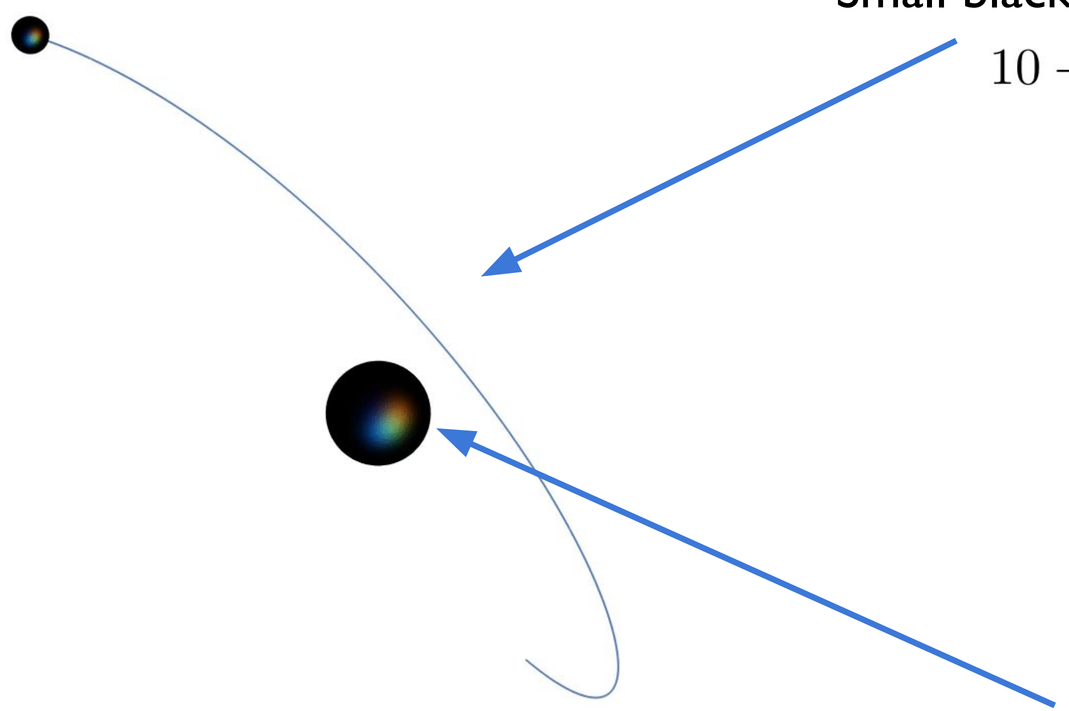
Dynamic tidal resonances in EMRIs

Béatrice Bonga - 21 September 2023

Bonga, Yang & Hughes [1905.00030, PRL] + Gupta, Bonga, Chua & Tanaka [2104.03422, PRD] + Gupta, Speri, Bonga, Chua & Tanaka [2205.04808, PRD] + Bonga, Gupta, Peters [23yy.xxxx]

Extreme Mass Ratio Inspirals (EMRI)



Small black hole/compact object

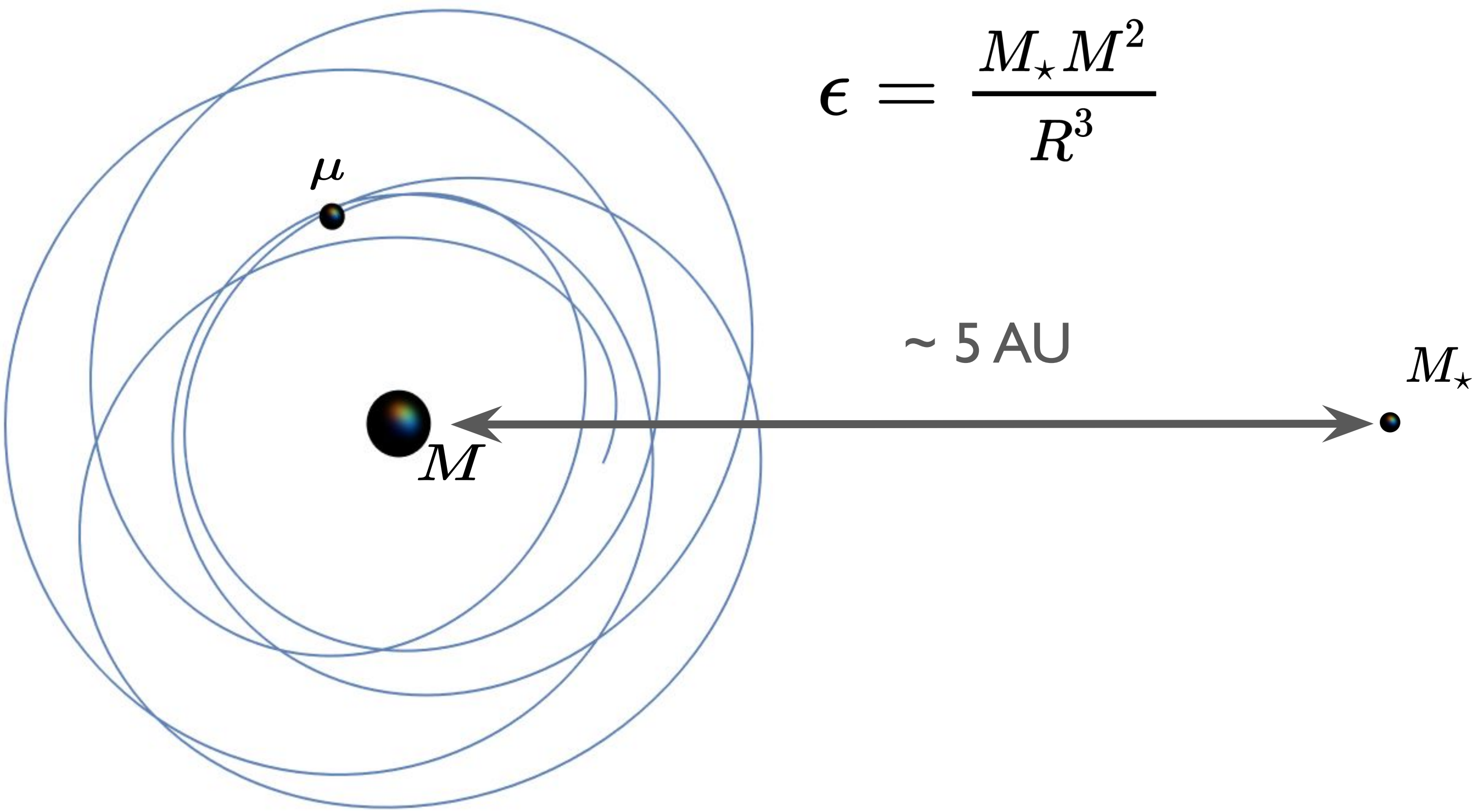
$10 - 100\, M_{\odot}$

Central massive black hole

$10^5 - 10^7 M_{\odot}$

EMRIs as isolated systems ?

$$\epsilon = \frac{M_\star M^2}{R^3}$$



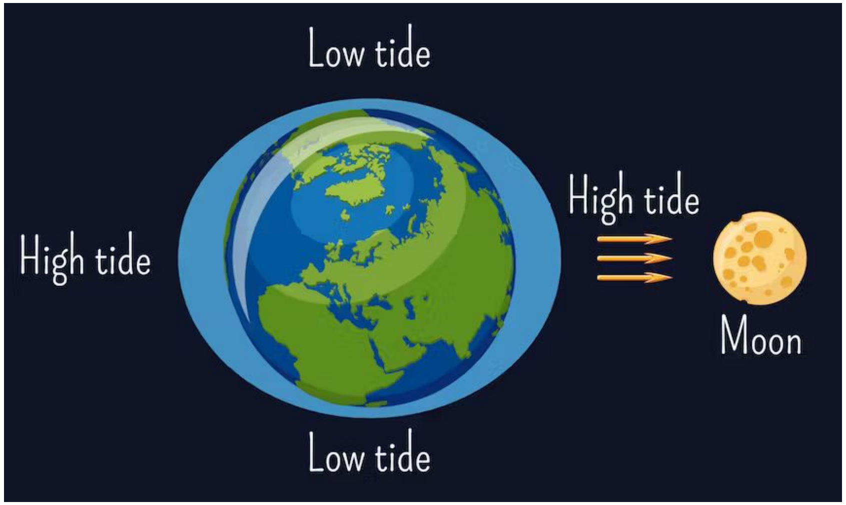
$$M = 4 \times 10^6 M_\odot, \mu = M_\star = 30 M_\odot, R = 5AU \Longrightarrow \epsilon \sim 2 \times 10^{-13}$$

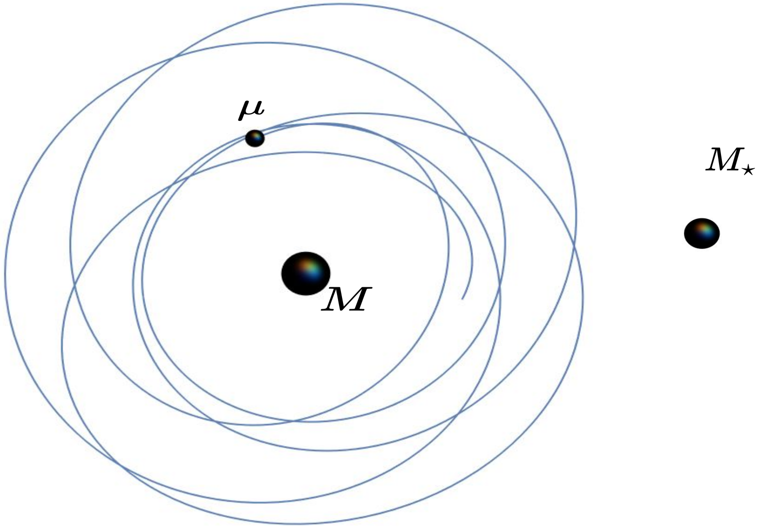
What is the effect of this ε?

