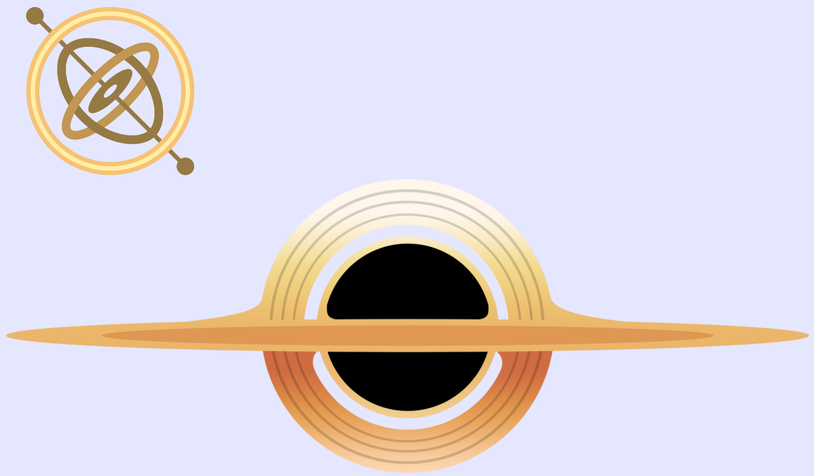
Secondary spin in asymmetric binaries

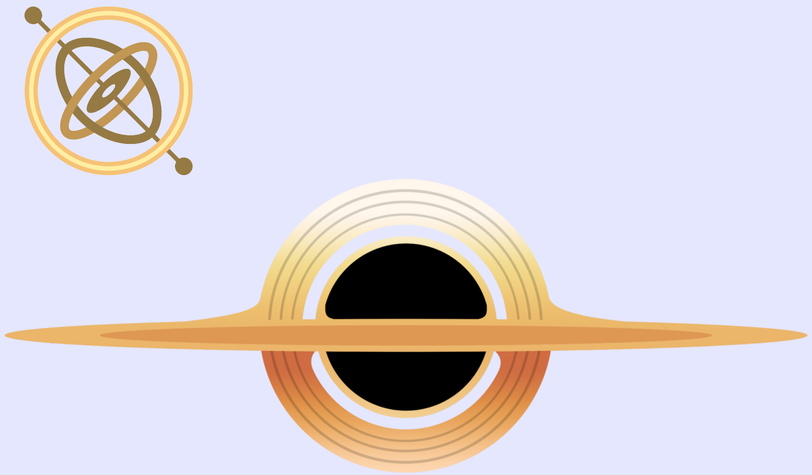
Accounting for the spin of *both* black holes in EMRI waveforms

Lisa V. Drummond

Massachusetts Institute of Technology



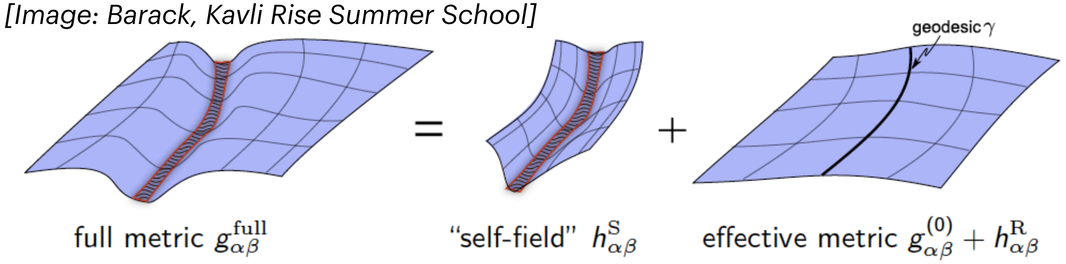
Asymmetric Binaries meet Fundamental Astrophysics, *GSSI, L'Aquila 2023*



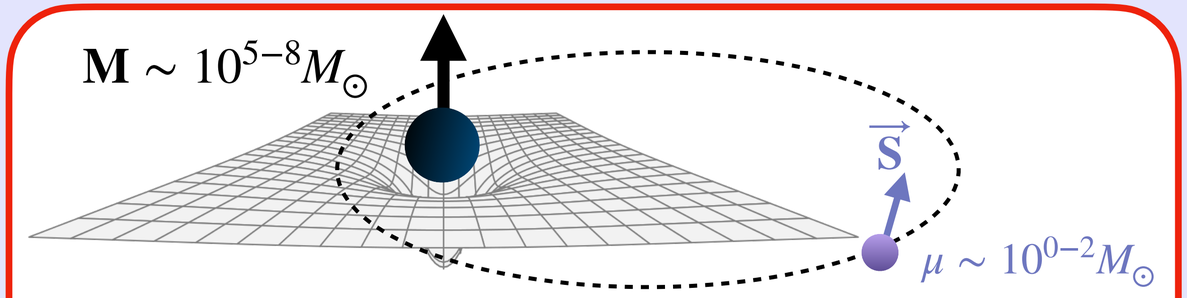
Overview of *secondary spin*

What about the *spin* of the small black hole?

[Image: Barack, Kavli Rise Summer School]



Gravitational-self force: *How the small body dimples spacetime, and how that backreacts on its motion*



Spin-curvature coupling: *How the small body's spin couples to curvature, and how that backreacts on its motion*

Accurate EMRI waveform models include ***two types of effects***:

1. ***gravitational self-force,*** and
2. ***spin-curvature force***

We must include the effect of the **spin of both black holes in EMRI models**

Secondary spin contributes to the 1PA term

$$\frac{Dp^\alpha}{d\tau} = f^\alpha \longrightarrow f^\alpha = f^{(1)\alpha} + f^{(2)\alpha} + \mathcal{O}(\varepsilon^3)$$

$$f^{(1)\alpha} = f^{(1)\alpha}_{mono} + f^\alpha_{SCF} \qquad f^{(2)\alpha} = f^{(2)\alpha}_{mono} + f^\alpha_{dipole}$$

GW phase:

$$\Phi = \underbrace{\varphi_0 \varepsilon^{-1}}_{\text{adiabatic: } f^\alpha_{\rm ad}} + \underbrace{\varphi_1 \varepsilon^{0}}_{\text{post-1-adiabatic: } f^\alpha_{\rm oscil} + \langle f^{(1)\alpha}_{\rm cons}\rangle + \langle f^{(2)\alpha}_{\rm diss}\rangle + \langle f^\alpha_{\rm SCF}\rangle + \langle f^\alpha_{\rm dipole,diss}\rangle}$$

Gravitational self force

Secondary spin

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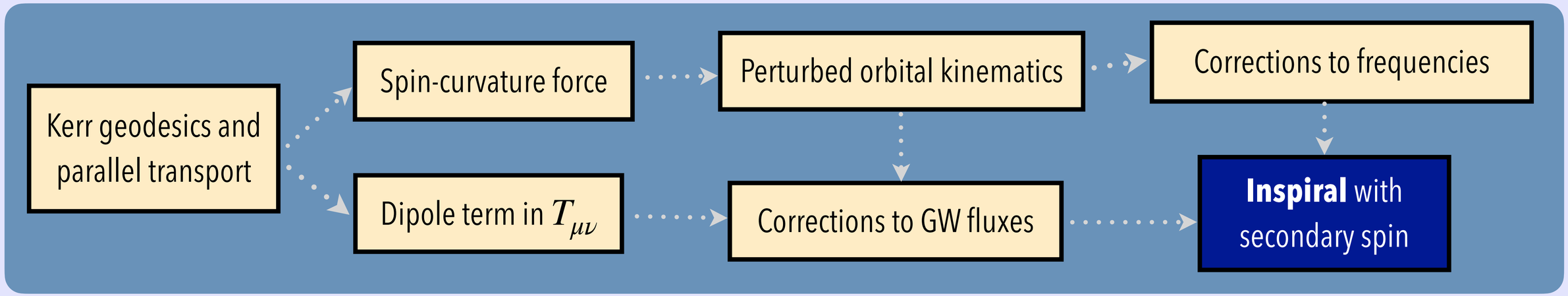
Secondary spin effects



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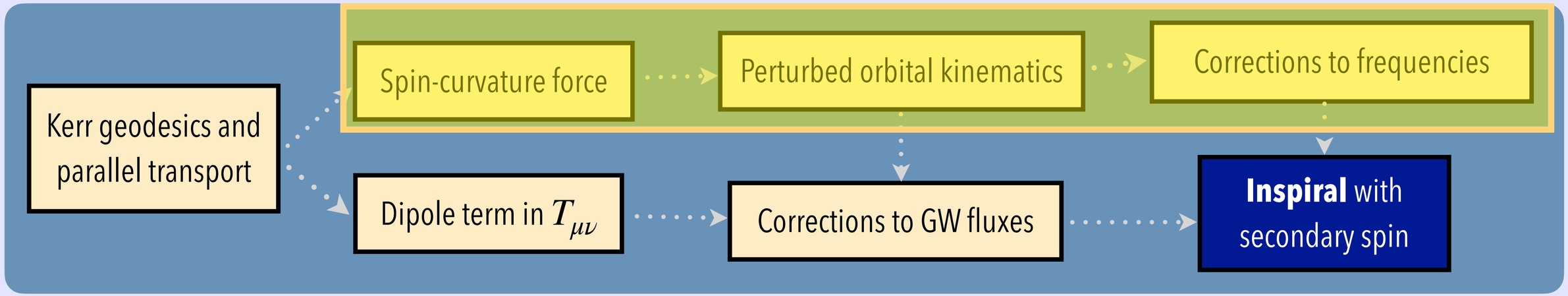
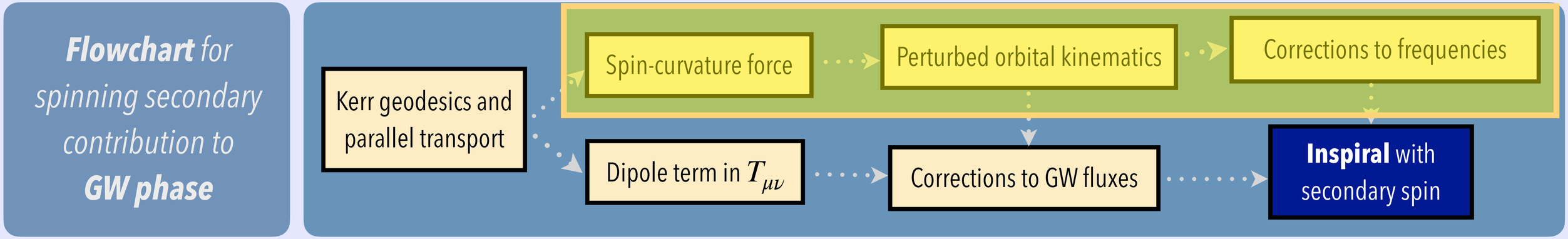
Secondary spin effects





