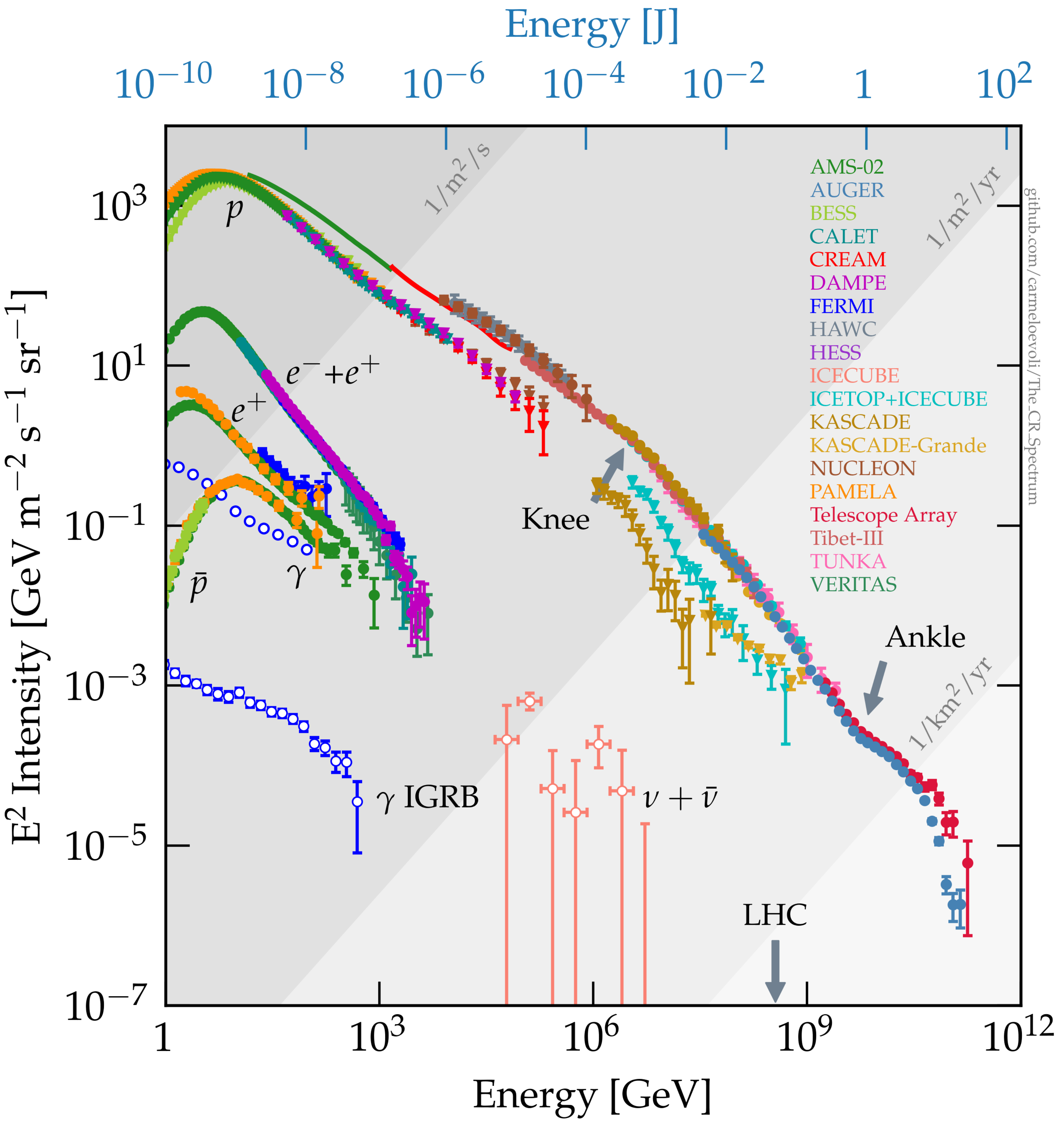
GRAN SASSO SCIENCE INSTITUTE

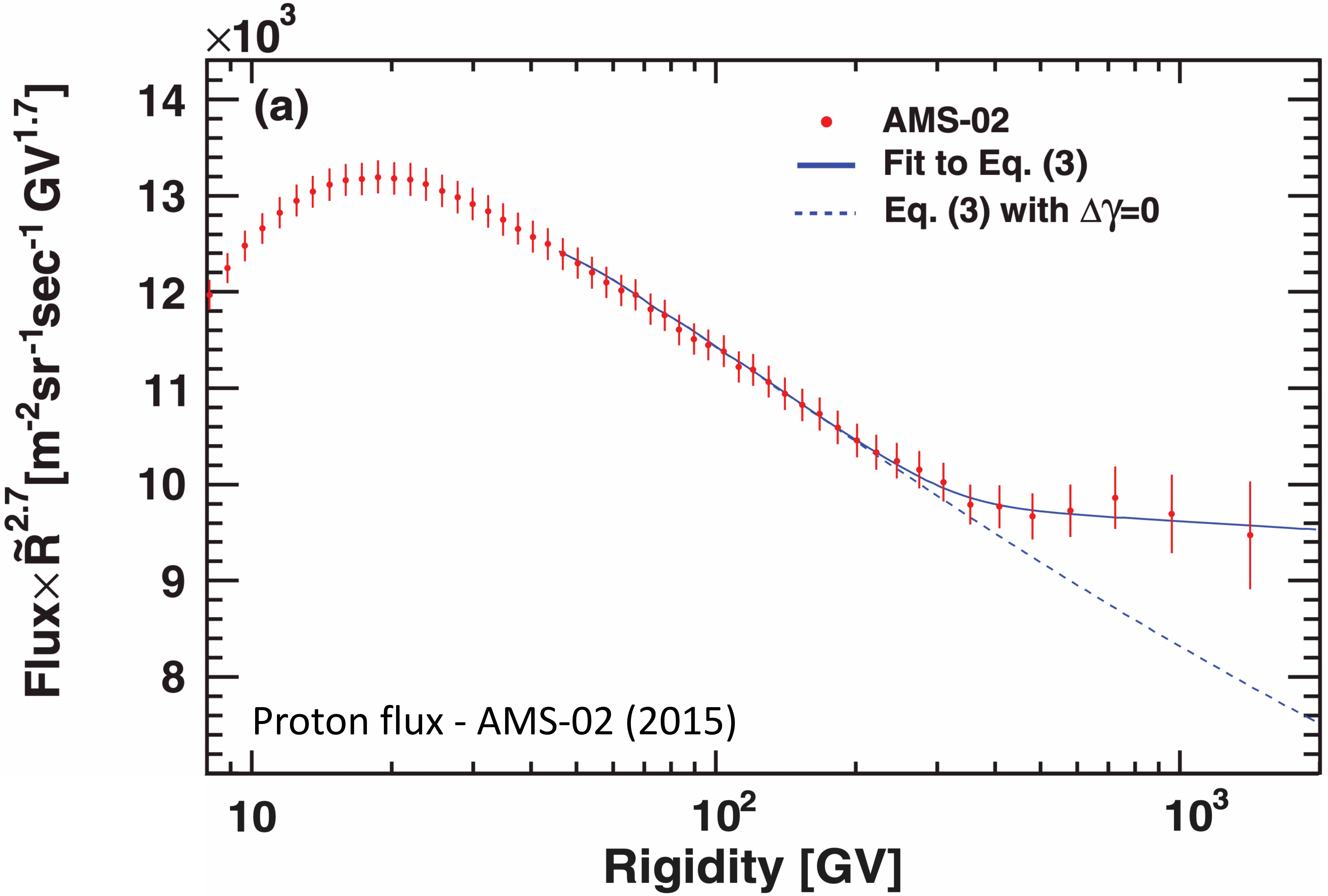
# CR transport in MHD turbulence

**Presented by: Ottavio Fornieri**

# The general picture



# Anomalies in the details

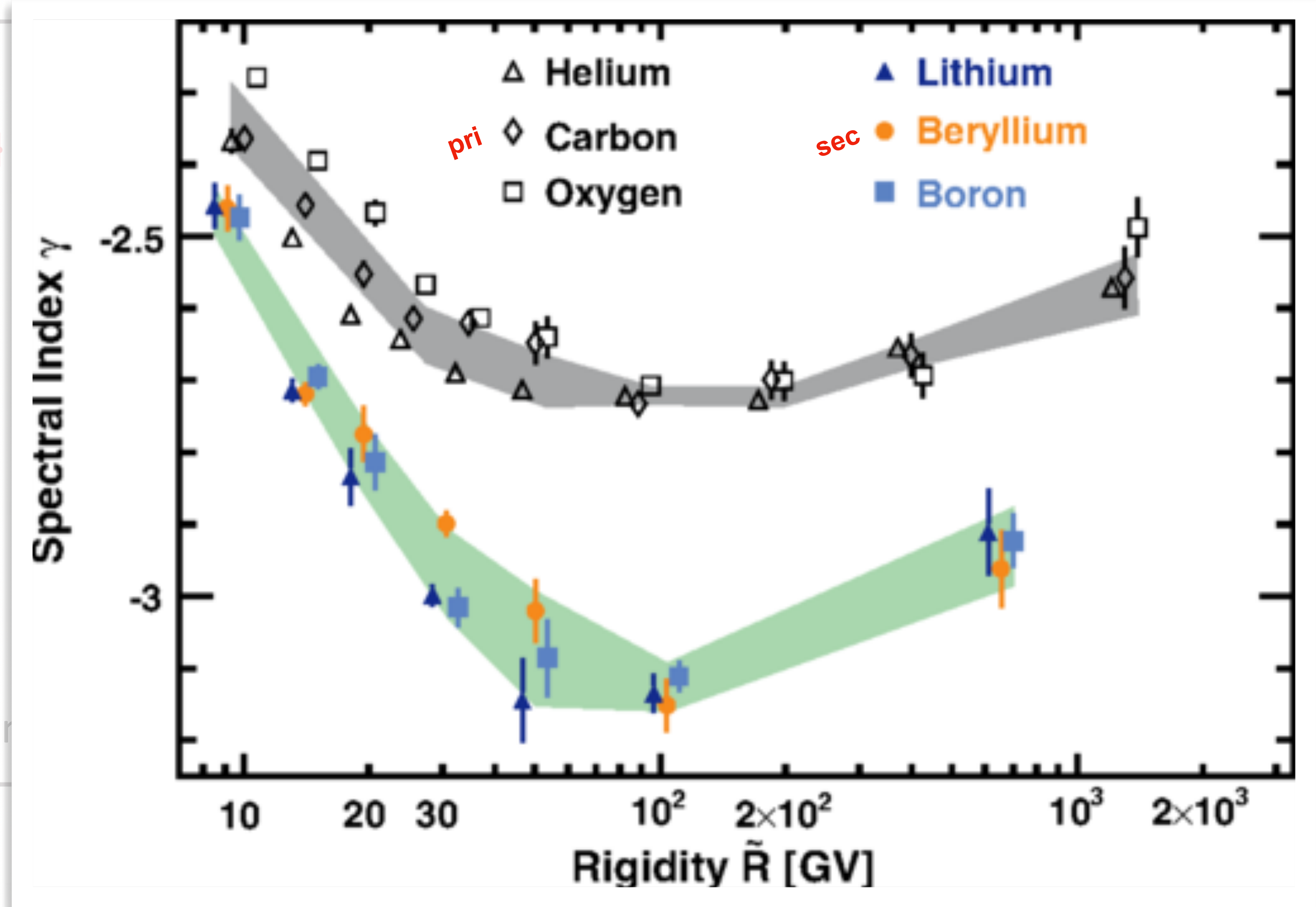


# Anomalies in the details

$$f_0(E) \sim N(E)/D(E)$$

$$N(E) \sim E^{-\Gamma_{inj}}$$
$$D(E) \sim E^{\delta_{diff}}$$

$$f_0^{pri}(E) \sim E^{-\Gamma_{inj}-\delta_{diff}}$$
$$f_0^{sec}(E) \sim E^{-\Gamma_{inj}-\delta_{diff}-\delta_{diff}} \sim E^{-\Gamma_{inj}-2\delta_{diff}}$$



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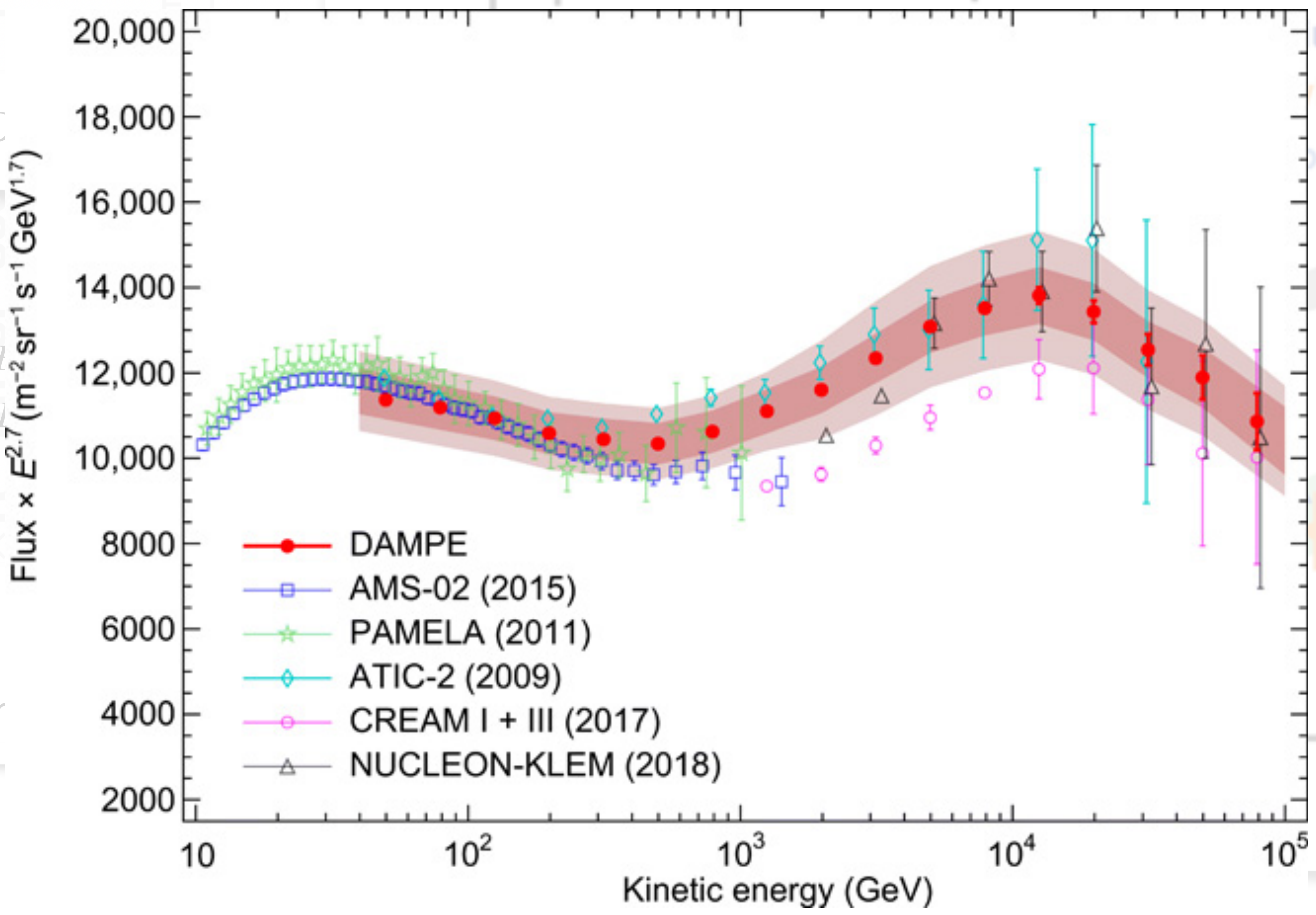

