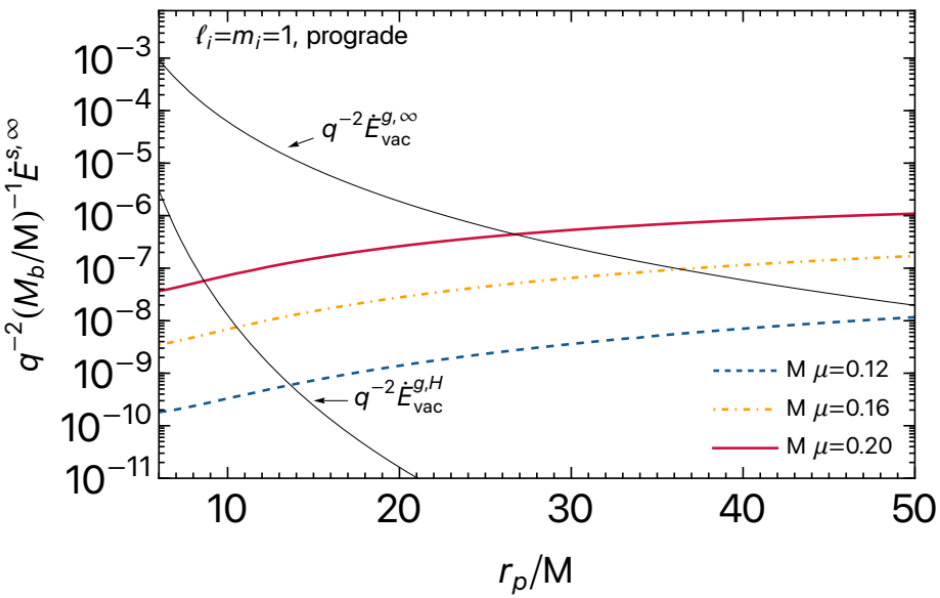
What's next?

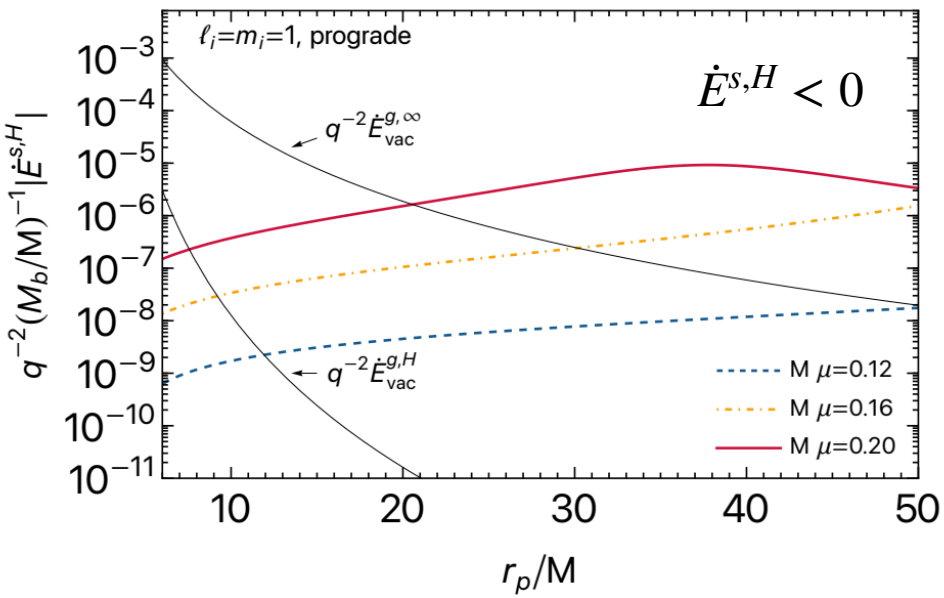
- **Comparisons** with Newtonian approximations
- **Orbital evolution** and **detectability**?
- **Generic** inclined/eccentric **orbits**.
- **Extension to spinning black-hole case:** perturbative formalism is generic so should be feasible.

