UNIVERSITÀ
DEGLI STUDI
DELL'AQUILA

DISIM
Dipartimento di Ingegneria
e Scienze dell'Informazione
e Matematica

INTERMATHS

BOOK OF ABSTRACTS AND CONFERENCE SCHEDULE

INTERNATIONAL CONFERENCE ON PARTIAL DIFFERENTIAL EQUATIONS AND APPLICATIONS

in honor of the 70th birthday of Pierangelo Marcati

GRAN SASSO SCIENCE INSTITUTE, L'AQUILA, 19-23 JUNE, 2023

: https://indico.gssi.it/event/486/

The conference focuses on Partial Differential Equations and their applications to several branches of Science. The aim of the meeting is to gather leading experts and researchers to propose and discuss new ideas in the field of Partial Differential Equations.

The conference is in honor of our dear friend and colleague Pierangelo Marcati for his 70th birthday and aims at celebrating his scientific contributions and his constant academic commitment to the community.

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SPEAKERS

Fabio Ancona (University of Padova, Italy)
Hugo Beirão da Veiga (University of Pisa, Italy)
Stefano Bianchini (SISSA, Trieste, Italy)
Yann Brenier (CNRS, ENS, Paris, France)
Alberto Bressan (Penn State University of Pennsylvania, USA)
Piermarco Cannarsa (Tor Vergata Univerisity of Roma, Italy)
Gui-Qiang G. Chen (University of Oxford, UK)
Constantine Dafermos (Brown University, USA)
Camillo De Lellis (IAS Princeton, USA)
Carlo Doglioni (INGV - National Institute of Geophysics and Volcanology)
Eduard Feireisl (Czech Academy of Science, Czech Republic)
Vladimir Georgiev (University of Pisa, Italy)
Tai-Ping Liu (Academia Sinica, Taiwan and Stanford University, USA)
Peter Markowich (KAUST, Saudi Arabia)
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Giuseppe Mingione (University of Parma, Italy)
Roberto Natalini (CNR, Rome)
Ronghua Pan (Georgia Tech, USA)
Mario Pulvirenti (Sapienza University of Roma, Italy)
Paolo Secchi (University of Brescia, Italy)
Gigliola Staffilani (MIT Boston, USA)
Athanasios Tzavaras (KAUST, Saudi Arabia)
Alberto Valli (University of Trento, Italy)
Zhouping Xin (The Chinese University of Hong Kong, Hong Kong)

Program Overview

	Monday		Tuesday		Wednesday
9:30 - 10:15	Registration	9:15 - 10:00	**Mingione**	9:00 - 9:45	**De Lellis**
10:15 - 10:30	Opening	10:00 - 10:10	Q&A	9:45 - 9:55	Q&A
10:35 - 11:20	**Dafermos**	10:10 - 10:55	**Brenier**	9:55 - 10:40	**Tzavaras**
11:20 - 11:30	Q&A	10:55 - 11:05	Q&A	10:40 - 10:50	Q&A
11:30 - 12:15	**Beirão da Veiga**	11:05 - 11:25	☕	10:50 - 11:10	☕
12:15 - 12:25	Q&A	11:25 - 12:10	**Doglioni**	11:10 - 11:55	**Bianchini**
12:25 - 14:00	🍴	12:10 - 12:20	Q&A	11:55 - 12:05	Q&A
14:00 - 14:45	**Staffilani**	12:20 - 14:30	🍴	12:05 - 12:50	**Liu**
14:45 - 14:55	Q&A	14:30 - 15:15	**Pulvirenti**	12:50 - 13:00	Q&A
14:55 - 15:40	**Georgiev**	15:15 - 15:25	Q&A		
15:40 - 15:50	Q&A	15:25 - 16:10	**Feireisl**		
15:50 - 16:20	☕	16:10 - 16:20	Q&A		
16:20 - 17:05	**Pan**	16:20 - 17:35	☕ and Posters		
17:05 - 17:15	Q&A				
17:15 - 18:00	**Markowich**				
18:00 - 18:10	Q&A				
20:00 - 23:30					Social Dinner 🍴