Osculating elements code

Geodesics + 5PN fluxes + tidal force

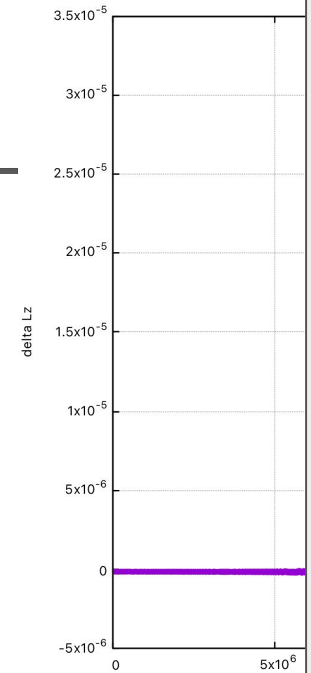
Tidal force weak and averages out

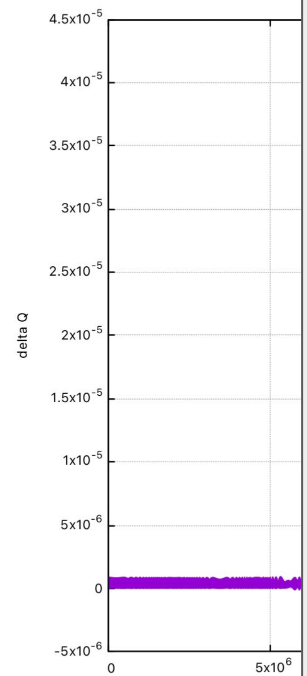




Evolution comparison

Tidal force is important during resonances!



