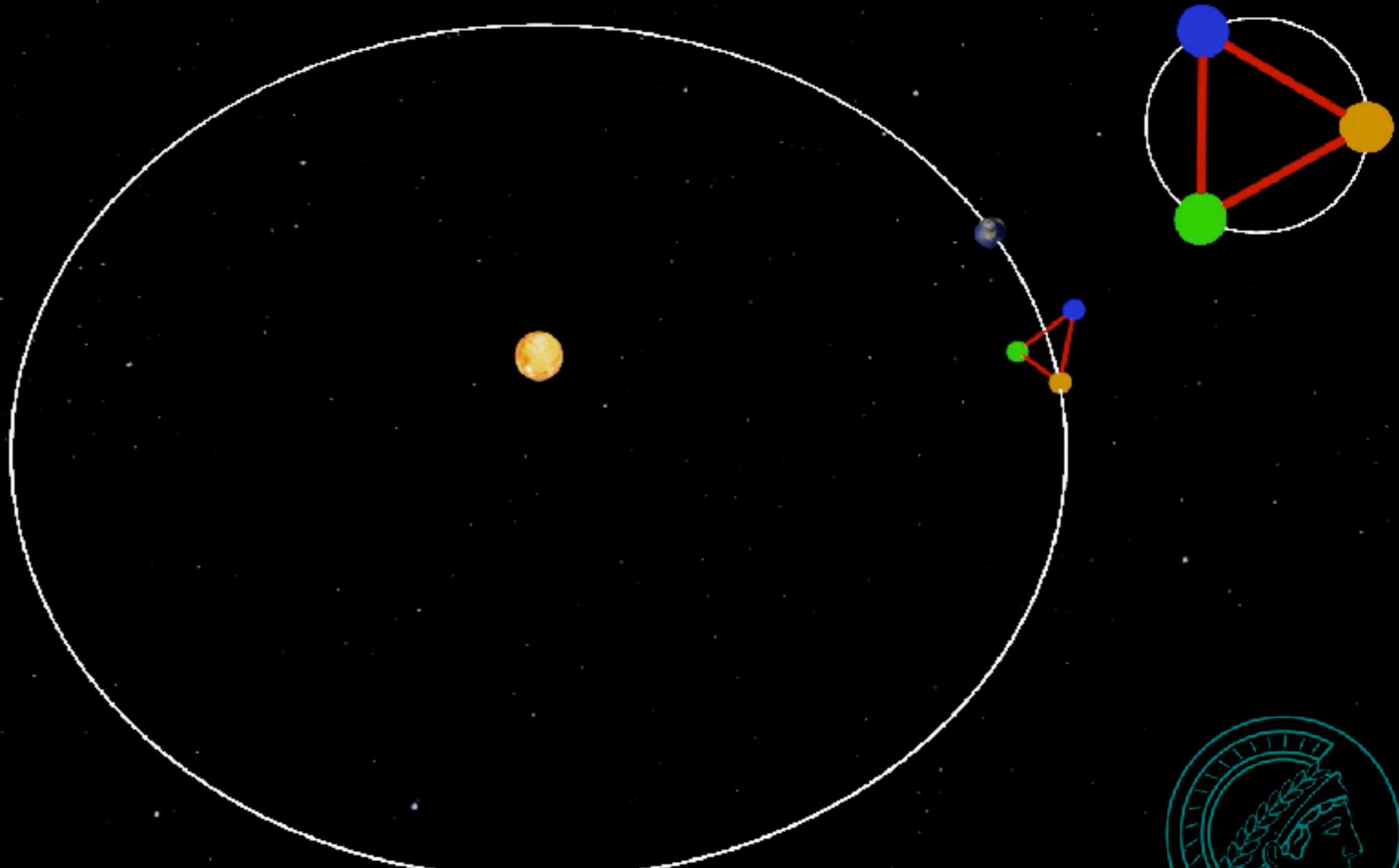


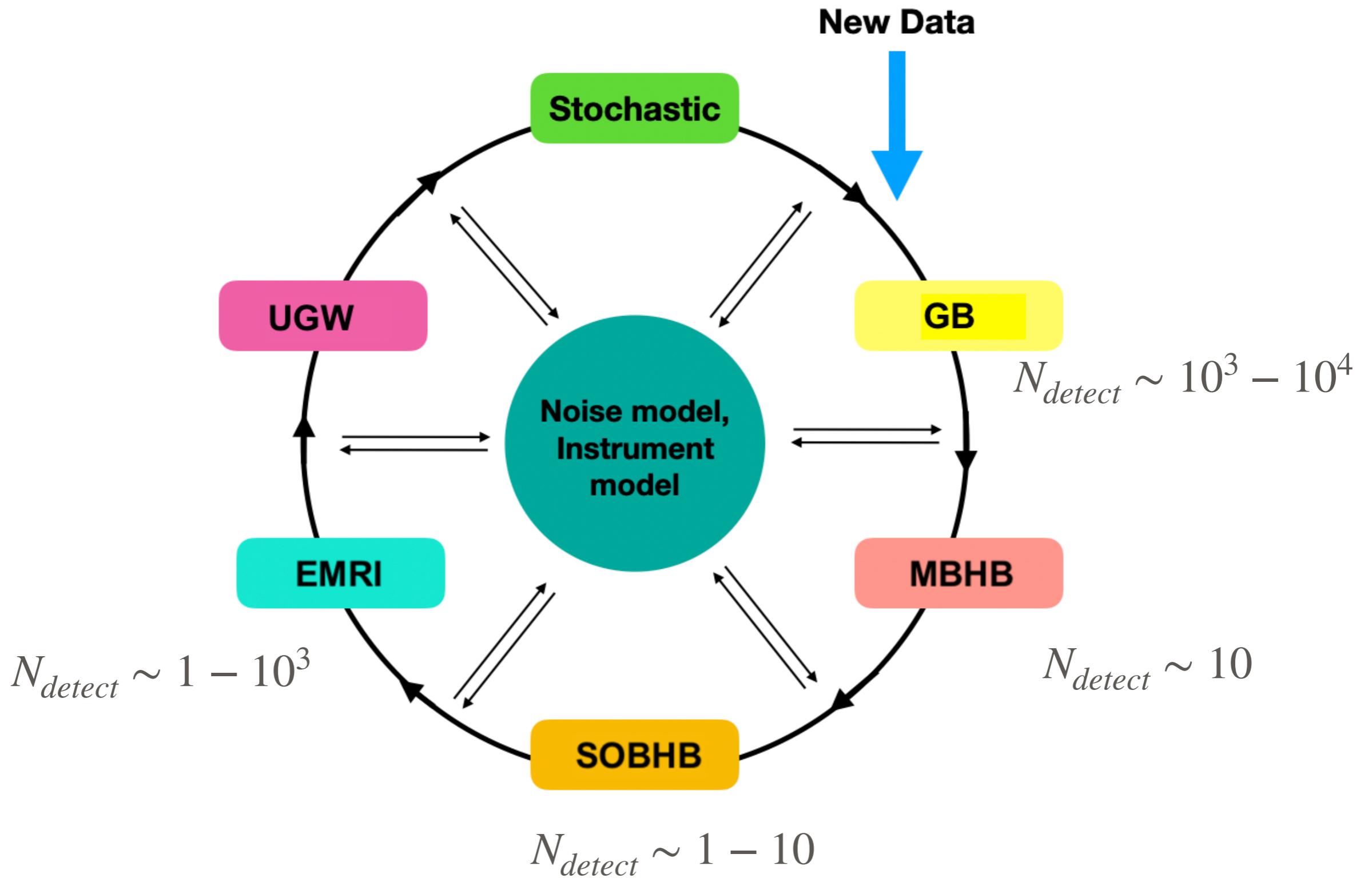
Testing General Relativity with LISA observations

Lorenzo Speri

Max Planck Institute for Gravitational physics (AEI Potsdam)