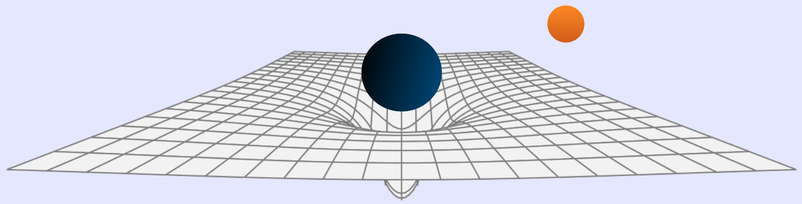
Kinematics of an orbiting small body

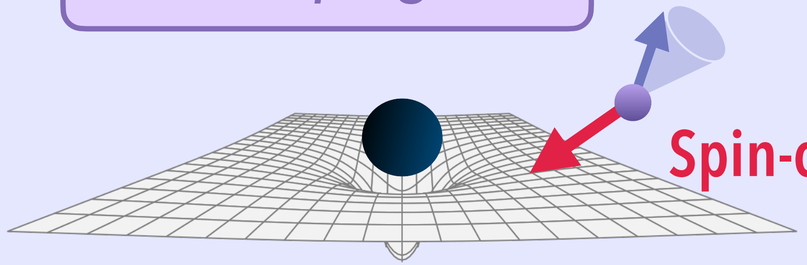
★ Coupling between curvature and small-body spin leads to **spin-curvature force**

★ Pushes the motion of the small body **away from** the geodesic orbit and causes small body's **spin to precess**

Non-spinning body: Geodesic equations

Spinning body: Spin-curvature coupling



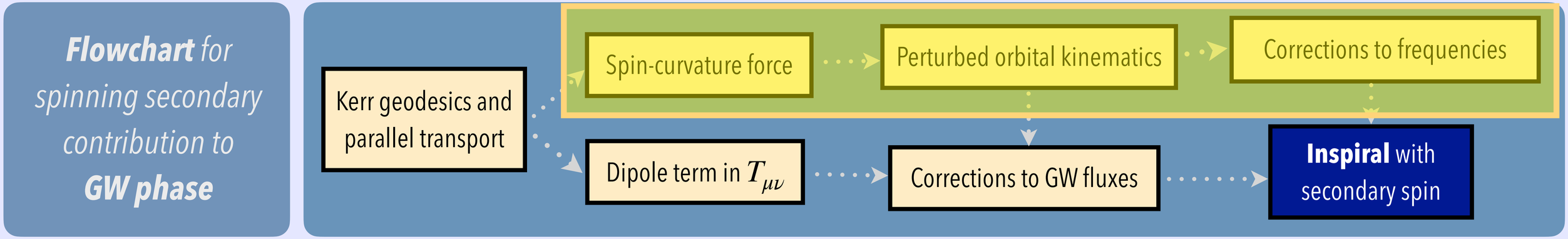


Spin-curvature force f^{α}_{SCF}

$$\ddot{x}^{\alpha} = f^{\alpha}_{geo}\,, \quad f^{\alpha}_{geo} \equiv -\Gamma^{\alpha}_{\beta\gamma}\frac{dx^{\beta}}{d\tau}\frac{dx^{\gamma}}{d\tau}$$

$$\ddot{x}^{\alpha} = f^{\alpha}_{geo} + f^{\alpha}_{SCF}\,, \quad f^{\alpha}_{SCF} \equiv -\frac{1}{2\mu}R^{\alpha}{}_{\nu\lambda\sigma}u^{\nu}S^{\lambda\sigma}$$

M. Mathisson, 1937; A. Papapetrou, 1951; W. G. Dixon, 1970



Mathisson-Papapetrou-Dixon equations

*Equations describing the motion of a **spinning test body** in curved spacetime*

$$\frac{Dp^\alpha}{d\tau} = -\frac{1}{2}R^\alpha_{\beta\gamma\delta}u^\beta S^{\gamma\delta} := f^\alpha_S/\mu \qquad (1)$$

Spin-curvature force

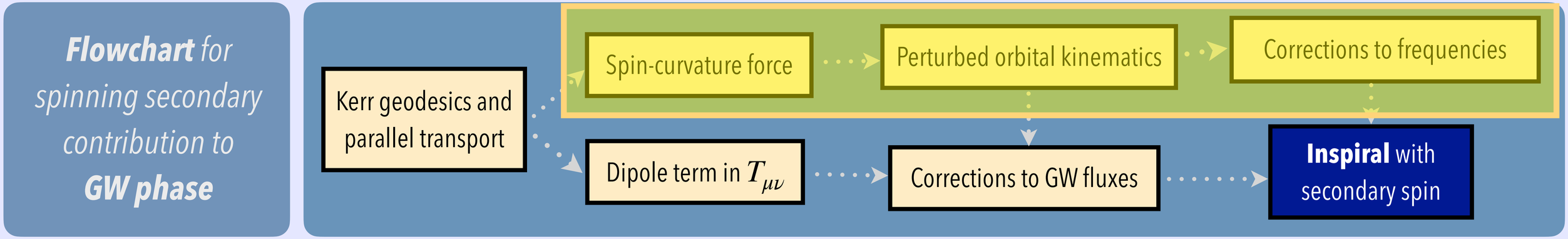
$$\frac{DS^{\alpha\beta}}{d\tau} = p^\alpha u^\beta - p^\beta u^\alpha \qquad (2)$$

$$p_\mu S^{\mu\nu} = 0 \qquad (3)$$

Tulczyjew-Dixon spin-supplementary condition

$S^{\mu\nu}$ *is the spin tensor of the secondary*

$S^\mu = -\frac{1}{2}\epsilon^{\mu\nu}{}_{\alpha\beta}p_\nu S^{\alpha\beta}$ *is the spin vector of the secondary*



Mathisson-Papapetrou-Dixon equations

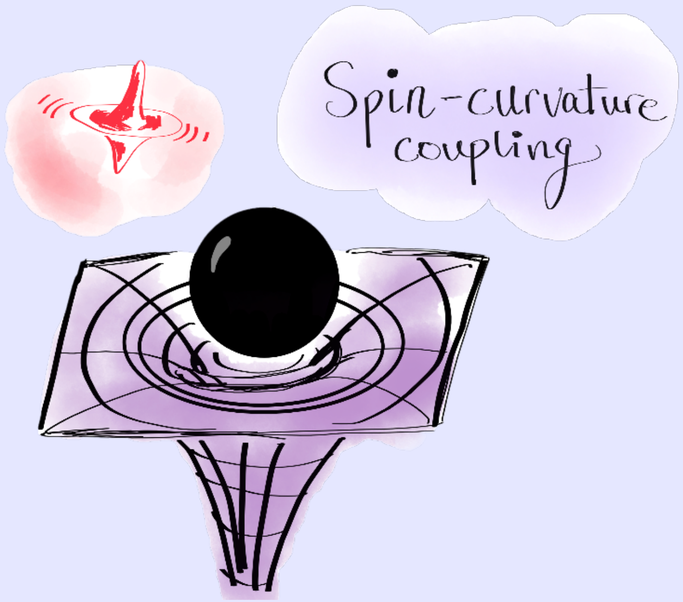
…to leading-order in spin

$$\frac{Du^\alpha}{d\tau} = -\frac{1}{2\mu} R^\alpha_{\beta\gamma\delta} u^\beta S^{\gamma\delta}$$

Motion of the small body

$$\frac{DS^\mu}{d\tau} = 0$$

Evolution of spin vector



Secondary spin contributes to the 1PA term

GW phase:

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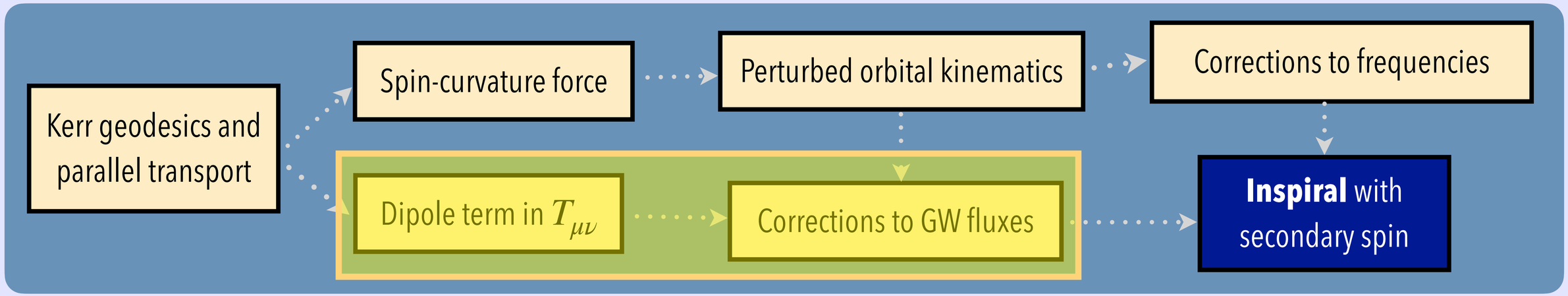
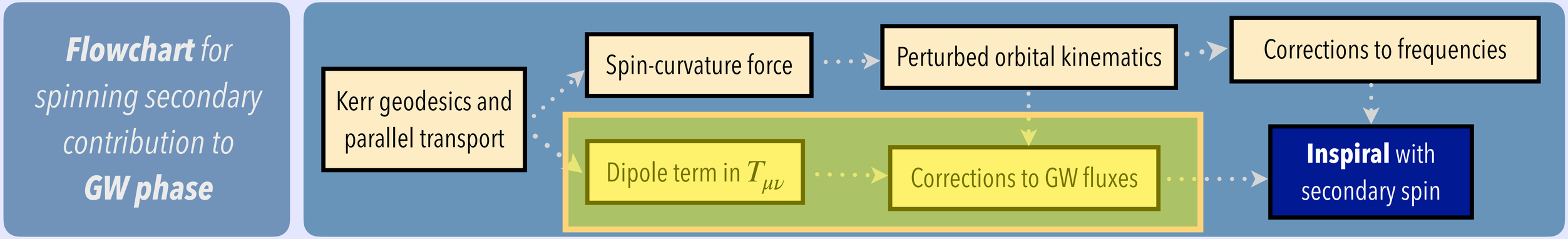
Secondary spin effects