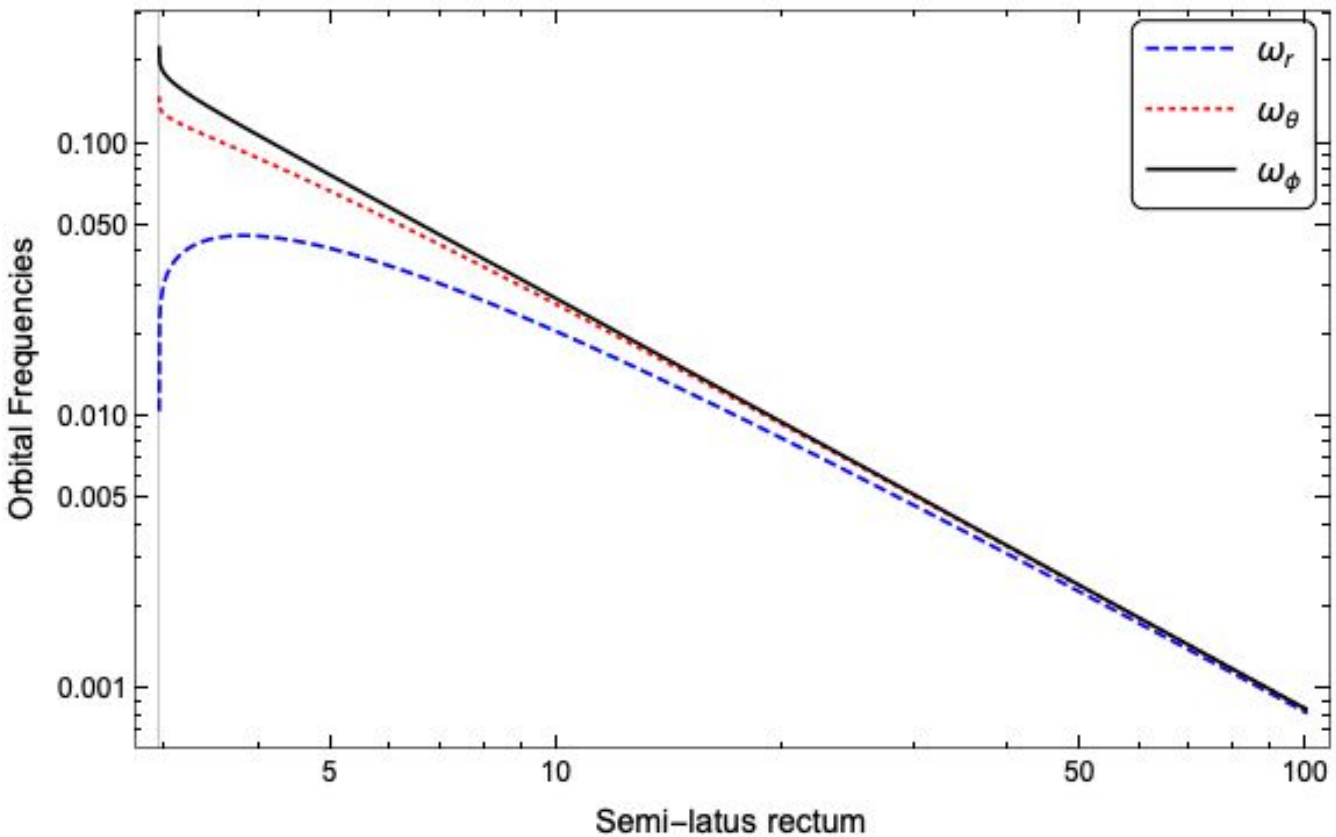


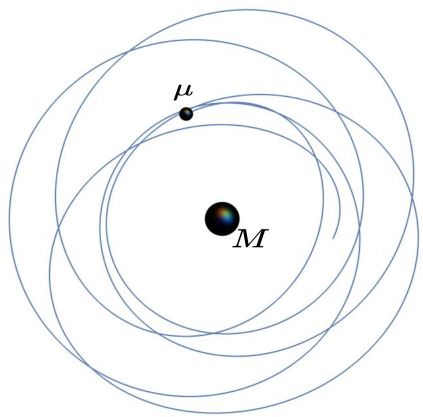
Resonances: matching of frequencies

Sound waves

Mechanical vibrations

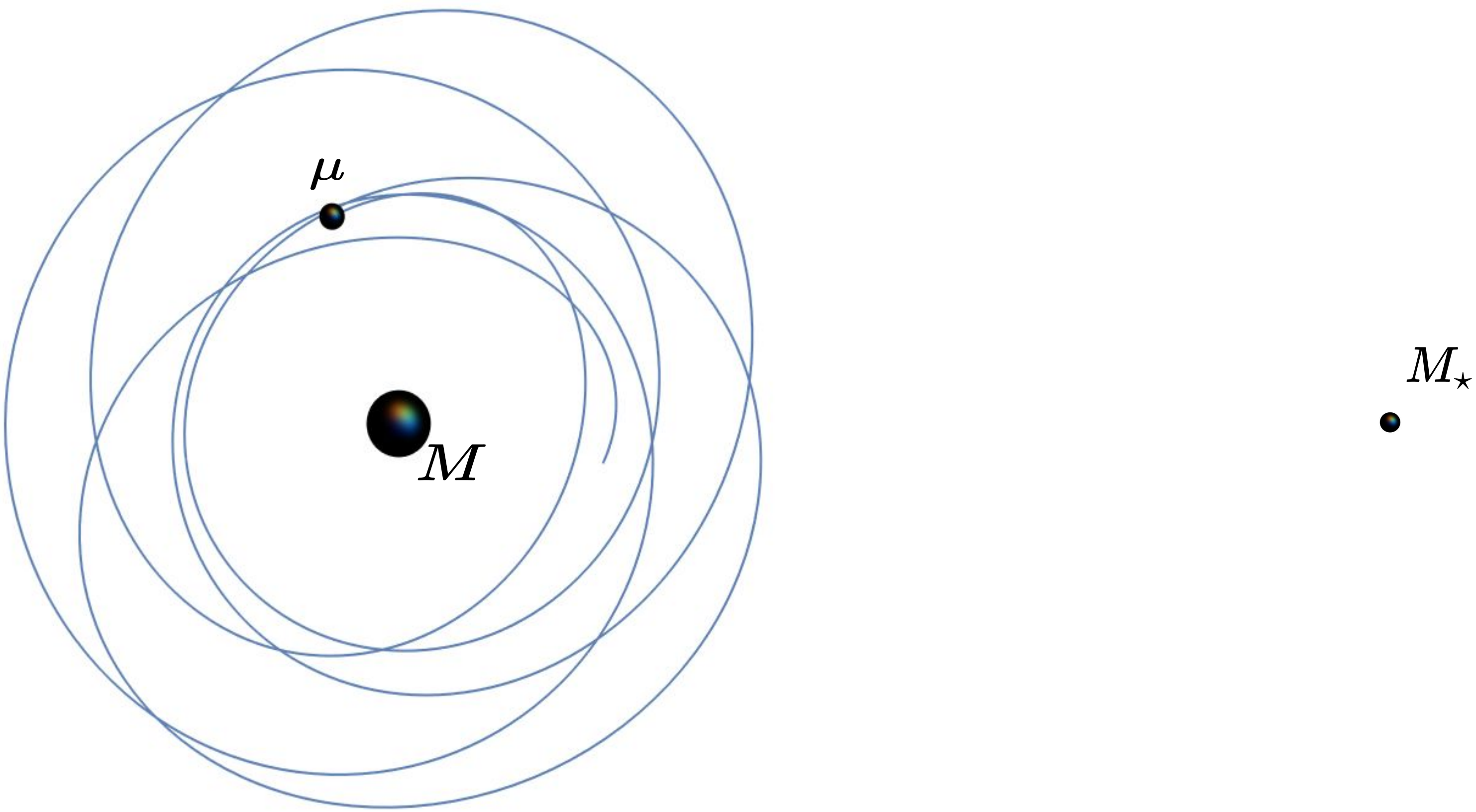
Orbital frequencies of EMRI





Orbital frequency perturber

Perturber is far → Keplerian frequency = Ω_ϕ



Resonance condition

$$n\,\omega_r + k\,\omega_\theta + m\,\omega_\phi + s\,\Omega_\phi = 0$$

Selection rule

only non-zero if $k + m + s$ is even

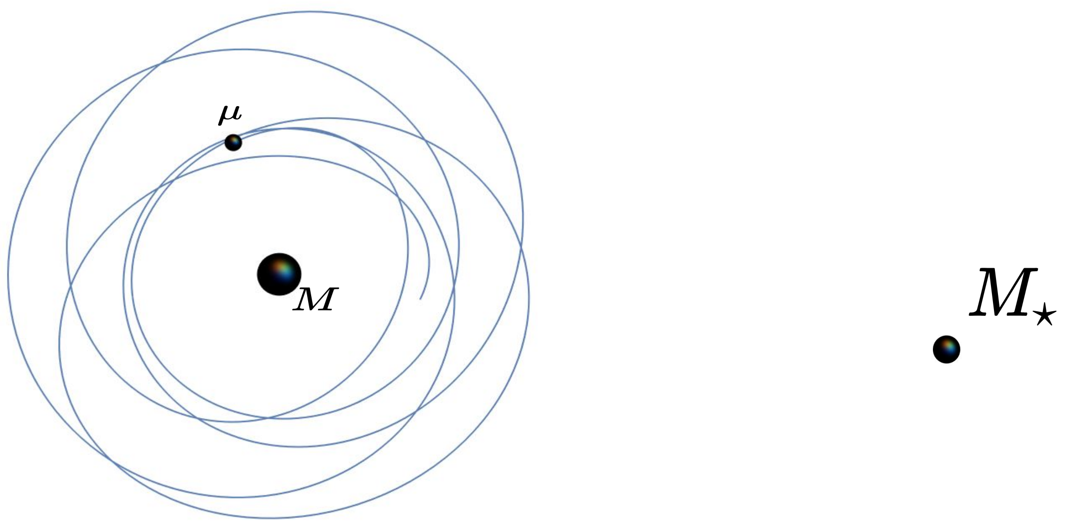
Key EMRI Waveform requirements

01	**Accurate**	Tidal focus should be included ✔
02	**Fast**	Osculating elements way soooooo slow!

Tidal force *only* important during resonance

In the past…

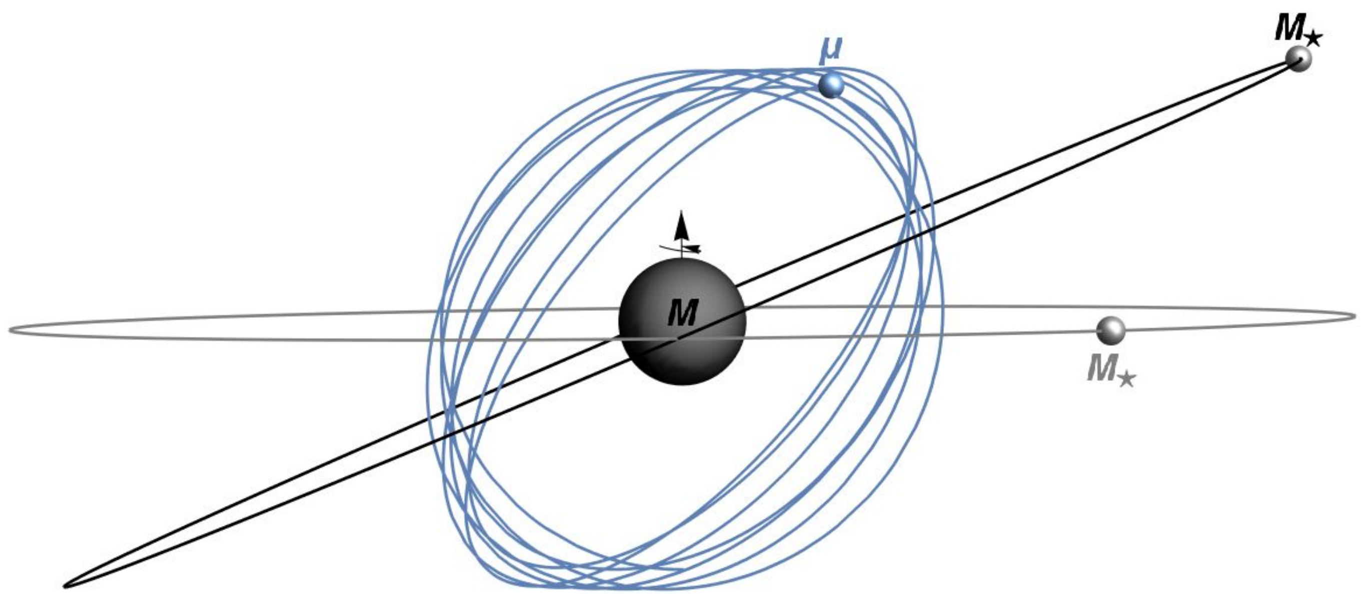
Perturber assumed to be *stationary*



$$\tau_{orbit} \ll \tau_{resonance} \ll \tau_{rad}, \tau_{tide}$$

Dynamic model

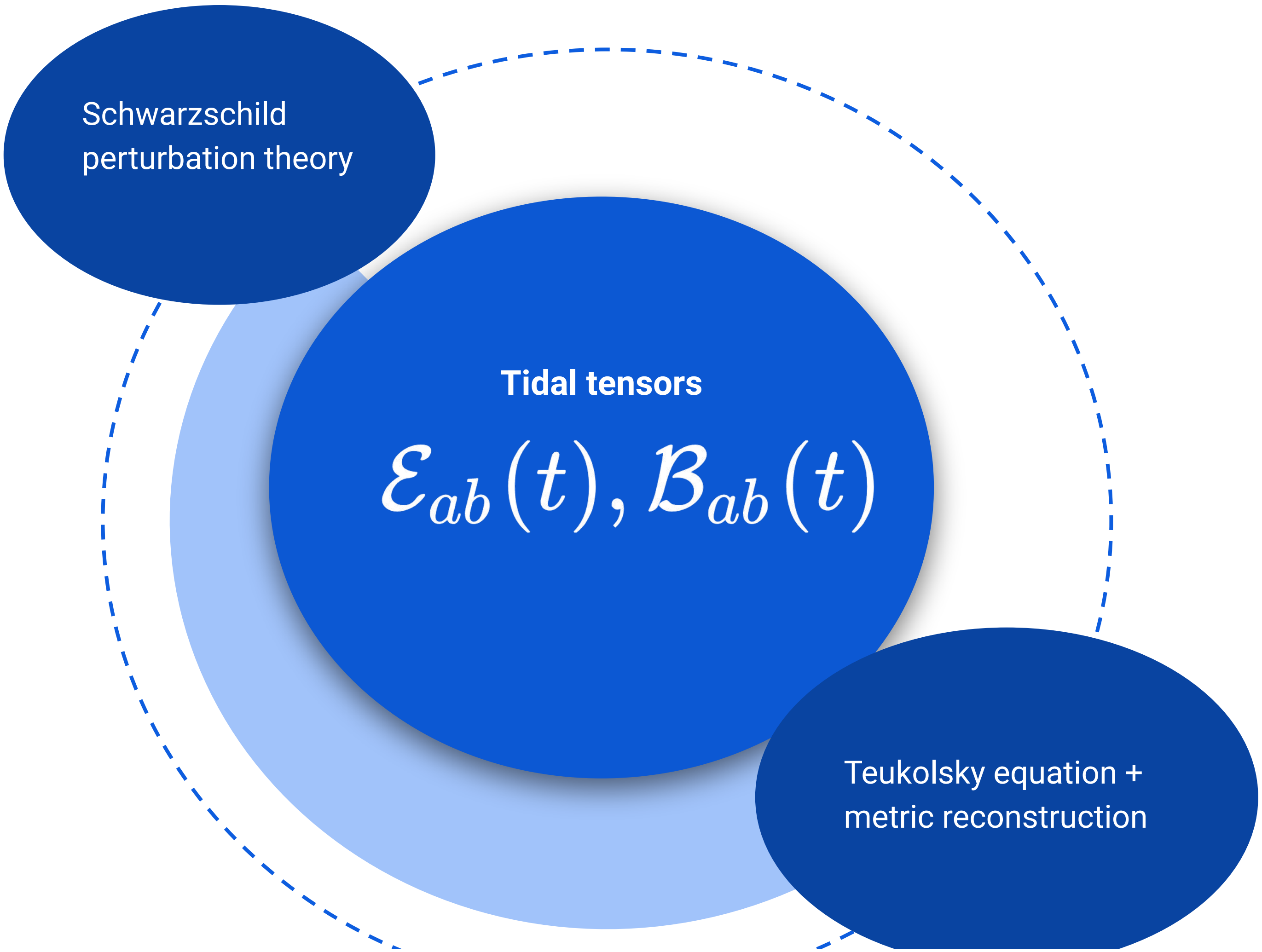
Perturber is in a circular Keplerian orbit → no extra parameter!