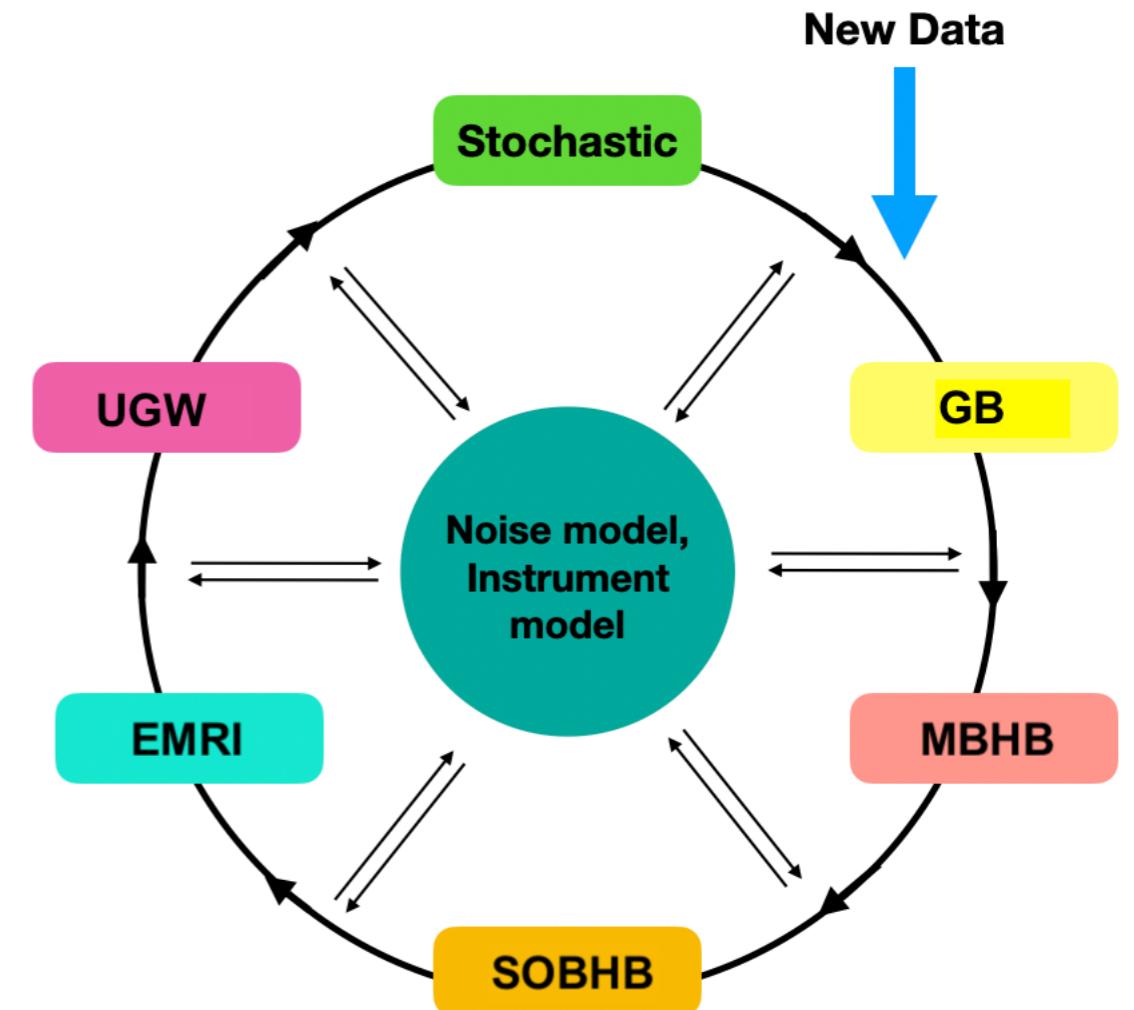


The LISA Global Fit



The LISA Global Fit

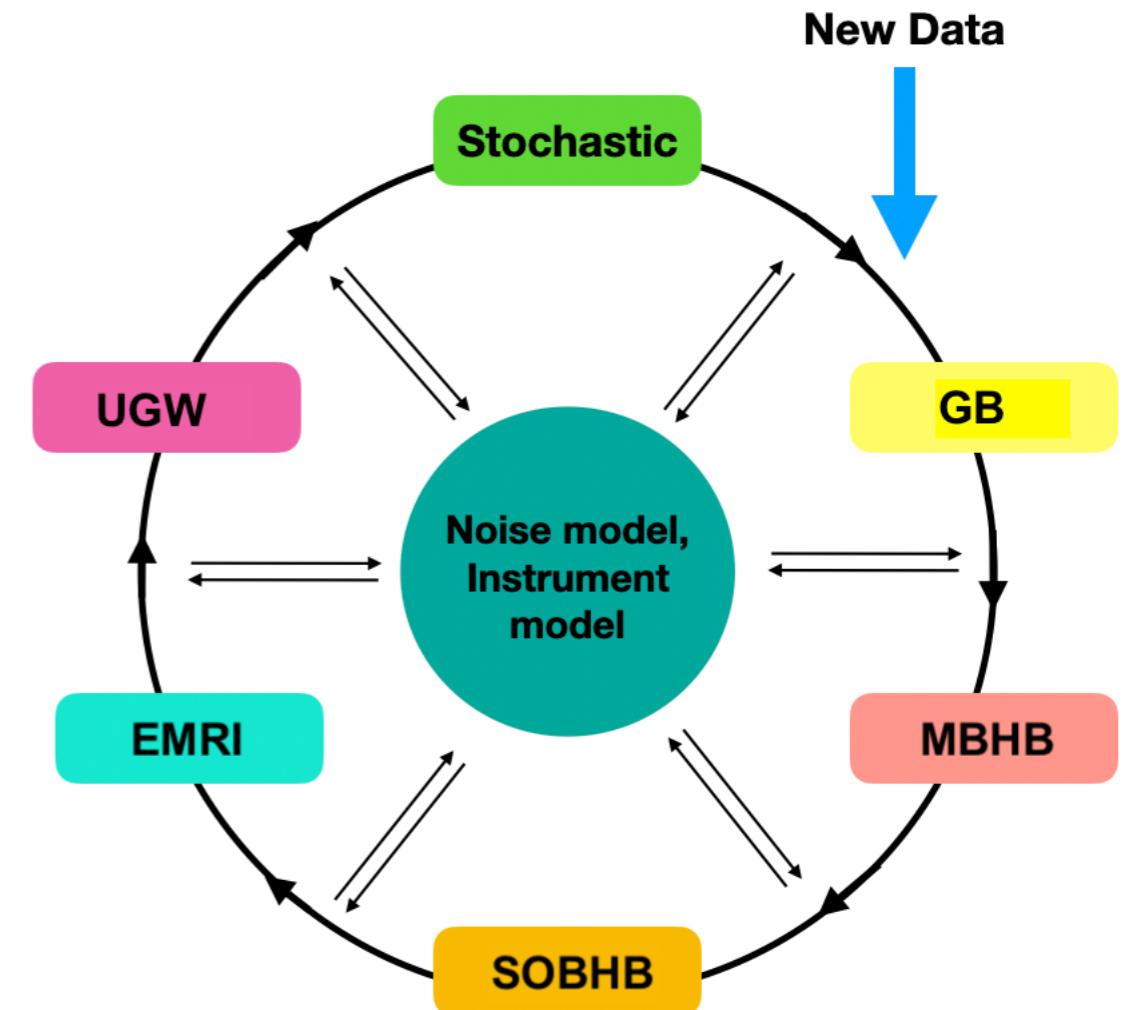
- Fit signals & noise



Littenberg+ 2020

The LISA Global Fit

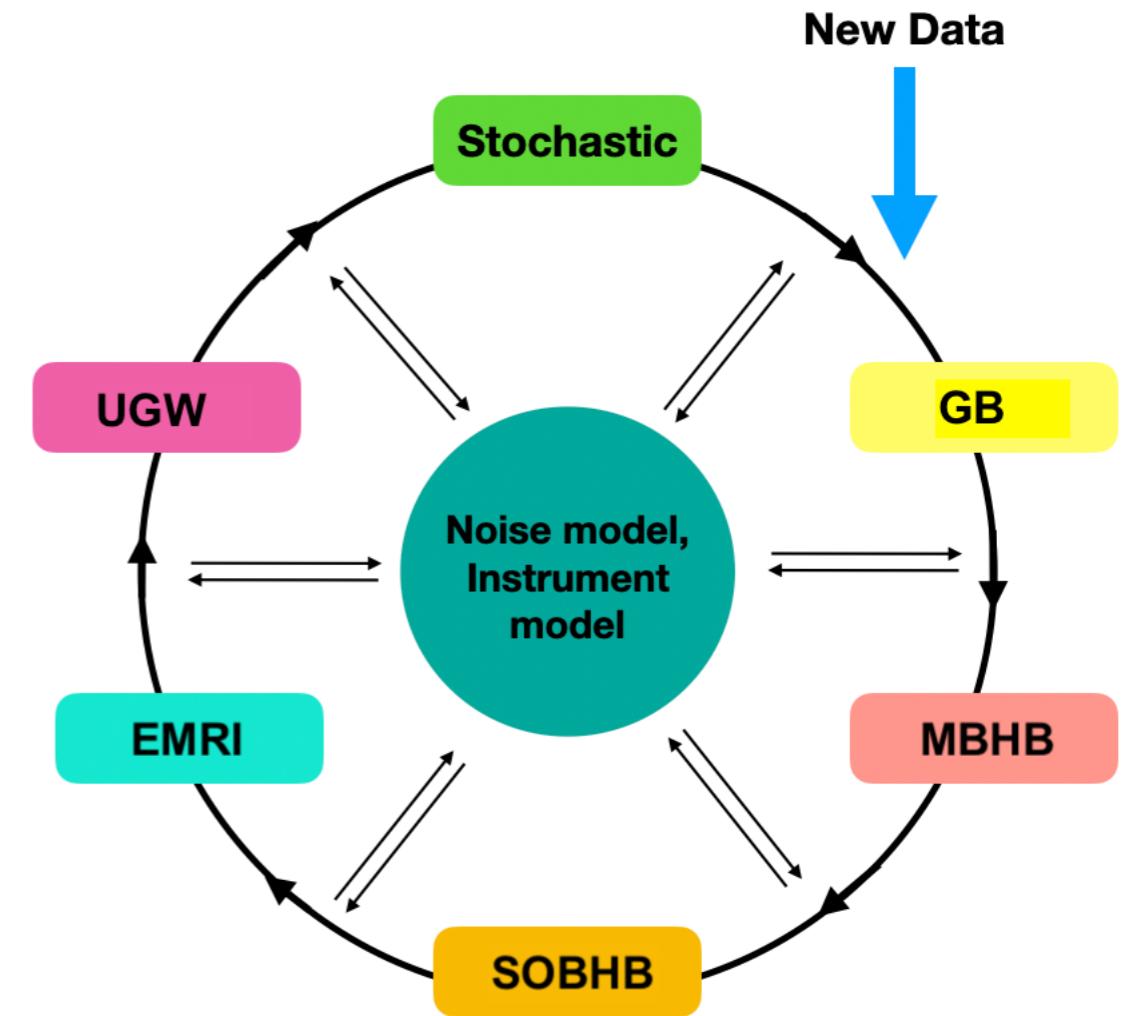
- Fit signals & noise
- Gibbs Sampling sources