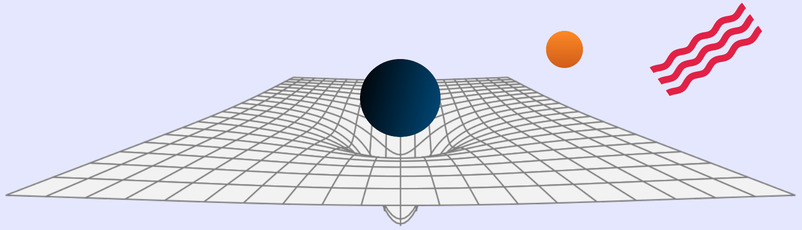


Radiation due to an orbiting small body

Compute GW radiation using the ***Teukolsky equation***

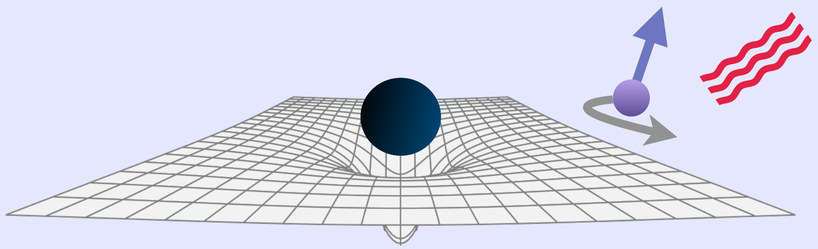
$${}_{-2}\mathcal{O}\ {}_{-2}\Psi = 4\pi\Sigma\mathcal{T}$$

Non-spinning body: Point-particle GW fluxes



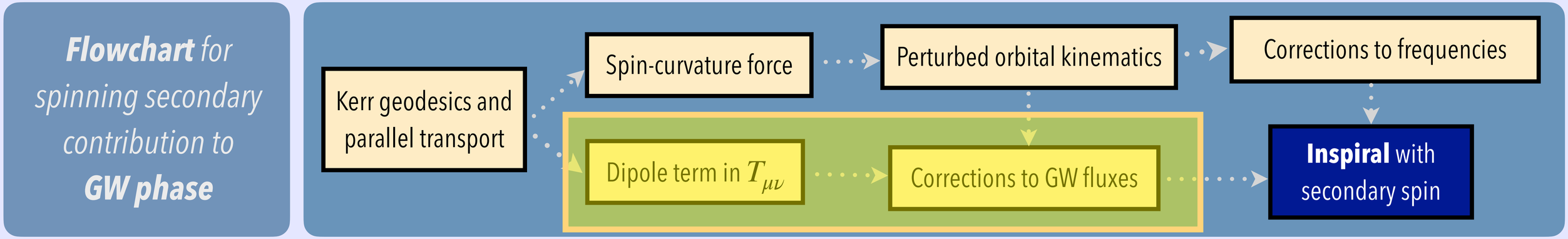
$$T^{\mu\nu}_{geo} = \int d\tau \left(\frac{\mu u^{\mu}_{geo} u^{\nu}_{geo}}{\sqrt{-g}} \delta^4 \left(x^{\rho} - z^{\rho}_{geo}(\tau) \right) \right)$$

Spinning body: Spinning-particle GW fluxes



$$T^{\mu\nu}_{spin} = \int d\tau \left(\frac{p^{(\mu} u^{\nu)}}{\sqrt{-g}} \delta^4 \left(x^{\rho} - z^{\rho}(\tau) \right) - \nabla_{\alpha} \left(\frac{S^{\alpha(\mu} u^{\nu)}}{\sqrt{-g}} \delta^3 \left(x^{\rho} - z^{\rho}(\tau) \right) \right) \right)$$

The source term $\mathcal{T}$ *in the* ***Teukolsky equation*** *can be found from the* ***stress-energy tensor*** $T^{\mu\nu}$ *describing the small body*



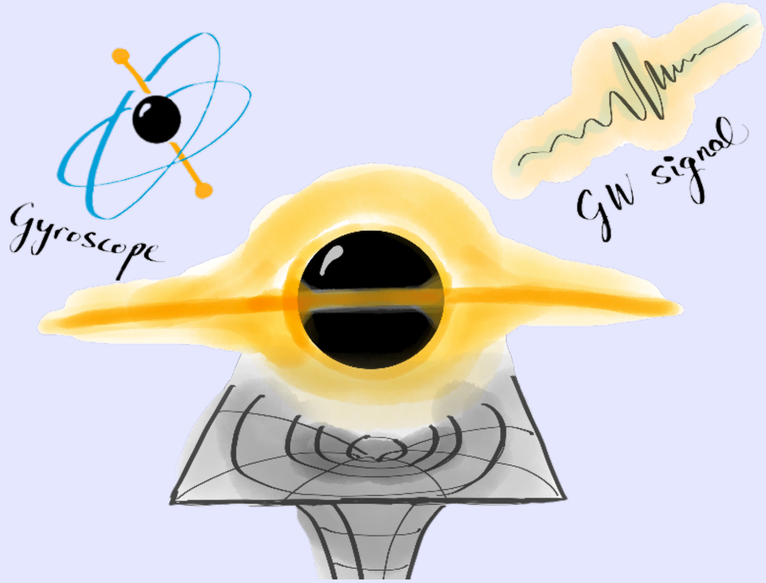
*Orbit of a **spinning body** around a black hole* → *Compute **GW fluxes** using Teukolsky equation*

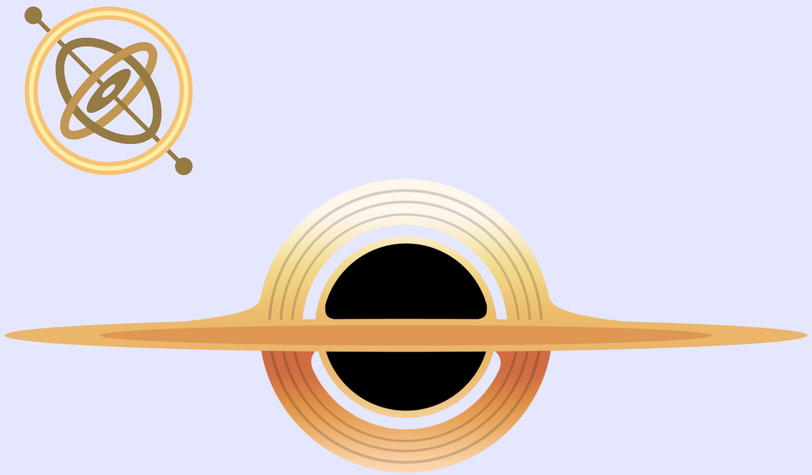
- Consider a spinning body orbit and compute **corresponding GW fluxes**

(e.g., Piovano, arXiv:2004.02654; Skoupý & Lukes-Gerakopoulos, arXiv:2102.04819)

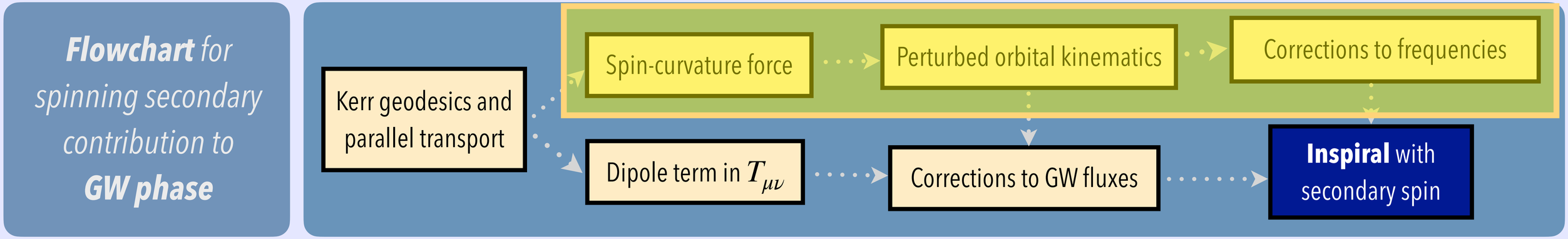
$$\langle \dot{E} \rangle = -q \left(\langle \mathcal{F}^{E\mathcal{I}^+} \rangle_S + \langle \mathcal{F}^{E\mathcal{H}^+} \rangle_S \right)$$

$$\langle \dot{J}_z \rangle = -q \left(\langle \mathcal{F}^{J_z\mathcal{I}^+} \rangle_S + \langle \mathcal{F}^{J_z\mathcal{H}^+} \rangle_S \right)$$



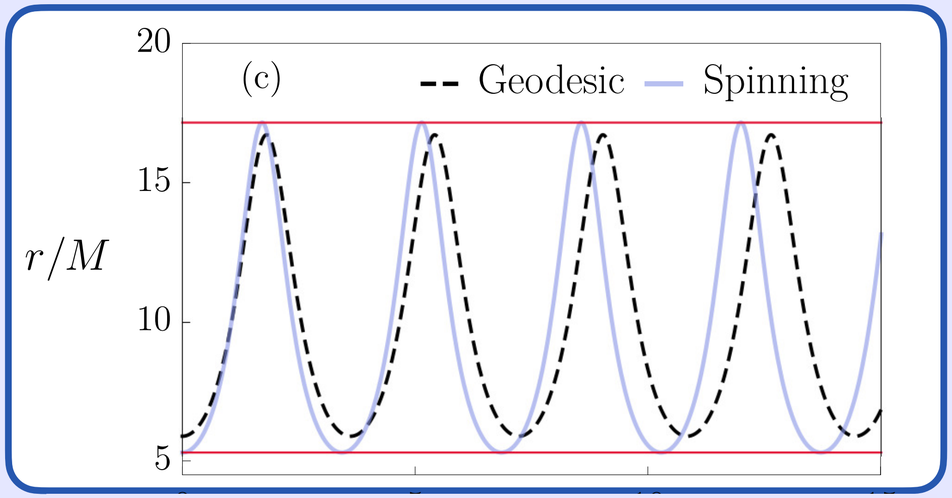


Recent progress in *secondary spin*



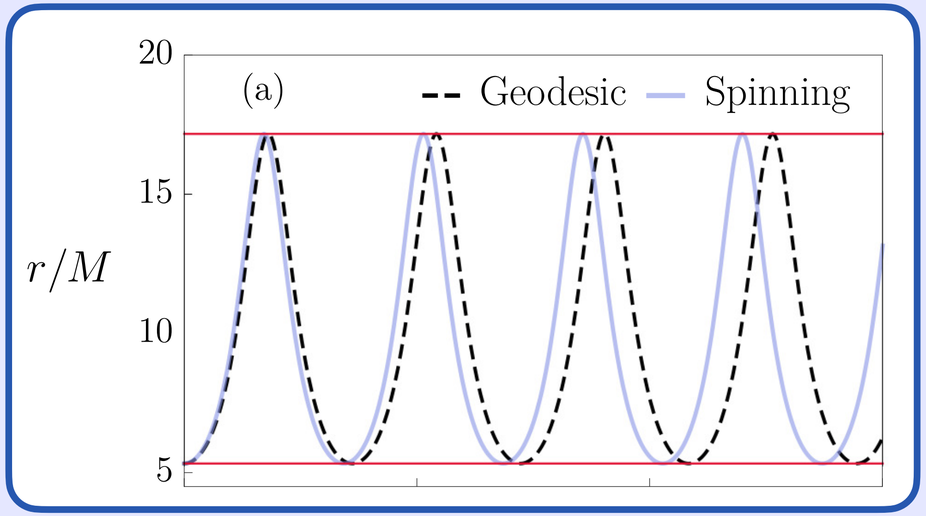
Mappings between *geodesics* and *spinning-body orbits*