New insights: dynamic model

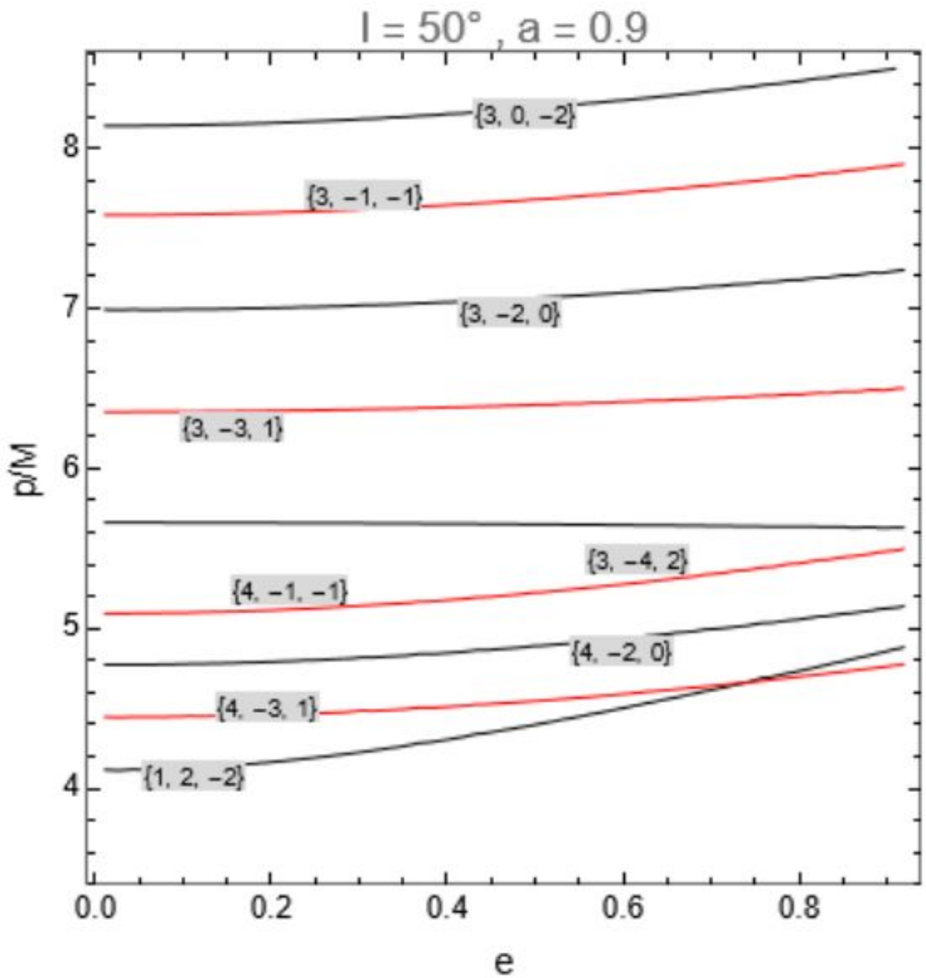
01	**More possible resonances**	
02	**More complicated tidal force**	Spin of central MBH does not affect the force much (jump sizes differ by less than 1%)

Tidal force

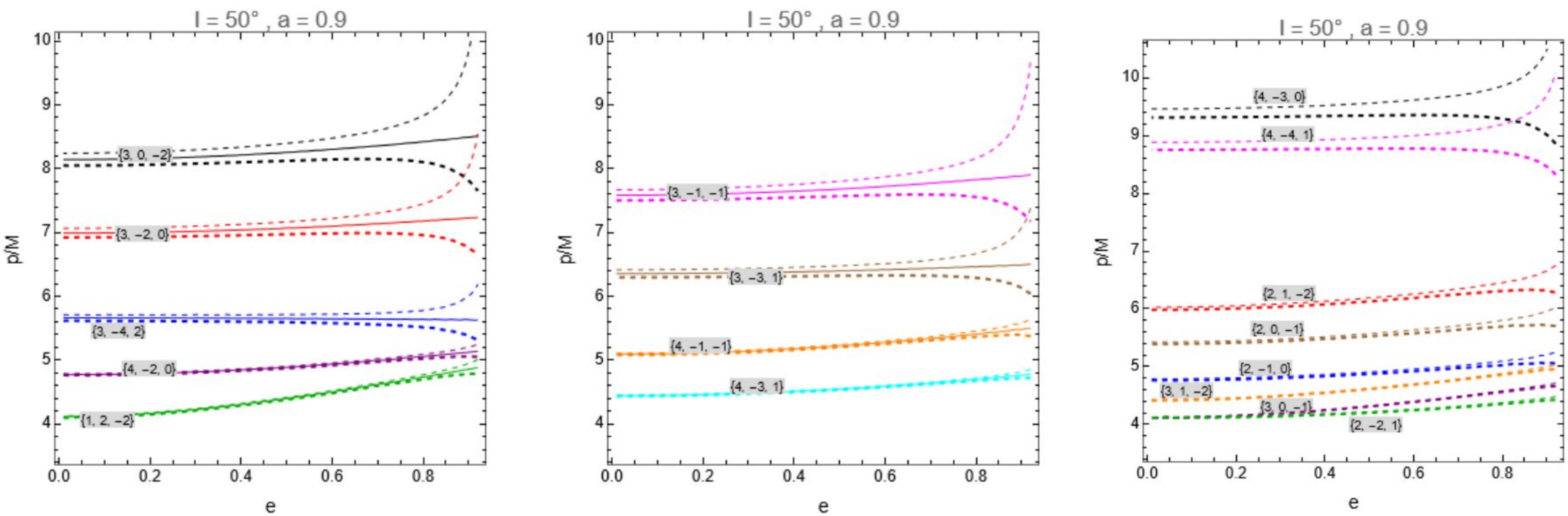


$$a^\alpha = -\frac{1}{2}(g^{\alpha\beta}_{\mathrm{Kerr}} + u^\alpha u^\beta)(2h_{\beta\lambda;\rho} - h_{\lambda\rho;\beta})u^\lambda u^\rho$$

Resonance contour stationary case



Resonance contour dynamic model



“Jump” size

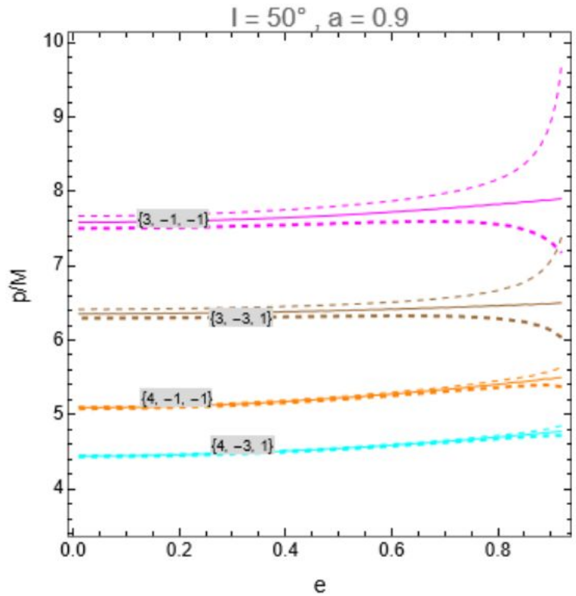
Jump = Re(duration x resonance strength x exp[i phase])