Littenberg+ 2020

The LISA Global Fit

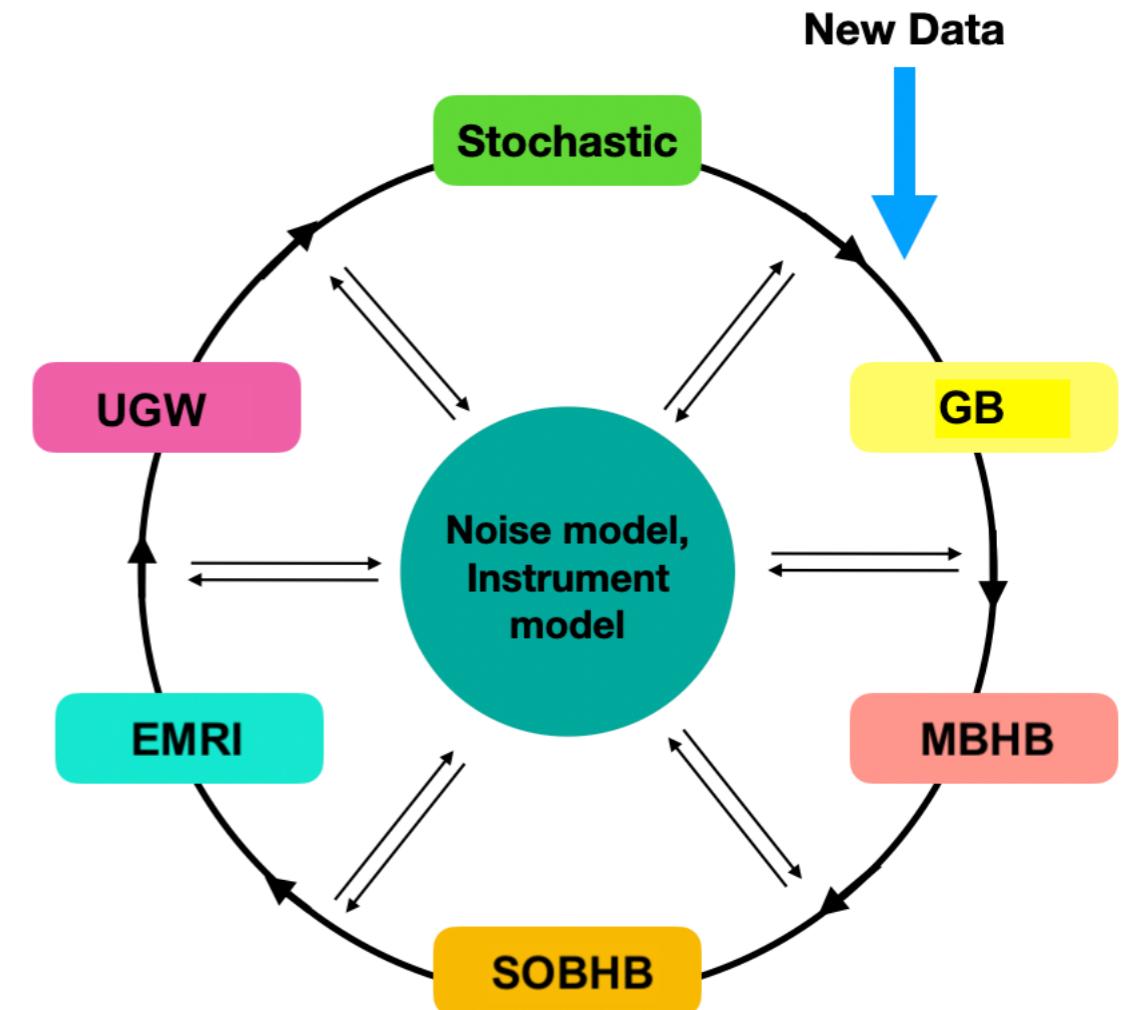
- Fit signals & noise
- Gibbs Sampling sources
- Source and noise variety requires different strategies



Littenberg+ 2020

The LISA Global Fit

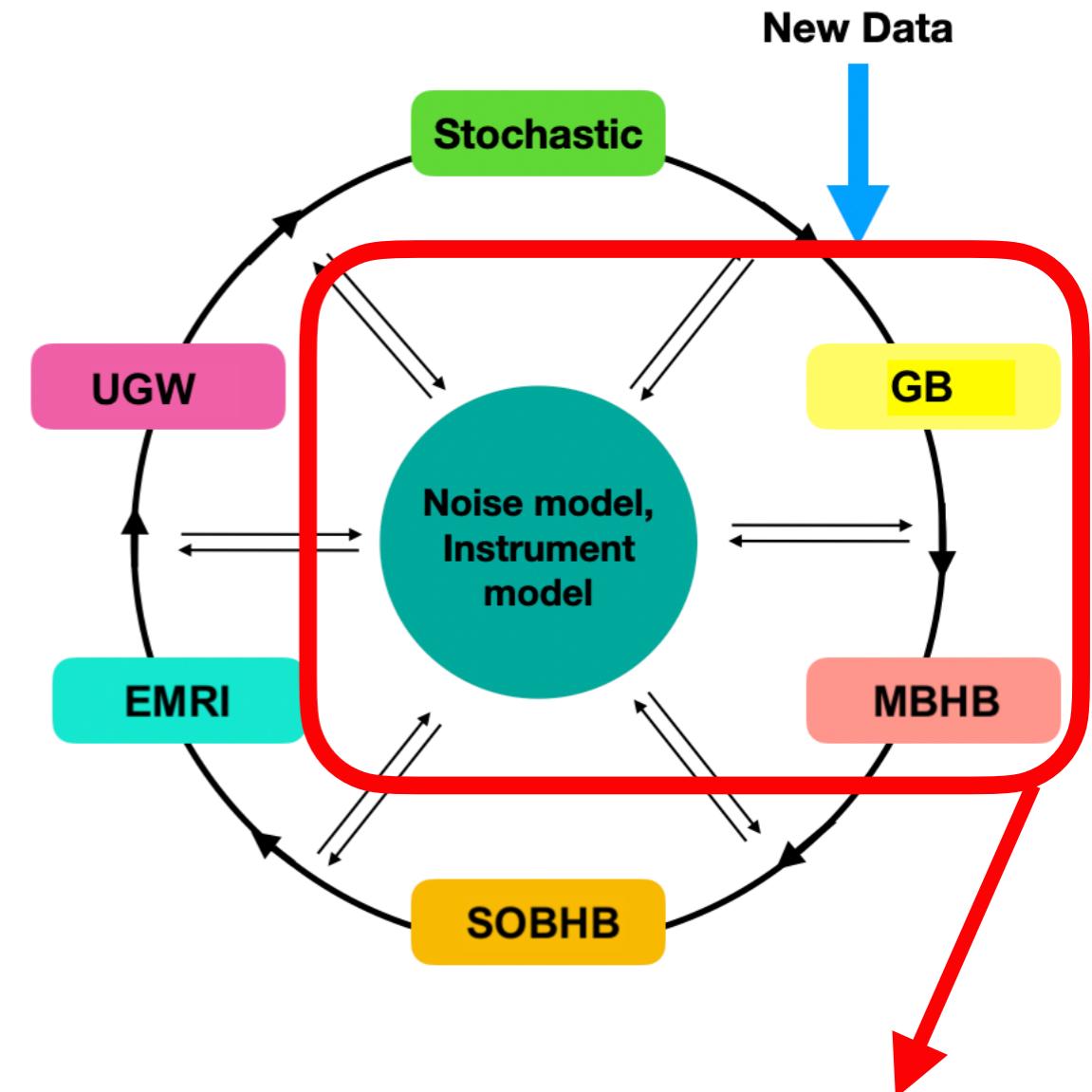
- Fit signals & noise
- Gibbs Sampling sources
- Source and noise variety requires different strategies
- Difficulty in finding the maximum Likelihood



Littenberg+ 2020

The LISA Global Fit

- Fit signals & noise
- Gibbs Sampling sources
- Source and noise variety requires different strategies
- Difficulty in finding the maximum Likelihood