Geodesic and spinning-body orbit have the same ***constants of motion***



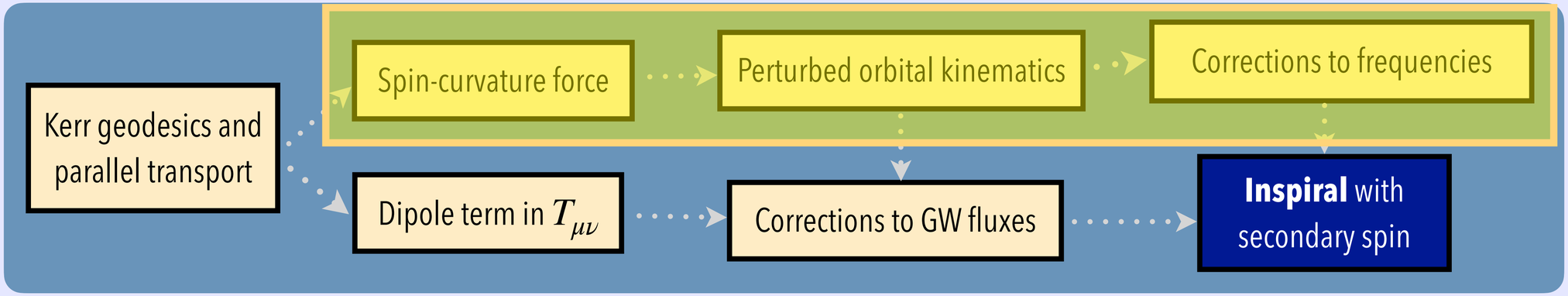
Time $M\lambda$

Geodesic and spinning-body orbit have the same ***turning points***



Time $M\lambda$

Flowchart** for spinning secondary contribution to **GW phase



"Reference"** geodesic and spinning-body orbit have the same **constants of motion

Vojtěch Witzany (**arXiv:1808.06582**) derived the Hamiltonian and canonical coordinates for the TD SSC

$$H_{TD} = \frac{1}{2}(g^{\mu\nu} - \gamma^{\mu\nu})U_\mu U_\nu = -1,$$

where $$\gamma^{\mu\nu} = \frac{4s^{\nu\gamma}R^{\mu}_{\gamma\kappa\lambda}s^{\kappa\lambda}}{4 + R_{\chi\eta\omega\xi}s^{\chi\eta}s^{\omega\xi}}$$

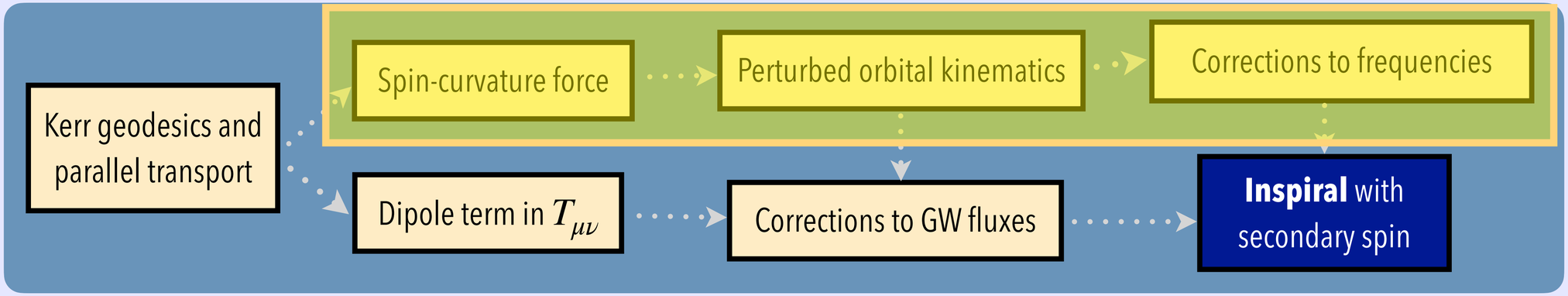
Hamiltonian for the Tulczyjew-Dixon condition

$$\mathcal{U}_\mu = U_\mu + \frac{1}{2}e_{C\nu;\mu}e^{\nu}_{D}s^{CD},$$

where $s_{CD} \equiv s^{\mu\nu}e_{C\mu}e_{D\nu}$

x^μ and $\mathcal{U}_\mu$ are canonically conjugate

Flowchart *for spinning secondary contribution to* **GW phase**



*"**Reference**" geodesic and spinning-body orbit have the same **constants of motion***

***Foundational work** on spinning body orbits using the Hamilton-Jacobi equation by Vojtěch Witzany (**arXiv:1903.03651**)*

$$\frac{dr}{d\lambda} = \pm\,\Delta\sqrt{w_r'^2 - e_{0r}e^{\kappa}_{C;r}e_{\kappa B}\tilde{s}^{CD}}\ ,$$

$$\frac{d\theta}{d\lambda} = \pm\sqrt{w_\theta'^2 - e_{0\theta}e^{\kappa}_{C;\theta}e_{\kappa B}\tilde{s}^{CD}}\ ,$$

$$\frac{d\psi_p}{d\lambda} = \sqrt{\hat{K}}\left(\frac{(r^2+a^2)\hat{E} - a\hat{L}_z}{\hat{K}+r^2} + a\frac{\hat{L}_z - a(1-z^2)\hat{E}}{\hat{K}-a^2z^2}\right)$$

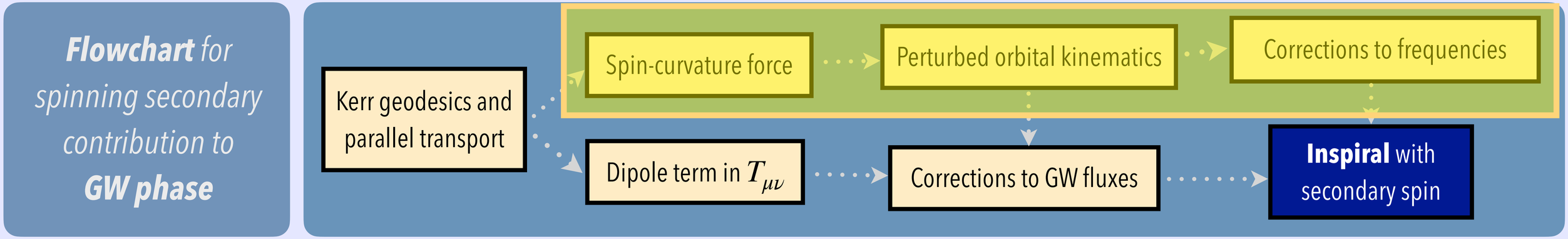
$(E, L_z, Q) \rightarrow (E^S, L_z^S, Q^S)$

↓

Compute $\delta r_t(E, L_z, Q)$ and $\delta\theta_t(E, L_z, Q)$

↓

Define angle-type coordinates using δr_t and $\delta\theta_t$



*"**Reference**" geodesic and spinning-body orbit have the same **turning points***

Alternative parameterisation: *Drummond & Hughes (**arXiv:2201.13334** and **arXiv:2201.13335**)*

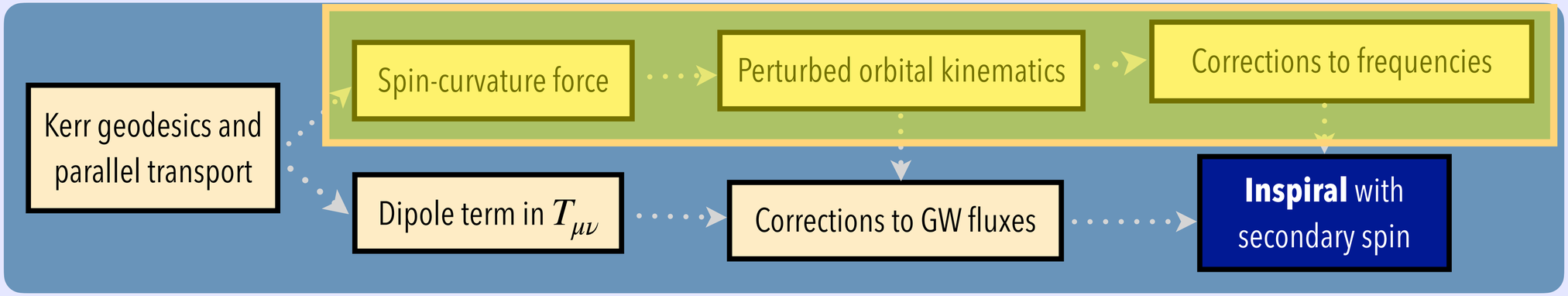
Frequency-domain treatment of spinning-body motion:

Orbital frequencies modified: $\Upsilon_r = \hat{\Upsilon}_r + \Upsilon_r^S$ and $\Upsilon_\theta = \hat{\Upsilon}_\theta + \Upsilon_\theta^S$

Spin precession frequency Υ_s introduced due to precessing spin vector

$$f[r(\lambda), \theta(\lambda), S(\lambda)] = \sum_{j=-1}^{1} \sum_{k,n=-\infty}^{\infty} f_{jkn} e^{-(ij\Upsilon_s + ik\Upsilon_\theta + in\Upsilon_r)\lambda}$$