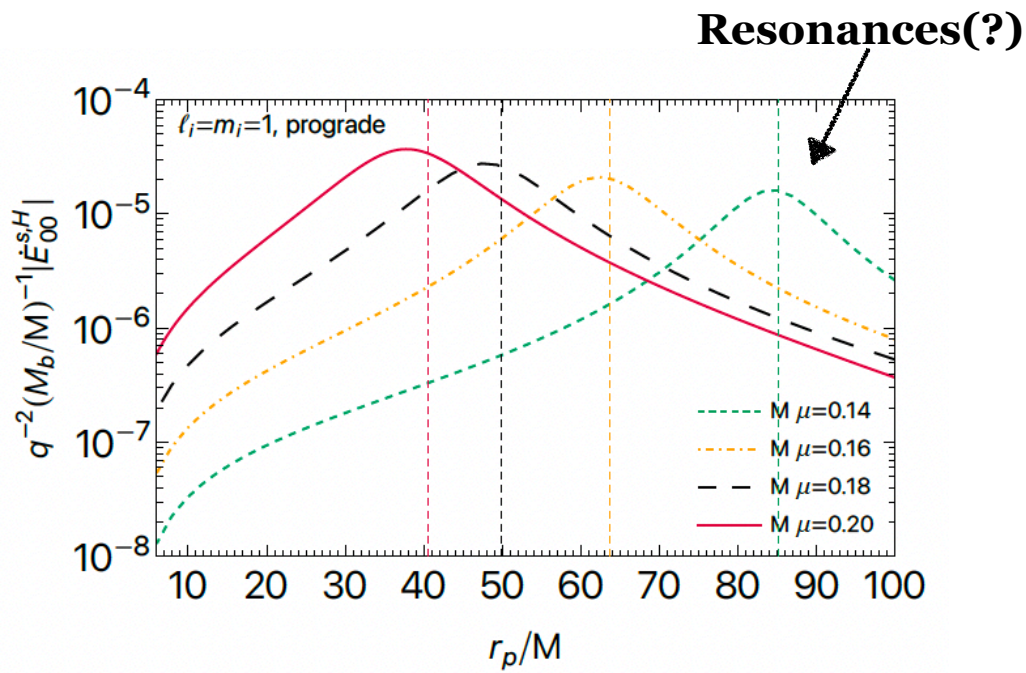
Thank you!

Backup slides

Power lost by a point particle in a **dipolar** cloud







- Possibility of **floating orbits** ($\dot{E}_{\rm particle} = 0$) due to transfer of energy/angular momentum from the cloud to the orbit?

Backreaction on the black hole geometry

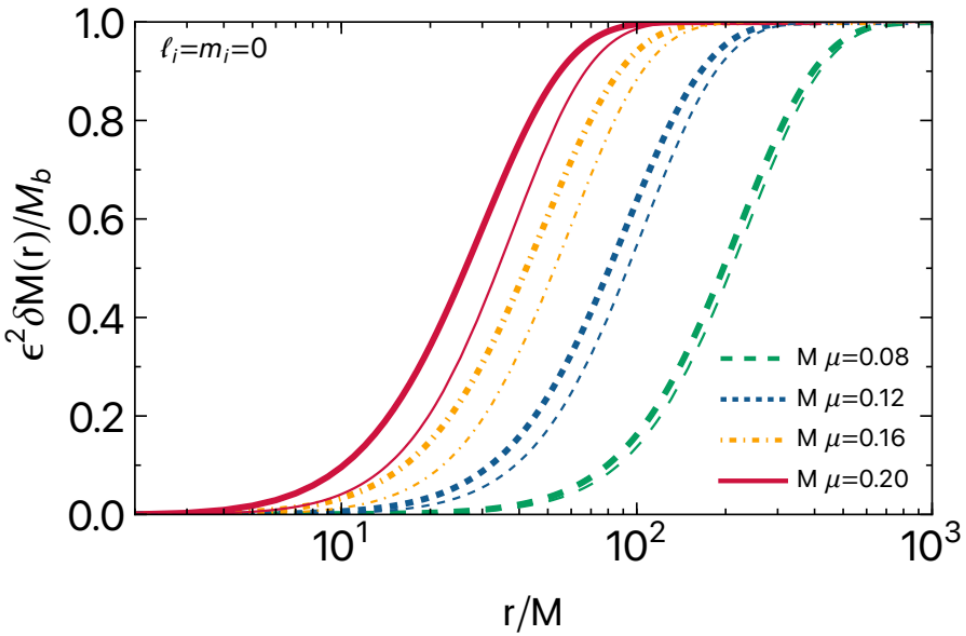
$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + \epsilon^2 g^{(2)}_{\mu\nu} + q h_{\mu\nu} + \dots\,, \qquad \Phi = \epsilon\Phi^{(1)} + q\epsilon\Phi^{(q)} + \dots$$

