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Search, recovery or destroy a waveform template: how important are eccentricity evolution and post-adiabatic terms for asymmetric binaries?

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Asymmetric Binaries meet Fundamental Astro-Physics
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Introduction-part 1 of 2

We can do wonder with EMRIs...
...if we use fast and accurate waveforms

- how fast? around 1-10ms
- how accurate? within 1 cycle $\Longrightarrow$ at 1-post-adiabatic order (1PA) [1]

Complications:

vast parameter space (14d) & long, slow inspirals

Data analysis perspective

- Can we use approximate models?
- What happen if the mismodeled adiabatic order?
- Can we ignore 1PA terms?

Try answer these questions with Bayesian analysis and MCMC
(GR, no environmental effects)

[1] Hinderer at al. PhysRevD.78.064028(2008)

Introduction - 2 of 2

According to two-times scale expansion

$$\Phi_{\rm GW} = \underbrace{q^{-1}\mathcal{C}^{(0)}}_{\text{adiabatic}} \quad + \quad \underbrace{q^{-1/2}\mathcal{C}^{(1/2)}}_{\text{resonances}} \quad + \quad \underbrace{q^{0}\mathcal{C}^{(1)}}_{\text{post-1-adiabatic}} \quad + \quad \mathcal{O}(q)$$

$\mathcal{C}^1$ contains second-order self-force+secondary spin corrections

Our state-of-the-art waveforms (Schwarzschild spacetime) :

- full 1PA for circular, equatorial orbits [2]
- 0PA eccentric

Compared with approximate PN inspired models.

All models implemented in FEW [3]

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[2] Wardell et al. PhysRevLett.130.241402(2023); Mathews et al. PhysRevD.105.084031 (2022); Piovano et al PhysRevD.104.124019(2021)

[3] Katz et al. PhysRevD.104.064047(2021); Chua et al. PhysRevLett.126.051102(2021)

Outline of the presentation

1. Bayesian analysis with EMRIs
2. How important are 1PA terms for circular orbits?
3. Mismodelling evolution eccentric orbits for adiabatic waveform
4. Conclusions

Bayesian analysis with EMRIs

Bayes theorem and how we use it

d the data stream of observations with parameters $\boldsymbol{\theta}$

$$\log p(\boldsymbol{\theta}|d) \propto \log p(d|\boldsymbol{\theta}) + \log p(\boldsymbol{\theta})$$

Priors $p(\boldsymbol{\theta})$: uniform for all parameters

Our samplers

- `emcee` for circular orbits
- `Eryn` with parallel tempering for eccentric orbits

Likelihood

Three TDI observable $X = \{A, E, T\}$, $h_{\mathrm{e}}^{(X)}(t;\boldsymbol{\theta})$ true waveform

$$d^{(X)}(t) = h_{\mathrm{e}}^{(X)}(t;\boldsymbol{\theta}) + n^{(X)}(t)$$

Noise $n^{(X)}(t)$ Gaussian and Stationary

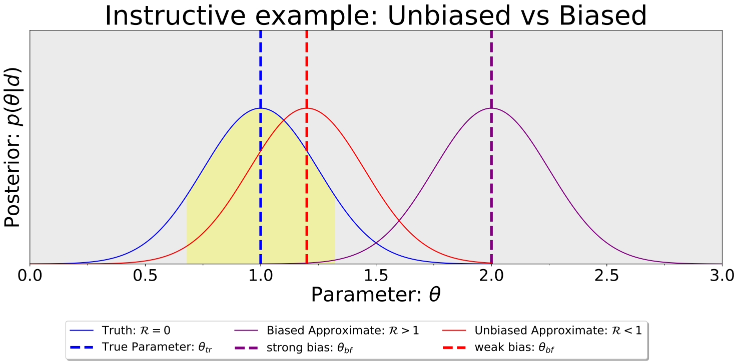
$$\implies \log p(d|\boldsymbol{\theta}) \propto -\frac{1}{2} \sum_{X=\{A,E,T\}} (d - h_m | d - h_m)_{(X)}$$

$$(a|b)_X = 4\mathrm{Re} \int_0^\infty \frac{\tilde{a}^{(X)}(f)(\tilde{b}^{(X)})^\star(f)}{S_n^{(X)}(f)} df$$