Flowchart for spinning secondary contribution to **GW phase**



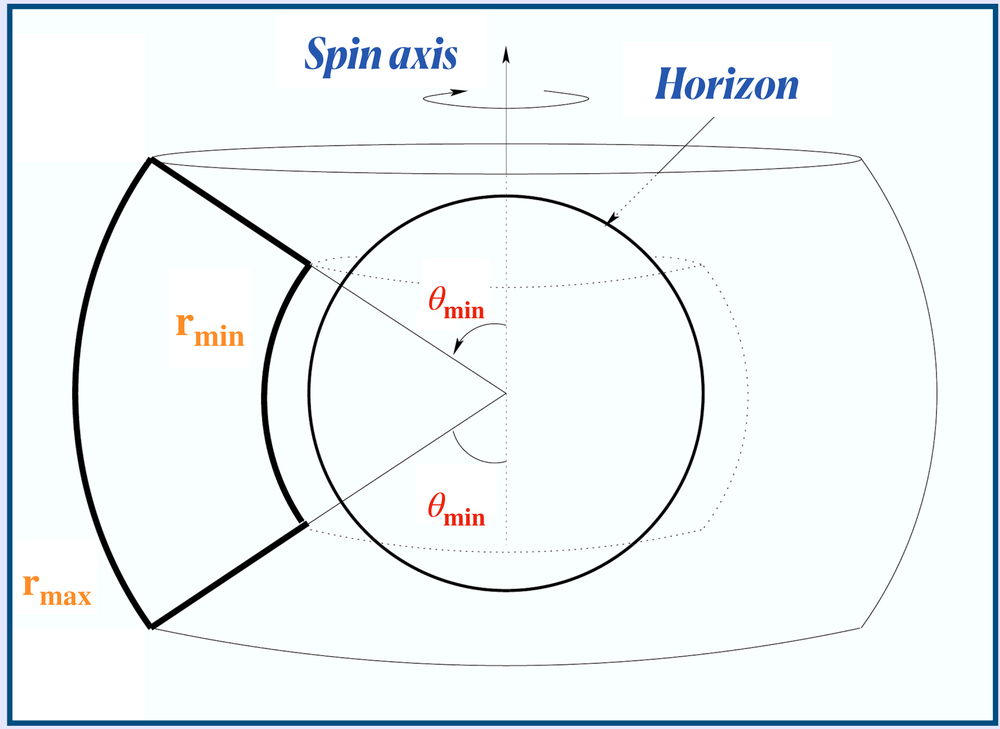
"Reference" *geodesic and spinning-body orbit have the same* ***turning points***

$$r_{min} = \frac{p}{1+e},\ r_{max} = \frac{p}{1-e} \text{ and } I = \pi/2 - sgn(L_z)\theta_{min}$$

$$r = \frac{pM}{1 + e\cos\left(\hat{\chi}_r + \chi_r^S\right)} + \delta r_S$$

Correction to libration region

Correction to true anomaly angle



Because are using a **frequency-domain formulation**; we can constrain all of the **purely radial** motion to remain inside the same radial turning points as the "reference" geodesic orbit.

Secondary spin contributes to the 1PA term

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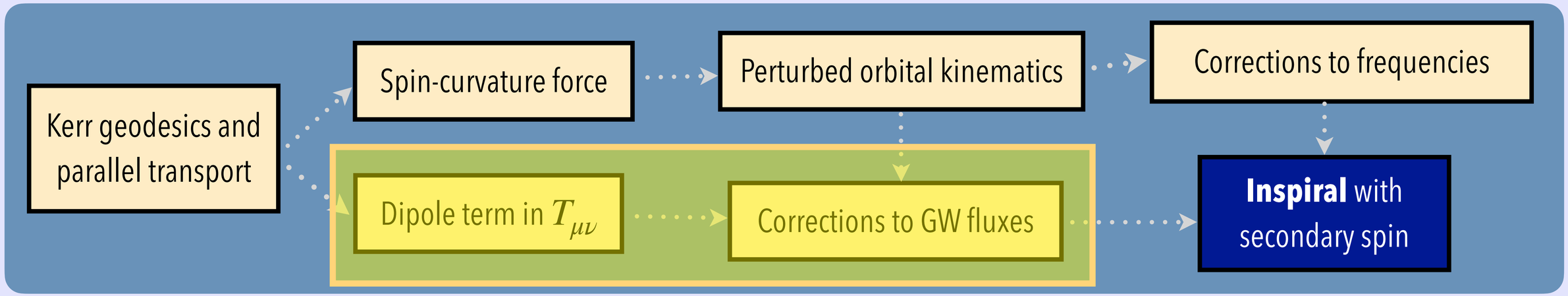
Secondary spin effects



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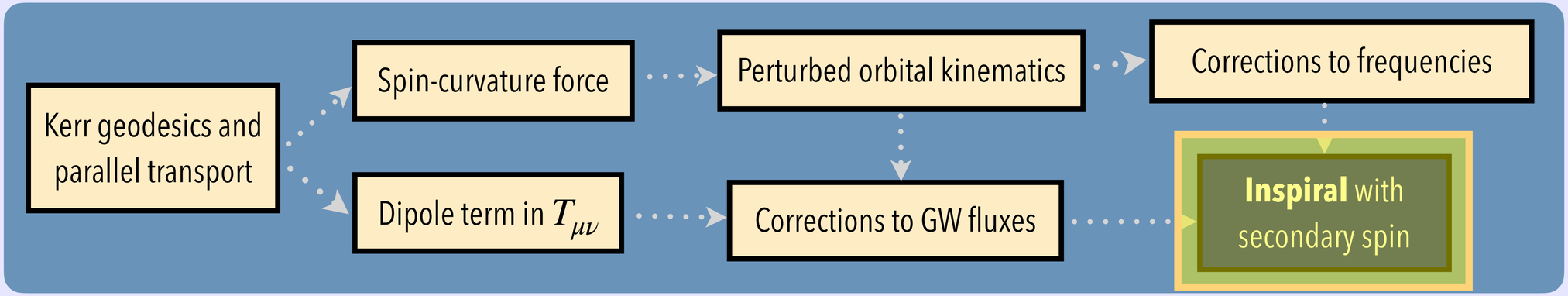
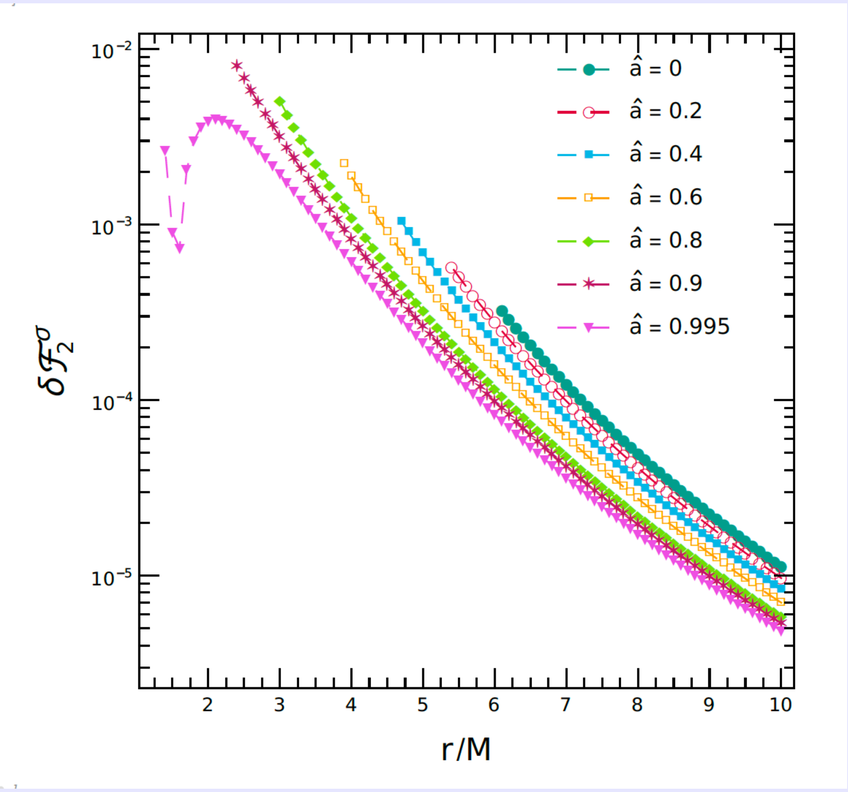
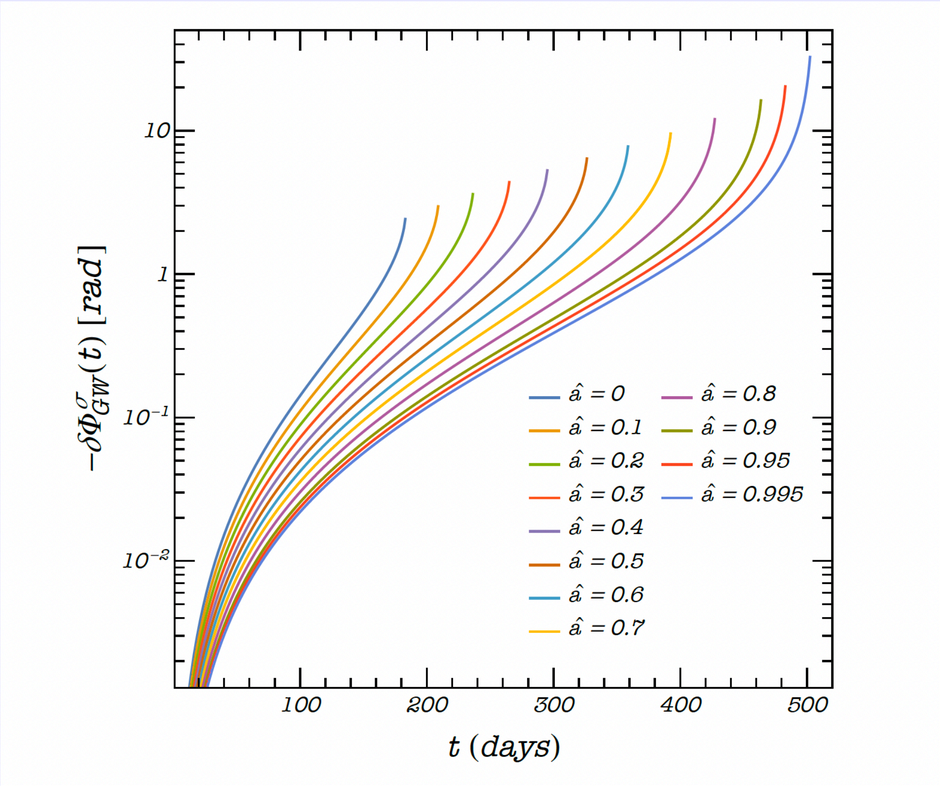


Circular aligned inspirals with spinning primary: Piovano et al., arXiv:2004.02654