	Thursday		Friday
9:15 - 10:00	**Cannarsa**	9:00 - 9:45	**Xin**
10:00 - 10:10	Q&A	9:45 - 9:55	Q&A
10:10 - 10:55	**Natalini**	9:55 - 10:40	**Mazzuccato**
10:55 - 11:05	Q&A	10:40 - 10:50	Q&A
11:05 - 11:25	☕	10:50 - 11:10	☕
11:25 - 12:10	**Chen**	11:10 - 11:55	**Bressan**
12:10 - 12:20	Q&A	11:55 - 12:05	Q&A
12:20 - 14:30	🍴	12:05 - 13:00	Conclusion
14:30 - 15:15	**Secchi**		
15:15 - 15:25	Q&A		
15:25 - 16:10	**Valli**		
16:10 - 16:20	Q&A		
16:20 - 16:40	☕		
16:40 - 17:25	**Ancona**		
17:25 - 17:35	Q&A		

MONDAY, June 19		
9:30 - 10:15	**Registration**	
10:15 - 10:30	**Opening**	
10:35 - 11:20	**Dafermos**	Hyperbolic systems of balance laws with stiff source
11:20 - 11:30	**Q&A**	
11:30 - 12:15	**Beirão da Veiga**	Poiseuille time periodic flows in space-periodic pipes
12:15 - 12:25	**Q&A**	
12:25 - 14:00	**Lunch Break**	
14:00 - 14:45	**Staffilani**	Some recent developments in wave turbulence theory
14:45 - 14:55	**Q&A**	
14:55 - 15:40	**Georgiev**	H^1 scattering for mass subcritical short-range NLS
15:40 - 15:50	**Q&A**	
15:50 - 16:20	**Coffee Break**	
16:20 - 17:05	**Pan**	Isentropic Approximation
17:05 - 17:15	**Q&A**	
17:15 - 18:00	**Markowich**	TBA
18:00 - 18:10	**Q&A**	

TUESDAY, June 20		
9:15 - 10:00	**Mingione**	Hopf, Caccioppoli and Schauder, reloaded
10:00 - 10:10	**Q&A**	
10:10 - 10:55	**Brenier**	Recovery of hyperbolic conservation laws by space-time optimization
10:55 - 11:05	**Q&A**	
11:05 - 11:25	**Coffee Break**	
11:25 - 12:10	**Doglioni**	Earth gradients
12:10 - 12:20	**Q&A**	
12:20 - 14:30	**Lunch Break**	
14:30 - 15:15	**Pulvirenti**	Is the BGK equation only a toy model ?
15:15 - 15:25	**Q&A**	
15:25 - 16:10	**Feireisl**	Glimm's method, convex integration, and density of the wild data for the Euler system of gas dynamics
16:10 - 16:20	**Q&A**	
16:20 - 17:35	**Coffee break and Poster Session**	

WEDNESDAY, June 21		
9:00 - 9:45	**De Lellis**	Area-minimizing integral currents: singularities and structure
9:45 - 9:55	**Q&A**	
9:55 - 10:40	**Tzavaras**	From Euler flows with friction to gradient flows
10:40 - 10:50	**Q&A**	
10:50 - 11:10	**Coffee Break**	
11:10 - 11:55	**Bianchini**	On spiral strategies for blocking fire
11:55 - 12:05	**Q&A**	
12:05 - 12:50	**Liu**	Coupling of Singular Layers for the Boltzmann Equation
12:50 - 13:00	**Q&A**	
13:00 - 14:30	**Free Lunch**	
14:30 - 20:00	**Free Afternoon**	
20:00 - 23:30	**Social Dinner**	

The social dinner will be held at the restaurant “Magione Papale”
Via Porta Napoli, 67/1 , 67100 L’Aquila (Italy)

THURSDAY, June 22		
9:15 - 10:00	**Cannarsa**	Direct and inverse problems for a class of degenerate parabolic equations arising in climate science
10:00 - 10:10	**Q&A**	
10:10 - 10:55	**Natalini**	A multiscale approach to cell movements
10:55 - 11:05	**Q&A**	
11:05 - 11:25	**Coffee Break**	
11:25 - 12:10	**Chen**	On the Compressible Euler-Poisson Equations and Related Nonlinear PDEs
12:10 - 12:20	**Q&A**	
12:20 - 14:30	**Lunch Break**	
14:30 - 15:15	**Secchi**	The two-dimensional plasma-vacuum interface problem in ideal MHD
15:15 - 15:25	**Q&A**	
15:25 - 16:10	**Valli**	On the proof of Taylor's conjecture: helicity is conserved for a magnetically-closed plasma
16:10 - 16:20	**Q&A**	
16:20 - 16:40	**Coffee Break**	
16:40 - 17:25	**Ancona**	Hard congestion limit of one-dimensional Euler equations with singular pressure in the BV setting
17:25 - 17:35	**Q&A**	