Set $n^{(X)} = 0$ to maximize the impact of systematic biases

$$\implies \log p(d|\boldsymbol{\theta}) \propto -\frac{1}{2} \sum_{X=\{A,E,T\}} (h_e - h_m | h_e - h_m)_{(X)}$$

Statistical vs systematic errors



Credit: Ollie Burke.

$$\mathcal{R} = \frac{\Delta\boldsymbol{\theta}_{\text{sys}}}{\Delta\boldsymbol{\theta}_{\text{stat}}}$$

$\mathcal{R}$ is SNR dependent since $\Delta\boldsymbol{\theta}_{\text{sys}} \propto SNR^0$ and $\Delta\boldsymbol{\theta}_{\text{stat}} \propto SNR^{-1}$

How important are 1PA terms for circular orbits?

Orbital evolution for circular orbits at post-adiabatic order

$$
\begin{aligned}
\frac{\mathrm{d}\Phi_\phi}{\mathrm{d}t} &= \Omega_\phi(r) \\
\frac{\mathrm{d}r}{\mathrm{d}t} &= -\nu\left[F_0(r) + \nu F_1(r)\right]
\end{aligned}
$$

ν symmetric mass ratio, χ spin of the secondary

Our models: same waveform structure, different evolution[4]

- cir1PA

$$F_1(r) = F_1^{\mathsf{SF}}(r) + \chi F_1^{\chi}(r).$$

- cir0PA+1PA-3PN

$$F_1(r) = F_1^{\mathsf{3PN}}(r) + \chi F_1^{\chi}(r).$$

$F_1^{\mathsf{3PN}}(r)$ given by resummed, PN series at 3rd order (credit: C. Kavanagh)

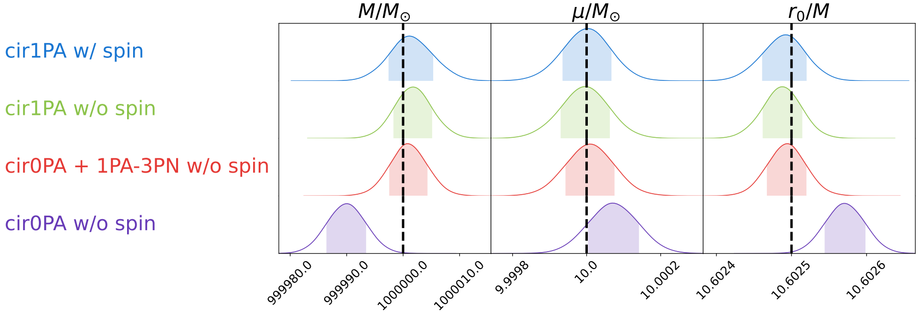
[4] Wardell et al. PhysRevLett.130.241402(2023); Mathews et al. PhysRevD.105.084031 (2022); Piovano et al PhysRevD.104.124019(2021)

Orbital evolution for circular orbits at post-adiabatic order

Caveats: what we neglected

- evolution mass and spin primary. (1PA but numerically small)
- 2SF flux at horizon (unknown; small contribution)
- 1PA correction waveform amplitude (neglible, extremely slow variation)

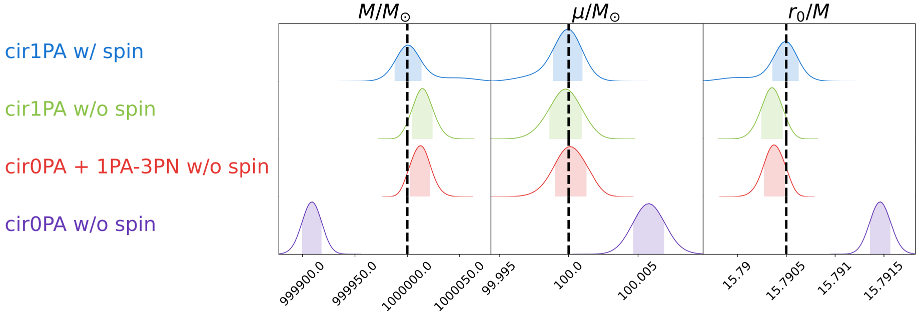
Neglecting 1PA term for circular orbit - EMRI



Parameters: $q = 10^{-5}$, $M = 10^6 M_\odot$, SNR ~ 70, $D_L = 1\,\mathrm{Gpc}$, $T = 2$ years.