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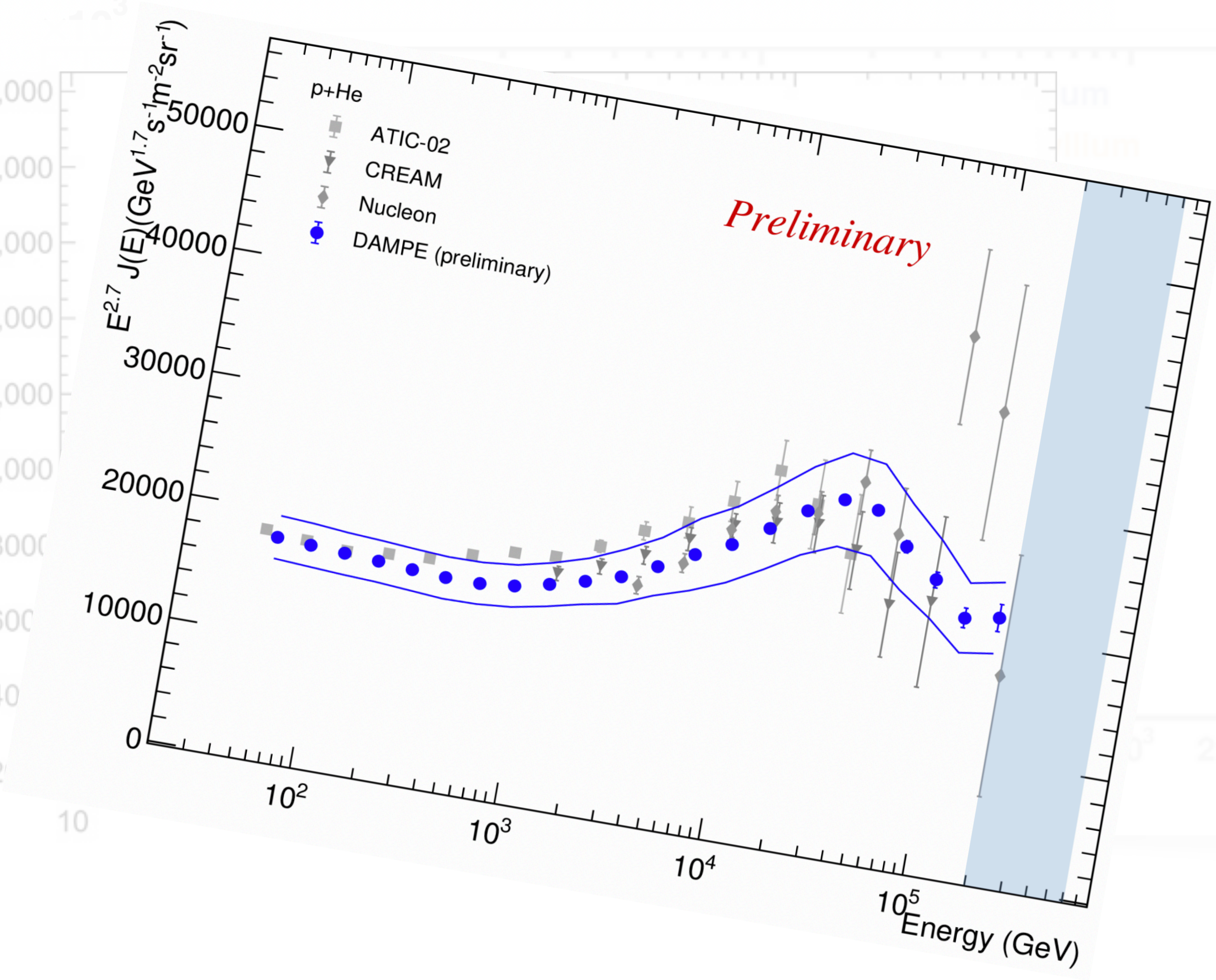

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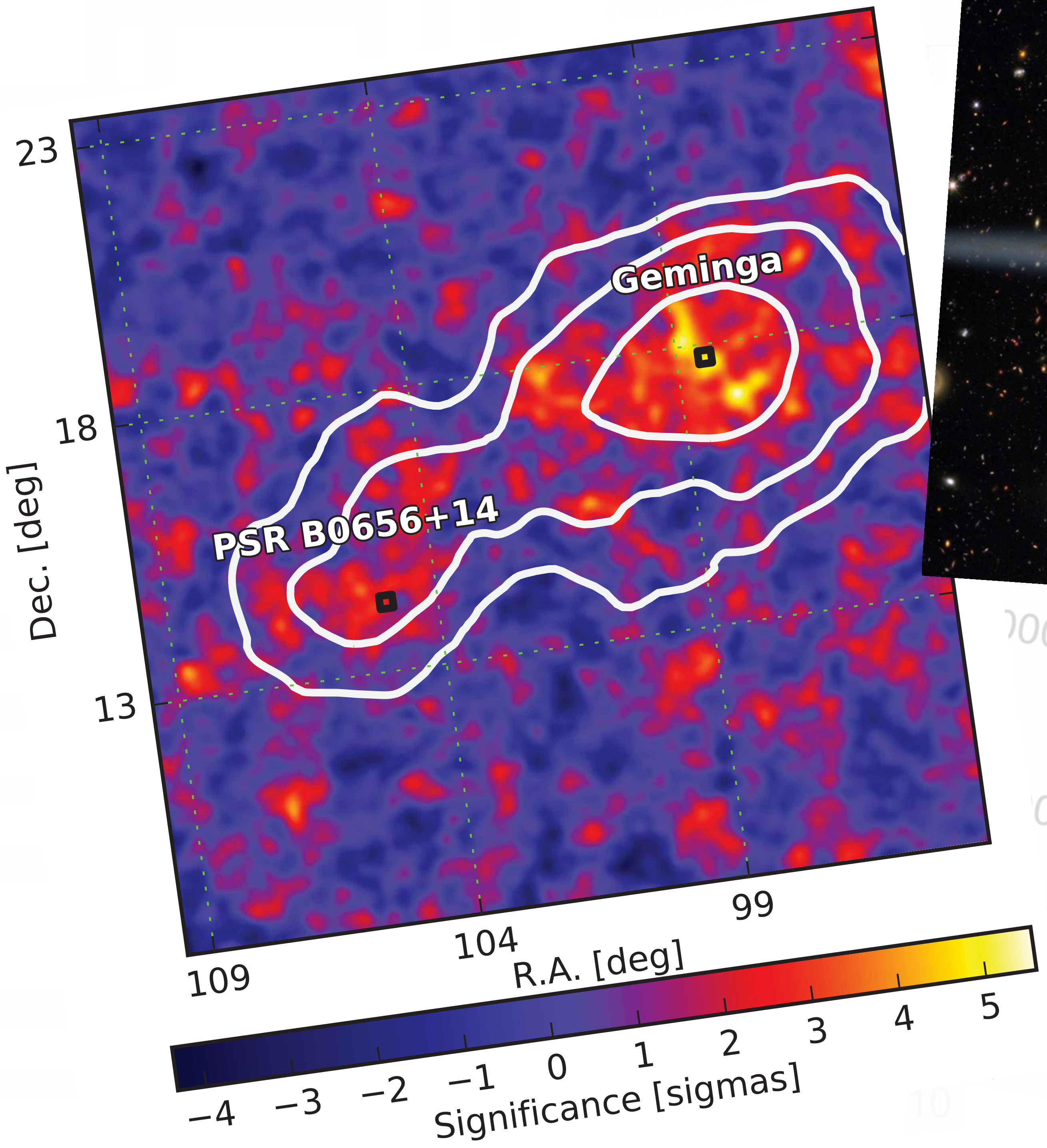