FRIDAY, June 23		
9:00 - 9:45	**Xin**	On The Existence of Multi-dimensional Compressible MHD Contact Discontinuities
9:45 - 9:55	**Q&A**	
9:55 - 10:40	**Mazzuccato**	Irregular transport and loss of regularity for transport equations
10:40 - 10:50	**Q&A**	
10:50 - 11:10	**Coffee Break**	
11:10 - 11:55	**Bressan**	Uniqueness and error estimates for hyperbolic conservation laws
11:55 - 12:05	**Q&A**	
12:05 - 13:00	**Conclusion**	

ABSTRACTS

FABIO ANCONA

University of Padova

HARD CONGESTION LIMIT OF ONE-DIMENSIONAL EULER EQUATIONS WITH SINGULAR PRESSURE IN THE BV SETTING

Abstract. The Euler equations with a maximal density constraint (hard congestion model) can be approximated by the system of gas dynamics with a singular pressure law (soft congestion model). I will present a rigorous justification of this singular limit in the setting of BV solutions. We will consider small BV perturbations of reference solutions constituted by (possibly interacting) large shock waves, which represent free/congested interfaces (in fact, this is a free boundary problem). The analysis is based on a front-tracking algorithm and on the introduction of appropriate rescaling of the singular pressure. This is a work in collaboration with R. Bianchini (IAC-CNR, Rome) and C. Perrin (CNRS, Aix Marseille Univ.).

References

[1] F. Ancona, R. Bianchini, C. Perrin, *Hard-congestion limit of the p-system in the BV setting*, ESAIM Proc. Surveys, Vol. 72, (2023), pp 1-23.

[2] F. Ancona, R. Bianchini, C. Perrin, *Soft-congestion approximation to the constrained Euler equations in the BV setting*, in preparation.

HUGO BEIRÃO DA VEIGA

University of Pisa

POISEUILLE TIME PERIODIC FLOWS IN SPACE-PERIODIC PIPES

Abstract. We study the motion of a viscous incompressible fluid in an $(n+1)$-dimensional double-infinite pipe Λ with an L-periodic shape in the $z = x_{n+1}$ direction. Denote by Σ_z the cross-section of the pipe at the level z, and by v_z the $(n+1)$-th component of the velocity. We look for fully developed solutions $\mathbf{v}(x, z, t)$ with a given T-time periodic total flux $g(t) = \int_{\Sigma_z} v_z(x, z, t)\, dx$ which should be simultaneously T-periodic with respect to time and L-space-periodic with respect to z. We prove existence and uniqueness for the above problem. The problem of determining $\mathbf{v}$ and Γ requires to solve a *non-standard parabolic equation involving a non-local term* of the solution. The new results [2] (which extend those proved by the author in [1]) were obtained in collaboration with Jiaqi Yang, from the Northwestern Polytechnical University, Xi'an, China.

References

[1] H. Beirão da Veiga, *Time-periodic solutions of the Navier-Stokes equations in unbounded cylindrical domains-Leray's problem for periodic flows*, Arch. Ration. Mech. Anal. 178(3), 301–325 (2005)

[2] H. Beirão da Veiga and J. Yang, *Analysis of a two-layer energy balance model: long time behaviour and greenhouse effect*, J. Math. Phys. 64 (2023), no. 1, Paper No. 011515, 19 pp.

STEFANO BIANCHINI

SISSA, Trieste

ON SPIRAL STRATEGIES FOR BLOCKING FIRE

Abstract. The fire blocking problem can be stated as follows: the fire is propagating in all directions with speed 1 starting from a nonempty open set $\Omega_0 \subset \mathbb{R}^2$, and a barrier $\Gamma(t)$ is constructed with speed $\sigma > 0$, i.e. $L(\Gamma(t)) \leq \sigma t$. The question is if there a strategy to build a barrier $\Gamma(t)$ which encloses the fire in finite time.
It is known that for $\sigma \leq 1$ this is impossible, while for $\sigma > 2$ there is an admissible strategy. An open conjecture is that for $\sigma \leq 2$ the fire cannot be blocked.
A somewhat simplified conjecture is that if the blocking barrier is spiral-like, then one can block the fire only if the speed of building the barrier is $\sigma > 2.61....$ The positive part of this conjecture is known.