Prototype Global Analysis of LISA Data with Multiple Source Types

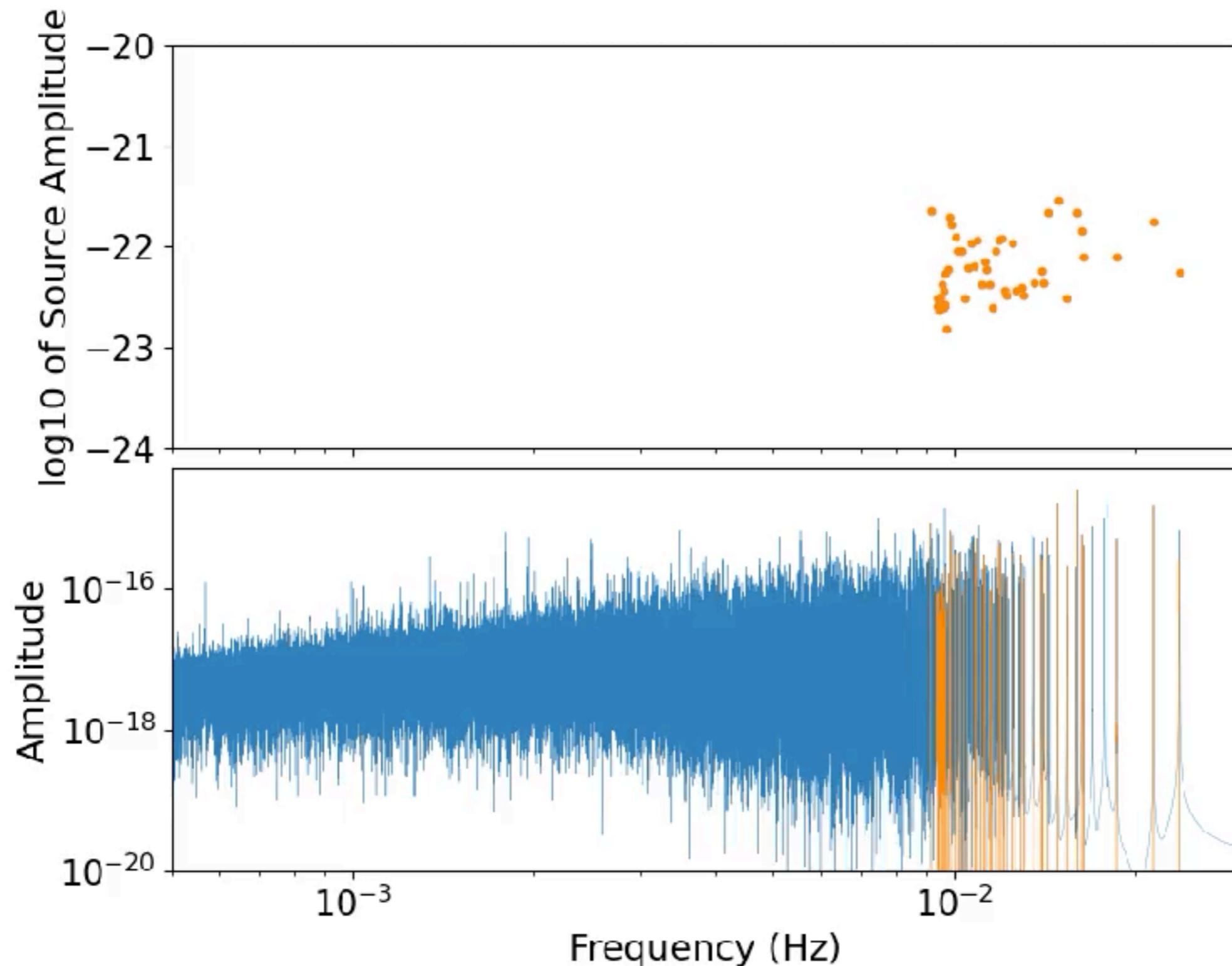
Tyson B. Littenberg

NASA Marshall Space Flight Center, Huntsville, Alabama 35811, USA

Neil J. Cornish

eXtreme Gravity Institute, Department of Physics,

The LISA Global Fit



credit: M. Katz

Data-analyst wish list for Beyond GR waveforms

Fast to evaluate

Data-analyst wish list for Beyond GR waveforms

Fast to evaluate



otherwise
it will not be used



Data-analyst wish list for Beyond GR waveforms

Fast to evaluate



otherwise
it will not be used

Unique features



Distinguish from
environments

Data-analyst wish list for Beyond GR waveforms

Fast to evaluate



otherwise
it will not be used

Unique features



Distinguish from
environments

Few extra-dimensions
or effective models

Data-analyst wish list for Beyond GR waveforms



Few extra-dimensions
or effective models



easier sampling

Extreme Mass Ratio Inspirals

EMRIs are ideal signals to perform tests of General Relativity

Extreme Mass Ratio Inspirals

EMRIs are ideal signals to perform tests of General Relativity

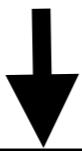
What do we need?

- GR Waveforms → Fast EMRI Waveforms
- Beyond GR Waveforms → Susanna & Thomas talks
- Bayesian inference pipeline → Eryn, MCMC

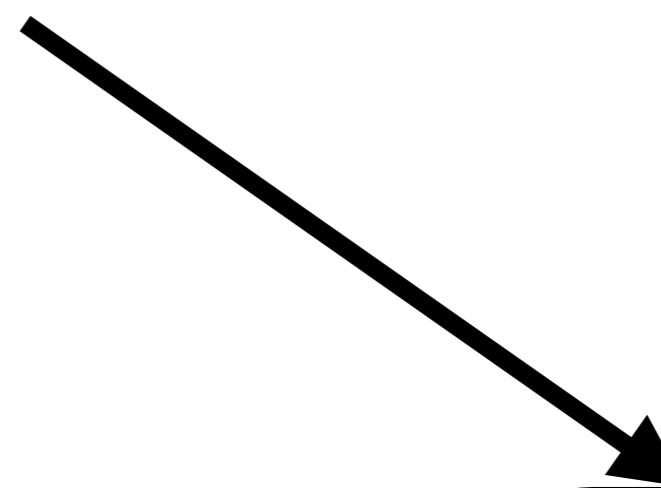
Fast EMRI Waveforms

$$h = \sum_{lmnk} S_{lmnk}(\theta, \phi) A_{lmnk}(p(t), e(t), x_I(t)) \times$$

$$\exp \left[-i\Phi_{mnk}(t) \right]$$



Trajectory



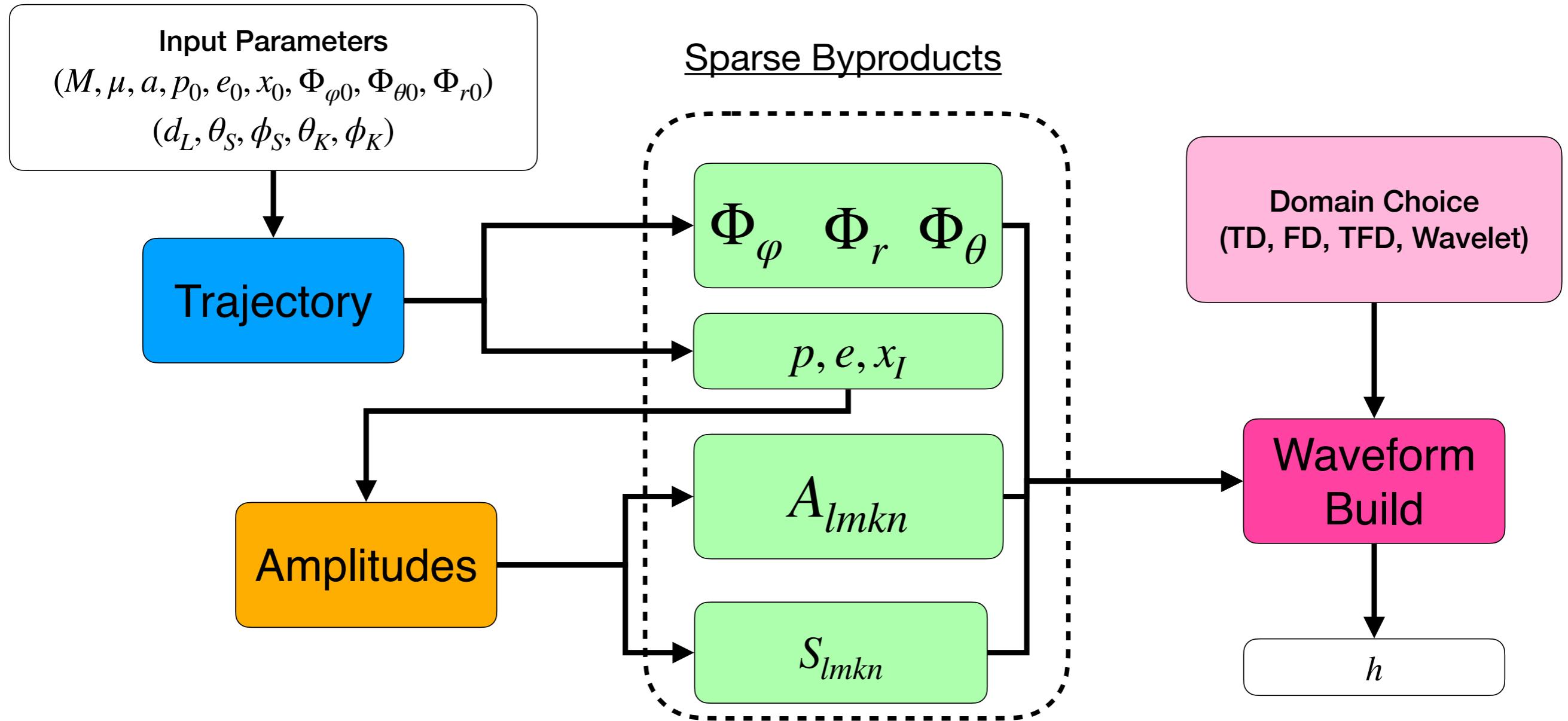
Amplitudes

Phase Evolution

$$\Phi_{mnk} = m \Phi_\varphi + n \Phi_r + k \Phi_\theta$$

Chua+ 2020
Katz+ 2021
Speri+ 2023

Fast EMRI Waveforms (FEW)