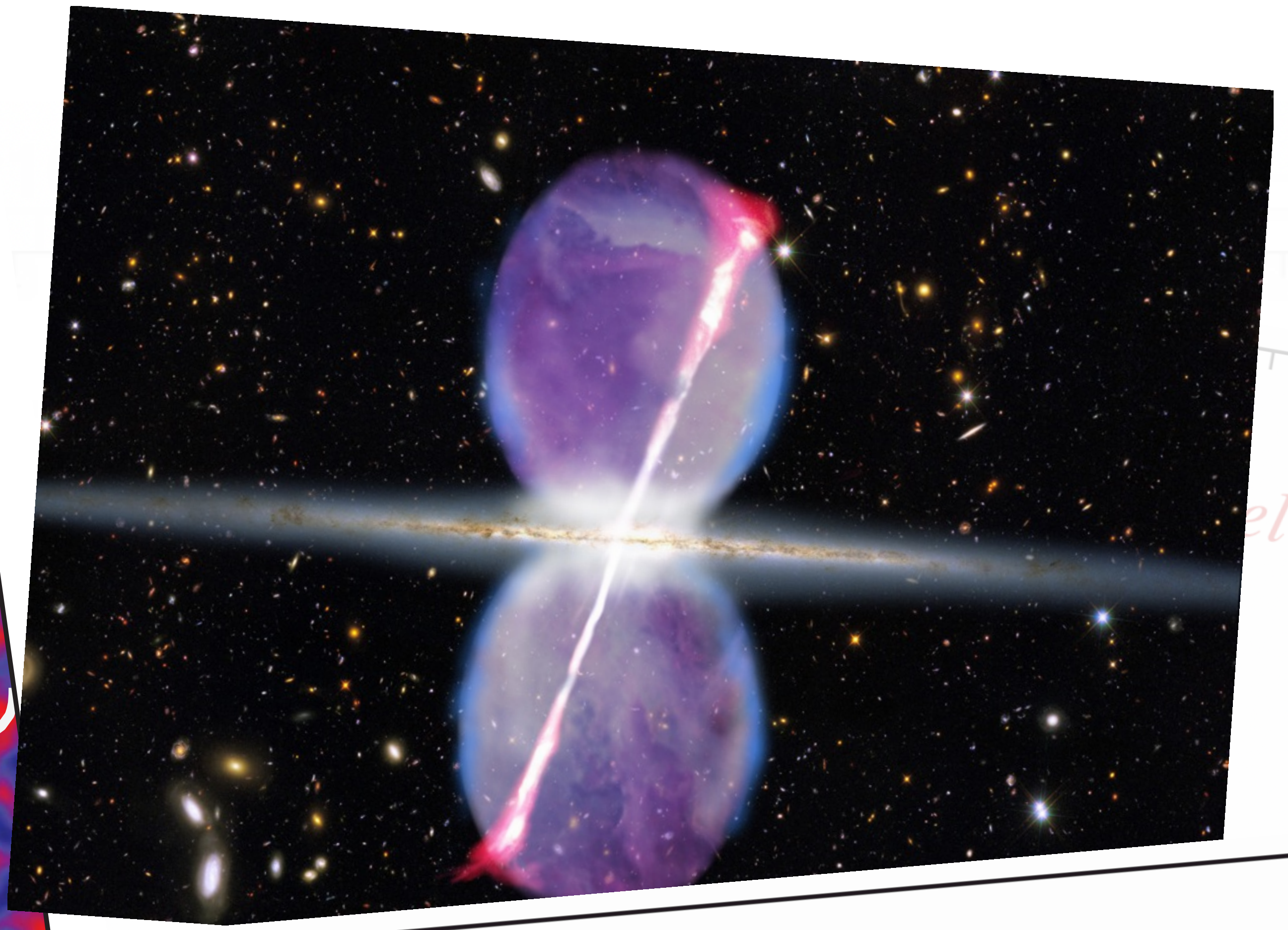

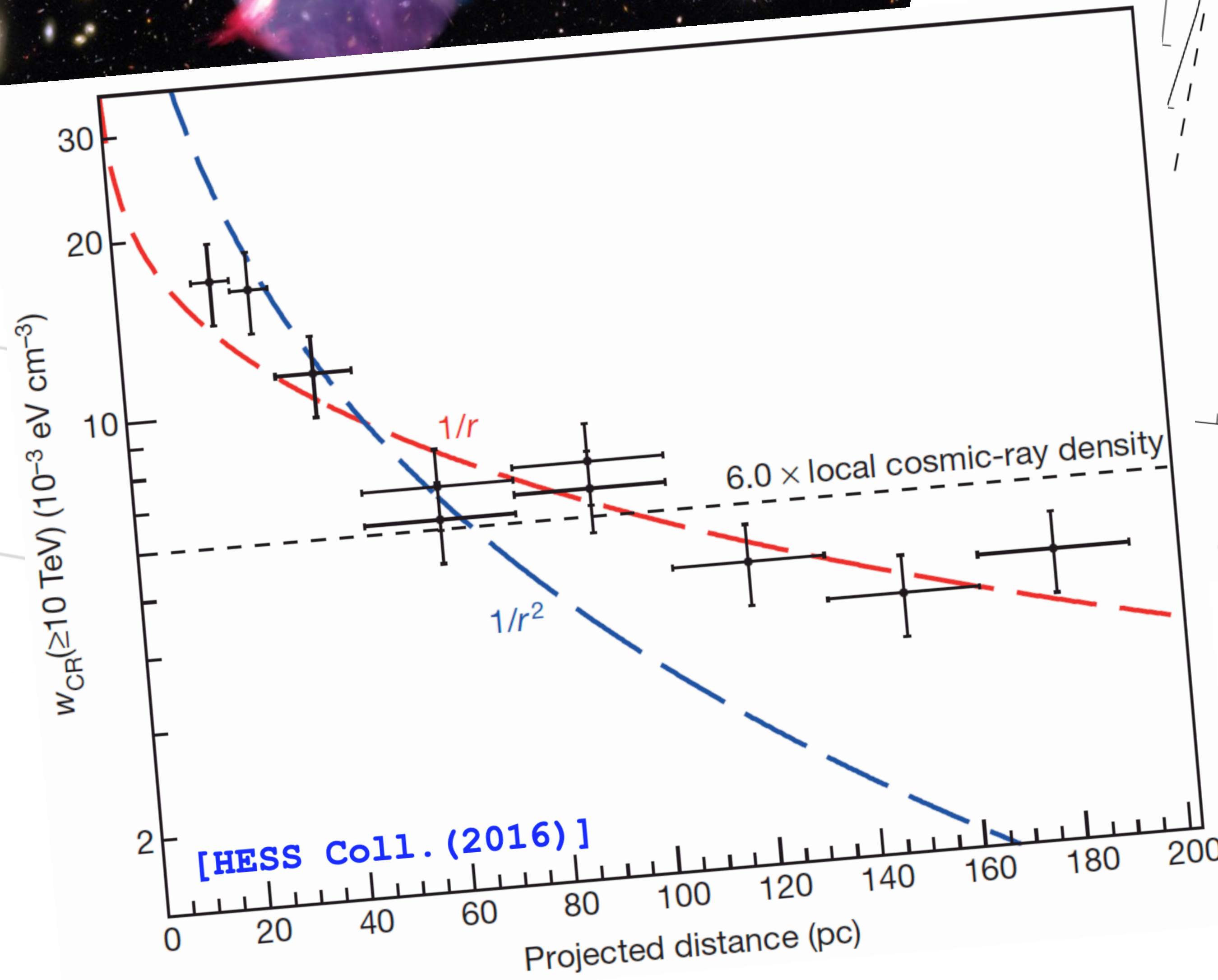


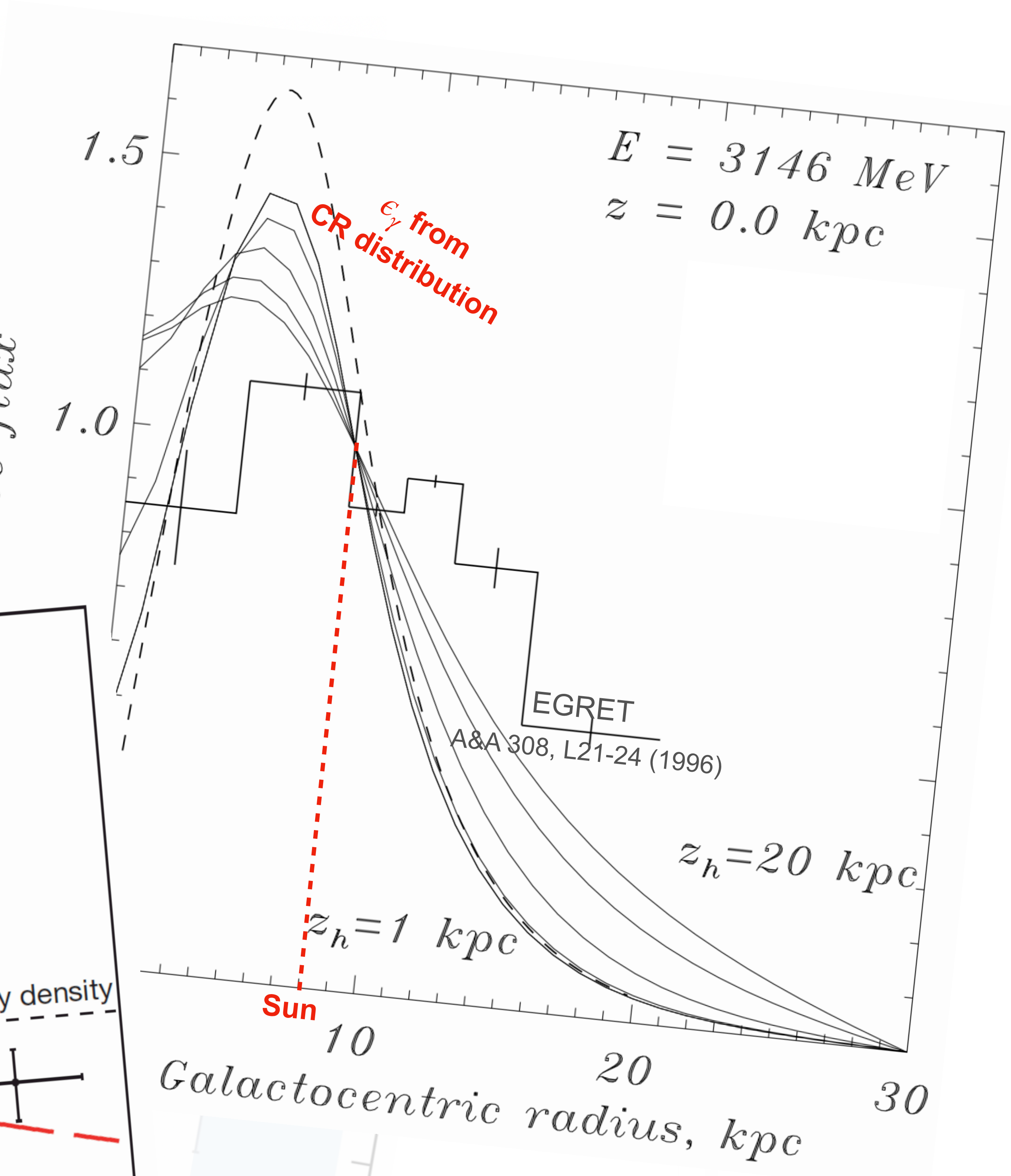

# CR spectrum at Earth =

**Acceleration at the Sources** ➕ **Transport across our Galaxy**

# CR production up to high energies

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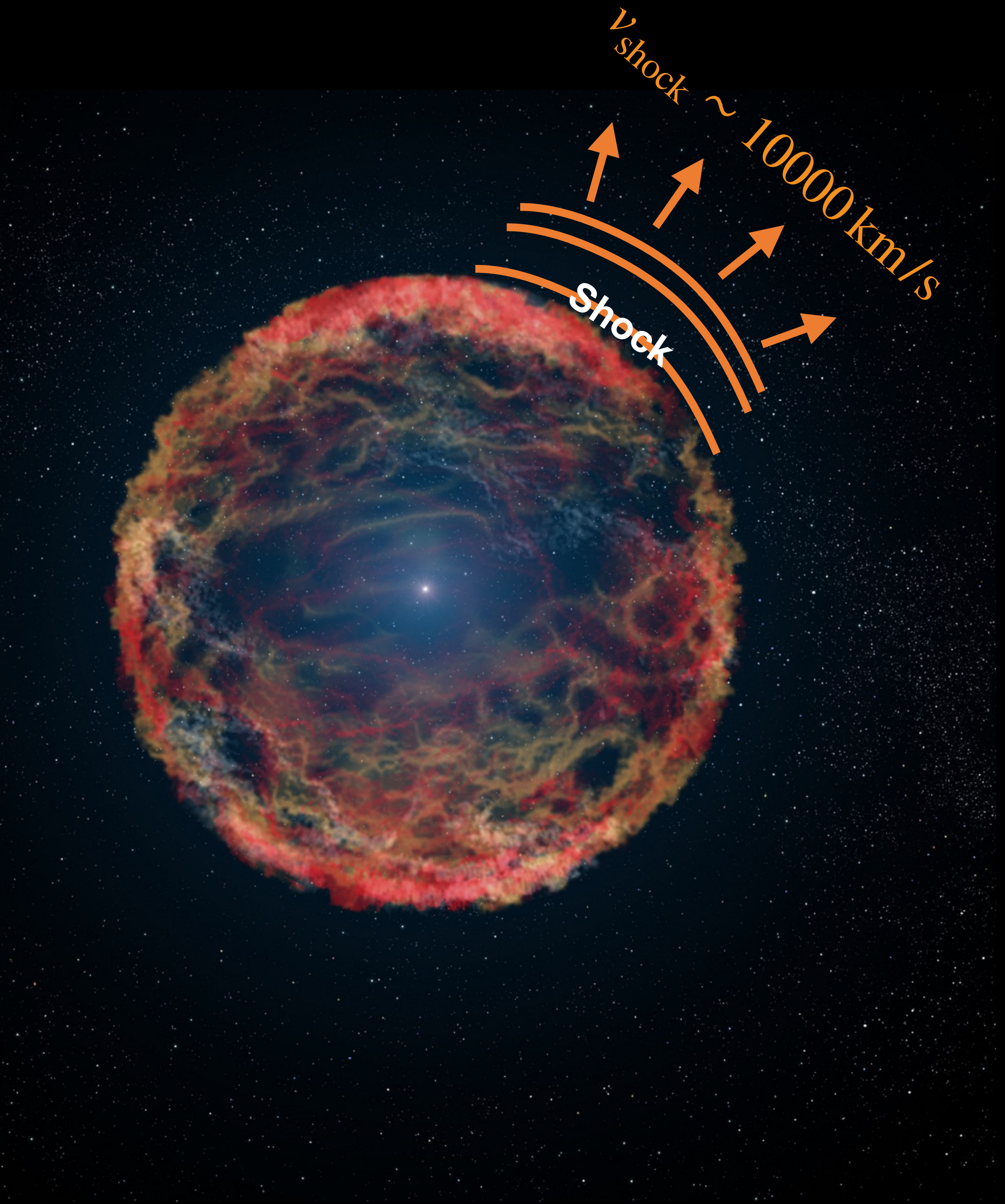

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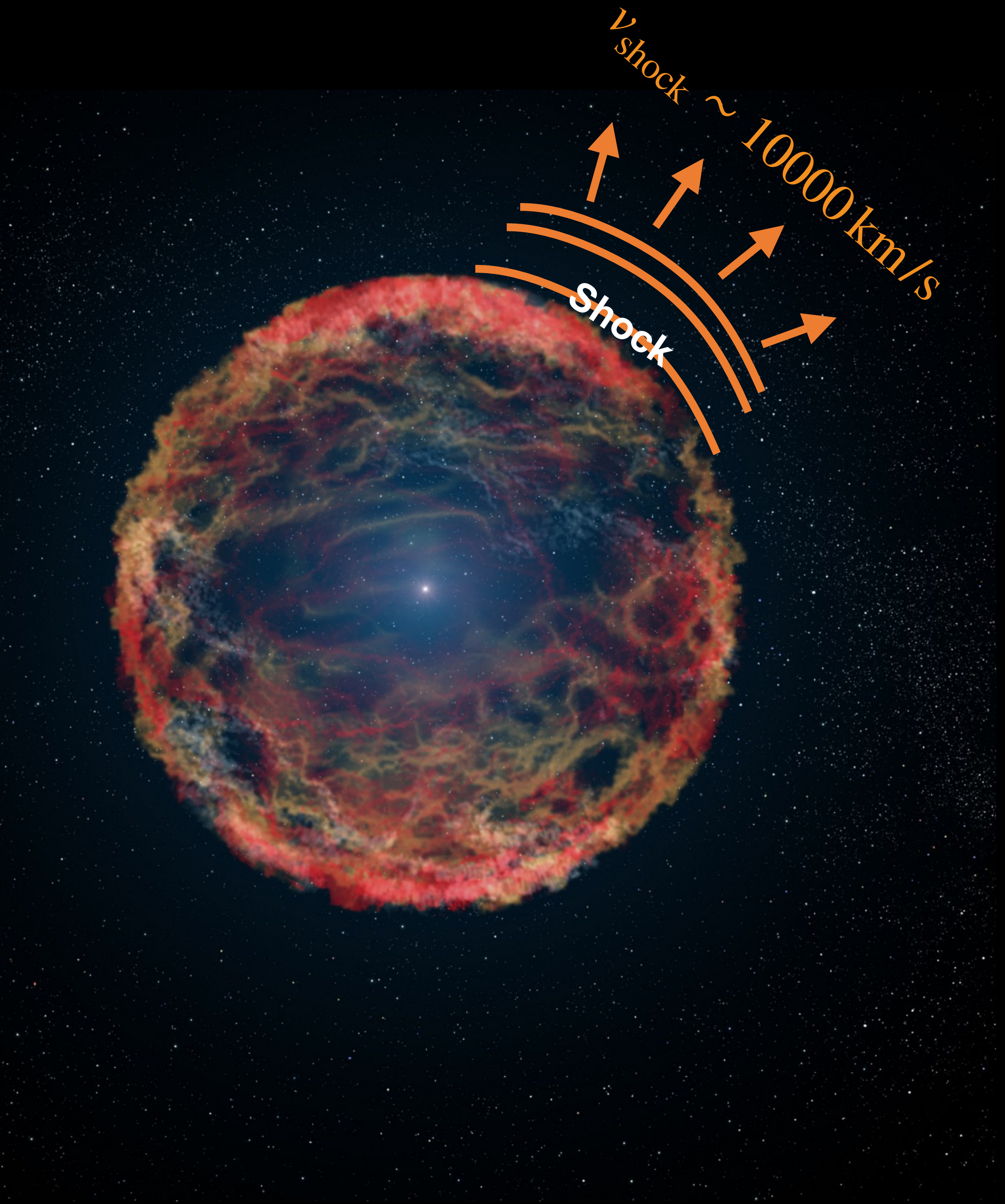
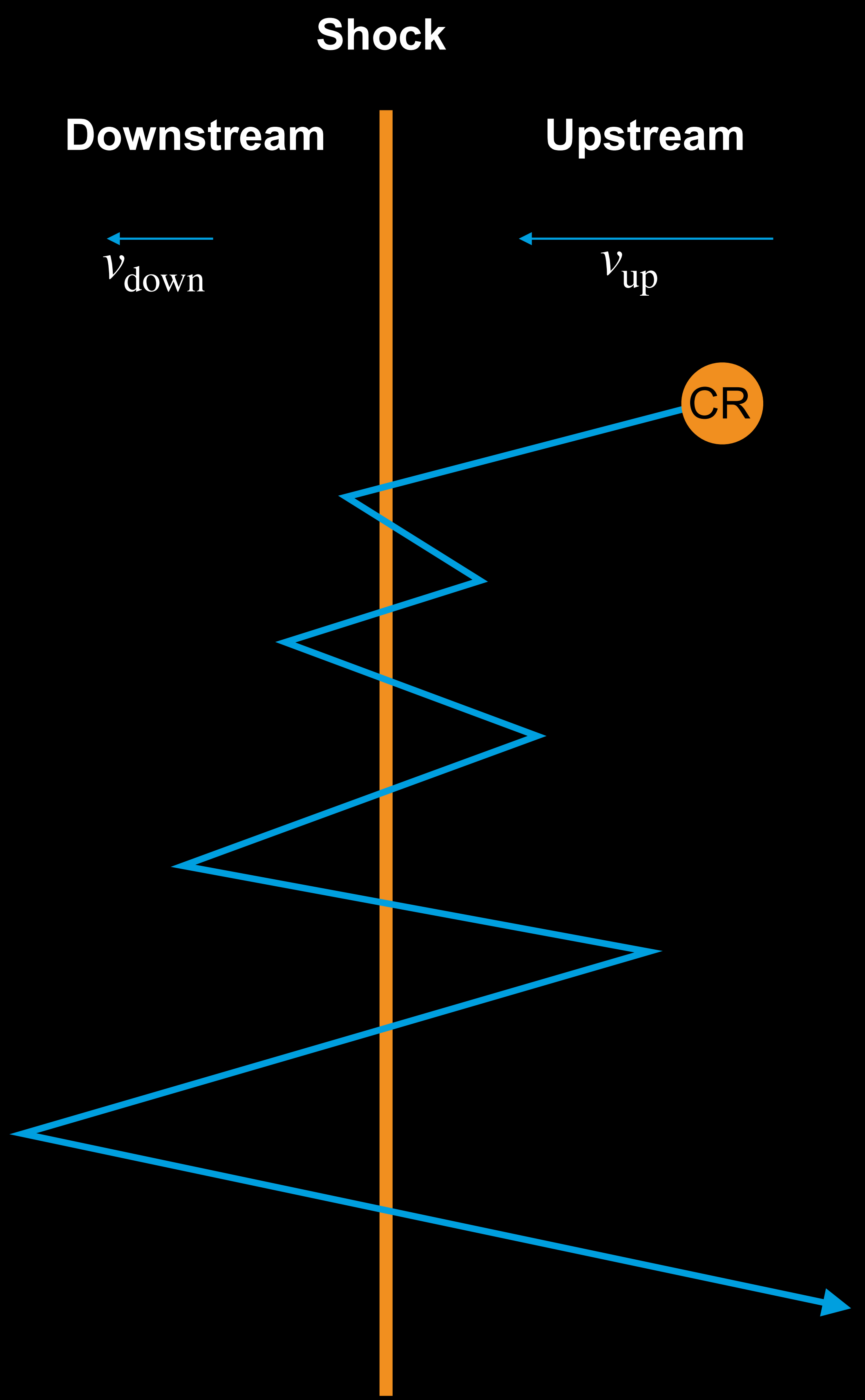
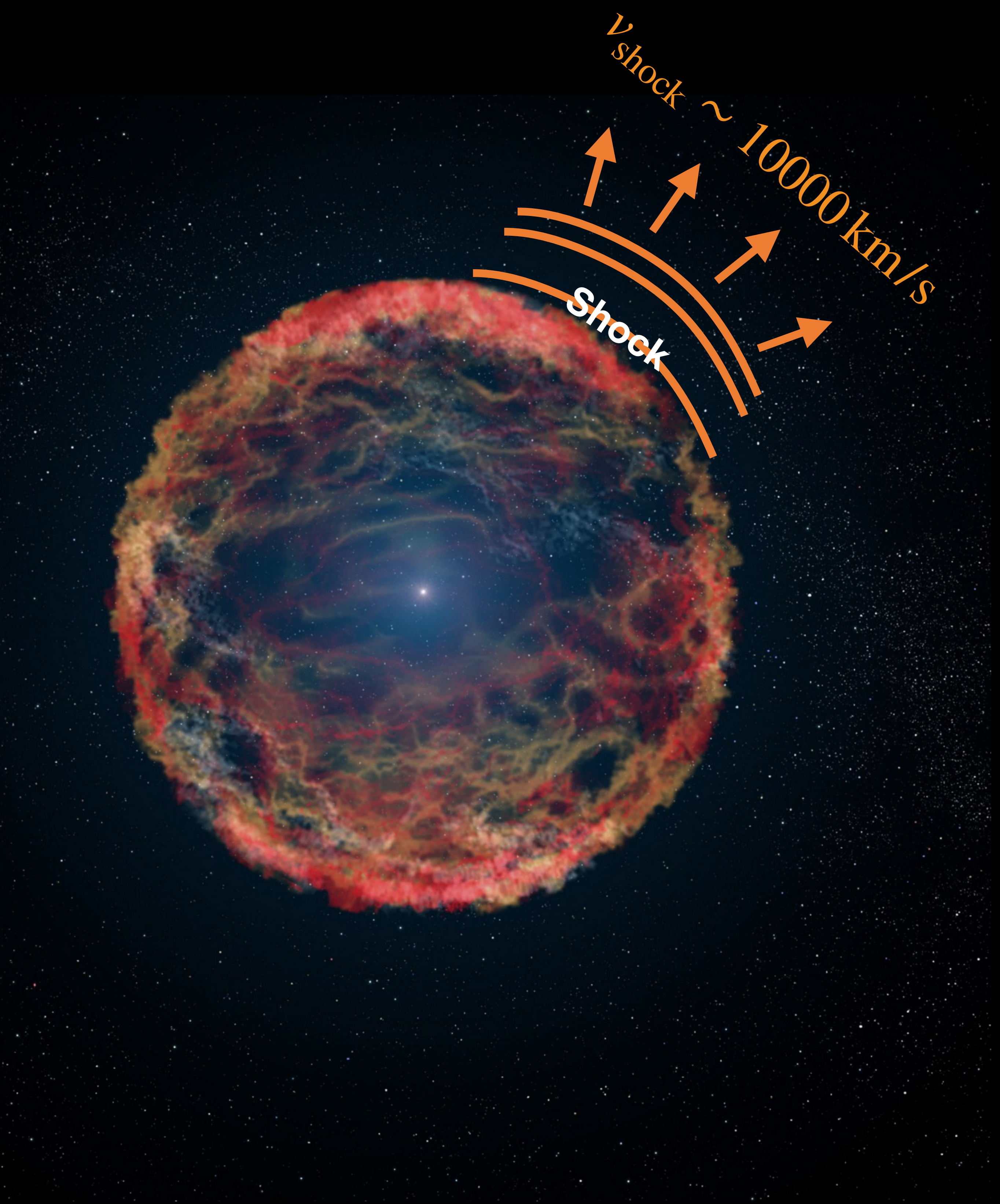
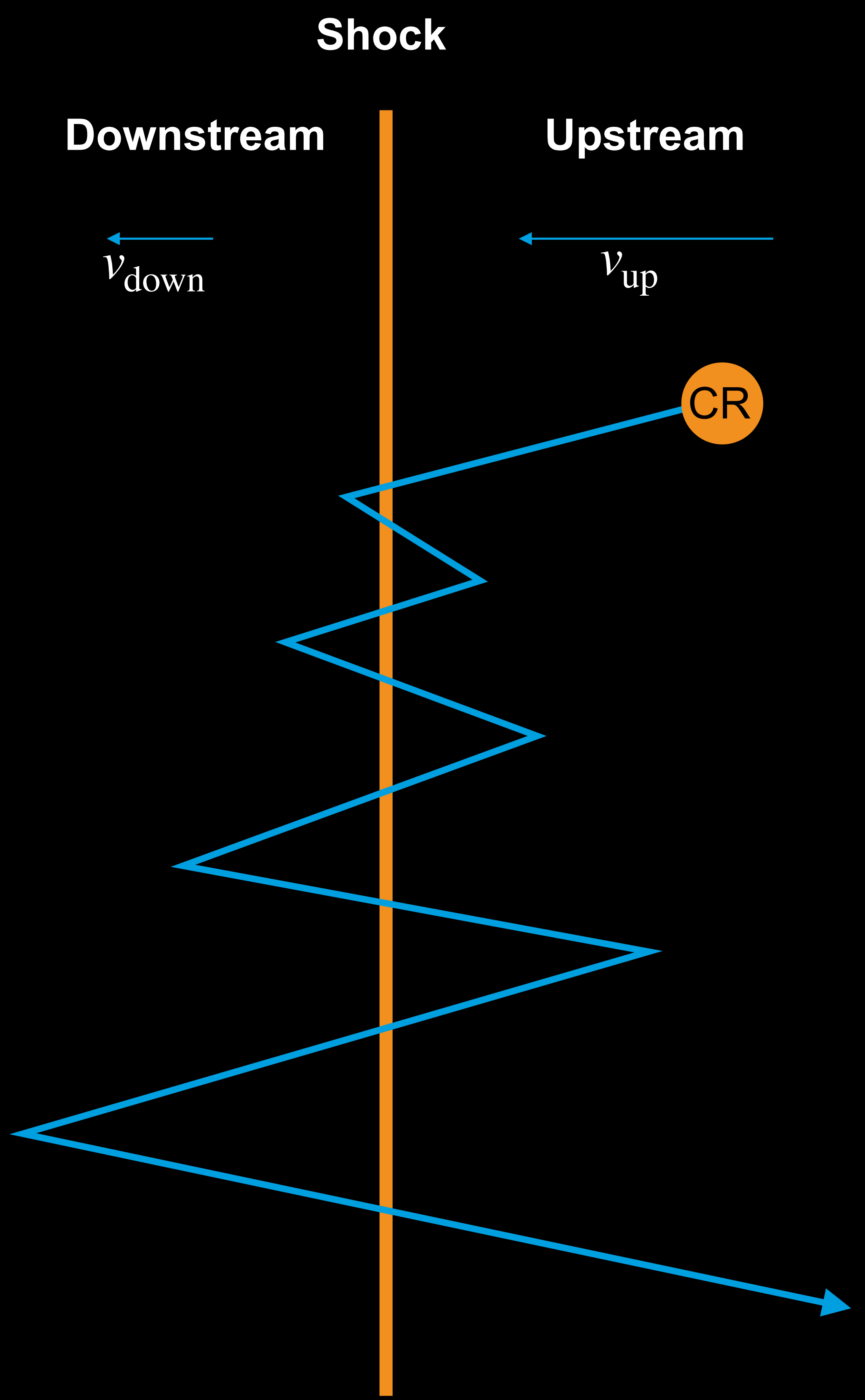
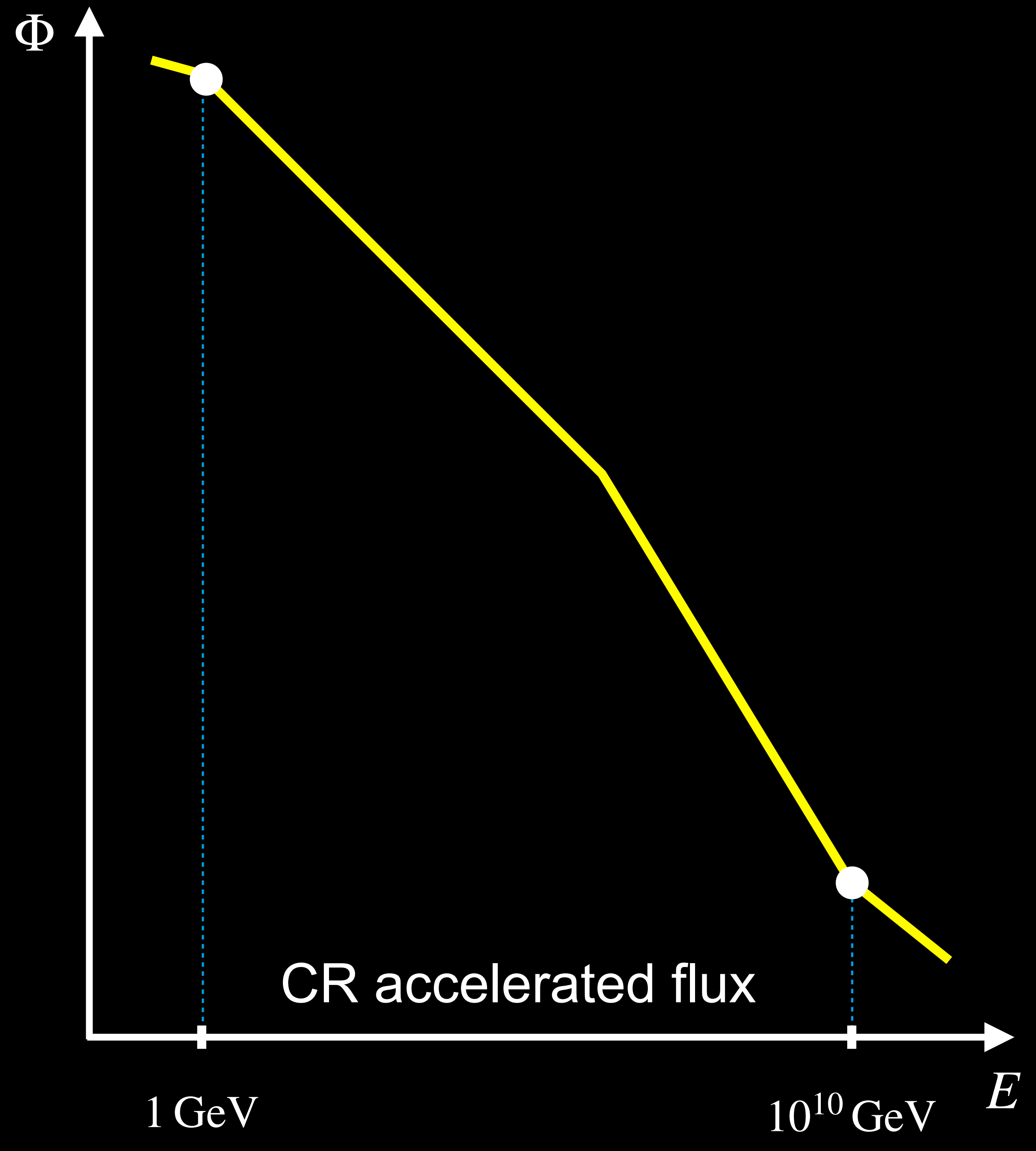