In this talk I will present how we prove the negative part, i.e. if $\sigma \leq 2.61$ then every admissible spiral is exponentially diverging, and in particular the fire cannot be blocked.

YANN BRENIER

CNRS, Laboratoire de Mathématiques d'Orsay, Université Paris-Saclay

RECOVERY OF HYPERBOLIC CONSERVATION LAWS BY SPACE-TIME OPTIMIZATION

Abstract. The principle of least action has been used for a long time to find important equations of physics and mechanics by space-time optimization techniques. Nevertheless, it is customary to say that recovering their solutions by space-time convex minimization doesn't make much sense. However, this is indeed the case for systems of conservation laws with convex entropy, at least in small time, in particular that of the dynamics of isentropic gases. We will also discuss Einstein's equations in vacuum, which can be formulated in a form close to those of Euler (provided that density and velocity fields with matrix values are used), but for which the approach does not yet apply, for lack of apparent convexity.

ALBERTO BRESSAN

Penn State University

UNIQUENESS AND ERROR ESTIMATES FOR HYPERBOLIC CONSERVATION LAWS

Abstract. In this talk I shall review some old and new results about uniqueness of solutions to hyperbolic conservation laws. In particular: for any n x n strictly hyperbolic system, any weak solution which takes values inside the domain of the semigroup of vanishing viscosity limits and satisfies the Liu admissibility conditions actually coincides with a semigroup trajectory. Implications of this result toward a posteriori error estimates will be discussed

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PIERMARCO CANNARSA

University of Rome Tor Vergata

DIRECT AND INVERSE PROBLEMS FOR A CLASS OF DEGENERATE PARABOLIC EQUATIONS ARISING IN CLIMATE SCIENCE

Abstract. A simple yet extremely valuable approach to the study of the climate system comes from the use of Energy Balance Models (EBMs), which had originally been introduced in the sixties independently by Budyko [1] and Sellers [5], and analysed in detail by Ghil [3]. Such models describe in a simplified yet effective way the evolution of the zonally averaged temperature on the Earth's surface, which turns out to be the solution of a nonlinear degenerate partial differential equation of parabolic type. After recalling the basic theory ensuring the well-posedness of the Cauchy problem for such equations, we will address some inverse problems, aiming at the determination of unknown quantities such as the insolation function.

The energy balance model can be improved by increasing vertical resolution. Indeed, considering various vertical layers it is possible to represent, at least approximately, the very important vertical exchange process occurring between surface and the atmosphere and, possibly, between different atmospheric levels (e.g. troposphere and stratosphere) (see [4]). Aiming at this, we will consider the two-layer energy balance model introduced in [2] discussing, as a first stage of the analysis, the resulting system of ordinary differential equations obtained by neglecting the latitudinal variation of the fields.

References

[1] M. I. Budyko, *The effect of solar radiation variations on the climate of the Earth*, Tellus, Vol. 21 (5), (1969), pp 611 - 619.

[2] P. Cannarsa, V. Lucarini, P. Martinez, C. Urbani, J. Vancostenoble, *Analysis of a two-layer energy balance model: long time behaviour and greenhouse effect*, preprint available on arXiv:2211.15430 [math.AP], 2022.

[3] M. Ghil, *Climate stability for a Sellers-type model*, J. Atmos Sci. 33, (1976), pp 3 - 20.

[4] D.L. Hartmann, *Global Physical Climatology*, 2nd edn, Academic Press, San Diego, CA, (411p), 2016.

[5] W. D. Sellers, *A Global Climatic Model Based on the Energy Balance of the Earth-Atmosphere System*, Journal of Applied Meteorology, 8(3), (1969), pp 392 - 400.

GUI-QIANG G. CHEN

University of Oxford

ON THE COMPRESSIBLE EULER-POISSON EQUATIONS AND RELATED NONLINEAR PDES

Abstract. We will discuss some recent developments in the analysis of the compressible Euler-Poisson equations and related nonlinear hyperbolic systems of balance laws. In particular, we will describe some recent results for the construction of global solutions, bounded in the energy norm, for the compressible Euler-Poisson equations with large initial data of spherical symmetry, including the general pressure law case, as the vanishing viscosity limits. Further related topics, perspectives, and open problems will also be addressed.