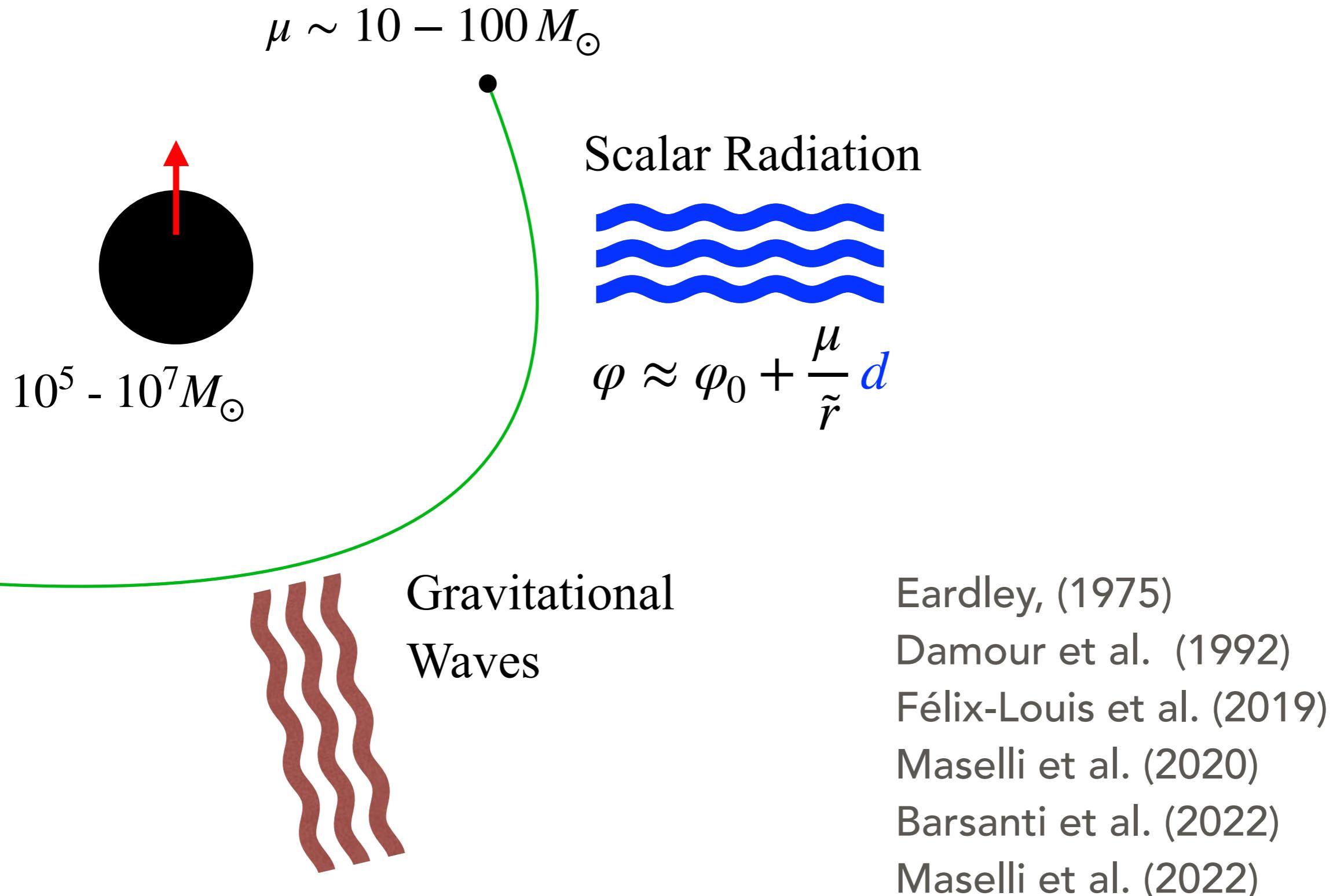


You can easily build your own beyond vacuum Waveform!

bhptoolkit.org/FastEMRIWaveforms

https://github.com/lorenzsp/GRAPPA_EMRI_tutorial/blob/main/BeyondVacuum-EMRIs.ipynb

Scalar fields in Extreme Mass Ratio Inspirals



Trajectory in FEW

$$\dot{\mathbf{j}} = \frac{\mu}{M} \left(\mathbf{j}_{\text{GW}} + d^2 \mathbf{j}_{\text{scalar}} \right), \quad J = (E, L)$$