Determined by radiation reaction timescale

Perturber: M_*/R^3, sky location
Central massive black hole: M, a
EMRI: p, e, i

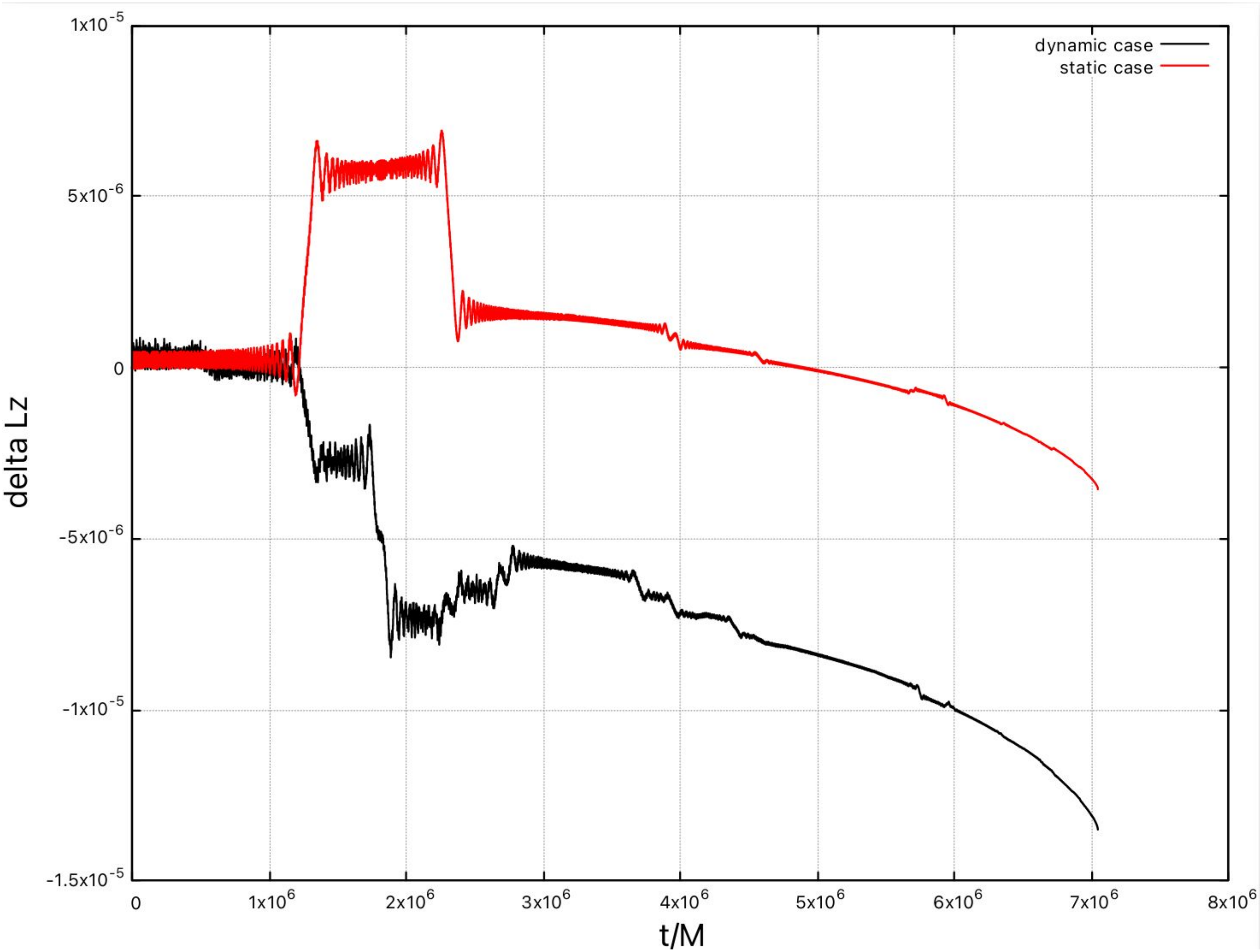
Dynamic versus stationary

1. Jump sizes are similar (<10% difference generically, <30% when e>0.7)

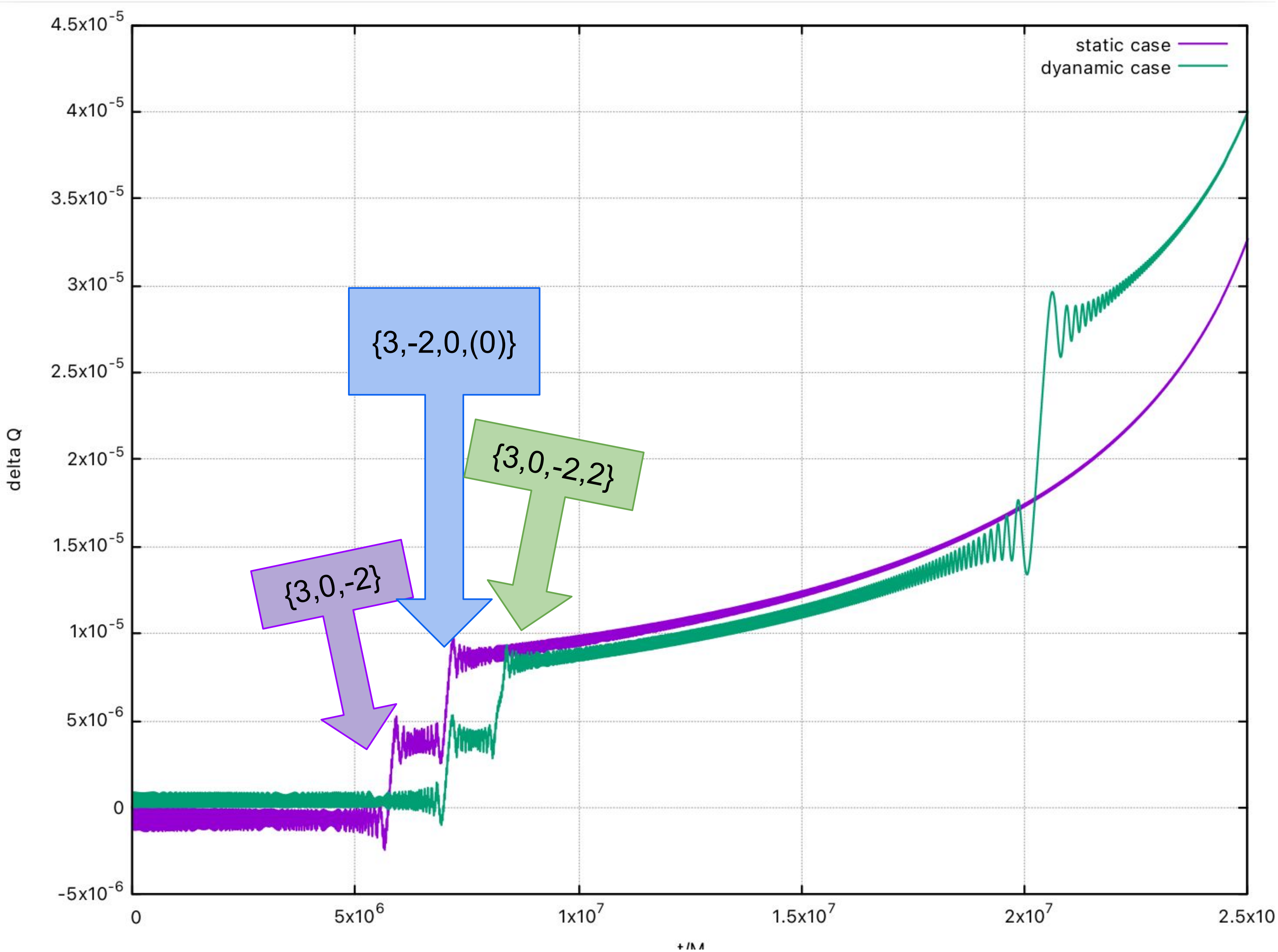


2. But… resonance condition is different ⟶ phase at resonance is different ⟶ actual jump is very different!

Dynamic versus stationary

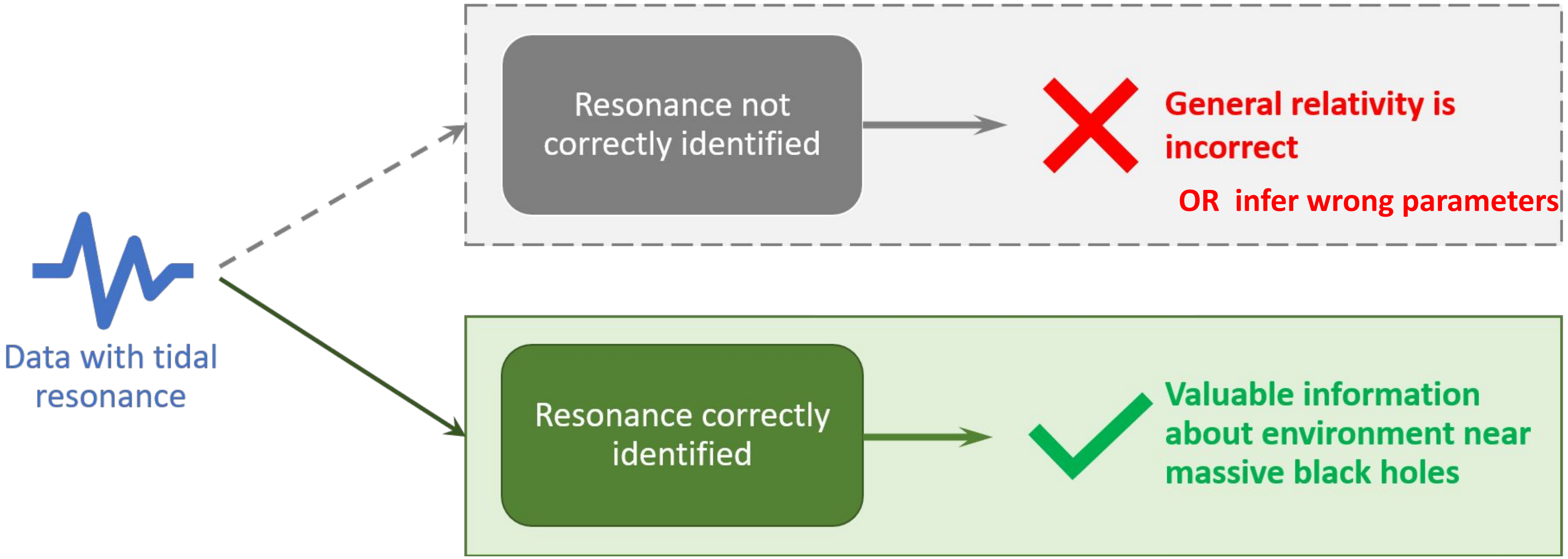


Ordering resonances different!



Summary

Dynamic tidal resonances will likely occur in EMRIs



Challenges for the future

1. How to deal with multiple resonances?

2. Upgrade dynamic model to eccentric orbit? (Is this needed?)

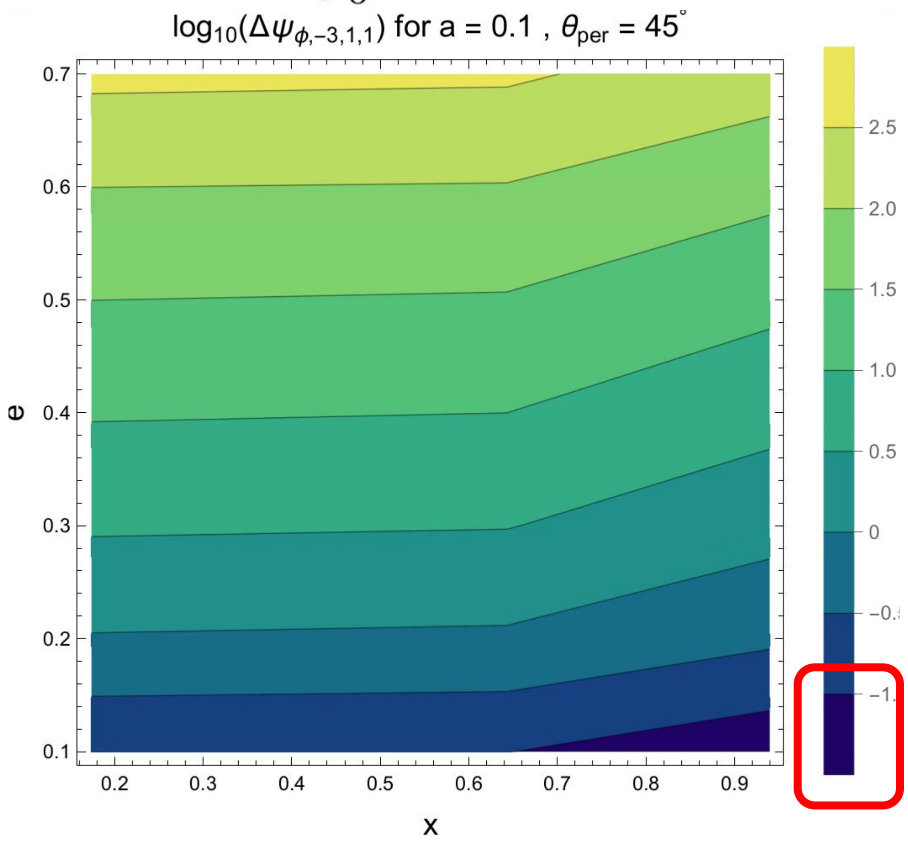
3. Are there additional physical effects that should be taken into account?