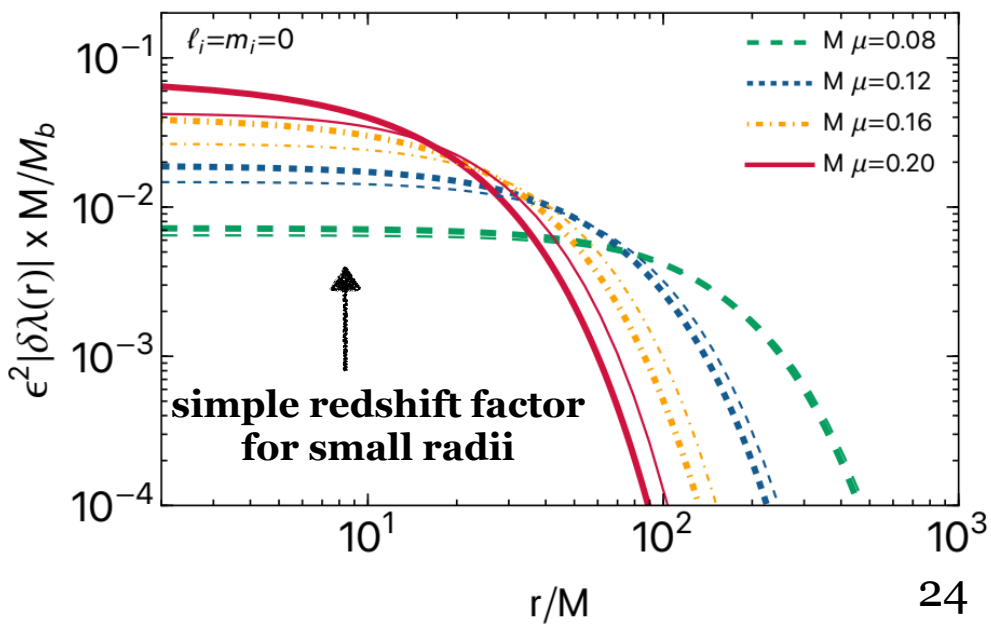
$$\mathcal{O}(q^0, \epsilon^2):\;\; \delta G^{(0)}_{\mu\nu}[g^{(2)}] = 8\pi T^{\Phi,(0)}_{\mu\nu}[\Phi^{(1)}, \Phi^{(1)\,*}]$$

In a non-spinning BH background, the **accretion of the cloud** at the horizon requires using ingoing Eddington-Finkelstein coordinates. Assuming a **spherical cloud**:

$$ds^2 = -\left(1 - \frac{2M + 2\epsilon^2\delta M(v,r)}{r}\right) e^{2\epsilon^2\delta\lambda(v,r)} dv^2 + 2e^{\epsilon^2\delta\lambda(v,r)} dv dr + r^2 d\Omega^2$$

$$\partial_v \delta M = 2\Im(\omega)\delta M(v,r) \approx 0\,, \quad \delta M(r) \approx 4\pi \int_{2M}^{r} \rho(0,r') r'^2 dr'$$





Corrections to (axial) GW flux

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + \epsilon^2 g^{(2)}_{\mu\nu} + q h_{\mu\nu} + \dots , \qquad \Phi = \epsilon\Phi^{(1)} + q\epsilon\Phi^{(q)} + \dots$$

$$\delta G_{\mu\nu}[\bar{g}, h] = 8\pi \left(T^{p}_{\mu\nu}[\bar{g}] + S^{\Phi^{(1)}\Phi^{(q)*}}[\Phi^{(q)*}, \Phi^{(1)}] + S^{\Phi^{(1)*}\Phi^{(q)}}[\Phi^{(q)}, \Phi^{(1)*}] + S^{h\Phi^{(1)}\Phi^{(1)}}_{\mu\nu}[h, \Phi^{(1)}, \Phi^{(1)\,*}] \right)$$

$$\longrightarrow \left[\frac{d^2}{d\bar{r}_*^2} + \sigma^2 - \bar{V}_{\rm ax} \right] \bar{\psi}^{lm}_{\rm ax}(r) = \bar{S}^{lm}_{\rm ax}(r)$$

$$\bar{V}_{\rm ax} = F_* \left(\frac{(l-1)(l+2)}{r^2} e^{\epsilon^2 \delta\lambda} - \frac{F'_*}{r} + \frac{2F_*}{r^2} \right)$$

$$F_*(r) := \left(f(r) - \frac{2\epsilon^2 \delta M(r)}{r} \right) e^{\epsilon^2 \delta\lambda(r)}$$

- For a spherical cloud, axial metric perturbations decouple from scalar perturbations ($\Phi^{(q)}$)
- For non-compact clouds, corrections at small orbital radii can be understood in terms of a **redshift effect**

[Cardoso+'22]