# CR production up to high energies



# Transport in the Galaxy



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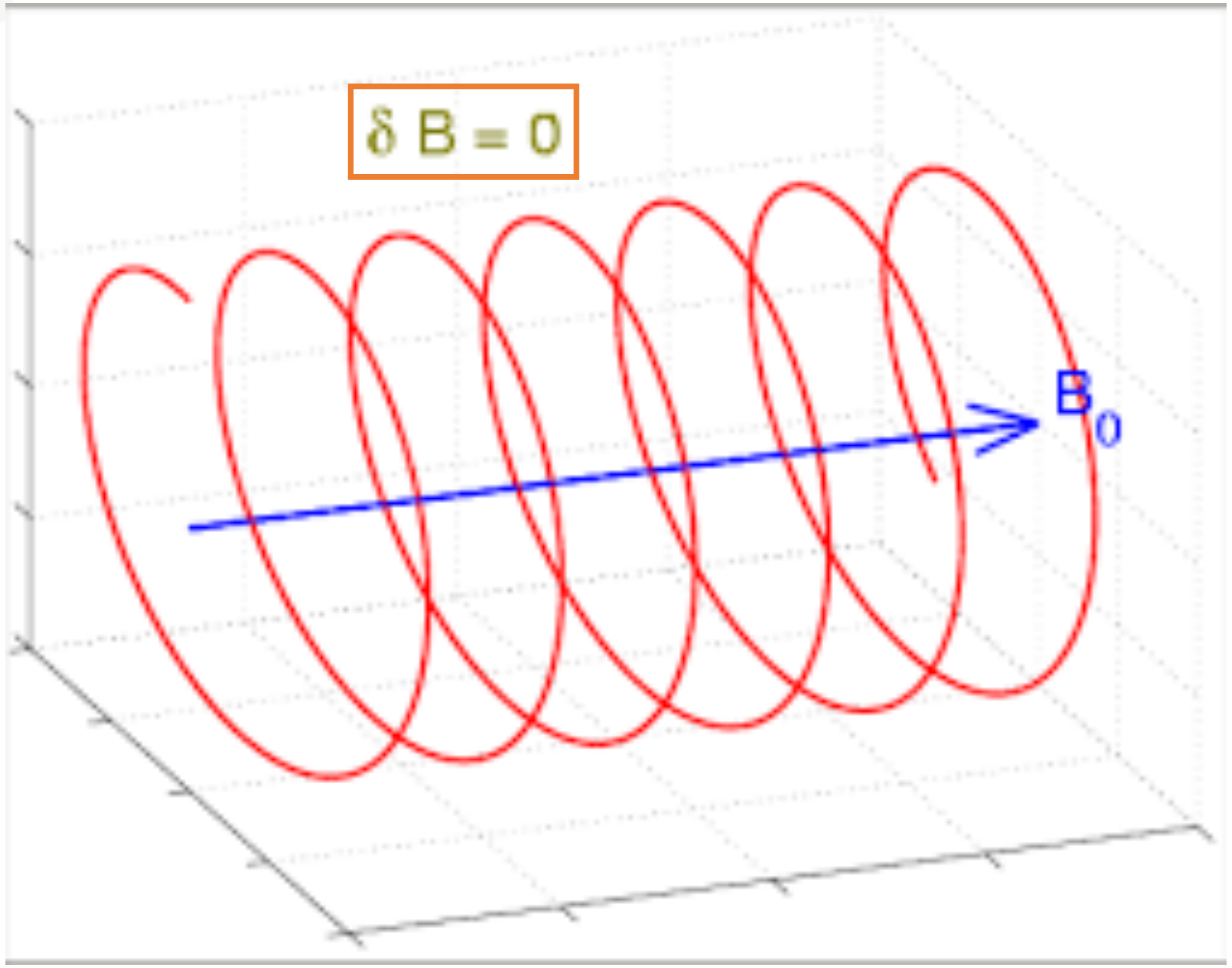
# Gyro-motion of charged particles

$$m\gamma \frac{d\mathbf{v}}{dt} = \frac{q}{c}\left(\mathbf{v} \wedge \mathbf{B}_{\text{tot}}\right)$$

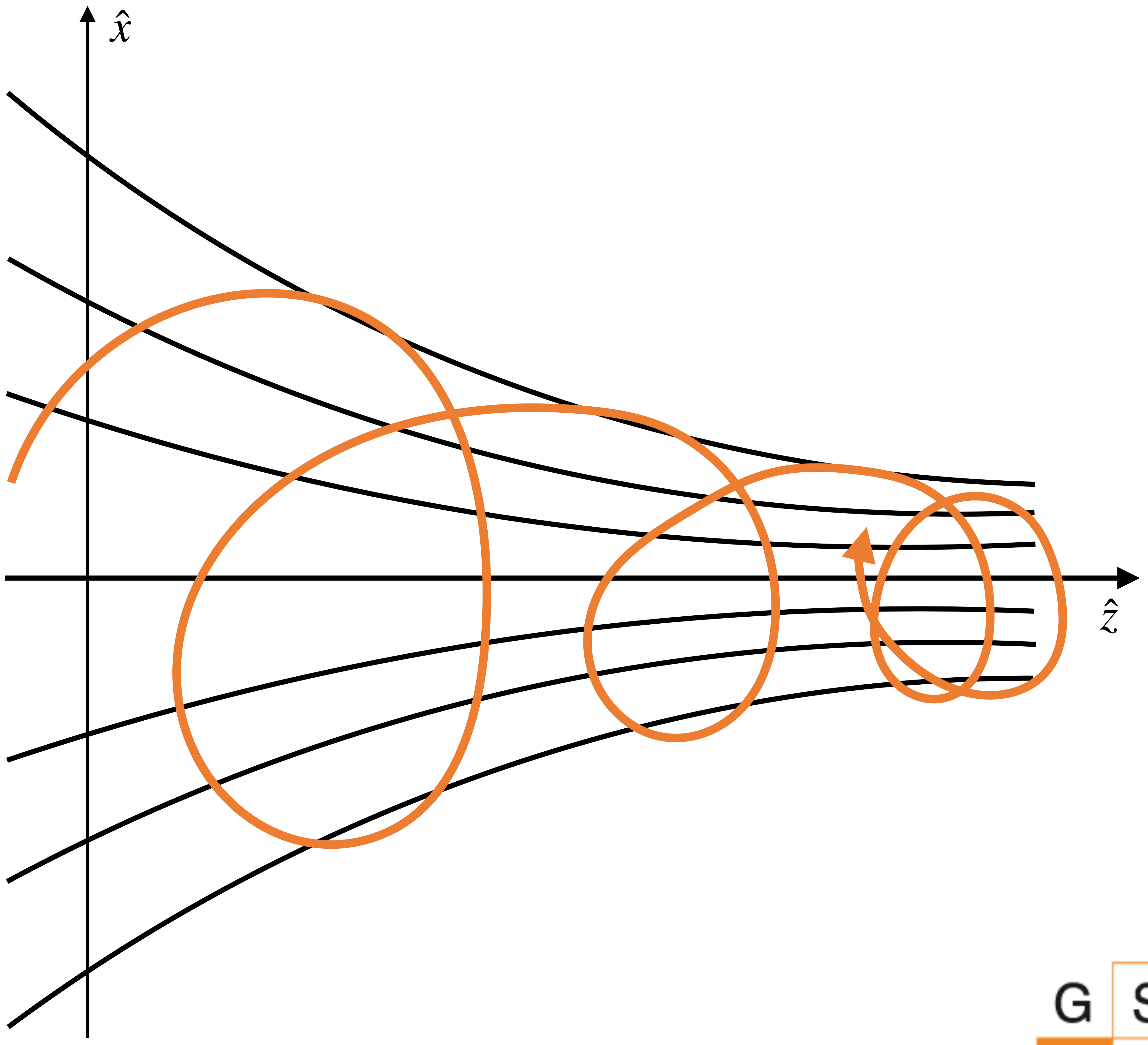


# A little complication: magnetic bottles

$$v_0^2 = v_{\parallel,0}^2 + v_{\perp,0}^2 \underset{\forall z}{=} v_\parallel^2 + v_\perp^2$$

$$\frac{v_{\perp,0}^2}{B} \underset{\forall z}{=} \frac{v_\perp^2}{B(z)}$$

$$\Delta(v_\parallel^2) = v_{\perp,0}^2 \left(B_0 - B(z)\right) \cdot \frac{1}{B_0}$$