Mismatch between cir0PA and cir1PA, is $\mathcal{M}(h^{(0PA)}(\boldsymbol{\theta}_{\rm bf}), h^e(\boldsymbol{\theta}_{\rm tr})) \sim 10^{-5}$

Neglecting 1PA term for circular orbit - EMRI

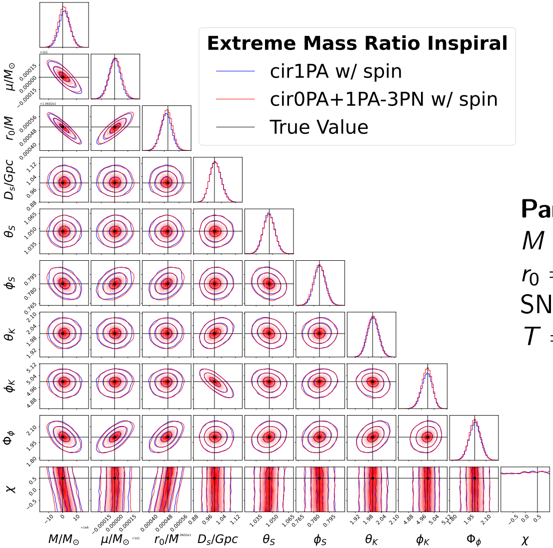


Parameters: $q = 10^{-4}$, $M = 10^6 M_\odot$, SNR ~ 65, $D_L = 2\,\mathrm{Gpc}$, $T = 1.5$ year.

Neglecting 1PA term for circular orbit

And the secondary spin?

Neglecting 1PA term for circular orbit - EMRI $q = 10^{-5}$



Parameters:

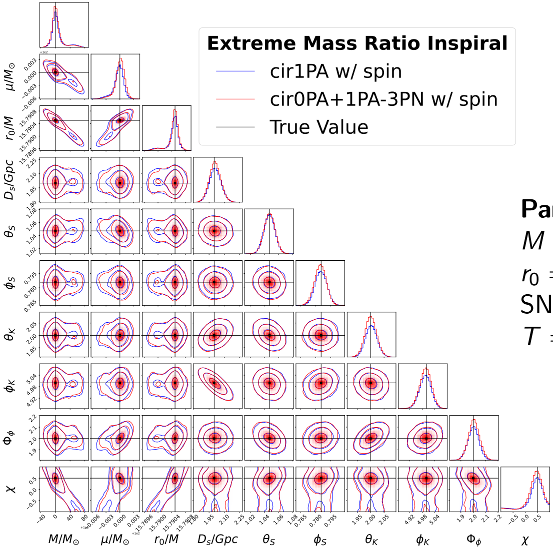
$M = 10^6 M_\odot$,

$r_0 = 10.60M$

SNR ~ 70, $D_L = 1\,\text{Gpc}$

$T = 2$ years.

Neglecting 1PA term for circular orbit - EMRI $q = 10^{-4}$



Parameters:

$M = 10^6 M_\odot$,

$r_0 = 15.79M$

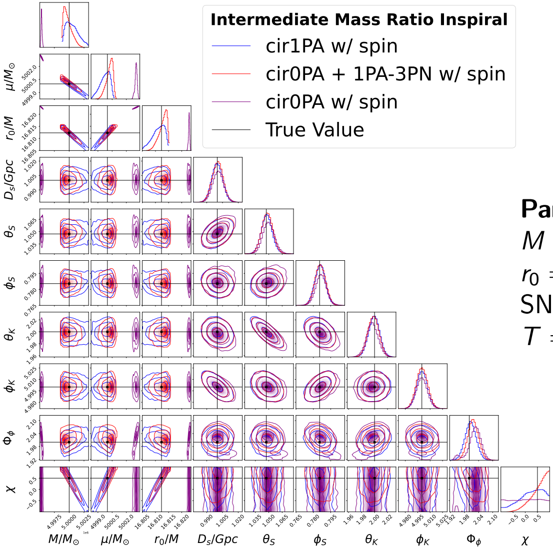
SNR ~ 65, $D_L = 2\,\mathrm{Gpc}$

$T = 1.5$ year.

Neglecting 1PA term for circular orbit

What happen for larger mass ratio?