Chebyshev interpolation

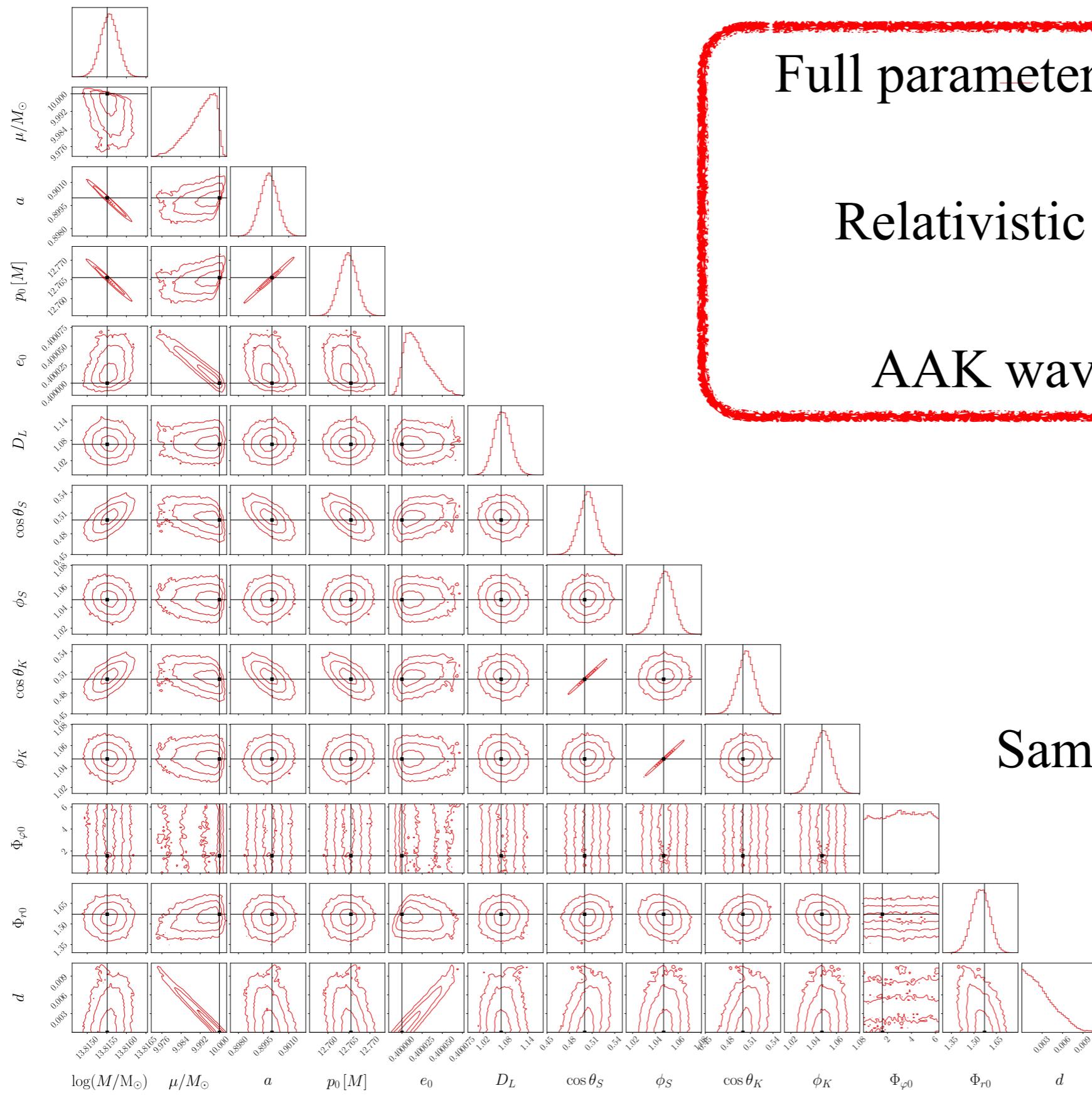
$a \in [-0.99, +0.99]$, 12 points

$e \in [0.0, 0.5]$, 12 points

$p \sim [p_{sep}, 100M]$, 16 points

relative precision of $\sigma[\dot{E}]/\dot{E} \sim 10^{-4}$

FEW + Bayesian analysis



Full parameter space: 13 dimensions

Relativistic adiabatic trajectory

+

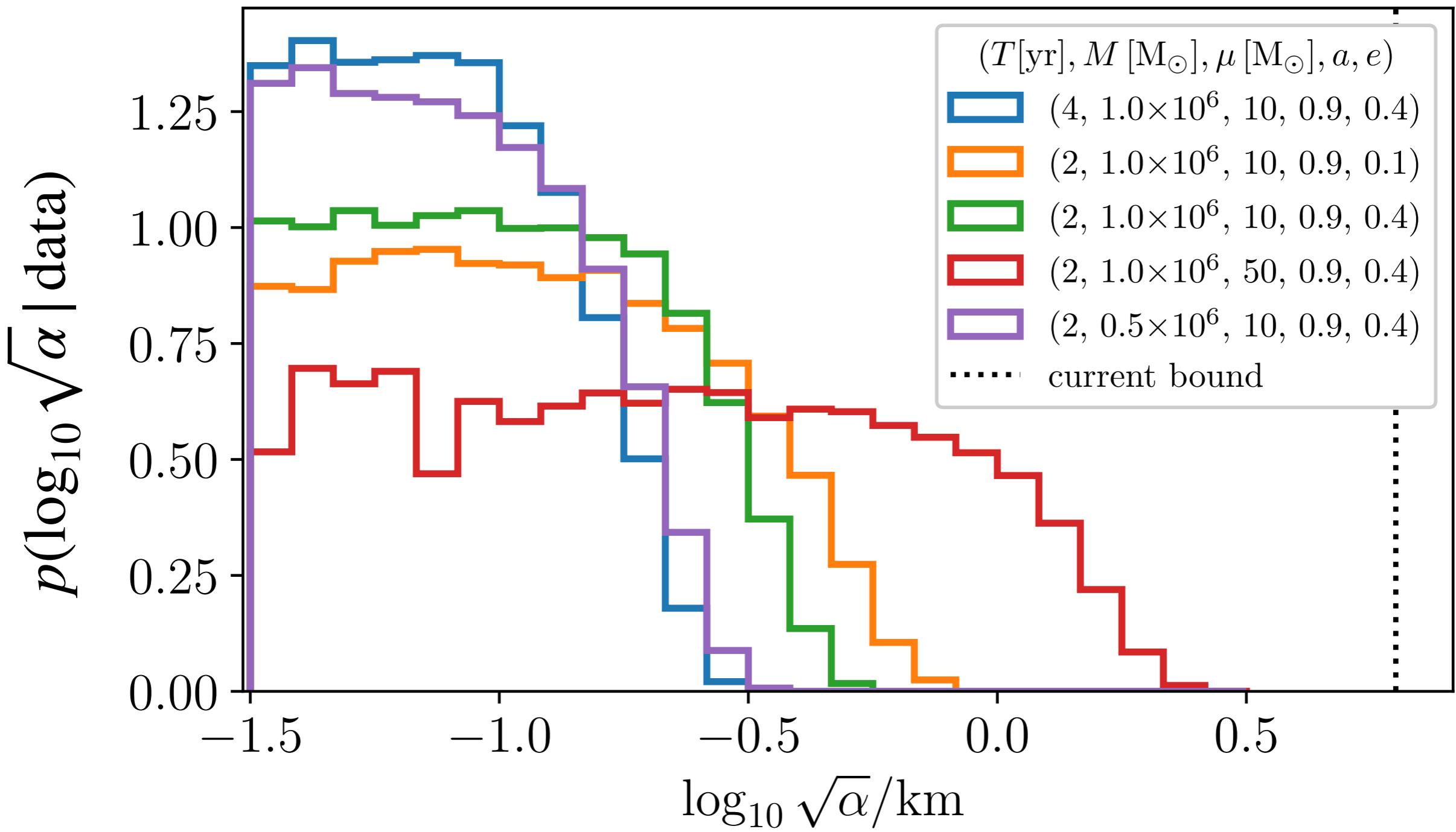
AAK waveform summation

Sample in scalar charge



Are EMRIs ideal for tests of GR?

SNR = 50



scalar-Gauss-Bonnet $\textcolor{blue}{d} \rightarrow \alpha$

Conclusions & Future outlook

- Fast EMRI Waveforms flexible to incorporate beyond GR effects
- EMRIs are indeed ideal for testing GR
- Bayes factor for $d \neq 0$
- Fit environmental effects and Beyond GR effects
- Tests of GR in the presence of multiple sources

Conclusions & Future outlook

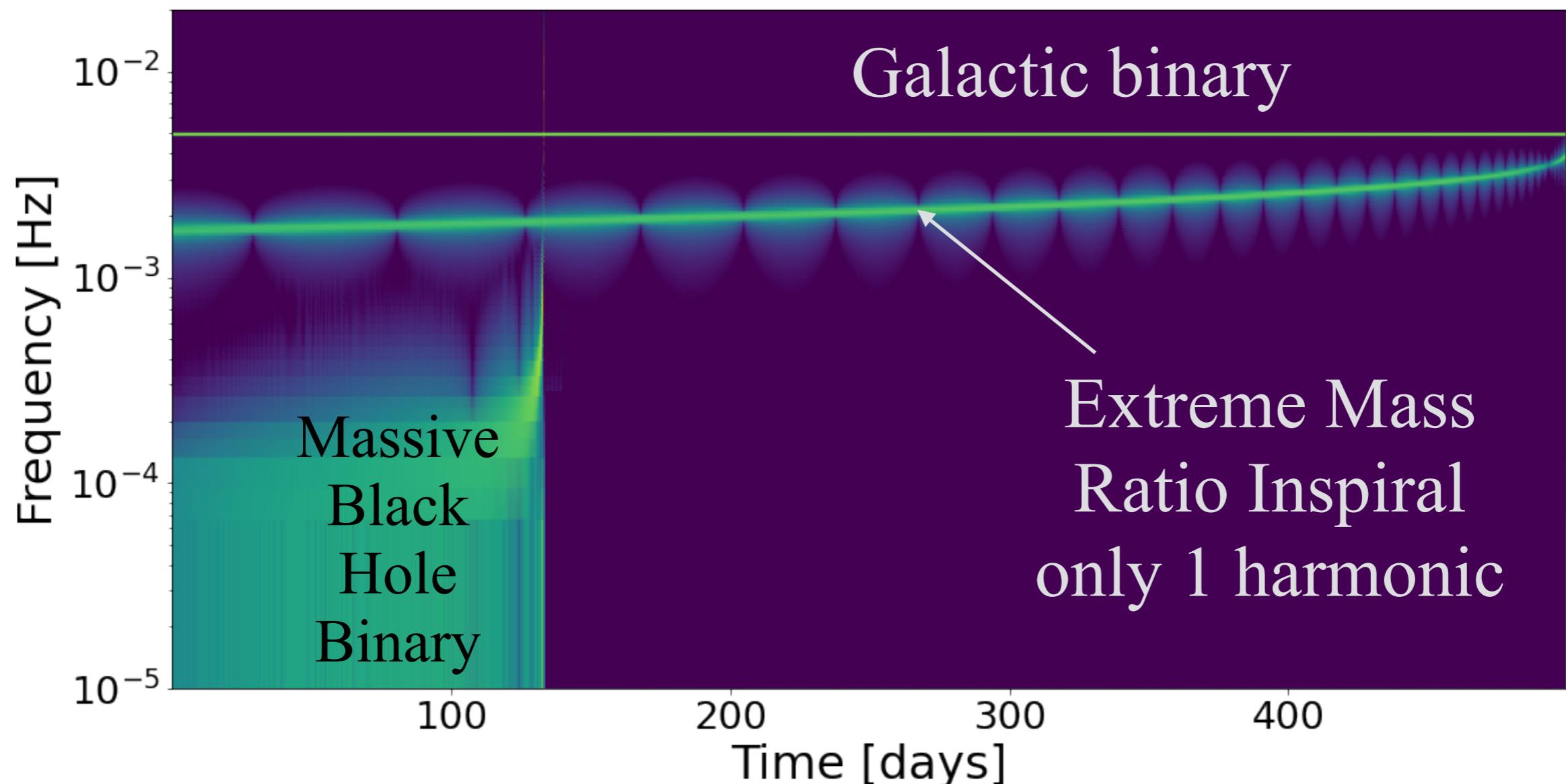
- Fast EMRI Waveforms flexible to incorporate beyond GR effects
- EMRIs are indeed ideal for testing GR
- Bayes factor for $d \neq 0$
- Fit environmental effects and Beyond GR effects
- Tests of GR in the presence of multiple sources

Remember my wish list!

- Fast to evaluate
- Unique features
- Few extra-dimensions

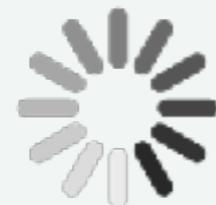
Extreme Mass Ratio Inspirals

EMRIs are ideal signals to perform tests of General Relativity



Status of EMRIs for Data analysis

Search



Parameter Estimation



Scalar fields in Extreme Mass Ratio Inspirals

$$S[g_{\mu\nu}, \varphi, \Psi] = S_0[g_{\mu\nu}, \varphi] + \alpha S_c[g_{\mu\nu}, \varphi] + S_m[g_{\mu\nu}, \varphi, \Psi]$$

$[S_0] = \text{Mass}^2$

$$\int d^4x \frac{\sqrt{-g}}{16\pi} \left(R - \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi \right)$$

Non-minimal coupling

$$- \int d\lambda m(\varphi) \sqrt{g_{\mu\nu} \frac{dy^\mu}{d\lambda} \frac{dy^\nu}{d\lambda}}$$

Eardley, ApJ 196 L59-62 (1975)

Damour &EF, CGQ 9, 9 (1992)

$$[S_c] = \text{Mass}^{2-n} \quad [\alpha] = \text{Mass}^n \quad n \geq 1$$

dimensionless deviation

$$\zeta = \frac{\alpha}{M^n} = \left(\frac{\mu}{M} \right)^n \frac{\alpha}{\mu^n}$$