4. Are these resonances degenerate with other physics?

Back up slides

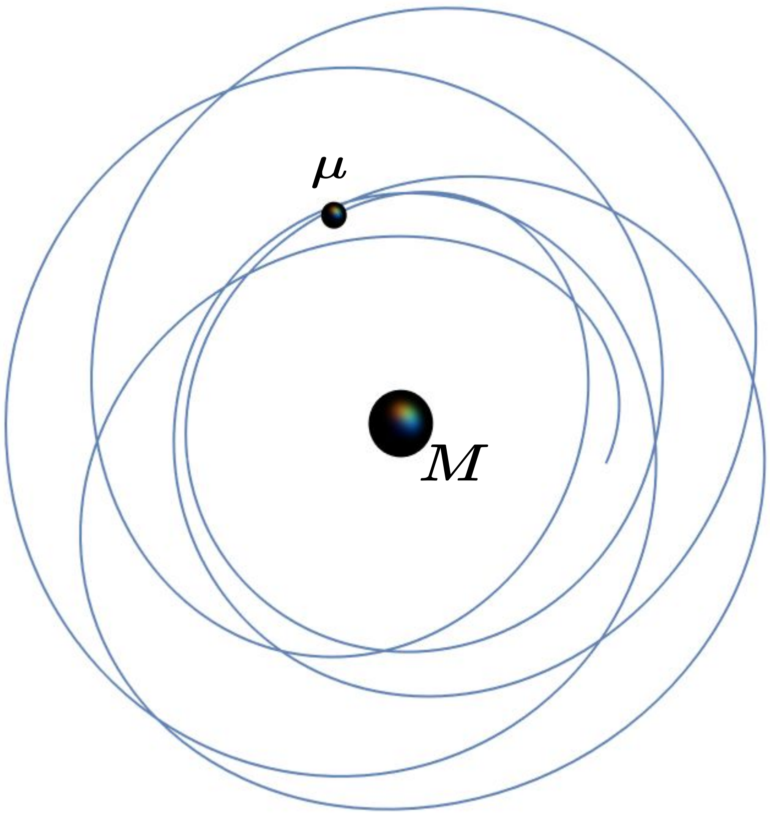
Effect on the waveform

$$\Delta\Psi := \int_0^{T_{\text{plunge}}} 2\Delta\omega_\phi dt$$



$$M = 4 \times 10^6 M_\odot, \mu = M_\star = 30 M_\odot \quad R = 10 AU \Longrightarrow \epsilon \sim 10^{-14}$$

Action-angle variables



Four constants of motion:
$\{\mu, E, L_z, Q\}$

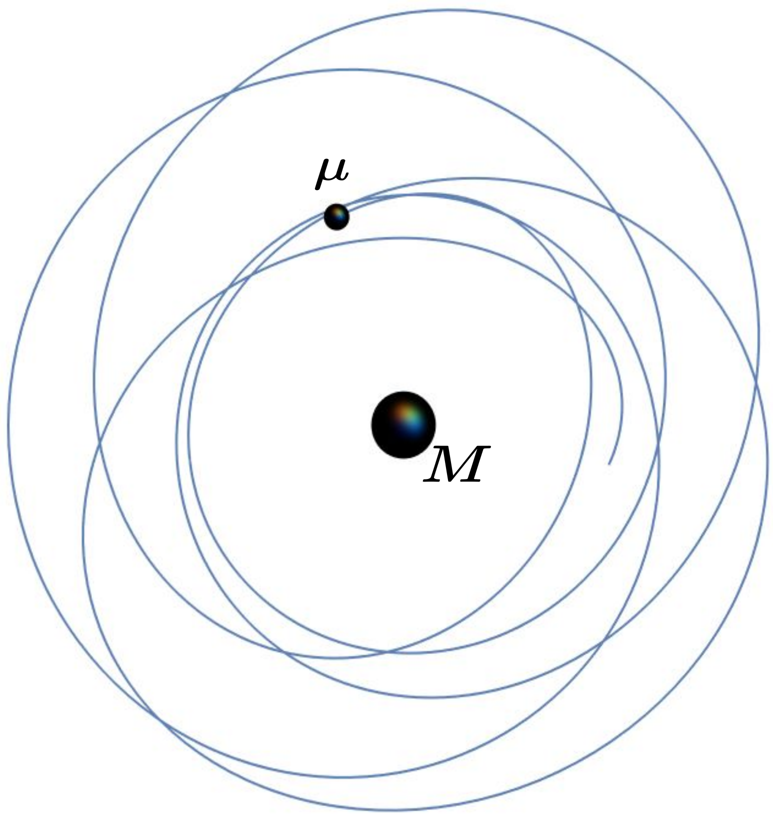
→ Geodesic equation is integrable

→ Action-angle variables are useful

$$\frac{dq_i}{d\tau} = \omega_i(\mathbf{J})$$
$$\frac{dJ_i}{d\tau} = 0$$

$$\mathbf{J} := \{J_r, J_\theta, J_\phi\}$$

Before introducing a perturber….



… motion is not geodesic!

Gravitational radiation changes description

$$\frac{dq_i}{d\tau} = \omega_i(\mathbf{J}) + \eta\, g^{(1)}_{i,\mathrm{sf}}(q_\theta, q_r, \mathbf{J}) + \mathcal{O}(\eta^2)$$

$$\frac{dJ_i}{d\tau} = \eta\, G^{(1)}_{i,\mathrm{sf}}(q_\theta, q_r, \mathbf{J}) + \mathcal{O}(\eta^2)$$

Mass ratio: μ/M

Self force 101: adiabatic approximation

$$\frac{dq_i}{d\tau} \approx \omega_i(\mathbf{J})$$
$$\frac{dJ_i}{d\tau} \approx \eta \left\langle G^{(1)}_{i,\mathrm{sf}}(q_\theta, q_r, \mathbf{J}) \right\rangle$$

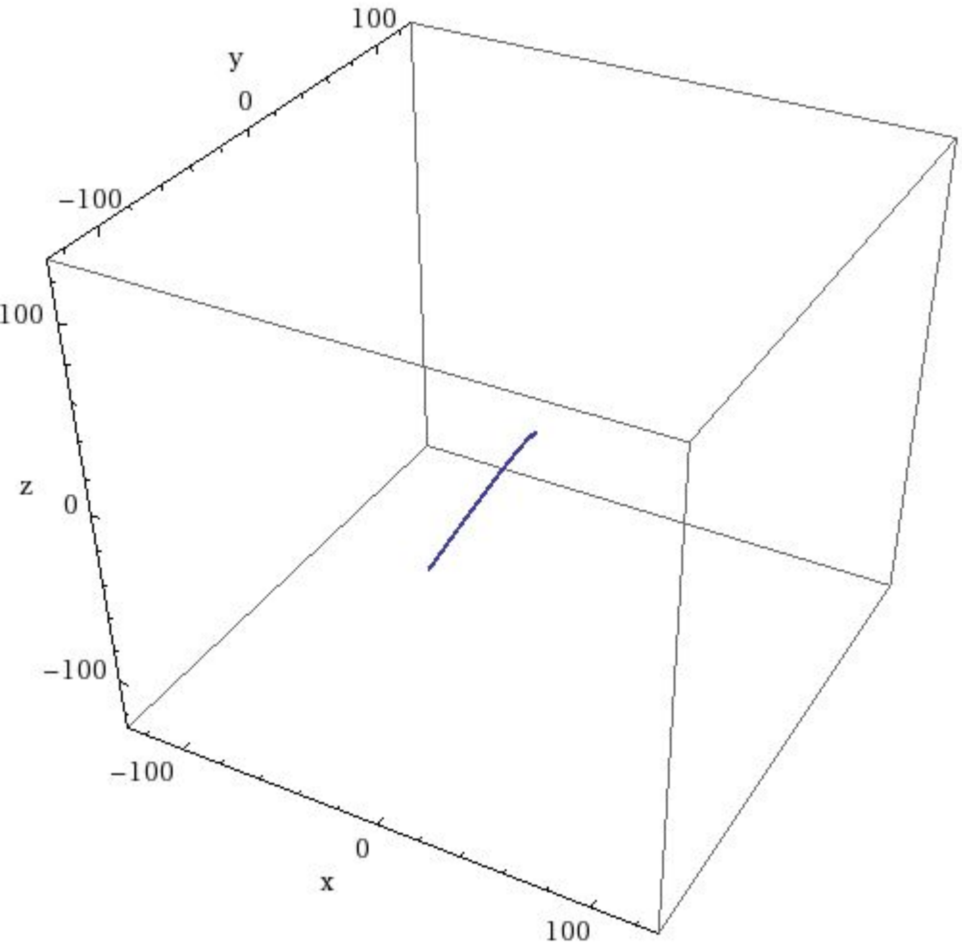
Averaging

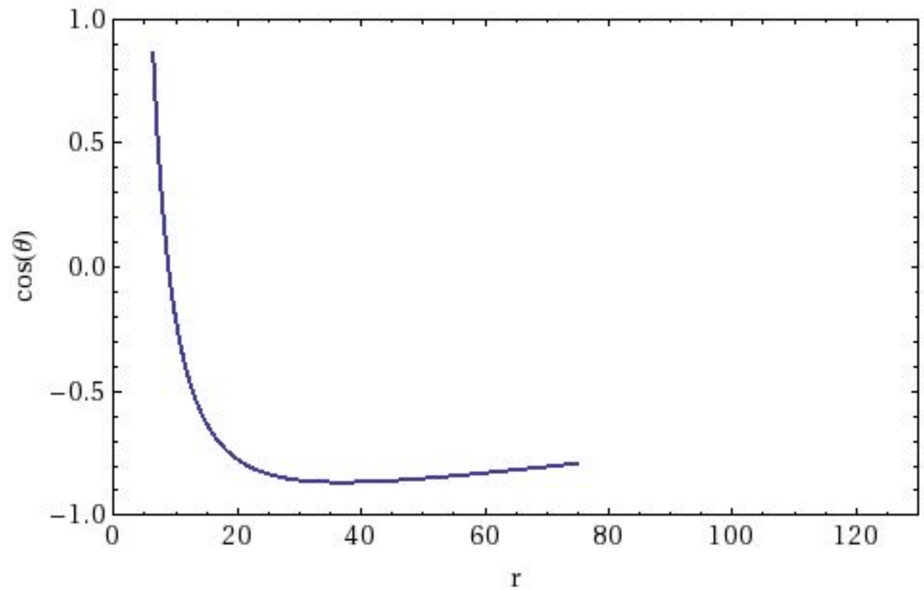
$$G^{(1)}_{\mathrm{sf}}(q_\theta, q_r, \mathbf{J}) = \sum_{k,n} G^{(1)}_{\mathrm{sf,kn}}(\mathbf{J}) e^{i(kq_\theta + nq_r)}$$

$$\Rightarrow \quad \left\langle G^{(1)}_{\mathrm{sf}}(q_\theta, q_r, \mathbf{J}) \right\rangle = G^{(1)}_{\mathrm{sf},00}(\mathbf{J})$$