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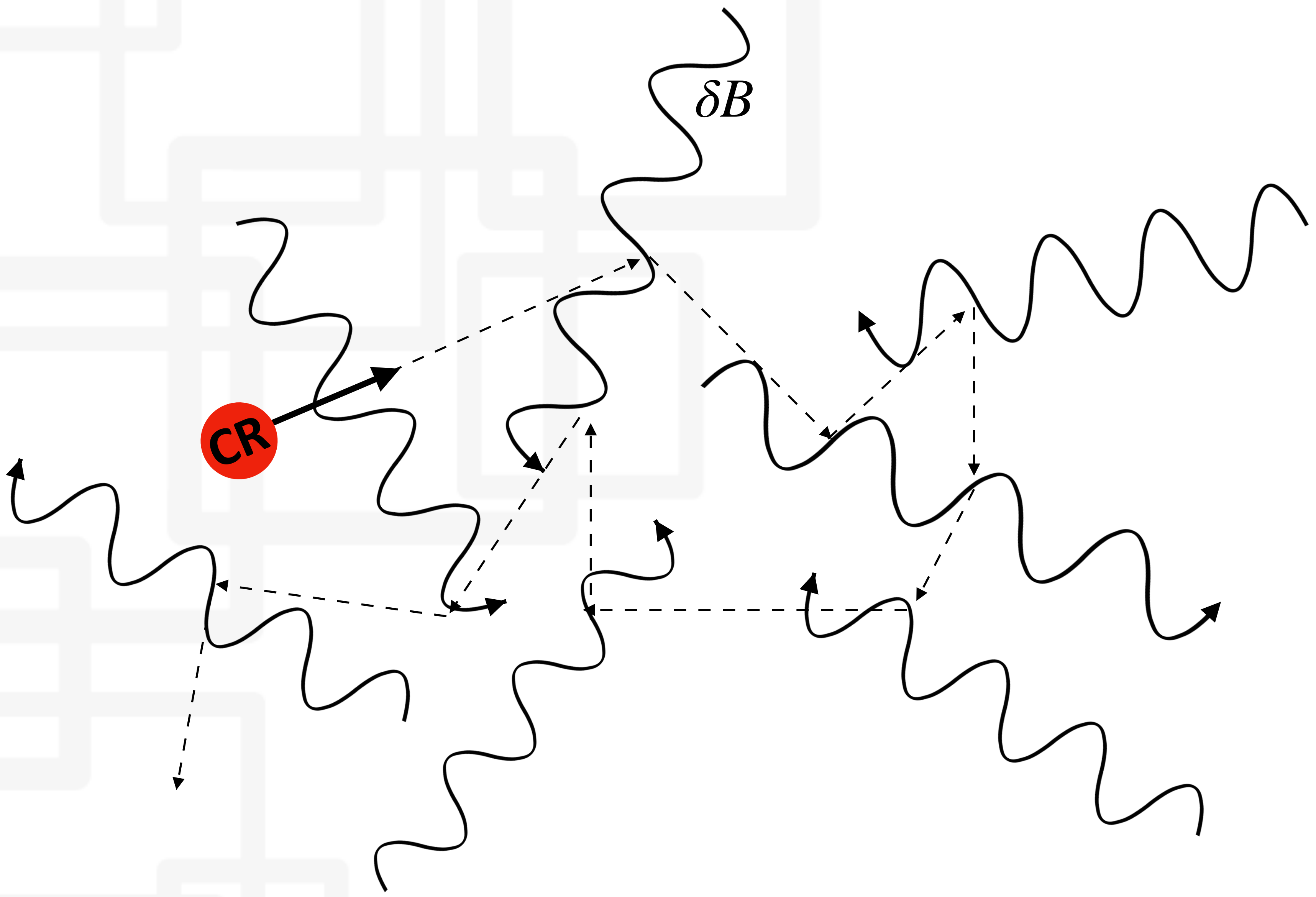
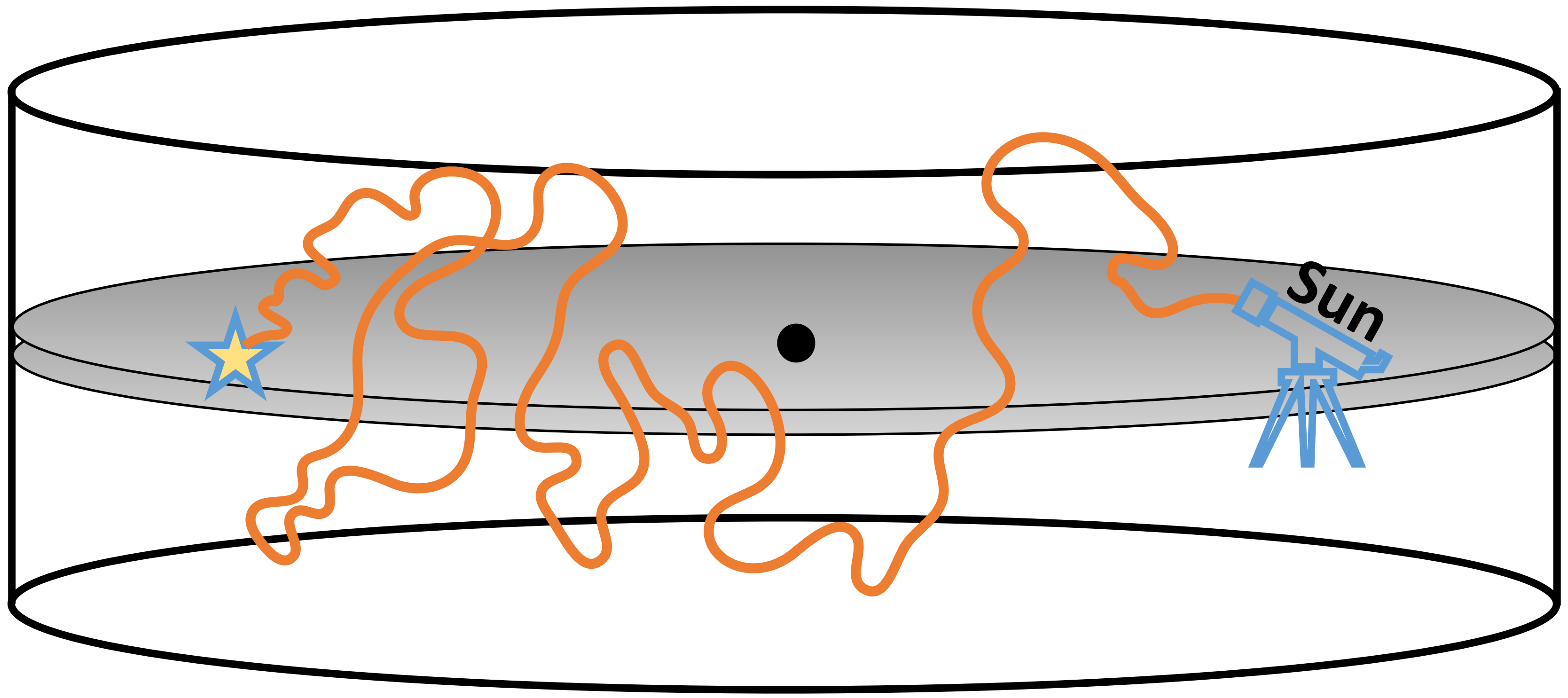
# What is CR diffusion?




*We call "CR diffusion" the random motion resulting from the scattering of charged particles against* ***turbulent magnetic fluctuations.***

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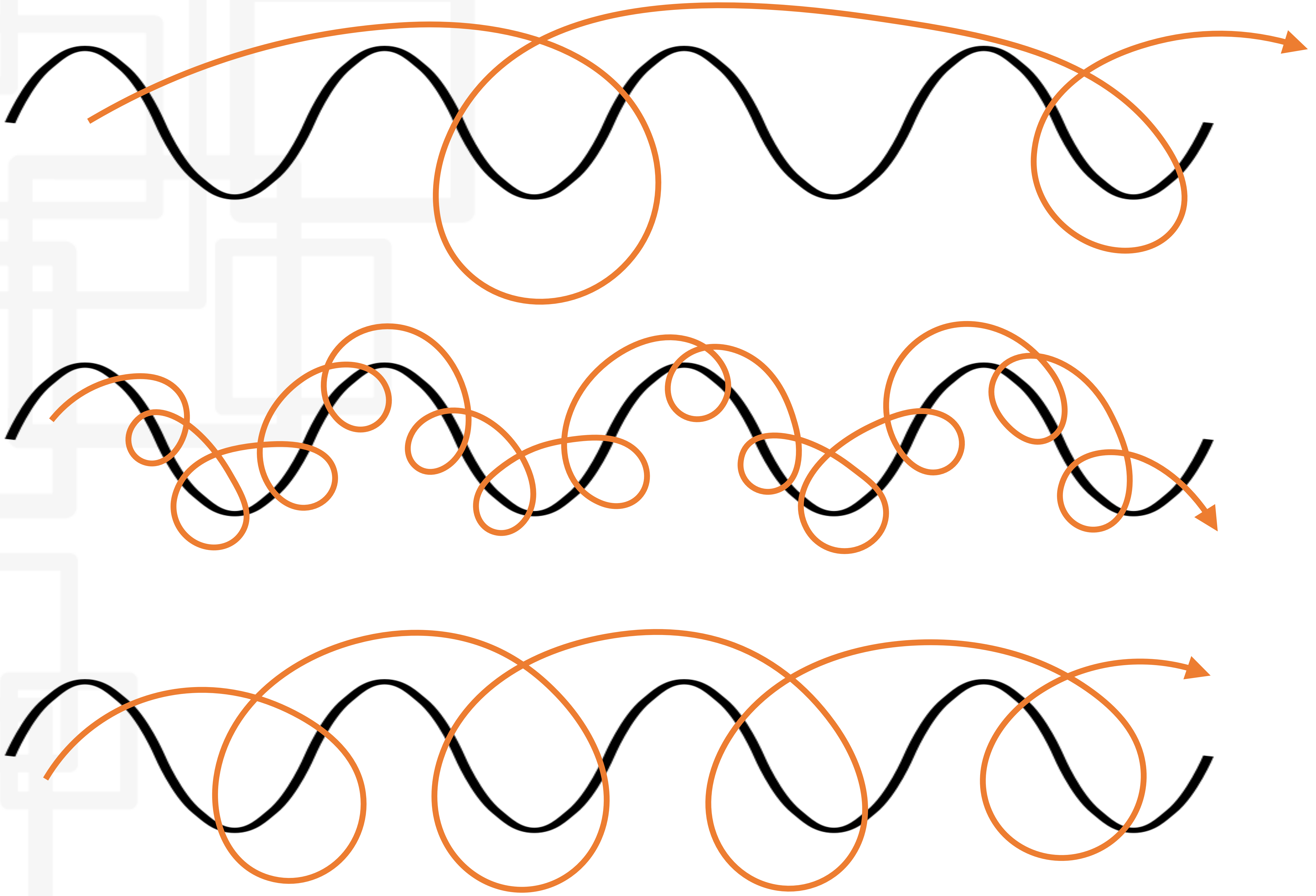
# Pitch-angle scattering on $\mathbf{B}$-fluctuactions



$$r_L \gg \frac{1}{k_{\text{fluctuation}}} \quad \rightarrow \quad \mathbf{B}_0$$

$$r_L \ll \frac{1}{k_{\text{fluctuation}}} \quad \rightarrow \quad \delta\mathbf{B}$$

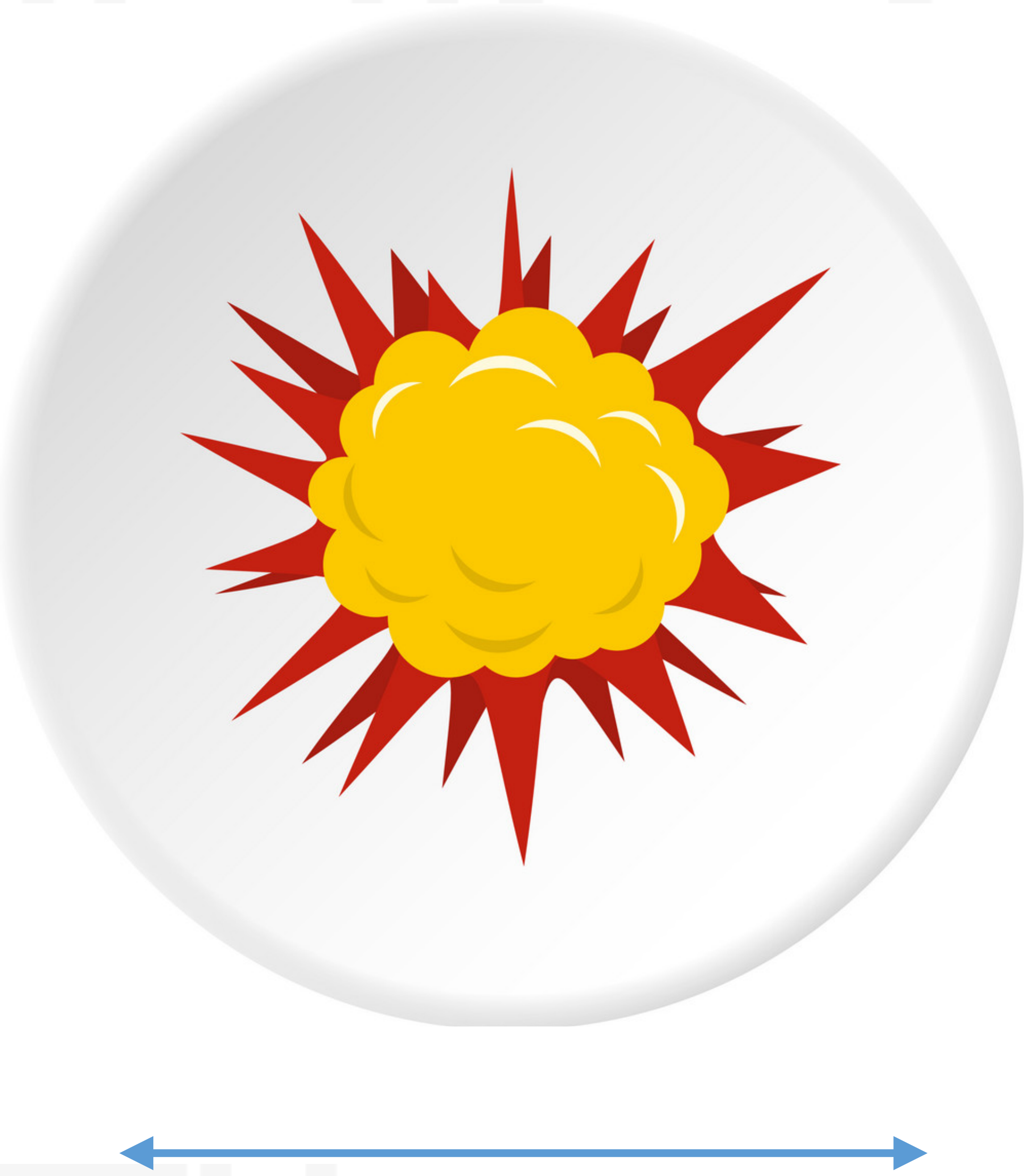
$$r_L \sim \frac{1}{k_{\text{fluctuation}}} \quad \rightarrow \quad \mathbf{B}_0 + \delta\mathbf{B}$$