Neglecting 1PA term for circular orbit - IMRI $q = 10^{-3}$



Parameters:

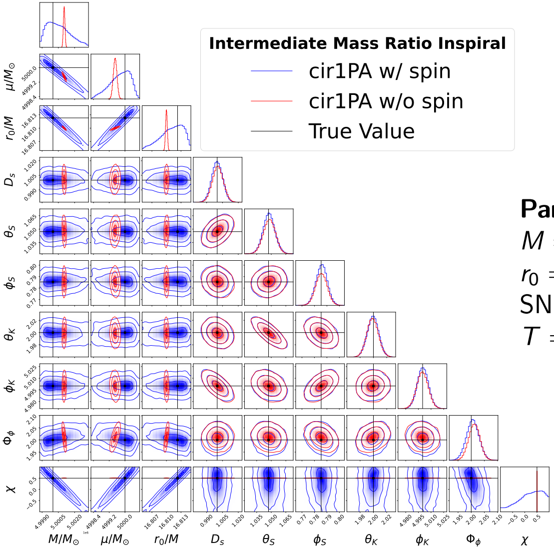
$M = 5 \cdot 10^6 M_\odot$

$r_0 = 16.81 M$

SNR ~ 340, $D_L = 1\,\mathrm{Gpc}$

$T = 1$ year.

Neglecting 1PA term for circular orbit - IMRI $q = 10^{-3}$



Parameters:

$M = 5 \cdot 10^6 M_\odot$,

$r_0 = 16.81 M$

SNR ~ 340, $D_L = 1$ Gpc

$T = 1$ year.

Mismodelling evolution eccentric orbits for adiabatic waveform

Orbital evolution for eccentric orbits at adiabatic order

$$\frac{\mathrm{d}\Phi_\phi}{\mathrm{d}t} = \Omega_\phi(p,e) \qquad \frac{\mathrm{d}\Phi_r}{\mathrm{d}t} = \Omega_r(p,e)$$

$$\frac{\mathrm{d}p}{\mathrm{d}t} = -q\left(\frac{\partial p}{\partial E_0}\mathcal{F}_0^E(p,e) + \frac{\partial p}{\partial J_0}\mathcal{F}_0^J(p,e)\right)$$

$$\frac{\mathrm{d}e}{\mathrm{d}t} = -q\left(\frac{\partial e}{\partial E_0}\mathcal{F}_0^E(p,e) + \frac{\partial e}{\partial J_0}\mathcal{F}_0^J(p,e)\right)$$

$\mathcal{F}_0^E(p,e)$ energy flux $\qquad$ $\mathcal{F}_0^J(p,e)$ angular momentum flux

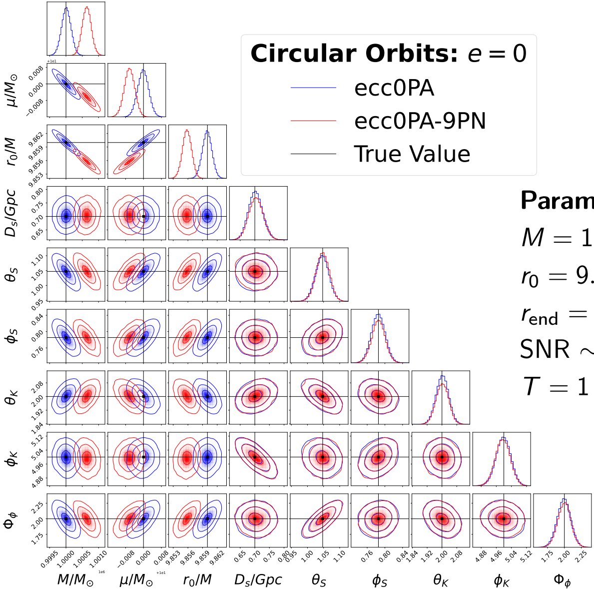
Two models:

same waveform structure (given by FEW)...

...but different orbital evolution

- ecc0PA: BH perturbation theory fluxes
- ecc0PA-9PN : 9PN series fluxes (credit: N. Warburton)

Systematic biases for eccentric orbits and adiabatic waveform



Parameters:

$M = 10^6 M_\odot$, $\mu = 10 M_\odot$

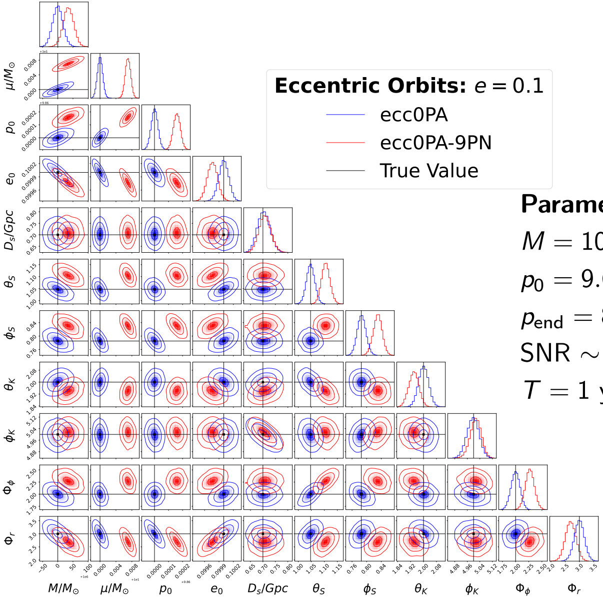
$r_0 = 9.6M$, $e_0 = 0$

$r_{\text{end}} = 8.1M$

SNR ~ 70, $D_L = 0.7\,\text{Gpc}$

$T = 1$ year

Systematic biases for eccentric orbits and adiabatic waveform



Parameters:

$M = 10^6 M_\odot, \ \mu = 10 M_\odot$

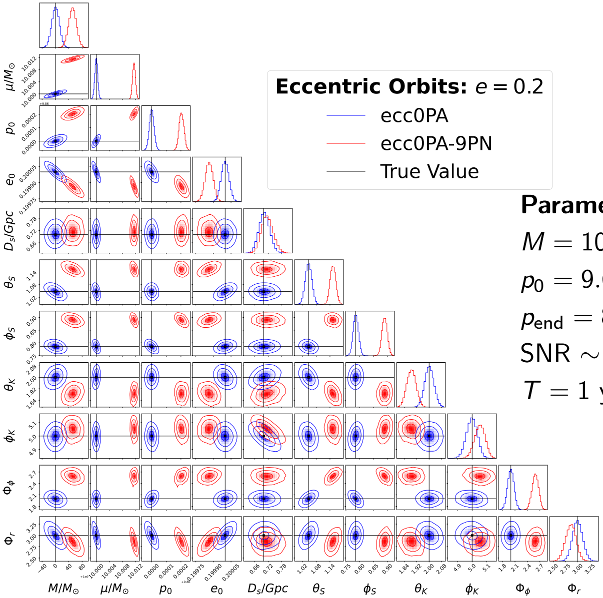
$p_0 = 9.6M, \ e_0 = 0.1$

$p_{\rm end} = 8.1M$

SNR $\sim 70, \ D_L = 0.7\,\rm Gpc$

$T = 1$ year

Systematic biases for eccentric orbits and adiabatic waveform



Parameters:

$M = 10^6 M_\odot,\ \mu = 10 M_\odot$

$p_0 = 9.6M,\ e_0 = 0.2$

$p_{\text{end}} = 8.1M$

SNR $\sim 70,\ D_L = 0.7\,\text{Gpc}$

$T = 1$ year

Conclusions

Conclusions: neglecting/mis modeling 1PA terms

Circular equatorial orbits in Schwarzschild

Neglecting 1PA terms

- only masses and r_0 are affected
- for $q \gtrsim 10^{-5}$ statistically significant biases...
- ...but tiny. Not relevant for (some) astrophysical applications
- 1PA-3PN correction works as well as 1PA-2GSF

Secondary spin

- strongly correlated with other parameters (especially masses, r_0)...
- ...must be included for $q \gtrsim 10^{-4}$
- ...can not be constrained for $q \lesssim 10^{-5}$

Conclusions: mismodeling 0PA terms

Eccentric orbits in Schwarzschild

Mismodeling 0PA term

- all parameters are affected
- larger biases for larger initial eccentricity e_0
- injection exact waveform & recovery with approximate waveform is much more difficult than circular orbits

But this the tip of the iceberg!

Analysis must be extended to Kerr, generic orbits, 1PA terms, resonances...

Final notes and acknowledgments

- thank to all collaborators for their time and contributions
- special thanks to O. Burke for amazing work on the data analysis front
- this work uses the FastEmriWaveform of the BHPToolkit https://bhptoolkit.org/FastEMRIWaveforms/html/index.html

- Feel free to contact me at gabriel.piovano@ucd.ie

Final notes and acknowledgments

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Thank you for you attention!