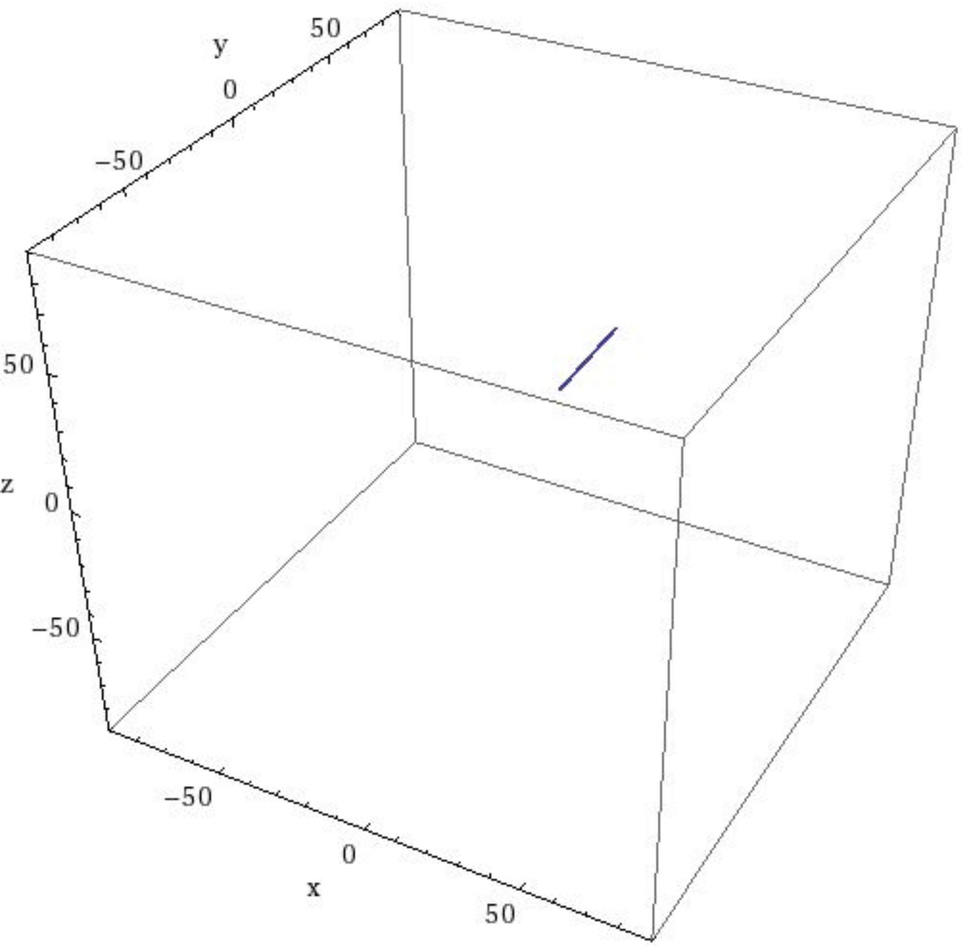
Not true when $k\omega_\theta + n\omega_r \approx 0$

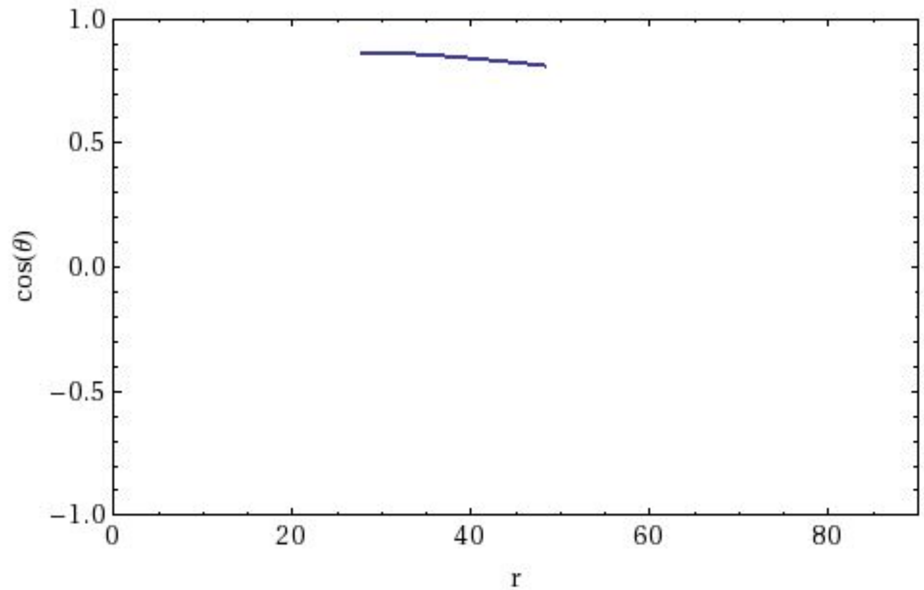
Generic evolution





Resonant orbit





Action-angle variables ***with*** tidal perturber

$$\frac{dq_i}{d\tau} = \omega_i(\mathbf{J}) + \epsilon\, g^{(1)}_{i,\mathrm{td}}(\underline{q_\phi}, q_\theta, q_r, \mathbf{J}) + \eta\, g^{(1)}_{k,\mathrm{sf}}(q_\theta, q_r, \mathbf{J}) + \mathcal{O}(\eta^2, \epsilon^2, \eta\epsilon)$$

$$\epsilon = M_\star M^2\, \mathrm{x}_\star / R^3$$

$$\frac{dJ_i}{d\tau} = \boxed{\epsilon\, G^{(1)}_{i,\mathrm{td}}(\underline{q_\phi}, q_\theta, q_r, \mathbf{J})} + \eta\, G^{(1)}_{i,\mathrm{sf}}(q_\theta, q_r, \mathbf{J}) + \mathcal{O}(\eta^2, \epsilon^2, \eta\epsilon)$$

Tidal resonance

$$G_i^{(1)}(q_\phi, q_\theta, q_r, \mathbf{J}) = \sum_{m,k,n} G_{i,mkn}^{(1)}(\mathbf{J}) e^{i(mq_\phi + kq_\theta + nq_r)}$$