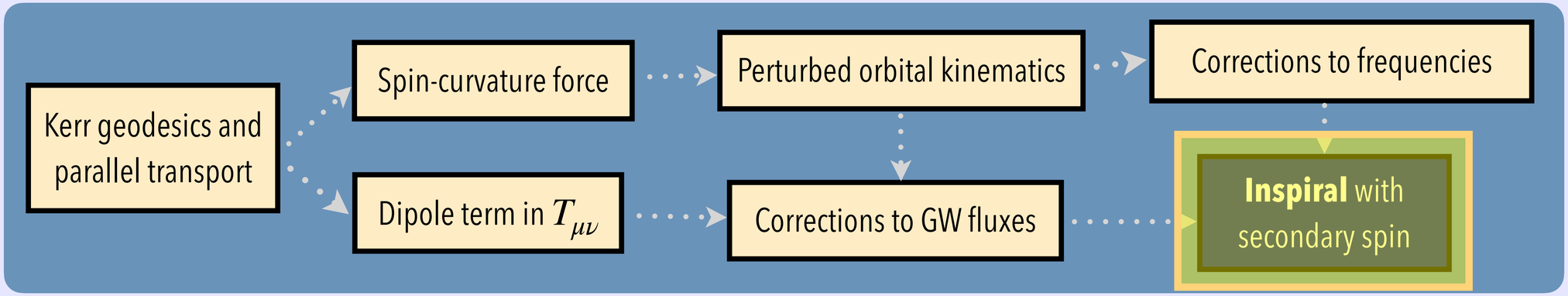


***GW fluxes** as a function of radius for different a values*

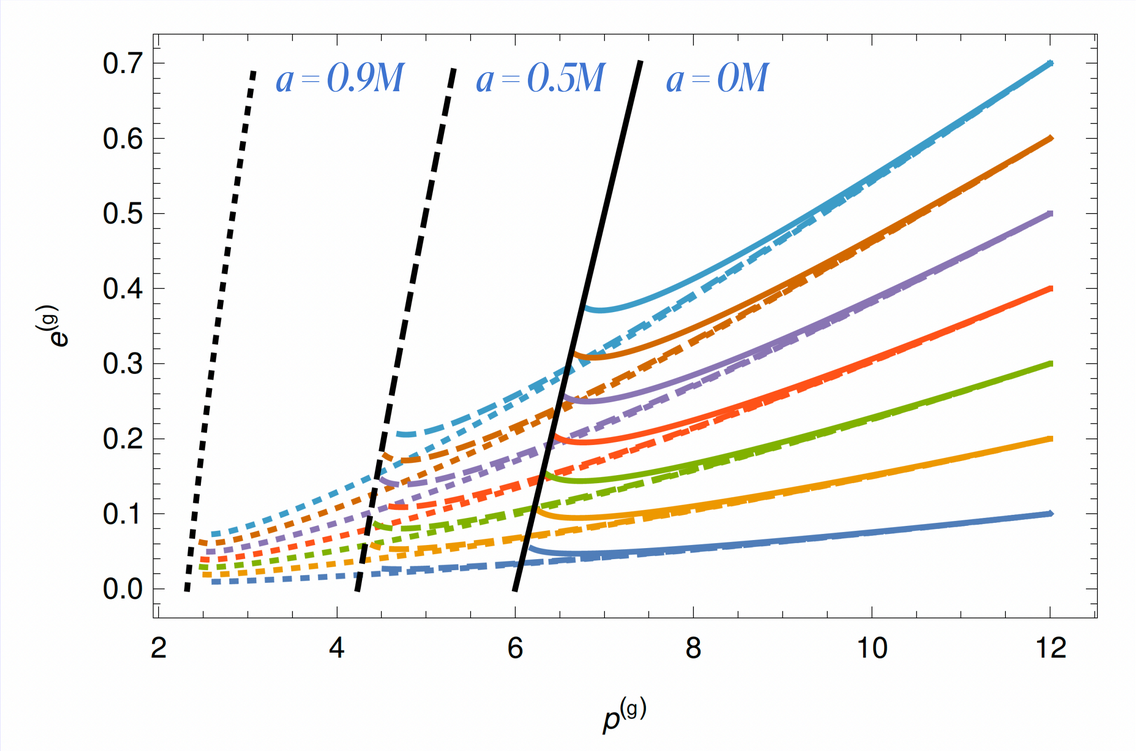


***GW phase shifts** as a function of radius for different a values*

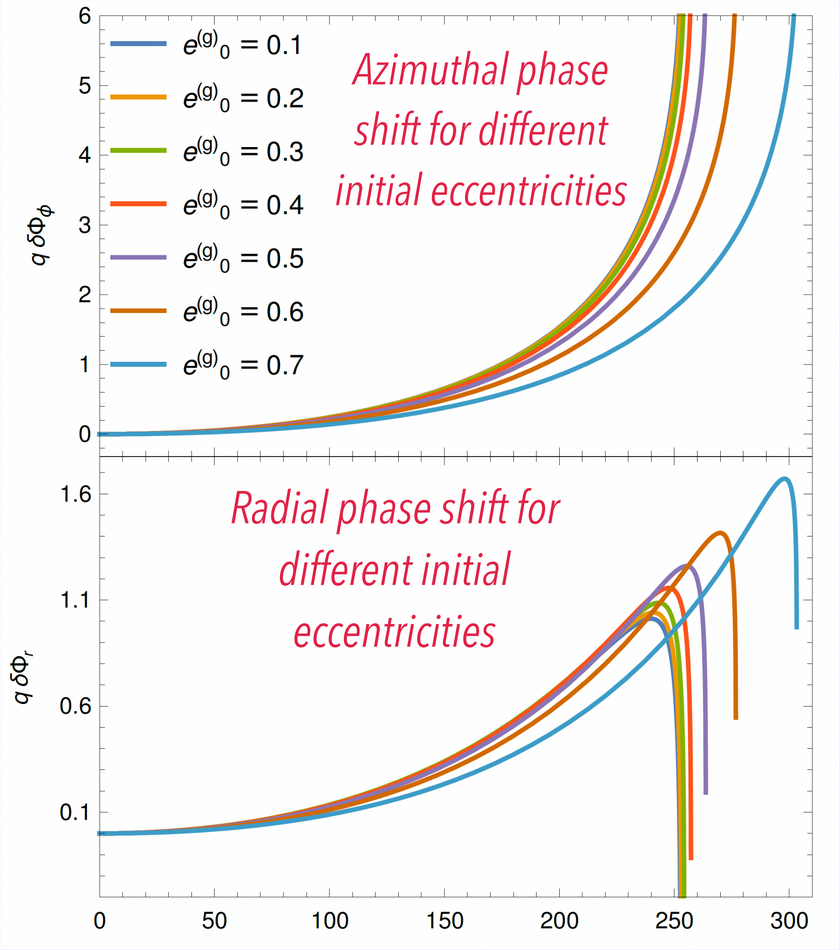
Flowchart *for spinning secondary contribution to* **GW phase**



Equatorial aligned inspirals, with spinning primary: Skoupý & Lukes-Gerakopoulos, arXiv:2102.04819 & arXiv:2201.07044