Field Equations

$$G_{\mu\nu} = T_{\mu\nu}^{scal} \longrightarrow \frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi - g_{\mu\nu}(\partial\varphi)^2 \sim \cancel{\mathcal{O}(\varphi^2)}$$

$$\cancel{+ \frac{\alpha}{\sqrt{-g}} \frac{\delta S_c}{\delta g^{\mu\nu}}} \longrightarrow \frac{\alpha S_c}{M^n} S_0$$

$$\frac{\alpha}{\sqrt{-g}} \frac{\delta S_c}{\delta g^{\mu\nu}} \sim \frac{\alpha}{M^n} G_{\mu\nu}$$

$$+ T_{\mu\nu}^p \longrightarrow \int m(\varphi) \frac{\delta^{(4)}(x - y_p(\lambda))}{\sqrt{-g}} \frac{dy_p^\mu}{d\lambda} \frac{dy_p^\nu}{d\lambda} d\lambda$$

Scalar Field Equations

$$\square \varphi = 16\pi \int m'(\varphi) \frac{\delta^{(4)}(x - y_p(\lambda))}{\sqrt{-g}} d\lambda - \alpha \frac{8\pi \delta S_c}{\sqrt{-g} \delta \varphi}$$

~~$\frac{8\pi \delta S_c}{\sqrt{-g} \delta \varphi}$~~

$$\sim \frac{\alpha}{M^n} \square \varphi$$

In the body frame

$$\varphi = \varphi_0 + \frac{\mu d}{\tilde{r}} + \mathcal{O}\left(\frac{\mu^2}{\tilde{r}^2}\right)$$

Matching with scalar field outside of
the worldtube

$$m(\varphi_0) = \mu \quad \frac{m'(\varphi_0)}{m(\varphi_0)} = -\frac{d}{4}$$

EMRI dynamics with scalar fields

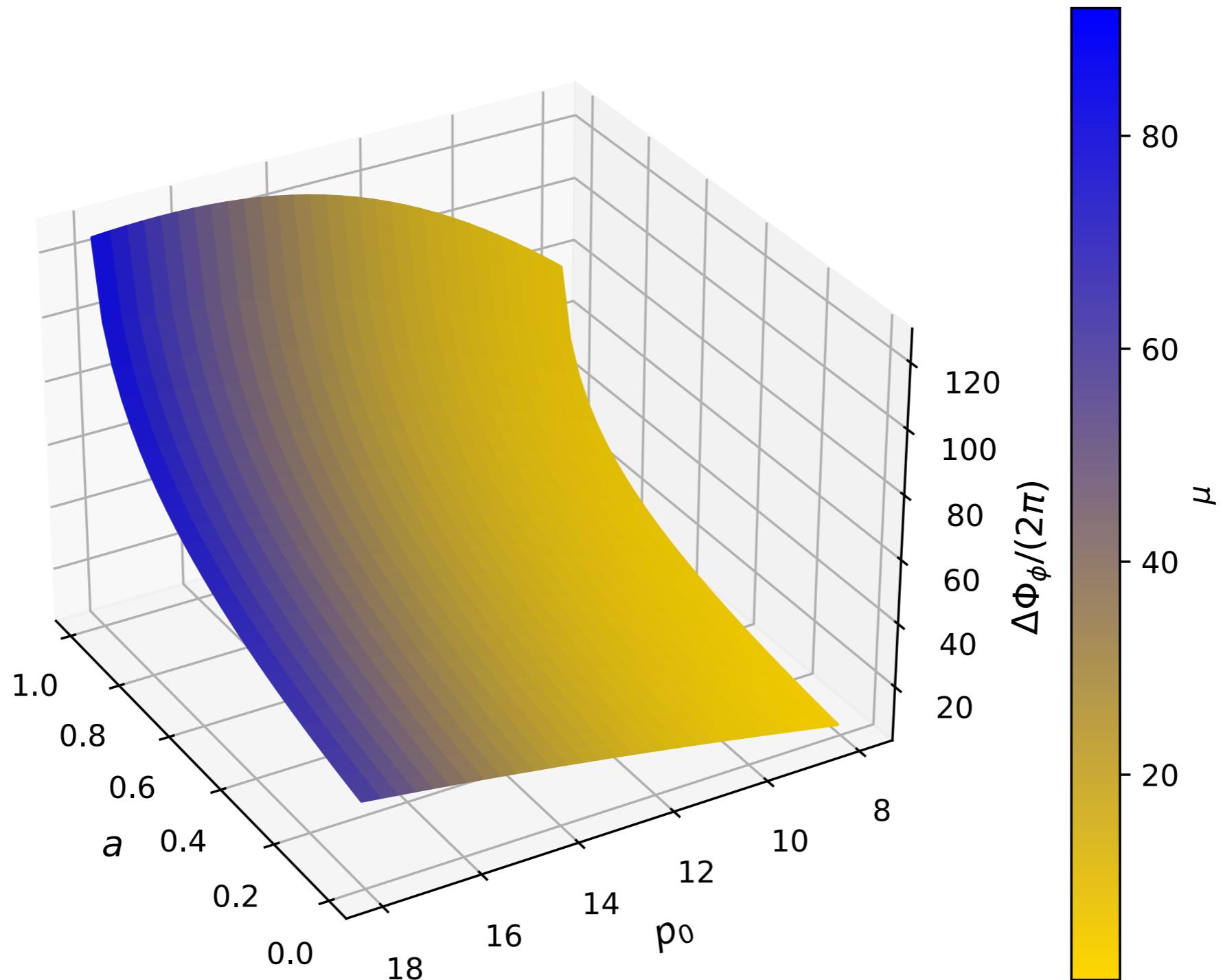
$$G_{\mu\nu} = 8\pi\mu \int \frac{\delta^{(4)}(x - y_p(\lambda))}{\sqrt{-g}} \frac{dy_p^\mu}{d\lambda} \frac{dy_p^\nu}{d\lambda} d\lambda$$

$$\square \varphi = -4\pi d\mu \int \frac{\delta^{(4)}(x - y_p(\lambda))}{\sqrt{-g}} d\lambda$$



$$\dot{E}_{\text{tot}} = \frac{\mu}{M} \left(\dot{E}_{\text{GW}} + d^2 \dot{E}_{\text{scalar}} \right)$$

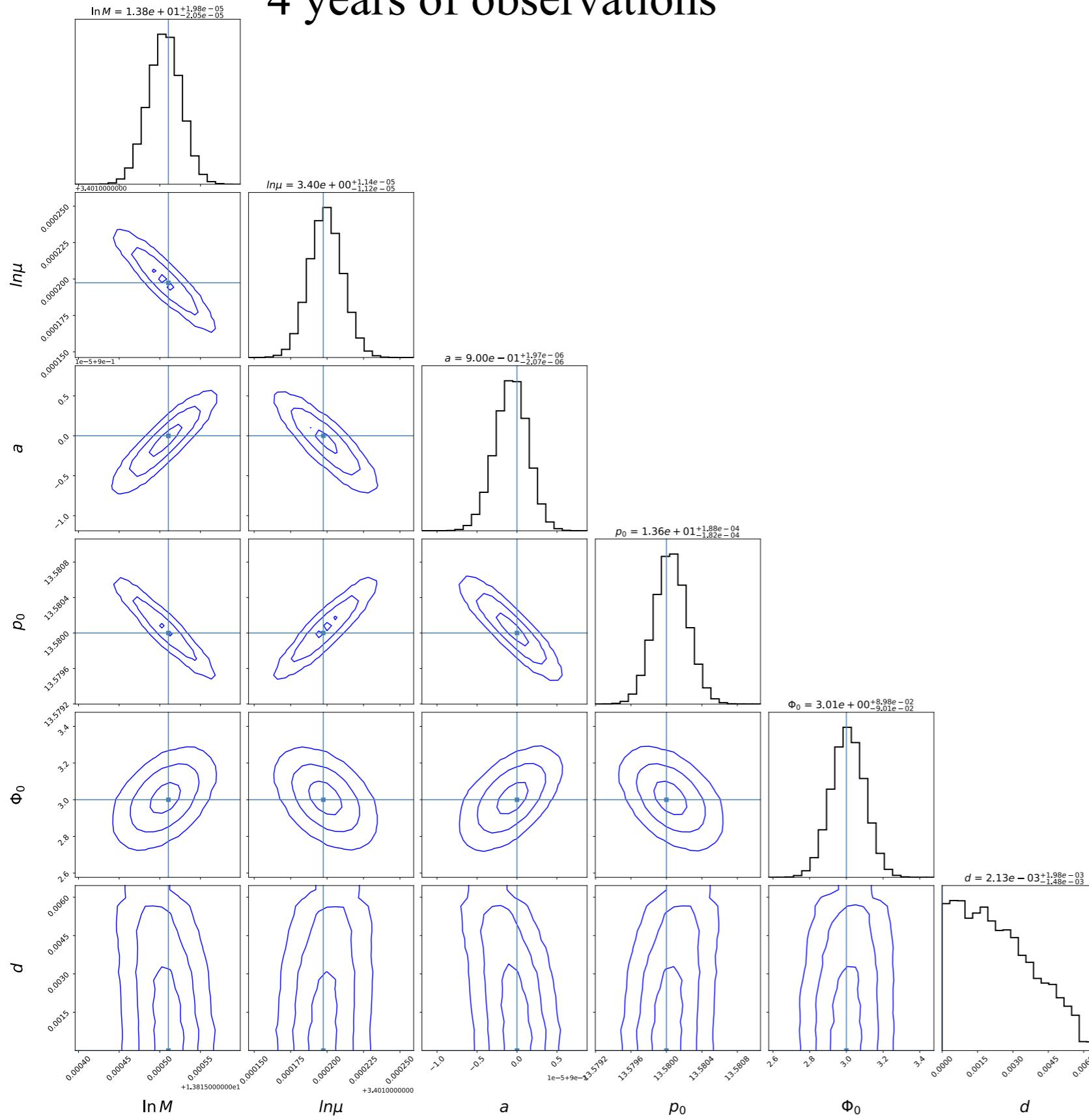
Circular Equatorial Kerr: Dephasing for fixed 4 yrs inspiral