CONSTANTINE DAFERMOS

Brown University

HYPERBOLIC SYSTEMS OF BALANCE LAWS WITH STIFF SOURCE

Abstract. The method of redistribution of damping is employed for solving the Cauchy problem, in the BV setting, for hyperbolic systems of balance laws with partially dissipative source that becomes stiff as the relaxation time shrinks to zero. One may then pass to the relaxation limit.

CAMILLO DE LELLIS

School of Mathematics, Institute for Advanced Study, Princeton

AREA-MINIMIZING INTEGRAL CURRENTS: SINGULARITIES AND STRUCTURE

Abstract. Area-minimizing integral currents were introduced by De Giorgi, Federer, and Fleming to build a successfull existence theory for the *oriented* Plateau problem. While celebrated examples of singular minimizers were discovered soon after, a first theorem which summarizes the work of several mathematicians in the 60es and 70es (De Giorgi, Fleming, Almgren, Simons, and Federer) and a second theorem of Almgren from 1980 give general dimension bounds for the singular set which match the one of the examples, in codimension 1 and in general codimension respectively.

In joint works with Anna Skorobogatova and Paul Minter we prove that in higher codimension the singular set is $(m-2)$-rectifiable and the tangent cone is unique at $\mathcal{H}^{m-2}$-a.e. point. Independently and at the same time, a proof of the same result has been discovered also by Krummel and Wickramasekera. This theorem is the counterpart, in general codimension, of a celebrated work of Leon Simon in the nineties for the codimension 1 case. Moreover, a recent theorem by Liu proves that the singular set can in fact be a fractal of any Hausdorff dimension $\alpha \leq m-2$, indicating that the above structure theorem is indeed close to optimal.

CARLO DOGLIONI

INGV - National Institute of Geophysics and Volcanology

EARTH GRADIENTS

EDUARD FEIREISL

Institute of Mathematics of the Czech Academy of Sciences

GLIMM'S METHOD, CONVEX INTEGRATION, AND DENSITY OF THE WILD DATA FOR THE EULER SYSTEM OF GAS DYNAMICS

Abstract. By pasting several solutions of the Riemann problem we show the wild data for the Euler system of gas dynamics are dense in the Lebesgue topology. Based on a joint work with E. Chiodaroli (Pisa).

VLADIMIR GEORGIEV

University of Pisa

H^1 SCATTERING FOR MASS SUBCRITICAL SHORT-RANGE NLS

Abstract. We consider short-range mass-subcritical nonlinear Schrodinger equations and we show that the corresponding solutions with initial data in Sigma scatter in H^1. Hence we up-grade the classical scattering result proved by Yajima and Tsutsumi from L^2 to H^1. We also provide some partial results concerning the scattering of the first order moments, as well as a short proof via lens transform of a classical result due to Tsutsumi and Cazenave-Weissler on the scattering in Sigma. The work is based on a joint work with N.Visciglia, N.Tzvetkov and N.Burq.

TAI-PING LIU

Academia Sinica, Taiwan, and Stanford University

COUPLING OF SINGULAR LAYERS FOR THE BOLTZMANN EQUATION

Abstract. To study the relationship between the Boltzmann equation in the kinetic theory and the fluid dynamics, it is essential to study the singular layers in the Boltzmann solutions. There are boundary, initial and shock layers. These layers connect the essentially kinetic regions to the fluid-like regions for a Boltzmann solution. Natural physical settings usually give rise to the nonlinear coupling of these layers. Rich phenomena result from these couplings. For the analysis of these couplings, strong quantitative techniques are required. One such technique makes use of the explicit construction of the Green's function for the linearized Boltzmann equation. We will illustrate this through specific examples.

PETER MARKOWICH

King Abdullah University of Science and Technology (KAUST)

TBA

ANNA MAZZUCATO

Penn State University

IRREGULAR TRANSPORT AND LOSS OF REGULARITY FOR TRANSPORT EQUATIONS

Abstract. I will present recent results concerning examples of loss of regularity for solutions to linear transport equations with advecting field in Sobolev spaces below the Lipschitz class. I will discuss how this loss is generic and can be made instantaneous and total (that is, there exists smooth initial data for which the solution leaves instantaneously any Sobolev space of positive order).
This is joint work with Giovanni Alberti, Gianluca Crippa, Gautam Iyer, and Tarek Elgindi [1, 2].