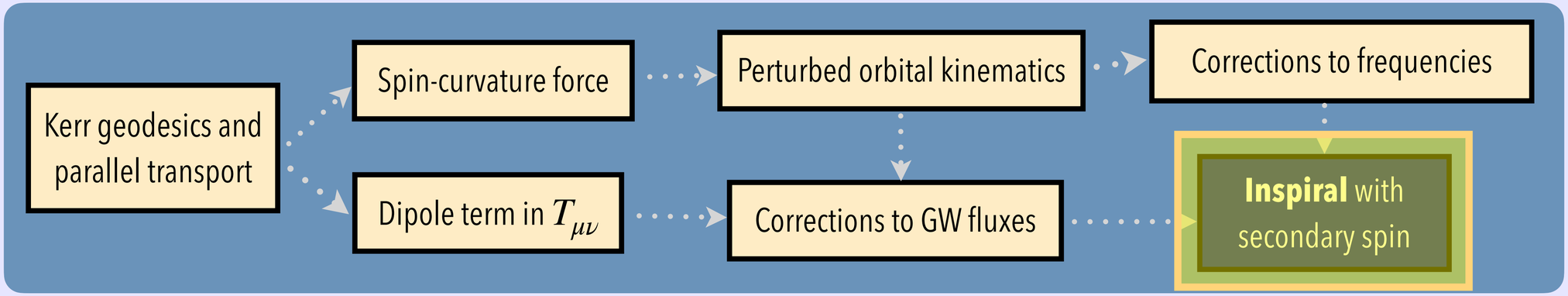


Trajectory in (p, e)-space

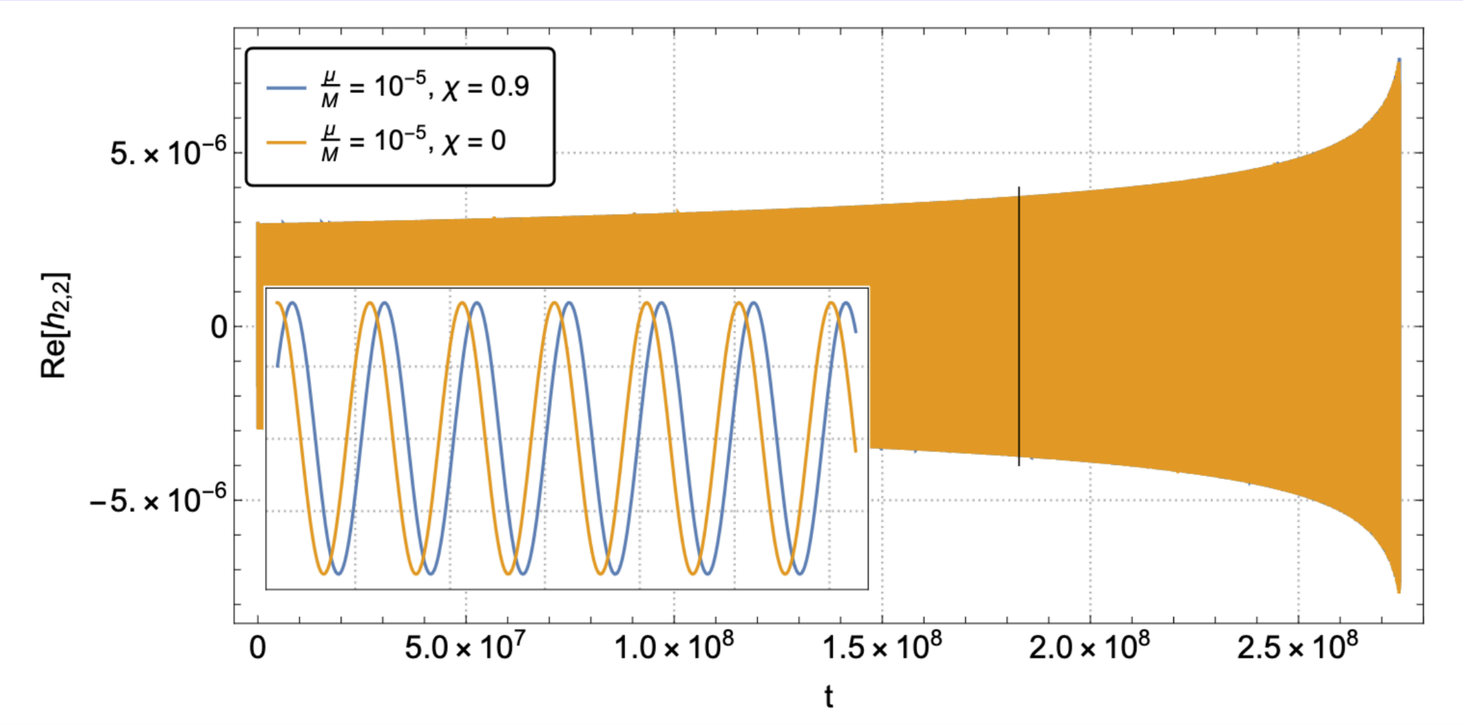


Time

Flowchart** for spinning secondary contribution to **GW phase



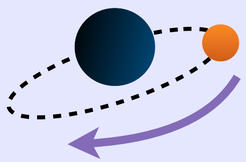
***Waveforms:** circular aligned orbit, non-spinning primary,* Mathews et al., arXiv:2112.13069



*Calculation of self-force **including spin of secondary** using RWZ formalism*

Components of the inspiral

Kinematics of an orbiting small body



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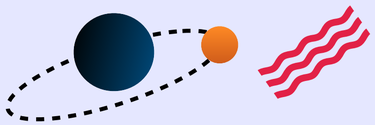
GW Radiation due to an orbiting small body



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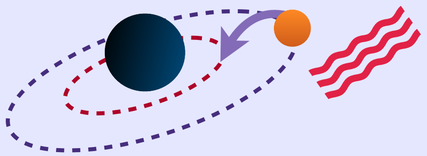
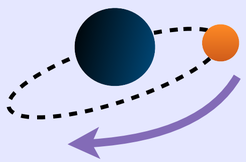
GW-driven inspiral



Components of the inspiral

Complexity of orbit

Kinematics of an orbiting small body



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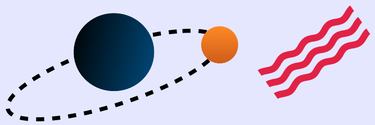
GW Radiation due to an orbiting small body



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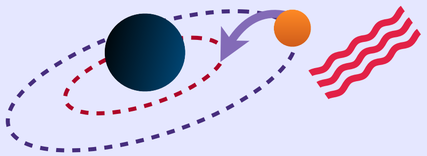
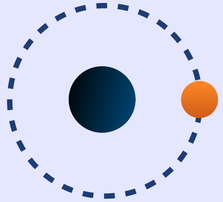
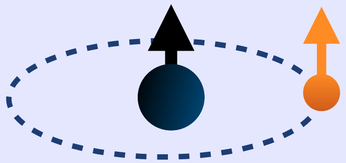
GW-driven **inspiral**

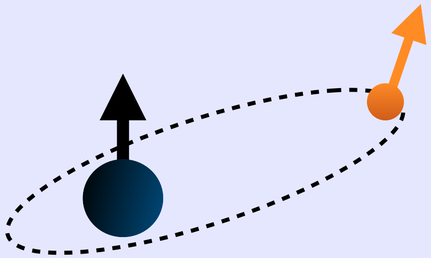




Circular equatorial inspiral with aligned spins



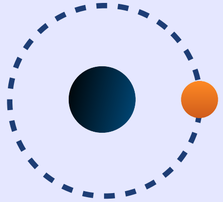
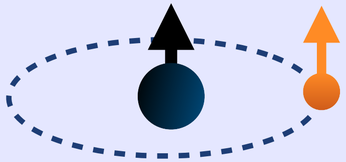
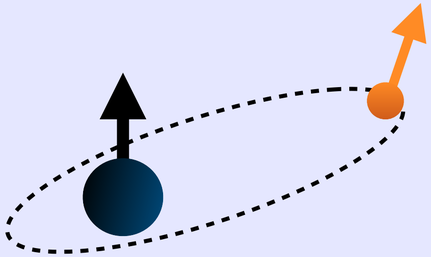
Eccentric equatorial inspiral with aligned spins



Generic inspiral with any spin orientation

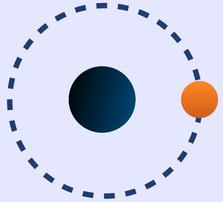
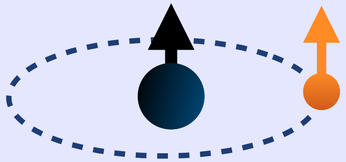
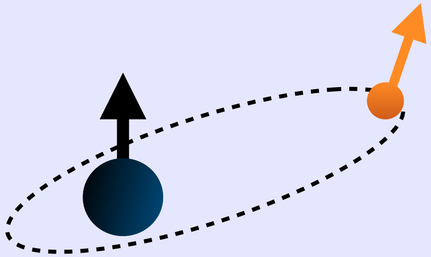
Components of the inspiral

Complexity of orbit

	Kinematics of an orbiting small body	+ GW Radiation due to an orbiting small body	= GW-driven inspiral
 Circular equatorial inspiral with aligned spins		Piovano, et al. 2020, **arXiv:**2004.02654 Mathews, et al. 2021, **arXiv:**2112.13069	Piovano, et al. 2020, **arXiv:**2004.02654 Mathews, et al. 2021, **arXiv:**2112.13069
 Eccentric equatorial inspiral with aligned spins		Skoupý, Lukes-Gerakopoulos, 2021, **arXiv:**2102.04819	Skoupý, Lukes-Gerakopoulos, 2022, **arXiv:**2201.07044
 Generic inspiral with any spin orientation	Witzany, 2019, **arXiv:**1903.03651 Witzany & Piovano, 2023, **arXiv:**2308.00021 Drummond & Hughes, 2022, **arXiv:**2201.13334, 2201.13335	Skoupý, Lukes-Gerakopoulos, Drummond & Hughes, 2023, **arXiv:**2303.16798	

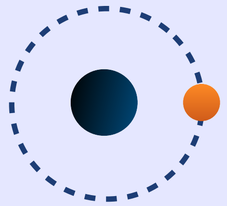
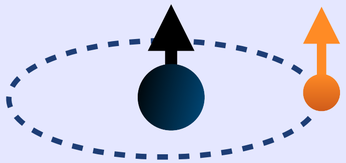
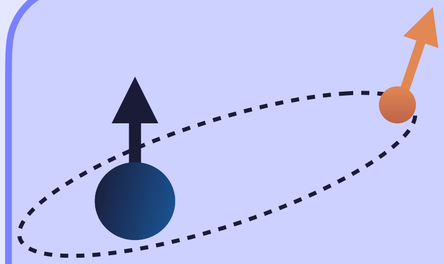
Components of the inspiral

Complexity of orbit

	Kinematics of an orbiting small body	+ GW Radiation due to an orbiting small body	= GW-driven inspiral
 Circular equatorial inspiral with aligned spins		Piovano, et al. 2020, **arXiv:**2004.02654 Mathews, et al. 2021, **arXiv:**2112.13069	Piovano, et al. 2020, **arXiv:**2004.02654 Mathews, et al. 2021, **arXiv:**2112.13069
 Eccentric equatorial inspiral with aligned spins		Skoupý, Lukes-Gerakopoulos, 2021, **arXiv:**2102.04819	Skoupý, Lukes-Gerakopoulos, 2022, **arXiv:**2201.07044
 Generic inspiral with any spin orientation	Witzany, 2019, **arXiv:**1903.03651 Witzany & Piovano, 2023, **arXiv:**2308.00021 Drummond & Hughes, 2022, **arXiv:**2201.13334, 2201.13335	Skoupý, Lukes-Gerakopoulos, Drummond & Hughes, 2023, **arXiv:**2303.16798	Generic inspiral: **work in progress**

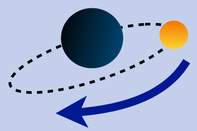
Components of the inspiral

Complexity of orbit

	Kinematics of an orbiting small body	+ GW Radiation due to an orbiting small body	= GW-driven inspiral
 Circular equatorial inspiral with aligned spins		Piovano, et al. 2020, **arXiv:**2004.02654 Mathews, et al. 2021, **arXiv:**2112.13069	Piovano, et al. 2020, **arXiv:**2004.02654 Mathews, et al. 2021, **arXiv:**2112.13069
 Eccentric equatorial inspiral with aligned spins		Skoupý, Lukes-Gerakopoulos, 2021, **arXiv:**2102.04819	Skoupý, Lukes-Gerakopoulos, 2022, **arXiv:**2201.07044
 Generic inspiral with any spin orientation	Witzany, 2019, **arXiv:**1903.03651 Witzany & Piovano, 2023, **arXiv:**2308.00021 Drummond & Hughes, 2022, **arXiv:**2201.13334, 2201.13335	Skoupý, Lukes-Gerakopoulos, Drummond & Hughes, 2023, **arXiv:**2303.16798	Generic inspiral: **work in progress**

How do we build a ***generic*** inspiral?