# Generation of turbulence

# Generation of turbulence in ISM

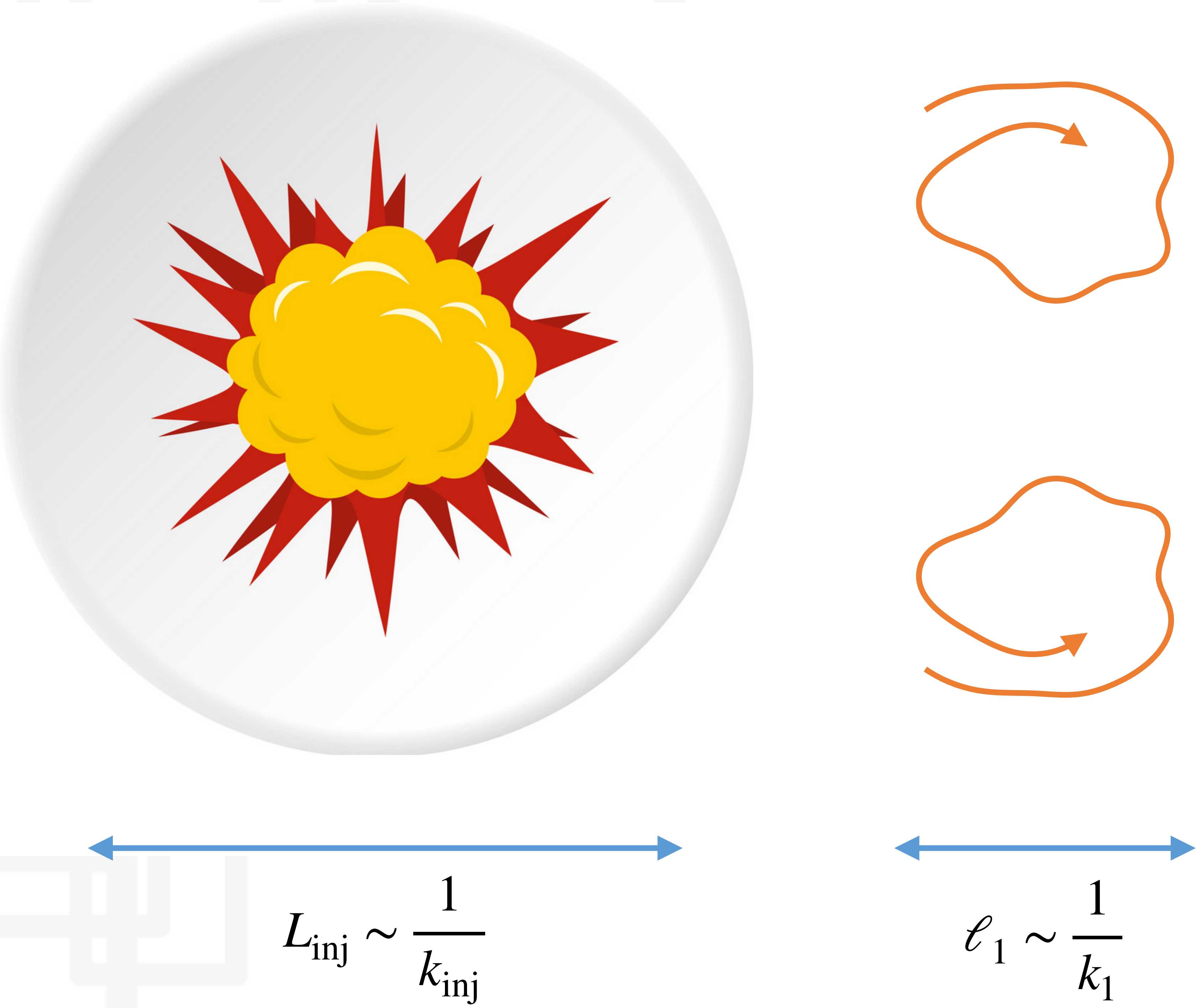
# Generation of turbulence in ISM

# Turbulent cascade in the inertial range



$$L_{\text{inj}} \sim \frac{1}{k_{\text{inj}}}$$

# Turbulent cascade in the inertial range



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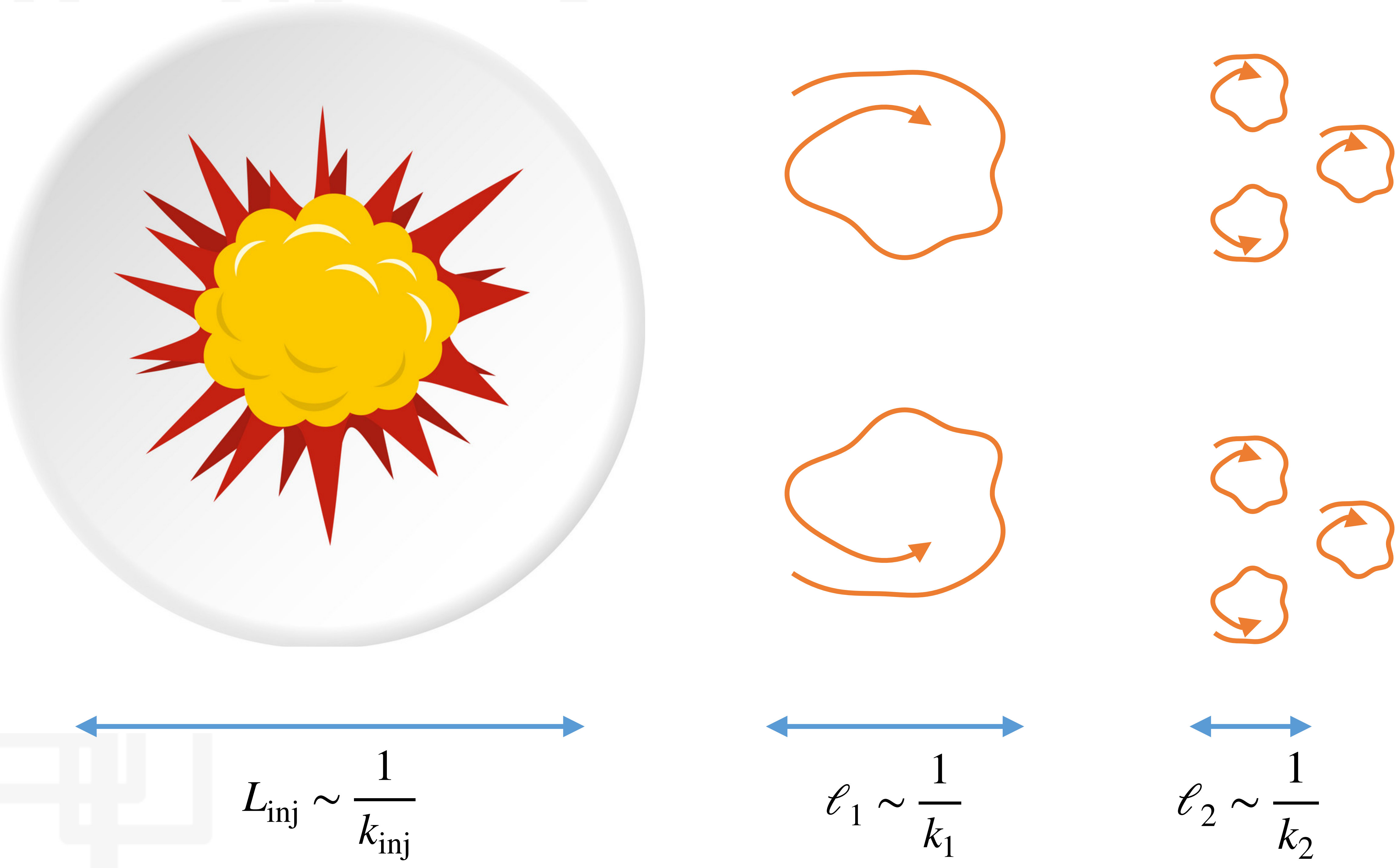

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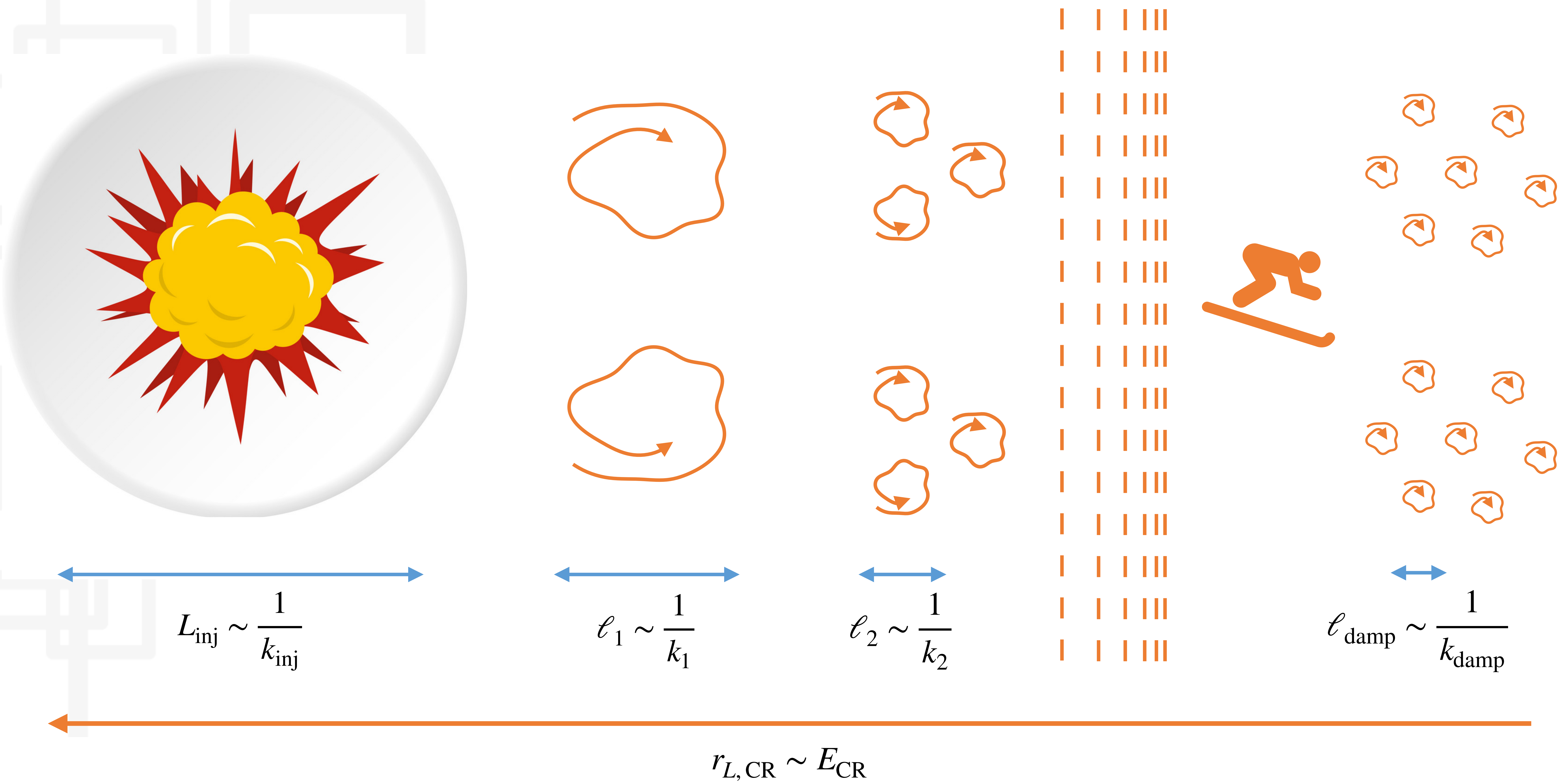

# MHD decomposition along the cascade

Magnetized medium ⇨ MHD equations ⇨

$$\begin{pmatrix} \omega^2 - k^2 v_A^2 - k_\perp^2 c_s^2 & 0 & -k_\perp k_\parallel c_s^2 \\ 0 & \omega^2 - k_\parallel^2 v_A^2 & 0 \\ -k_\perp k_\parallel c_s^2 & 0 & \omega^2 - k_\parallel^2 c_s^2 \end{pmatrix} \begin{pmatrix} \delta u_x \\ \delta u_y \\ \delta u_z \end{pmatrix} = 0$$

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Magnetized medium ⇒ MHD equations ⇒

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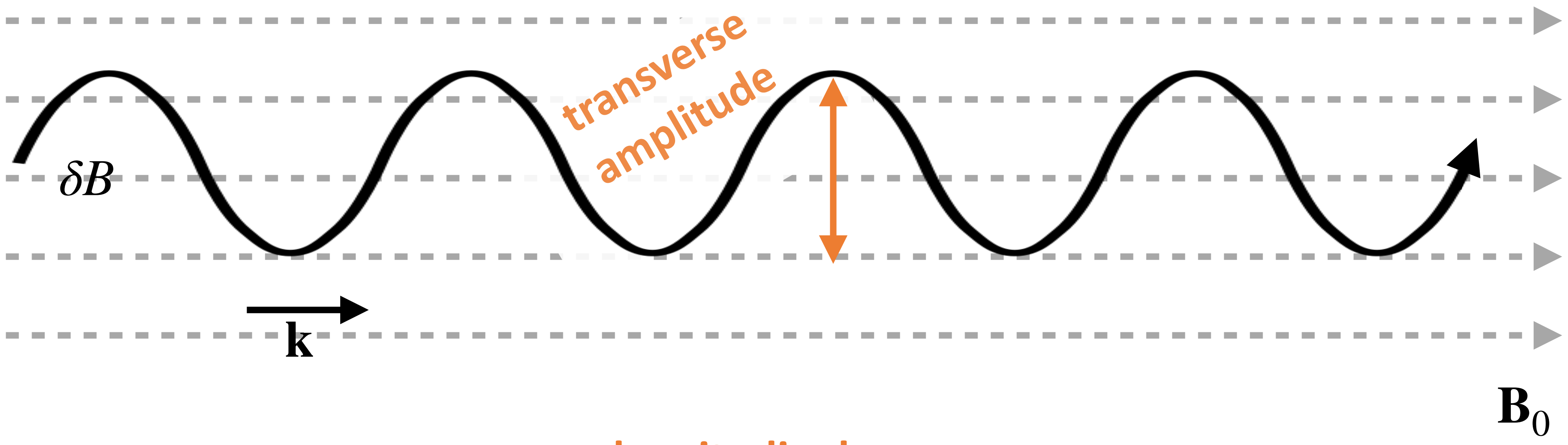
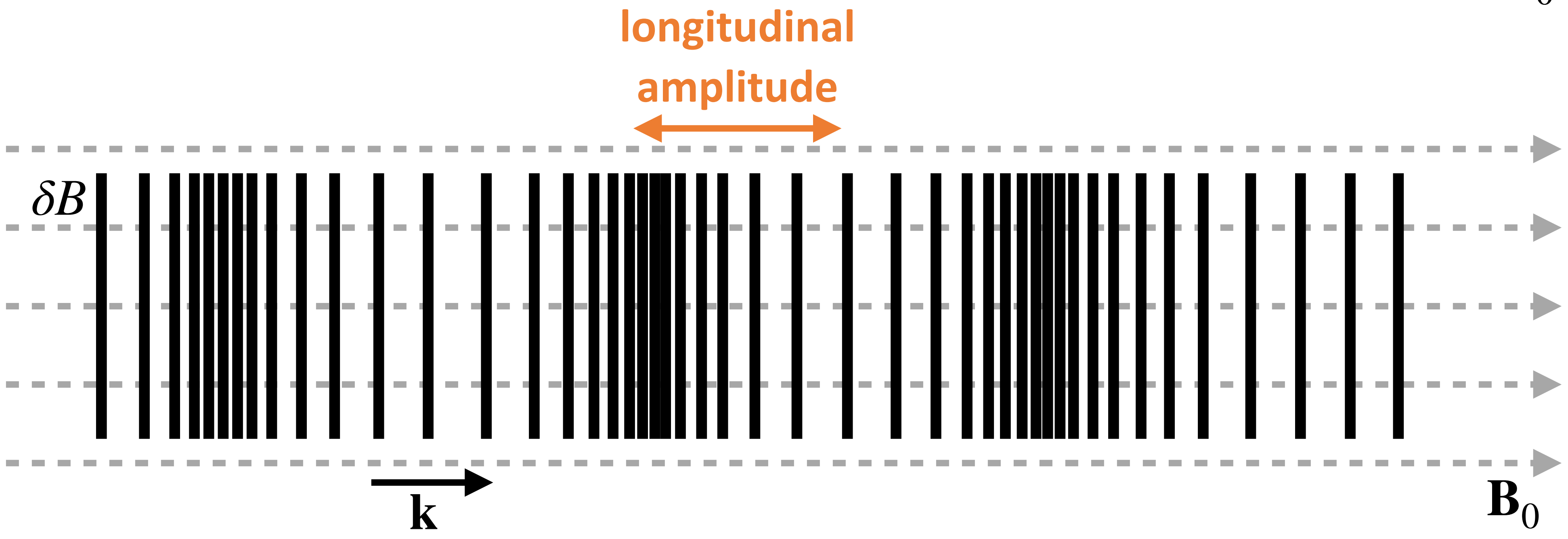
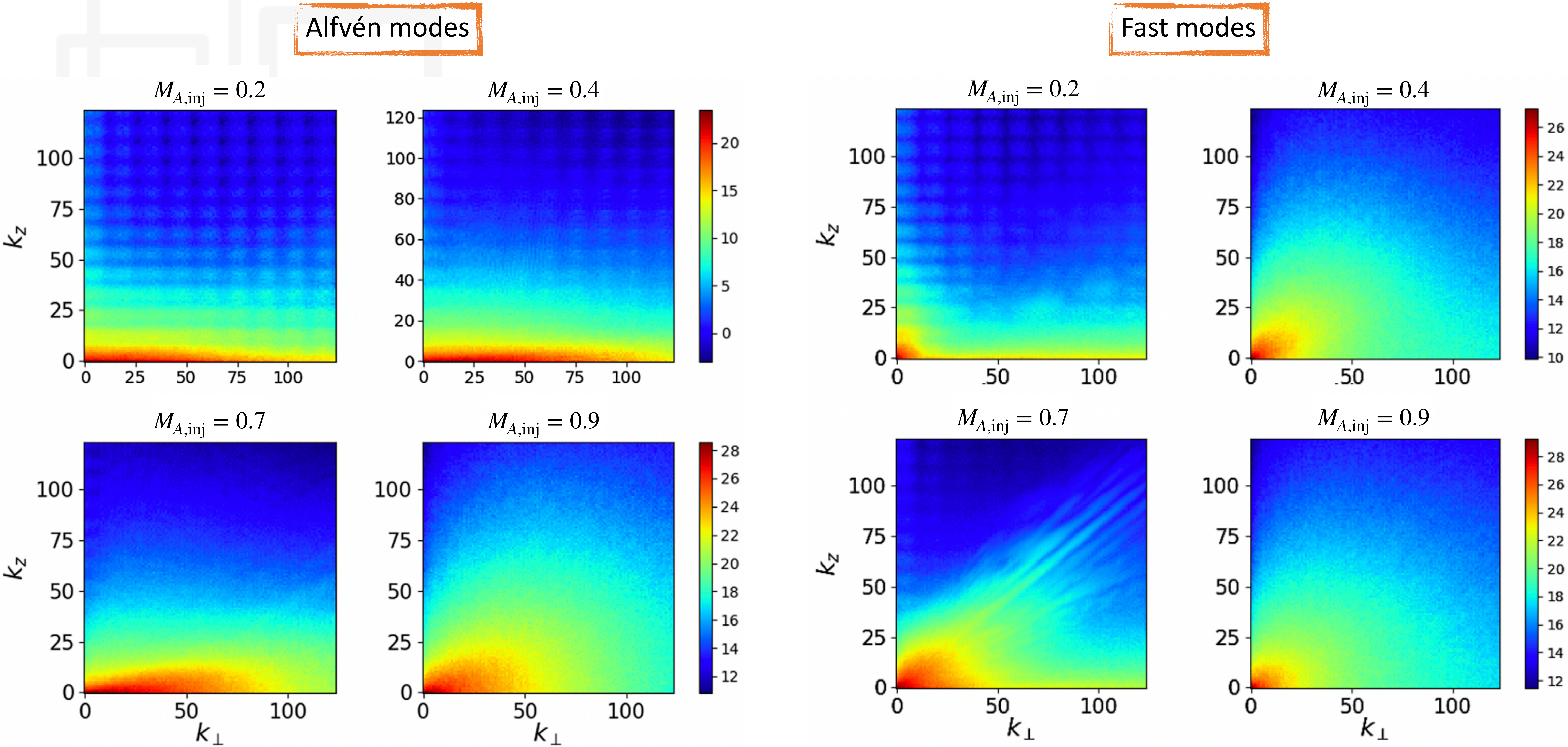
- **Alfvén modes**