References

[1] Giovanni Alberti, Gianluca Crippa, and Anna L. Mazzucato. *Loss of regularity for the continuity equation with non-Lipschitz velocity field*, Ann. PDE, 5(1):Paper No. 9, 19, 2019.

[2] Gianluca Crippa, Tarek Elgindi, Gautam Iyer, and Anna L. Mazzucato. *Growth of Sobolev norms and loss of regularity in transport equations*, Philos. Trans. Roy. Soc. A, 380(2225):Paper No. 24, 12, 2022.

GIUSEPPE MINGIONE

University of Parma

HOPF, CACCIOPPOLI AND SCHAUDER, RELOADED

Abstract. So called Schauder estimates are a classical fact in the theory of elliptic and parabolic PDEs. The basic problem they answer to is how the regularity of coefficients of equations reflects in the regularity of solutions. In the uniformly elliptic case this is a perturbative theory, in the sense that the main results can be obtained using as reference estimates those valid for equations without coefficients and making small perturbations. In the nonuniformly elliptic setting, as the one for instance fixed by Ladyzhenskaya and Uraltseva in the sixties and later on developed by many authors, Schauder estimates are not perturbative by counterexamples (there are elliptic problems with Hölder coefficients and discontinuous solutions). In this talk, we address the problem of validity of Schauder theory for nonuniformly elliptic equations and non-differentiable integral functionals. From recent, joint work with Cristiana De Filippis (Parma).

ROBERTO NATALINI

Consiglio Nazionale delle Ricerche

A MULTISCALE APPROACH TO CELL MOVEMENTS

Abstract. In this talk I focus on a quite general class of hybrid mathematical models of collective motions of cells under the influence of chemical stimuli. The models are hybrid in the sense that cells are discrete entities given by ODE, while the chemoattractant is considered as a continuous signal which solves a diffusive equation. For these models it is possible to prove the mean-field limit in the Wasserstein distance to a system given by the coupling of a Vlasov-type equation with the chemoattractant equation. This approach and results are not based on empirical measures, but rather on marginals of large number of individuals densities, and we show the limit with explicit bounds, by proving also existence and uniqueness for the limit system. In the monokinetic case we derive new pressureless nonlocal Euler-type model with chemotaxis, which will be compared with other macroscopic models of cell movement. Numerical simulation comparing the different scales are presented. These results have been obtained by Roberto Natalini and Thierry Paul in collaboration, for the numerical part, with Marta Menci.

References

[1] G. Bretti, A. De Ninno, R. Natalini, D. Peri, and N. Roselli, Estimation Algorithm for a Hybrid PDE–ODE Model Inspired by Immunocompetent Cancer-on-Chip Experiment, Axioms 2021, 10(4), 243.

[2] E. C. Braun, G. Bretti, R. Natalini, Parameter estimation techniques for a chemotaxis model inspired by Cancer-on-Chip (COC) experiments, International Journal of Non-Linear Mechanics, Volume 140, April 2022, 103895.

[3] R. Natalini, T. Paul, On The Mean Field limit for Cucker-Smale models, Discrete & Continuous Dynamical Systems – B, 2022, 27(5): 2873-2889.

[4] R. Natalini, T. Paul, The Mean-Field limit for hybrid models of collective motions with chemotaxis, SIAM J. Math. Anal. 55, No. 2, 900-928 (2023).

[5] M. Menci, R. Natalini, T. Paul, Microscopic, kinetic, and hydrodynamic models of collective motions with chemotaxis: a numerical study, preprint 2023.

RONGHUA PAN

Georgia Institute of Technology

ISENTROPIC APPROXIMATION

Abstract. In the study of compressible flows, the isentropic model was often used to replace the more complicated full system when the entropy is near a constant. This is based on the expectation that the corresponding isentropic model is a good approximation to the full system when the entropy is sufficiently close to the constant. We will discuss the mathematical justification of isentropic approximation in Euler flows and in Navier-Stokes-Fourier flows. This is based on the joint work with Y. Chen, J. Jia, and L. Tong.

MARIO PULVIRENTI

Sapienza University of Rome

IS THE BGK EQUATION ONLY A TOY MODEL ?