Kinematics *of an orbiting small body*



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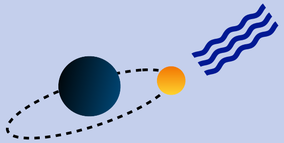
GW ***radiation*** *due to an orbiting small body*



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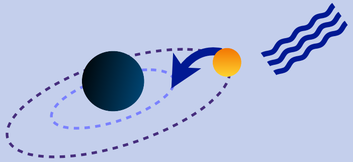
GW-driven ***inspiral***



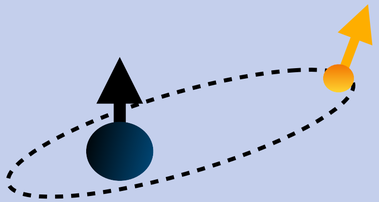
Spinning-body orbital kinematics

Drummond & Hughes, 2022a and 2022b

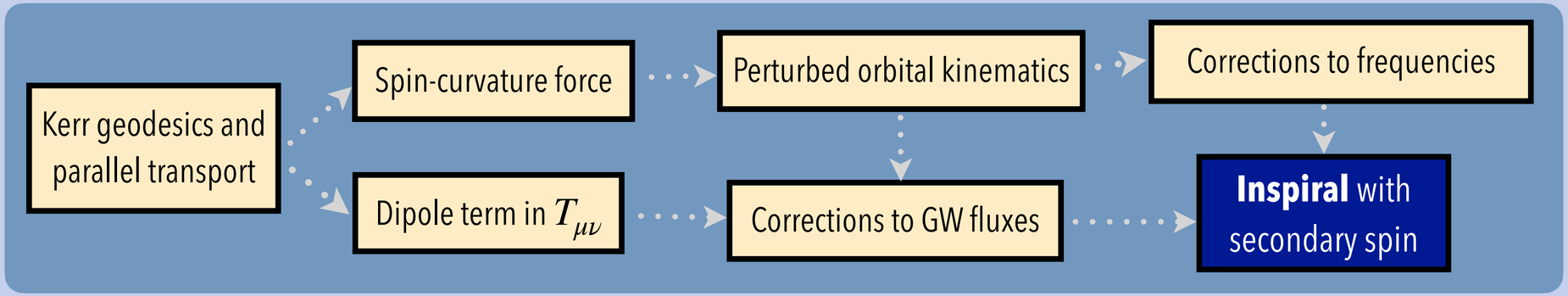
Spinning-particle GW fluxes

Skoupý, Lukes-Gerakopoulos, Drummond & Hughes, 2023

Generic ***inspiral*** *with any spin orientation*

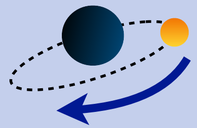


Work in progress; Carter-like constant evolution must be characterized



How do we build a ***generic*** inspiral?

Kinematics *of an orbiting small body*



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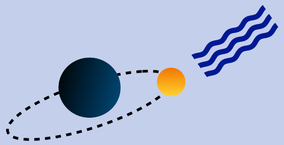
GW ***radiation*** *due to an orbiting small body*



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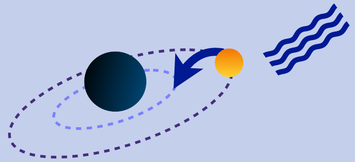
GW-driven ***inspiral***

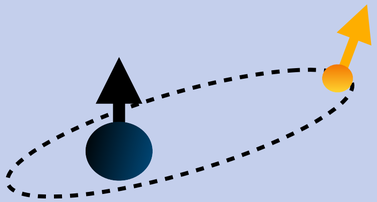


Spinning-body orbital kinematics

Non-spinning body: Point-particle GW fluxes

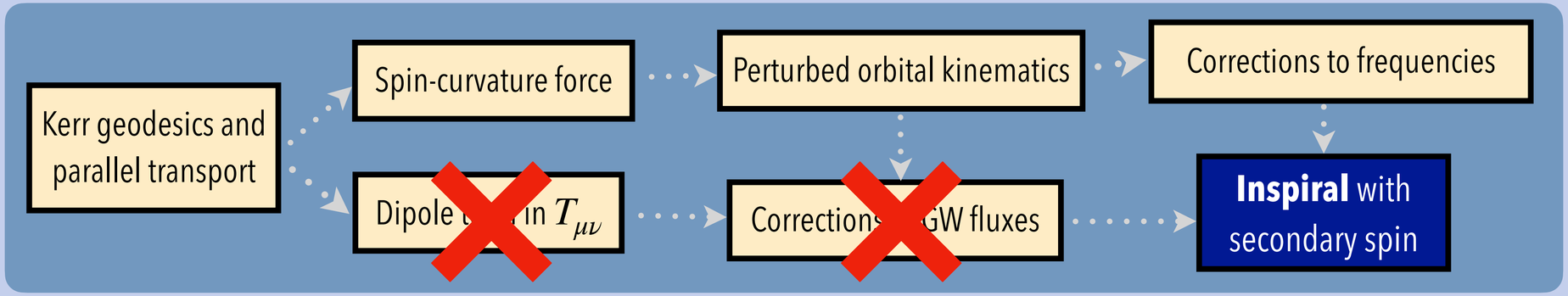
Hughes et al., 2021

Generic ***inspiral*** *with any spin orientation*

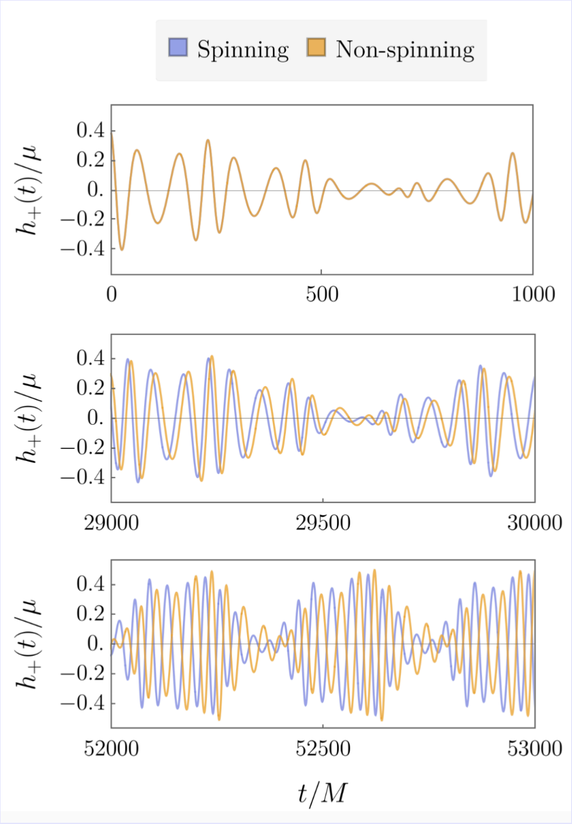


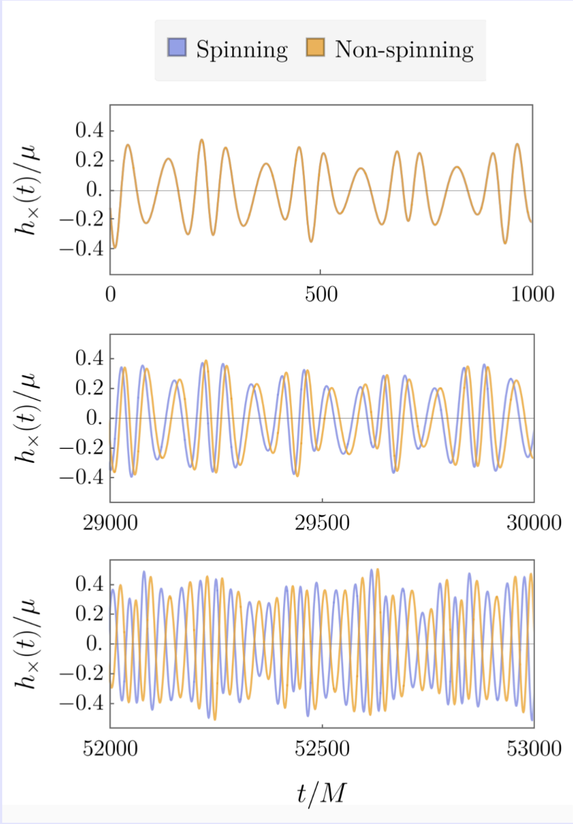
Drummond et al., 2023

a first pass only, incomplete!

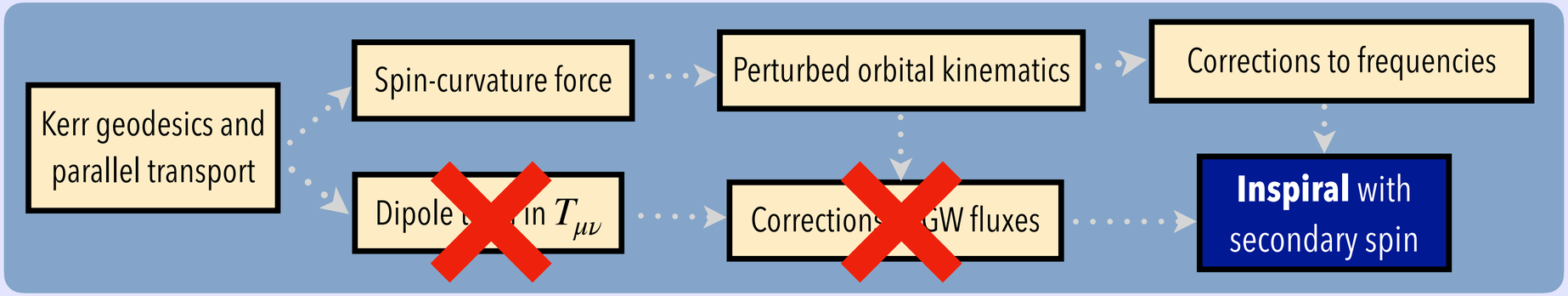


How do we build a ***generic*** inspiral?





Drummond et al., 2023



Review of recent progress (~2019 onwards)

★ **Fully generic spinning-body orbits** with arbitrary spin alignment are well-characterized; two different approaches have been used and compared (Witzany, arXiv:1903.03651, Drummond & Hughes, arXiv:2201.13334, arXiv:2201.13335)

★ **Self-force calculation including secondary spin** (Josh Mathews, arXiv:2212.13069)

★ **Equatorial inspirals including all spinning-body effects** have been computed (Skoupý et al., arXiv:2201.07044)

★**Fully generic spinning-body GW fluxes** computed (Skoupý et al., arXiv:2303.16798)

★ **First pass at generic spinning-body waveforms** (neglecting dipole stress-tensor effects) has been computed (Drummond et al., 2023)

★ The **detectability of small-body spin** has been assessed using a Fisher Matrix framework (Piovano et al., arXiv:2105.07083)

Open questions and *future work*

★ Compute generic **Carter-like constant evolution** for spinning secondary and generate generic inspirals

★ **Combine with self force** (for generic inspirals, need to take care that the parameterization of self-force and secondary spin effects is equivalent); study the interplay with self-force, do spinning secondary or self-force effects dominate in different regions of parameter space, do they cancel each other?

★ Compute GW flux data and phase shifts **across entire parameter space**

★ **Detectability** of small-body spin for generic orbital configurations? Full Bayesian infererence study, for example with ***few* (Fast EMRI waveforms)**

★ Incorporate these recent developments into the SpinningSecondary package of the **Black Hole Perturbation Toolkit**

Thank you!