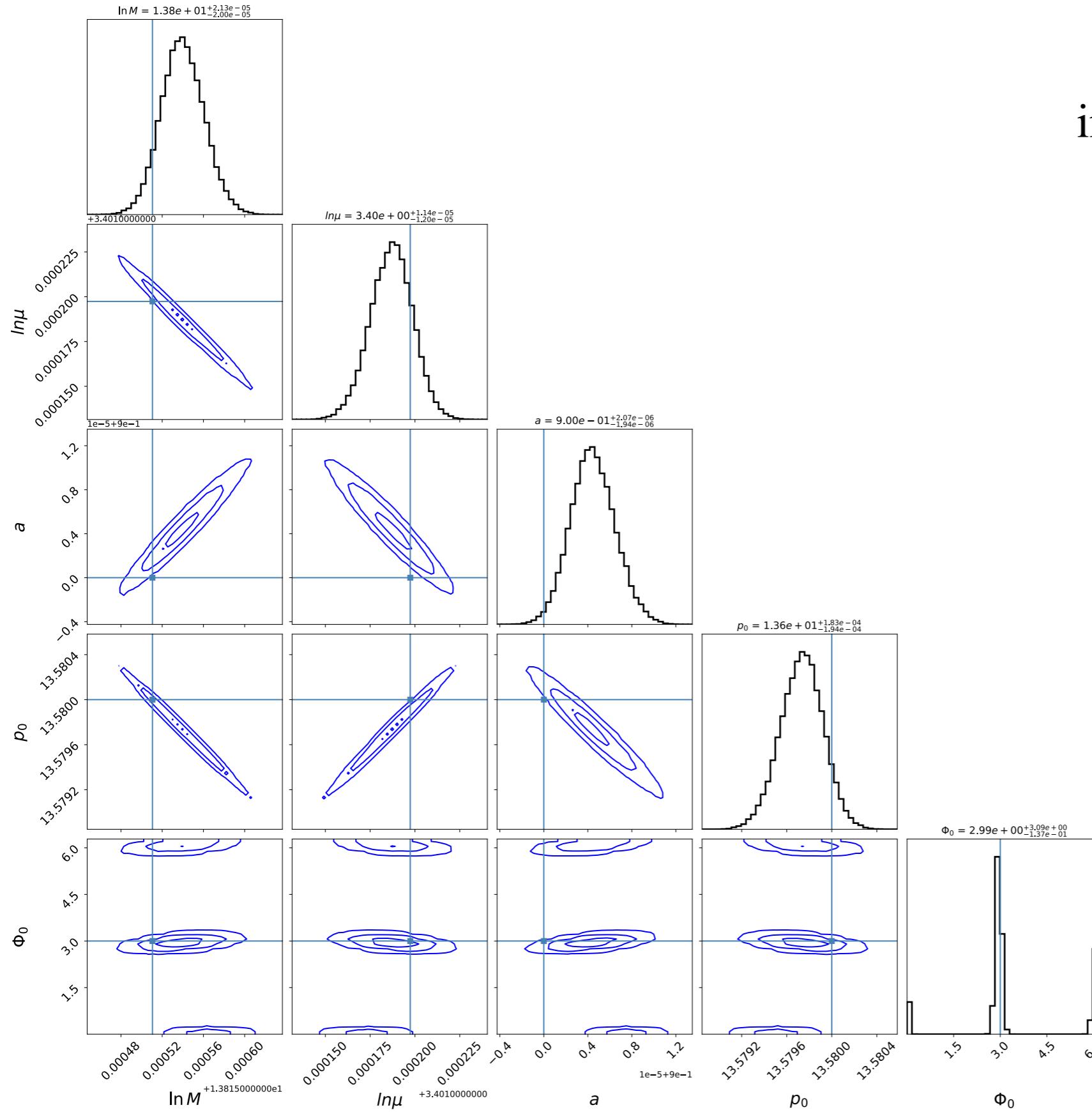
$$M = 10^6 \quad a = 0.9 \quad T_{\text{obs}} = 4\text{yr} \quad d = 0.03$$

Circular Equatorial Kerr: Constraints on the scalar charge

4 years of observations

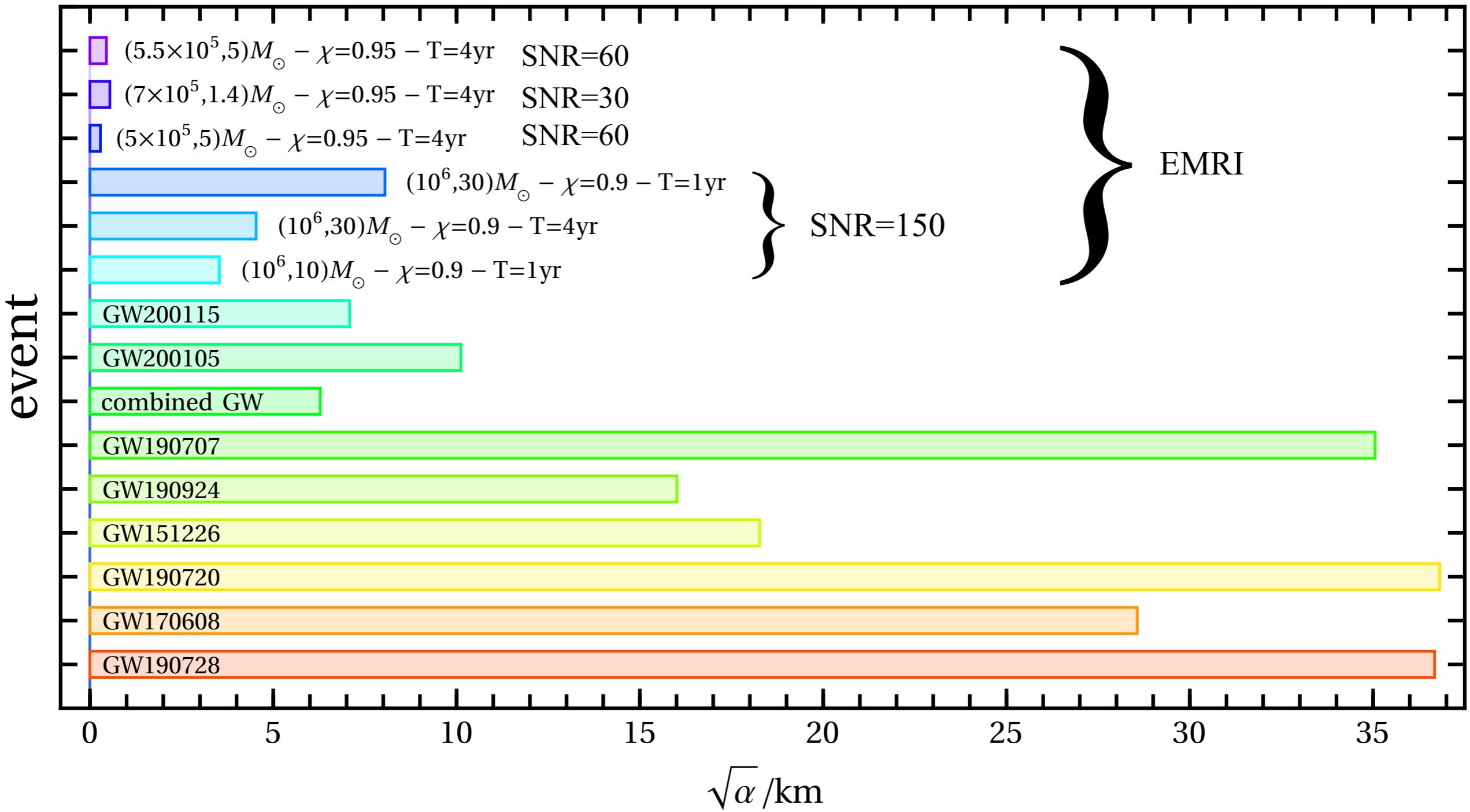


Biases induced by ignoring beyond GR effects



injected $d = 0.07$

Mapping to scalar-Gauss-Bonnet



Mapping to scalar-Gauss-Bonnet

Mapping to scalar Gauss-Bonnet gravity

$$\alpha S_c = \frac{\alpha}{4} \int d^4x \frac{\sqrt{-g}}{16\pi} f(\varphi) \mathcal{G}$$

$$\beta = \frac{\alpha}{\mu^2}$$

$$f(\varphi) = e^\varphi$$

$$f(\varphi) = \varphi$$

$$d \approx 2\beta + \frac{73}{30}\beta^2$$

$$d \approx 2\beta + \frac{73}{60}\beta^3$$

The LISA Global Fit

- Fit signals & noise