Abstract. In 1954 Bhatnagar, Gross and Krook, introduced a kinetic equation (the BGK equation), to handle physical situations where the Knudsen number is small compared to the scales where Boltzmann's equation can be applied, but not enough for using hydrodynamic equations. I consider a stochastic particle system, the inhomogeneous Kac model, which is a continuous version of the Bird's direct simulation Monte Carlo method (DSMC). By scaling suitably the space-time variables we can have either kinetic (Boltzmann) or hydrodynamic descriptions. Now the BGK equation cannot be obtained from a pure scaling, but it does follow from a simple modification of the dynamics, at least formally. I present this argument, developed in collaboration with P. Buttà and S. Simonella. This can be seen as a research plan.

PAOLO SECCHI

University of Brescia

THE TWO-DIMENSIONAL PLASMA-VACUUM INTERFACE PROBLEM IN IDEAL MHD

Abstract. In this talk we consider the two-dimensional plasma-vacuum interface problem in ideal compressible magnetohydrodynamics (MHD). This is a hyperbolic-elliptic coupled system with a characteristic free boundary. In the plasma region the 2D planar flow is governed by the hyperbolic equations of ideal compressible MHD, while in the vacuum region the magnetic field obeys the elliptic system of pre-Maxwell dynamics. At the free interface moving with the velocity of plasma particles, the total pressure is continuous and the magnetic field on both sides is tangent to the boundary. The plasma-vacuum system is not isolated from the outside world, since it is driven by a given surface current which forces oscillations onto the system. We present our result about the local-in-time existence and uniqueness of solutions to the nonlinear free boundary problem, provided that the plasma magnetic field or the vacuum magnetic field is non-zero at each point of the initial interface. The proof follows from the analysis of the linearized MHD equations in the plasma region and the elliptic system for the vacuum magnetic field, suitable tame estimates in Sobolev spaces for the full linearized problem, and a Nash-Moser iteration. This is a joint work with A. Morando (Brescia), Y. Trakhinin (Novosibirsk), P. Trebeschi (Brescia) and D. Yuan (Beijing Normal Univ.)

GIGLIOLA STAFFILANI

Massachusetts Institute of Technology

SOME RECENT DEVELOPMENTS IN WAVE TURBULENCE THEORY

Abstract. In this talk I will present two different approaches in the study of wave turbulence theory. The first, introduced by Bourgain, consists in analyzing the long time behavior of high Sobolev norms for the defocusing, cubic NLS equation on 2D tori (periodic solutions). In this context I will emphasize how the rationality or irrationality of the torus affects the analysis. The second approach deals with the rigorous derivation of the 3-wave kinetic equation from a weakly nonlinear multidimensional KdV type equation.

ATHANASIOS E. TZAVARAS

King Abdullah University of Science and Technology (KAUST)

FROM EULER FLOWS WITH FRICTION TO GRADIENT FLOWS

Abstract. I will review some works on the high-friction limit (or small mass approximation) from Euler flows to advection-diffusion systems that are gradient flows, and related asymptotic problems in fluid mechanics. The formulation at an abstract level exploits the variational structure of compressible Euler flows and is connected to the interpretation of nonlinear Fokker-Planck systems as gradient flows in Wasserstein distance. The technical tool is relative entropy formula for abstract Euler flows induced by the variational structure. Examples that fit into the framework is the emergence of porous media as the high friction limit from the compressible Euler equations with friction. Others examples include the high-friction limit form bipolar Euler-Poisson models to the bipolar drift-diffusion equations, as well as other asymptotic limit problems for electromechanical models like the zero-electron mass limit in plasmas. Finally, I discuss high-friction limits in multicomponent systems and the emergence of the Maxwell-Stefan diffusion system from systems describing multicomponent flows of compressible gases.

ALBERTO VALLI

University of Trento

ON THE PROOF OF TAYLOR'S CONJECTURE: HELICITY IS CONSERVED FOR A MAGNETICALLY-CLOSED PLASMA

Abstract. Taylor's conjecture in resistive magnetohydrodynamics states that the helicity of the entire domain is conserved in time (see Taylor [4], [5]).