- **Magnetosonic modes**



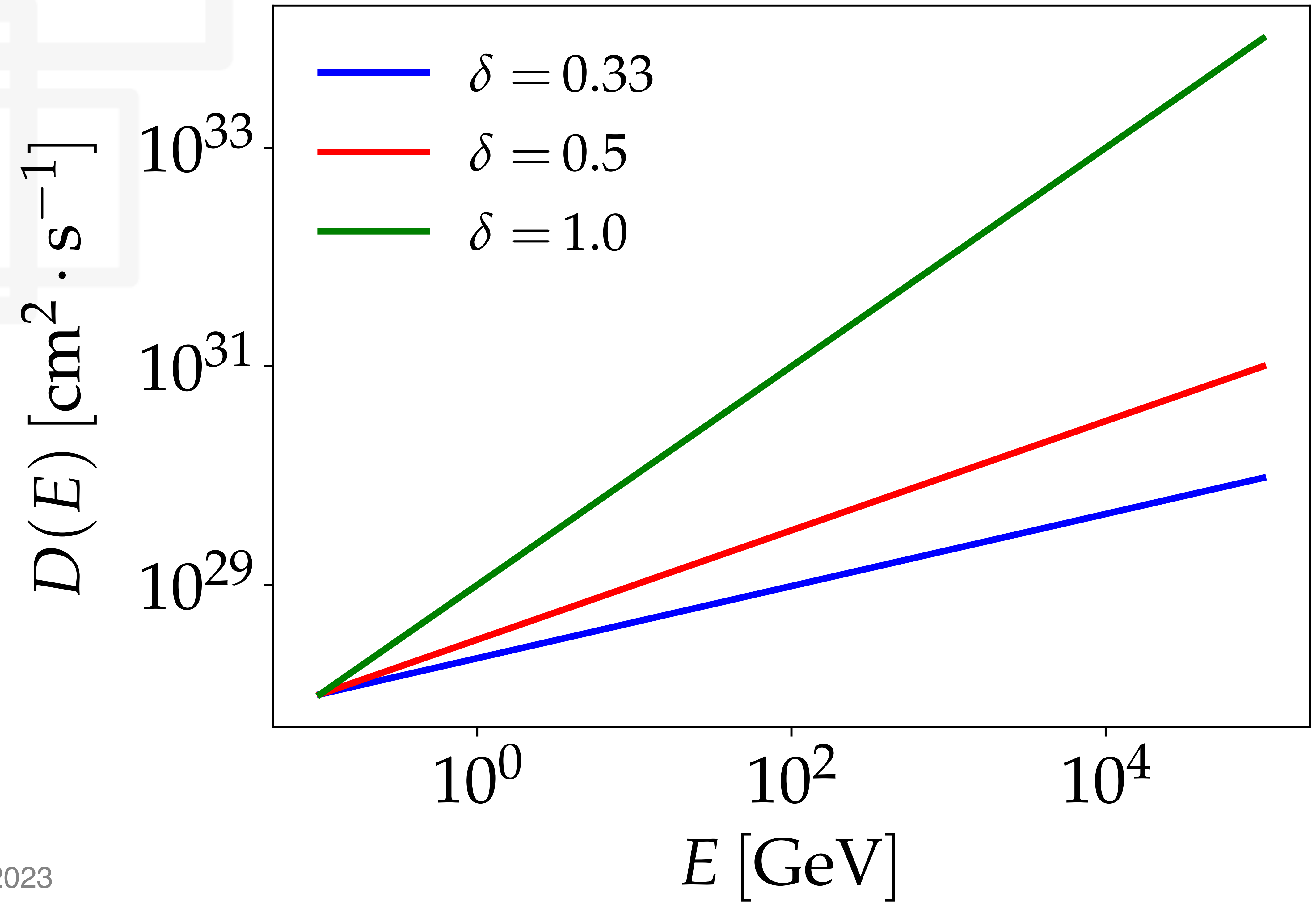
# Distribution of the turbulent power



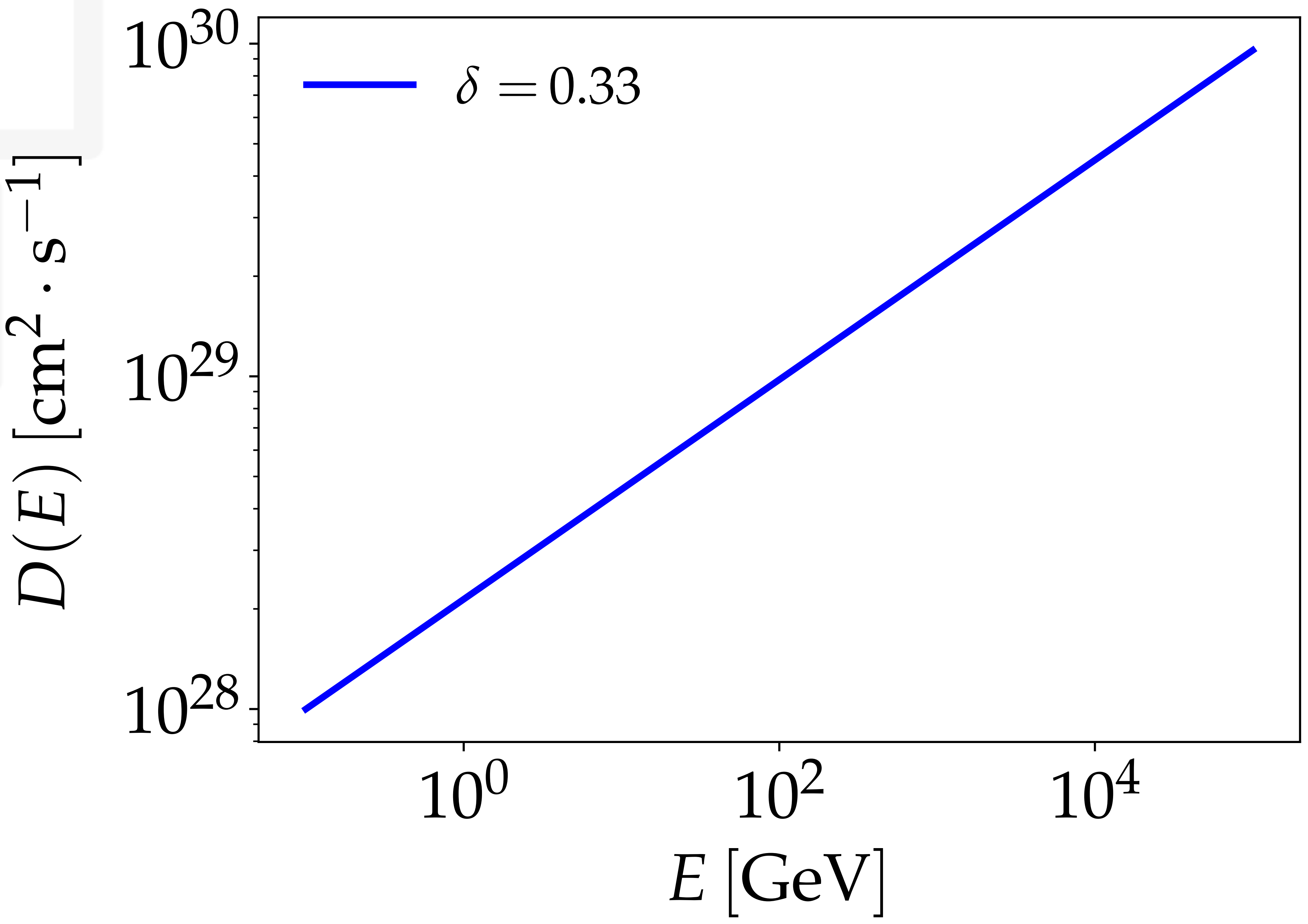
# From turbulence to CR diffusion

$$D(E) = \frac{1}{3} \cdot \frac{c\, r_L}{k_{\rm res} \cdot E(k_{\rm res})} \quad \Rightarrow \quad D(E) \sim \frac{r_L}{r_L^{-1} \cdot r_L^{\alpha}}$$

$$D(E) \sim E^{2-\alpha} \equiv E^{\delta}$$



# Damping of turbulence



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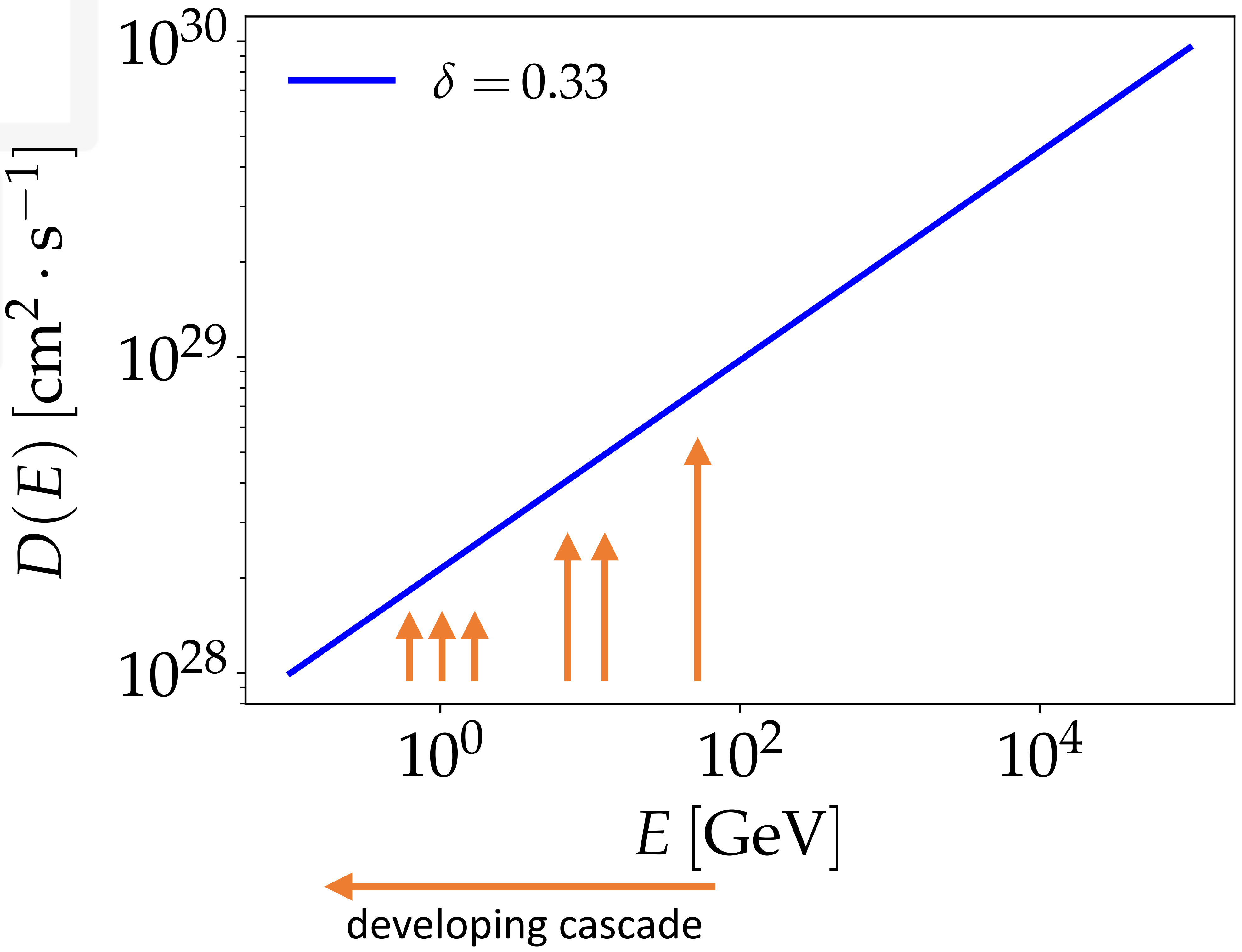




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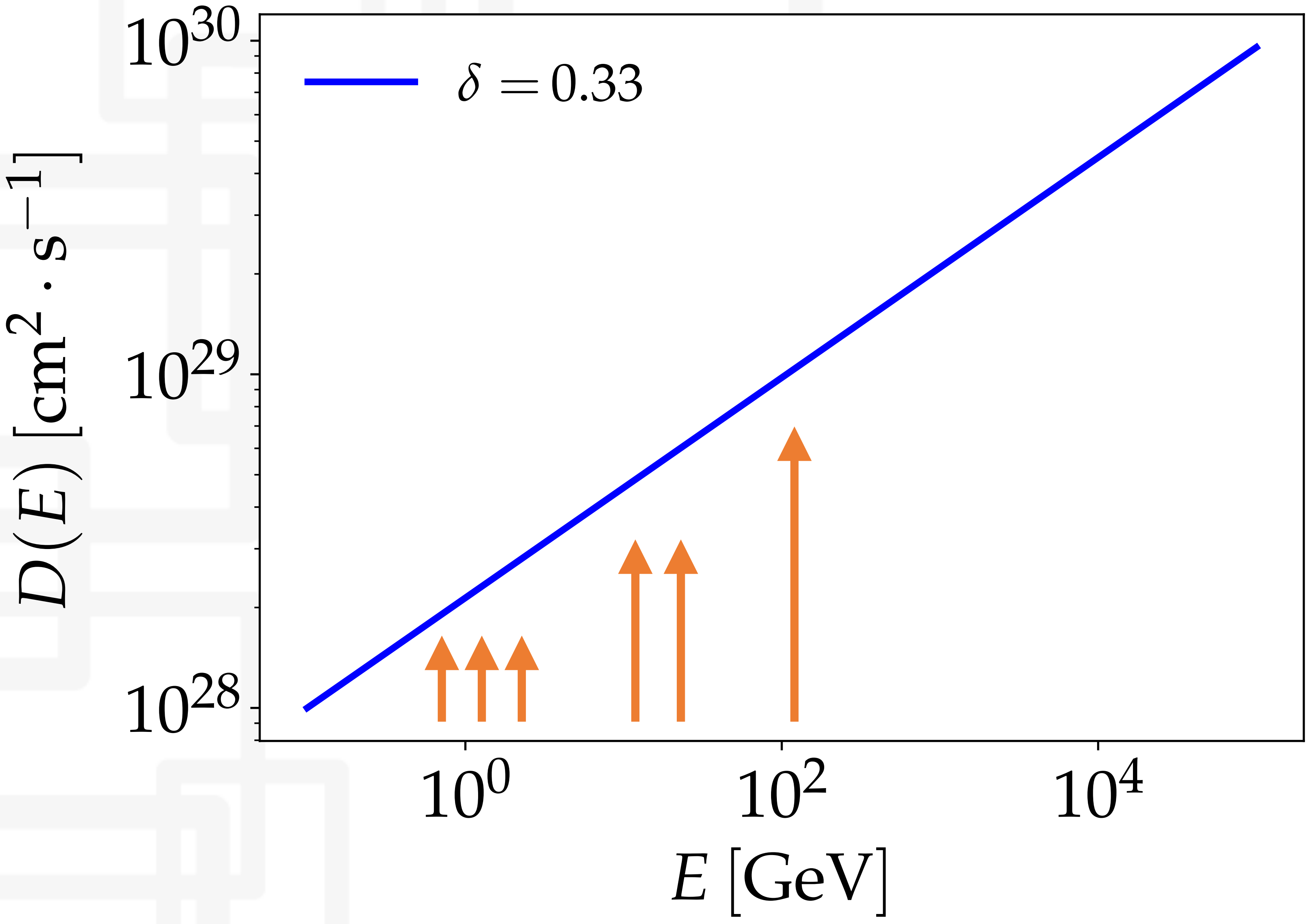
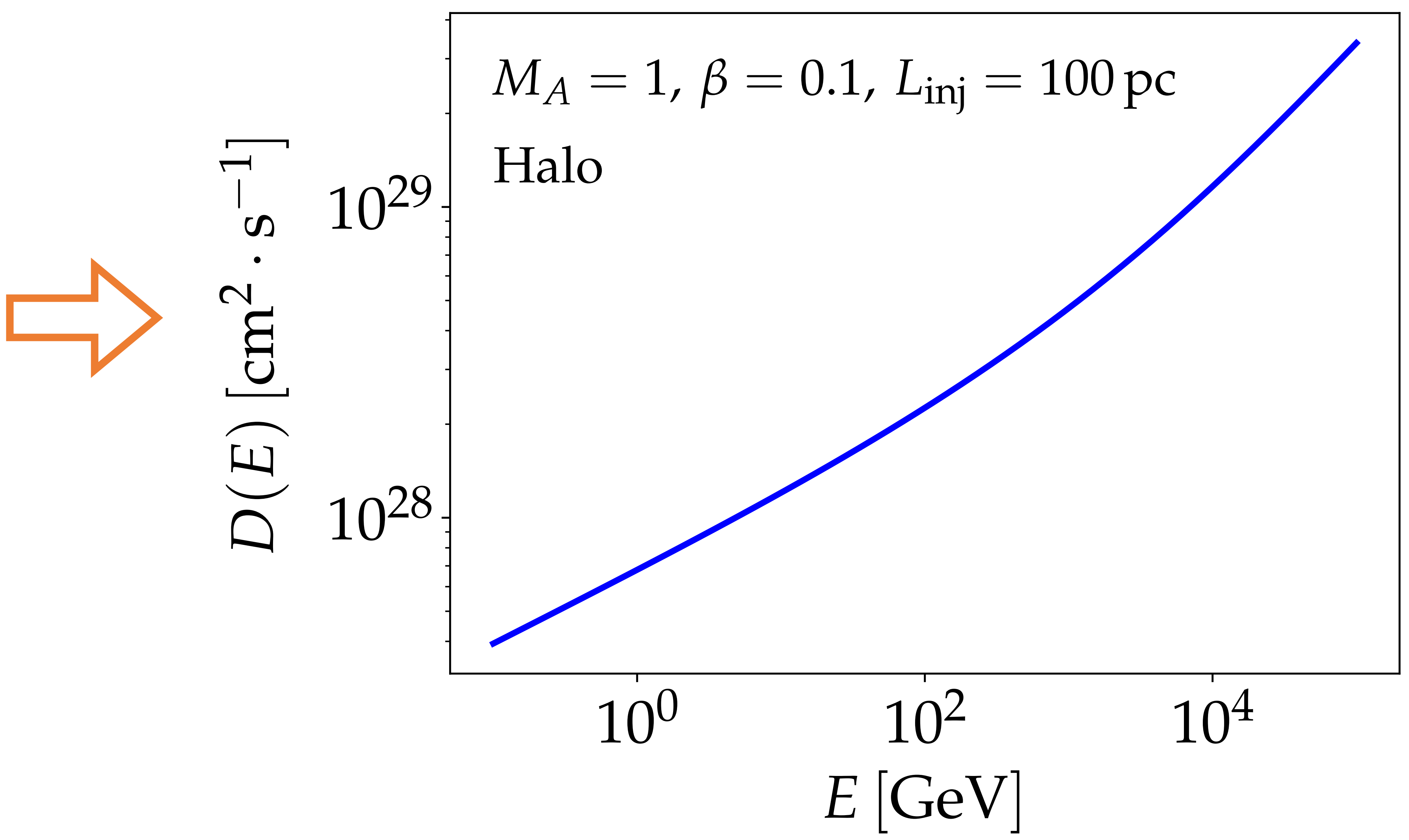
# Damping of turbulence