$$\omega_{mkn} := m\omega_\phi + k\omega_\theta + n\omega_r = 0$$

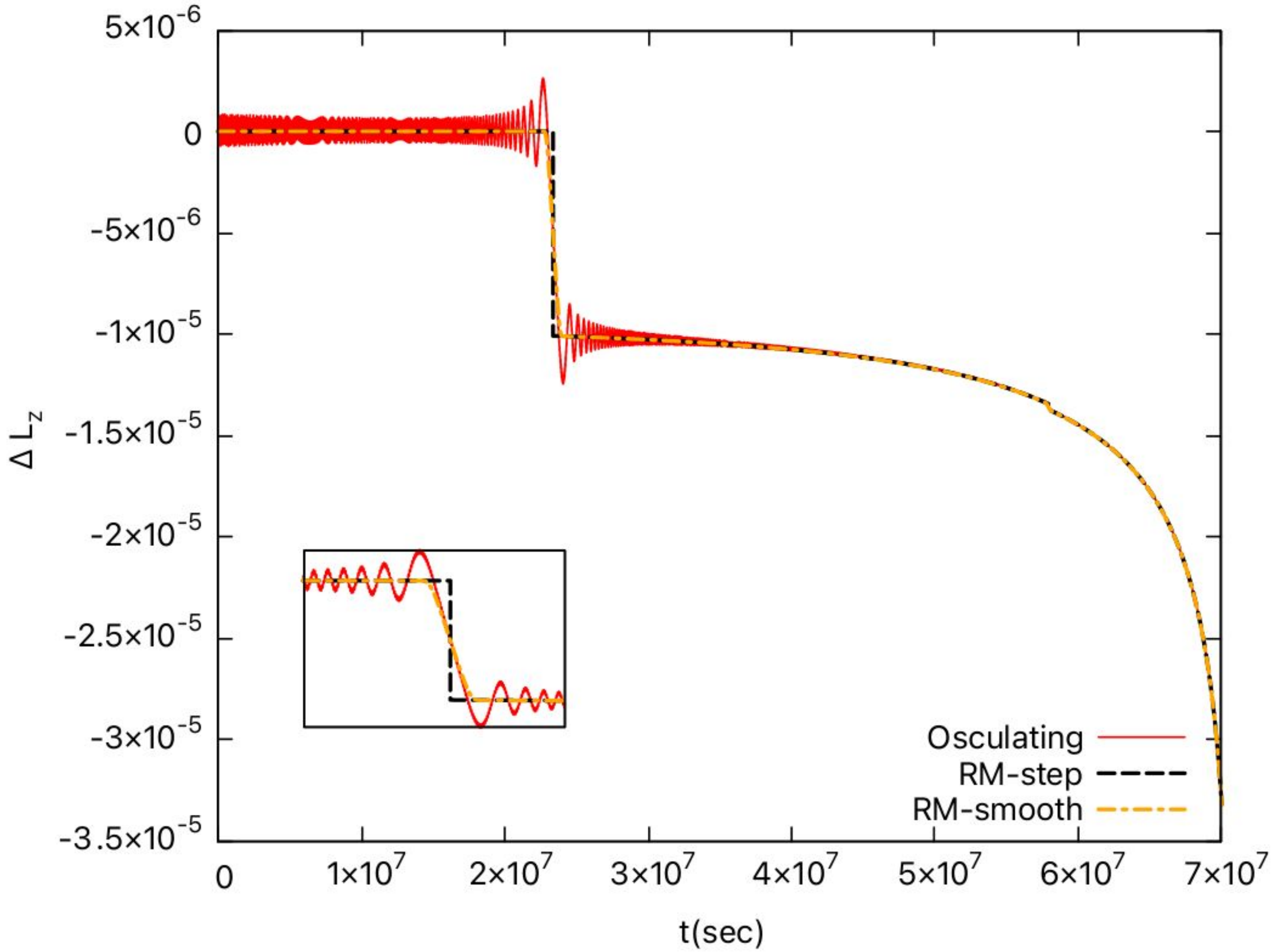
Trends

Jump sizes

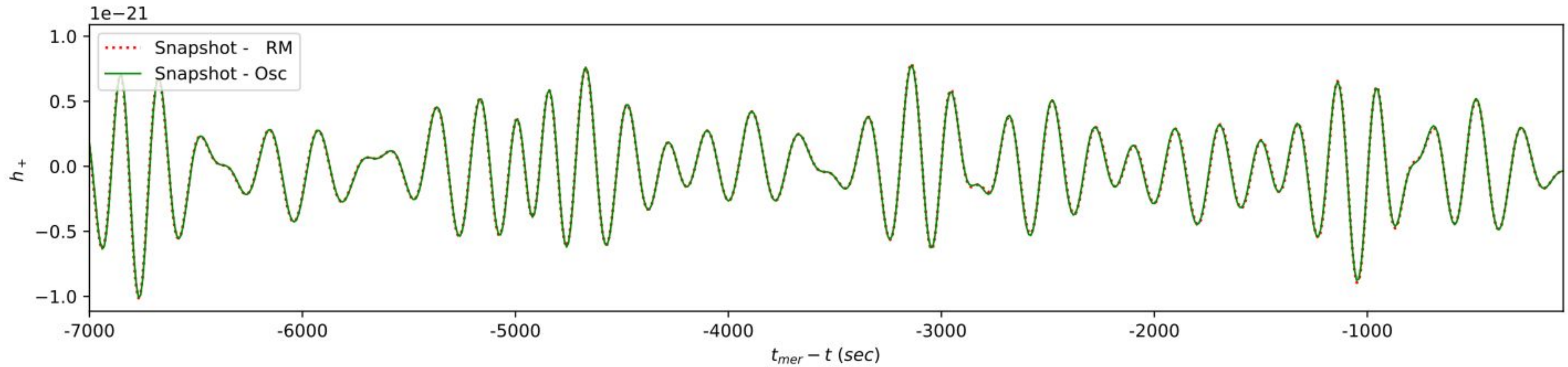
- larger eccentricities → larger jumps
- higher inclination → larger jump for Q, but jump for L decreases
- higher spin → larger jumps (if retrograde), smaller jumps (if prograde)

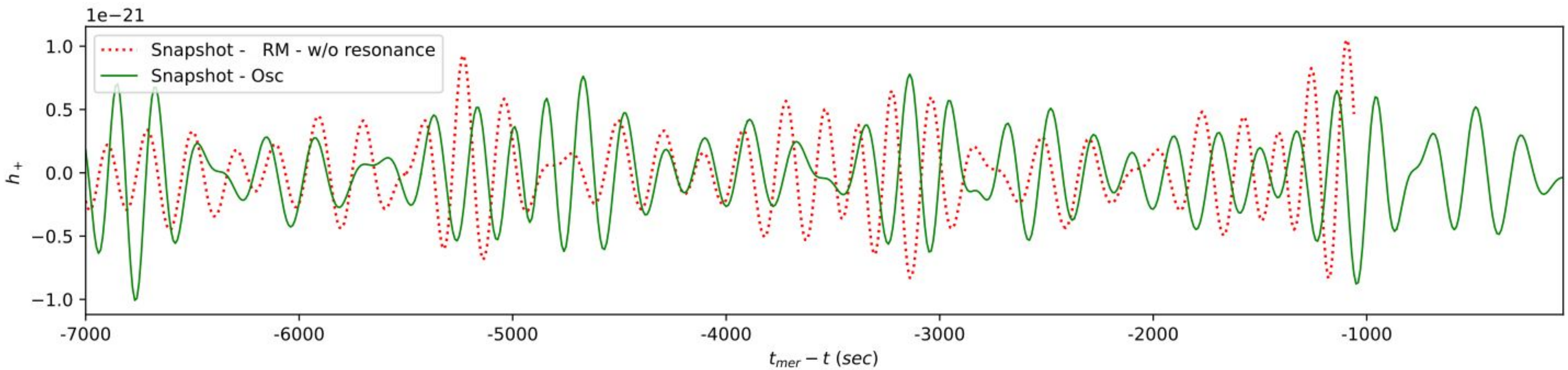
Effect on the waveform also determined when the resonance occurs: need signal before and after resonance!

Change in "constants of motion"



Waveform validation



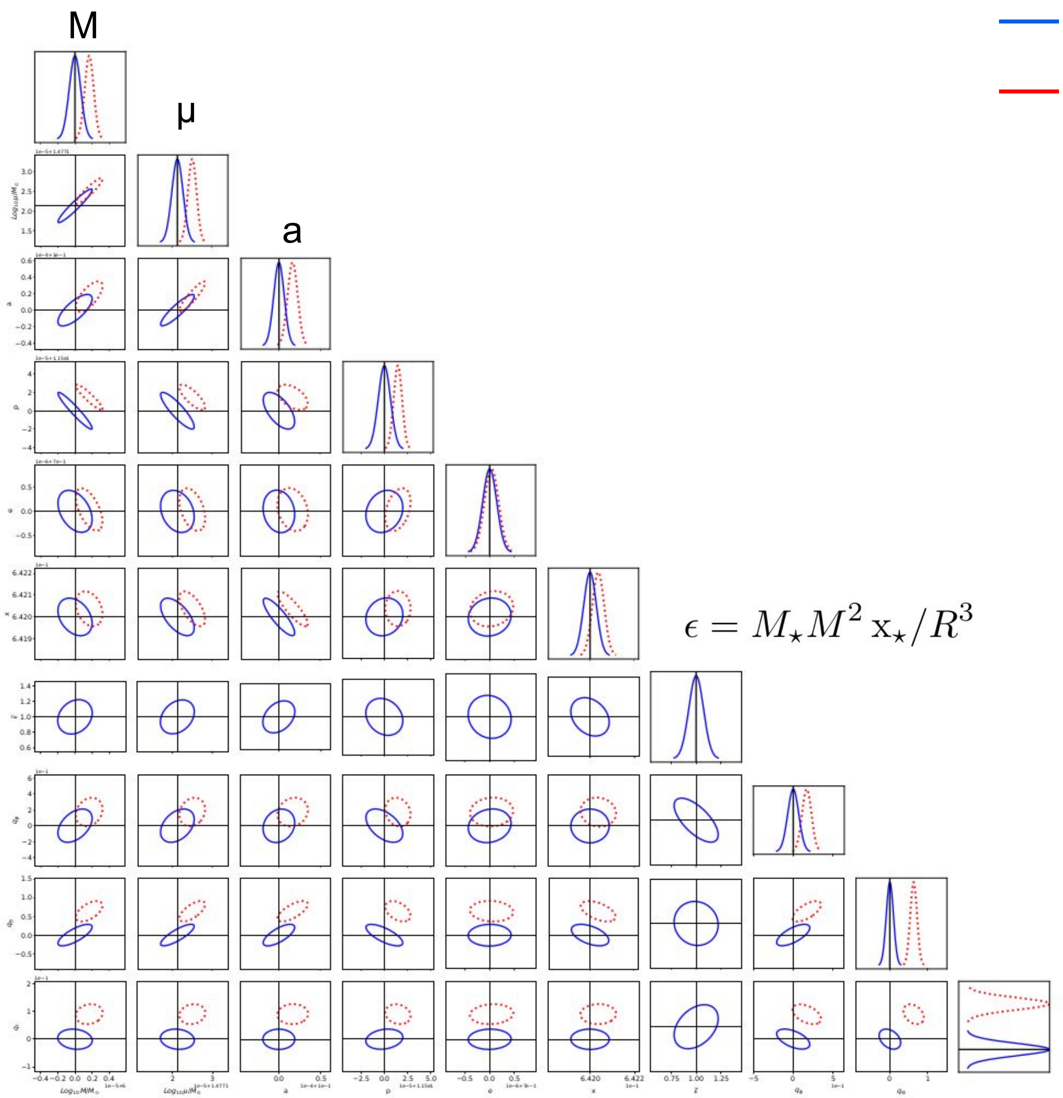


$M = 4 \times 10^6 M_\odot , \mu = M_\star = 30 M_\odot , R = 5AU \longrightarrow \epsilon \sim 2 \times 10^{-13}$

EMRIs have many parameters

$$\left(\log_{10} \frac{M}{M_\odot}, \log_{10} \frac{\mu}{M_\odot}, a, p, e, \mathrm{x}, q_r, q_\theta, q_\phi, \widetilde{\epsilon} \right)$$

If not accounted for, obtain wrong parameters



— Resonance Model

— Not accounting for resonance

$$\epsilon = M_{\star} M^2 \, \mathrm{x}_{\star} / R^3$$

Extra tidal parameter does not harm others