The helicity of a simply connected domain Ω is defined as follows: consider the space of divergence-free and tangential vector fields

$$V = \{\mathbf{v} \in (L^2(\Omega))^3 \,|\, \operatorname{div} \mathbf{v} = 0 \ \text{ in } \ \Omega, \ \mathbf{v} \cdot \mathbf{n} = 0 \ \text{ on } \ \partial\Omega\}.$$

The first step is defining the helicity of a vector field $\mathbf{v} \in V$, which is given by

$$H(\mathbf{v}) = \int_\Omega \mathbf{v} \cdot \mathbf{A},$$

where $\mathbf{curl}\,\mathbf{A} = \mathbf{v}$, namely, $\mathbf{A}$ is a vector potential of $\mathbf{v}$. Taking into account that Ω is simply connected, it is easily shown that this definition is gauge invariant: it does not depend on $\mathbf{A}$ but only on $\mathbf{v}$. Finally, the helicity of the domain Ω is defined by

$$H_\Omega = \sup_{\mathbf{v}\in V,\ \|\mathbf{v}\|_{L^2(\Omega)}=1} |H(\mathbf{v})|. \tag{1}$$

A gauge invariant definition of the helicity of a vector field $\mathbf{v} \in V$ in a multiply connected domain has been given by MacTaggart and Valli in [3], and reads as follows:

$$\Upsilon(\mathbf{v}) = \int_\Omega \mathbf{A}\cdot\mathbf{v} - \sum_{j=1}^{g} \Big(\oint_{\gamma_j} \mathbf{A}\cdot\mathbf{t}_j \Big) \Big(\int_{\Sigma_j} \mathbf{v}\cdot\mathbf{n}_j \Big), \tag{2}$$

where $\{\gamma_j\}_{j=1}^g$ are the cycles which generate the first homology group of $\overline{\Omega}$ (the tangent vector on γ_j is denoted by $\mathbf{t}_j$), and $\{\Sigma_j\}_{j=1}^g$ are connected orientable Lipschitz 'cutting' surfaces satisfying $\Sigma_j \subset \Omega$ and $\partial\Sigma_j \subset \partial\Omega$, such that every curl-free vector in Ω has a global potential in the 'cut' domain $\Omega^0 := \Omega \setminus \bigcup_{j=1}^g \Sigma_j$ (each surface Σ_j 'cuts' the corresponding cycle γ_j; the orientation of unit normal vector $\mathbf{n}_j$ agrees with that of $\mathbf{t}_j$).

Recently, Faraco and Lindberg [1] provided a rigorous proof of Taylor's conjecture by considering limits of Leray - Hopf solutions. Their proof is valid for simply connected and multiply connected domains. In the former, the choice of the vector potential in the magnetic helicity is arbitrary, but in the latter this is not the case; in particular, the magnetic flux through the cuts Σ_j must be zero (e.g., the toroidal magnetic flux in a tokamak would be zero, which is a non-physical condition).

Thus the problem of confirming Taylor's conjecture without reference to specific vector potentials remained to be resolved.

In this talk, following Faraco et al. [2], we bring together the results in [1] and [3] to provide a complete proof of Taylor's conjecture for arbitrary vector potentials of divergence-free and tangential magnetic fields with no restrictions on the magnetic flux.

References

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ZHOUPING XIN

The Chinese University of Hong Kong

ON THE EXISTENCE OF MULTI-DIMENSIONAL COMPRESSIBLE MHD CONTACT DISCONTINUITIES

Abstract. Contact discontinuities for the ideal compressible magnetohydrodynamics (MHD) are most typical interfacial waves for astrophysical plasmas and prototypical fundamental waves for systems of hyperbolic conservations laws. Such waves are characteristic discontinuities for which there is no flow across the discontinuity surface while the magnetic field crosses transversally, which lead to a two-phase free boundary problem where the pressure, velocity and magnetic field are continuous across the interface whereas the entropy and density may have discontinuities. Some of the major difficulties for the existence of the Multi-dimensional ideal MHD contact discontinuities are the possible nonlinear Rayleigh-Taylor instability and loss of derivatives due to the non-ellipticity of the associated linearized problem. In this talk, I will present the recent work where we have proved the local existence and uniqueness of MHD contact discontinuities in both 2D and 3D in Sobolev spaces without any additional constraints such as Rayleigh-Taylor sign condition or with surface tensions. The key ingredients of our analysis are the Cauchy formula for MHD, the transversality of the magnetic field, and an elaborate viscous approximation. This talk is based on a joint work with Professor Yanjin Wang of Xiamen University.