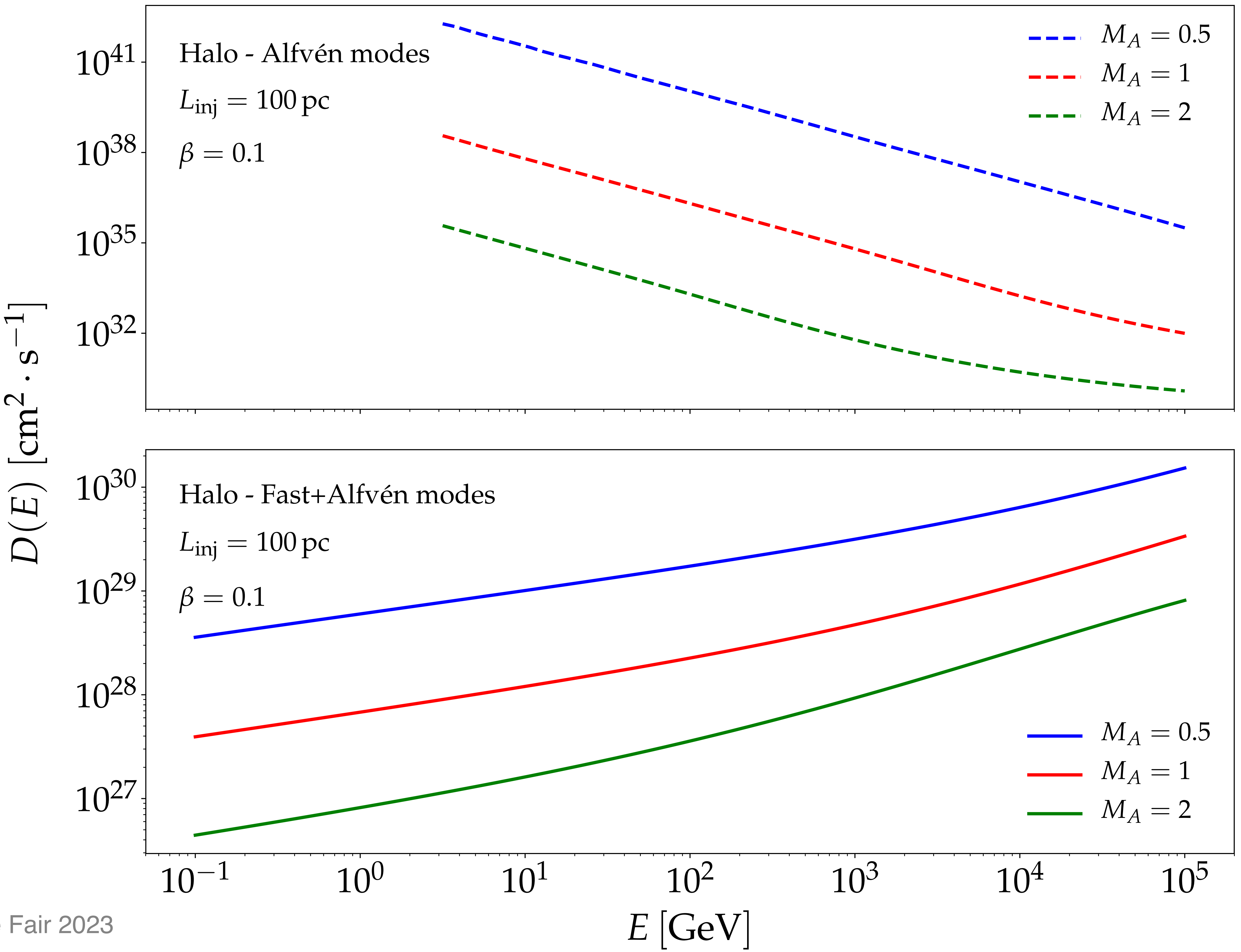


# Conclusion…

## Cosmic-ray phenomenology

# Resulting CR diffusivity



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# Take-home message

***Accurate*** *measurements require detailed knowledge of the* ***microphysics of CR transport*** *in our Galaxy.*

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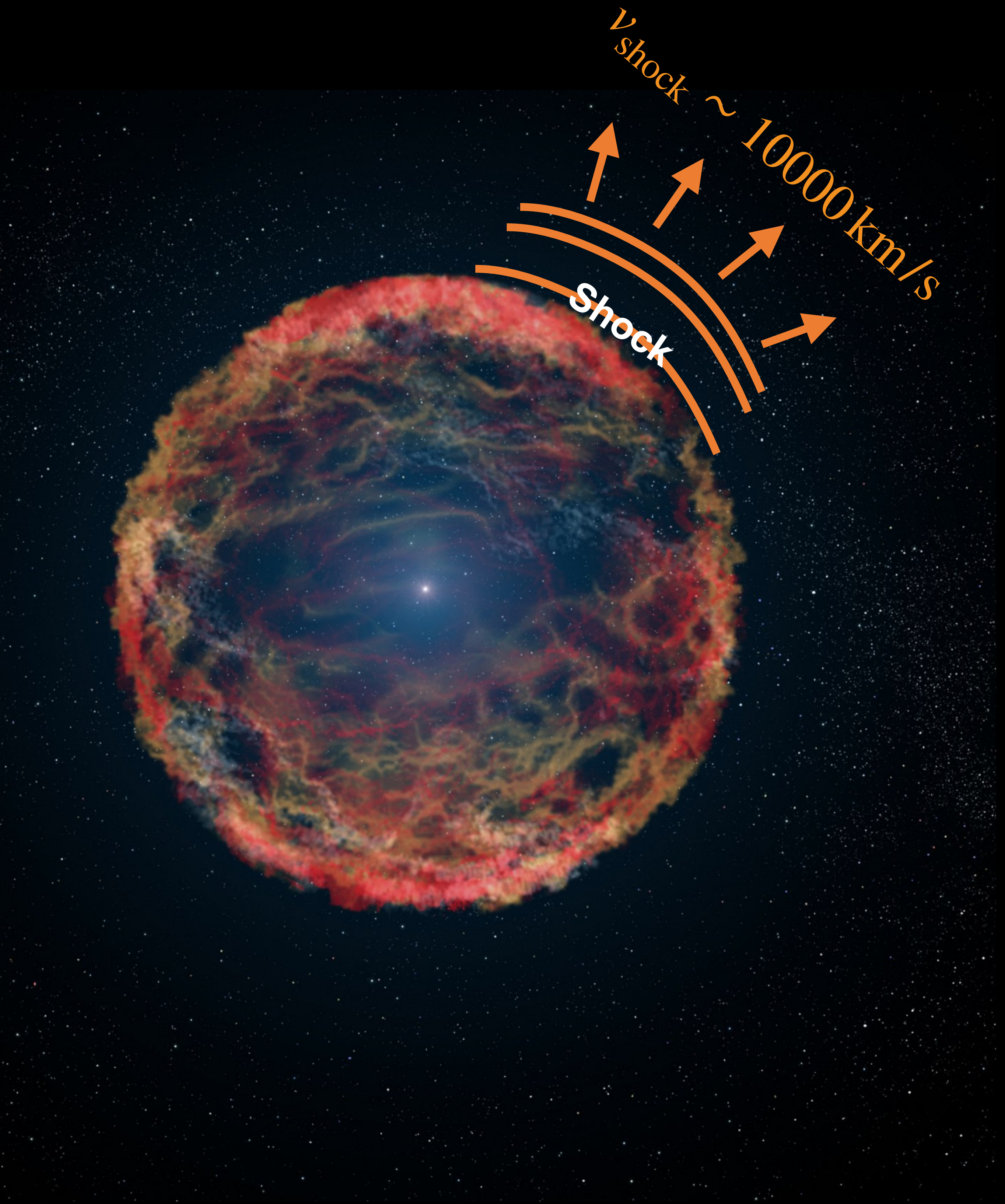
# Backup slides

# CR production up to high energies



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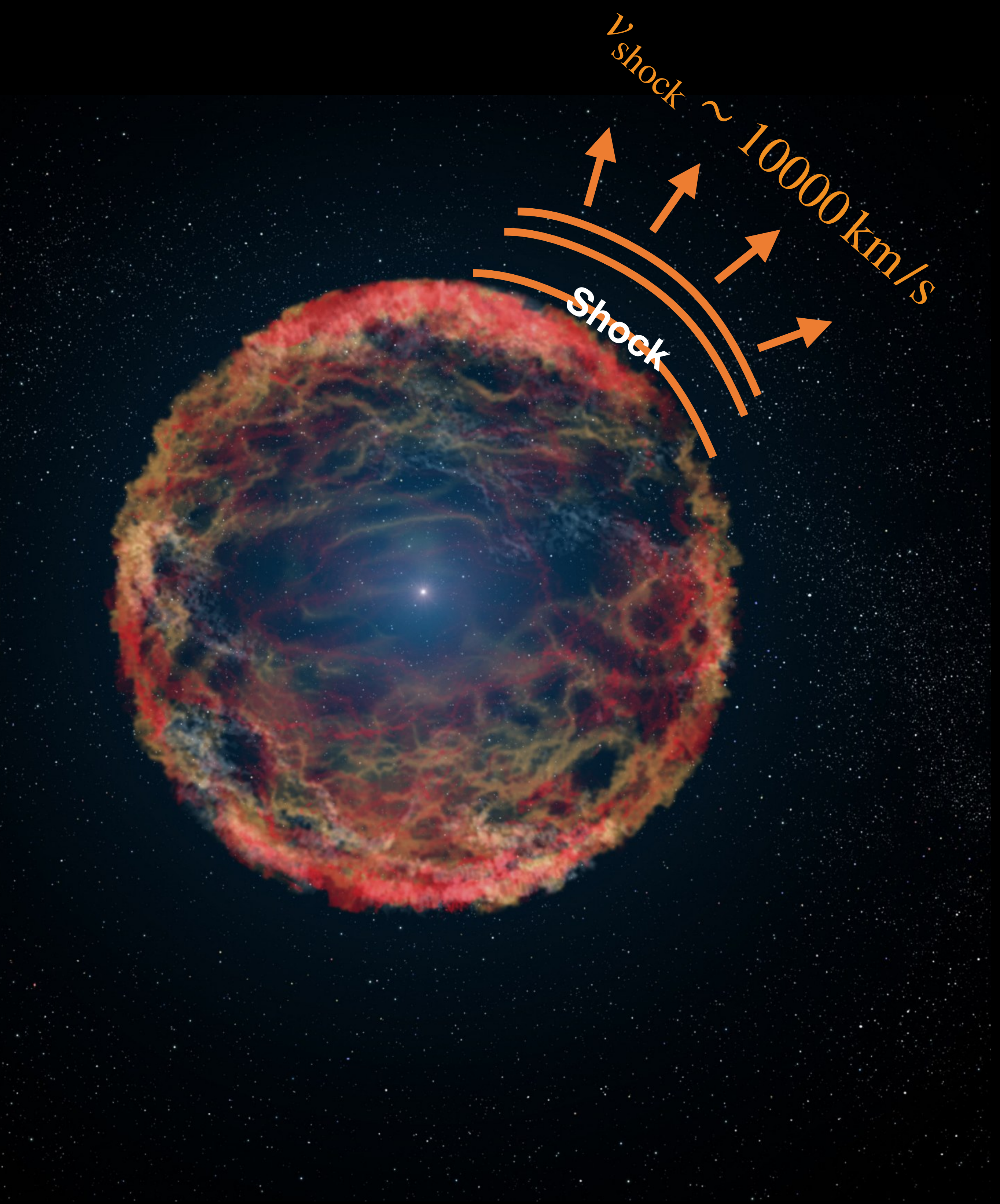


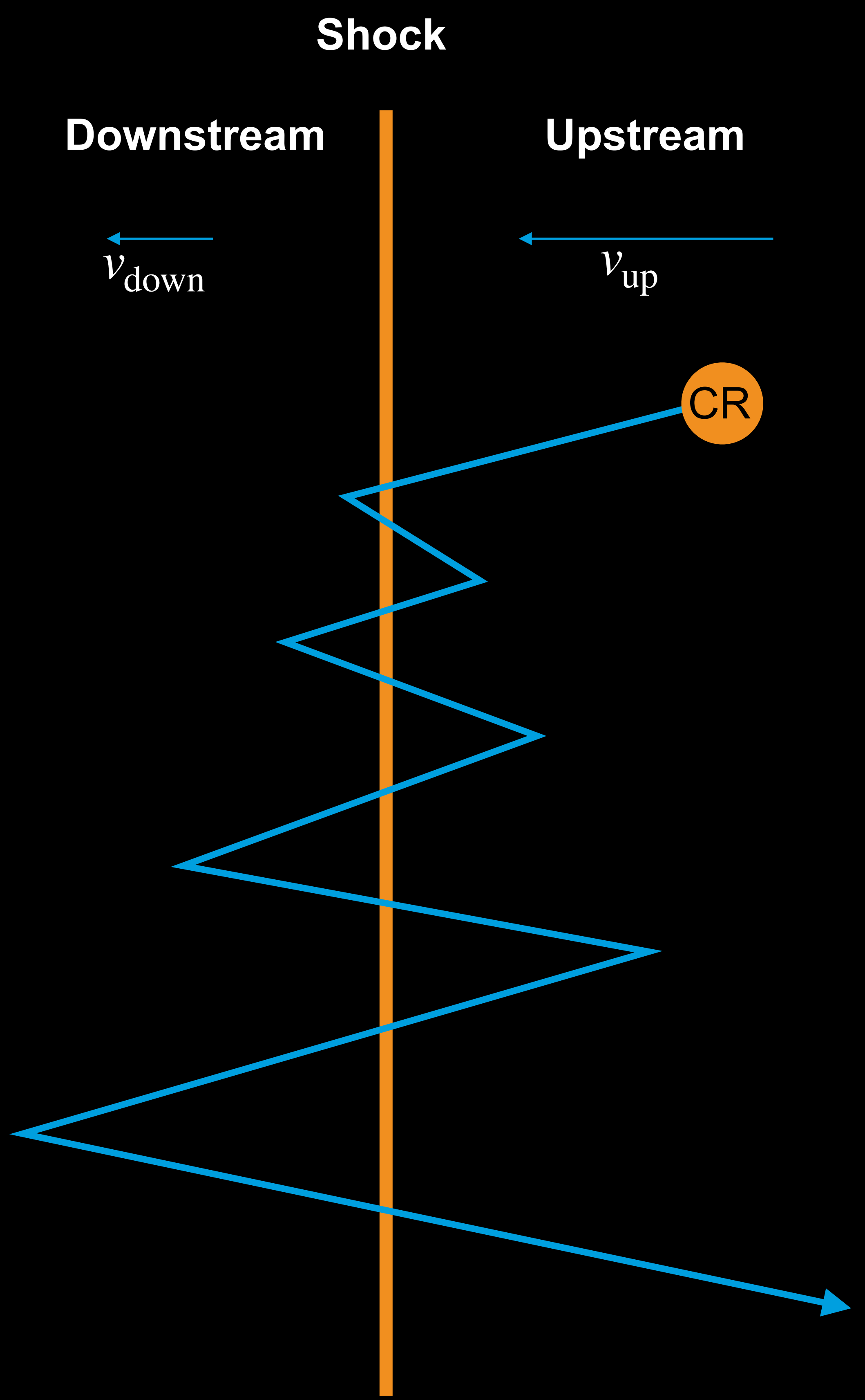

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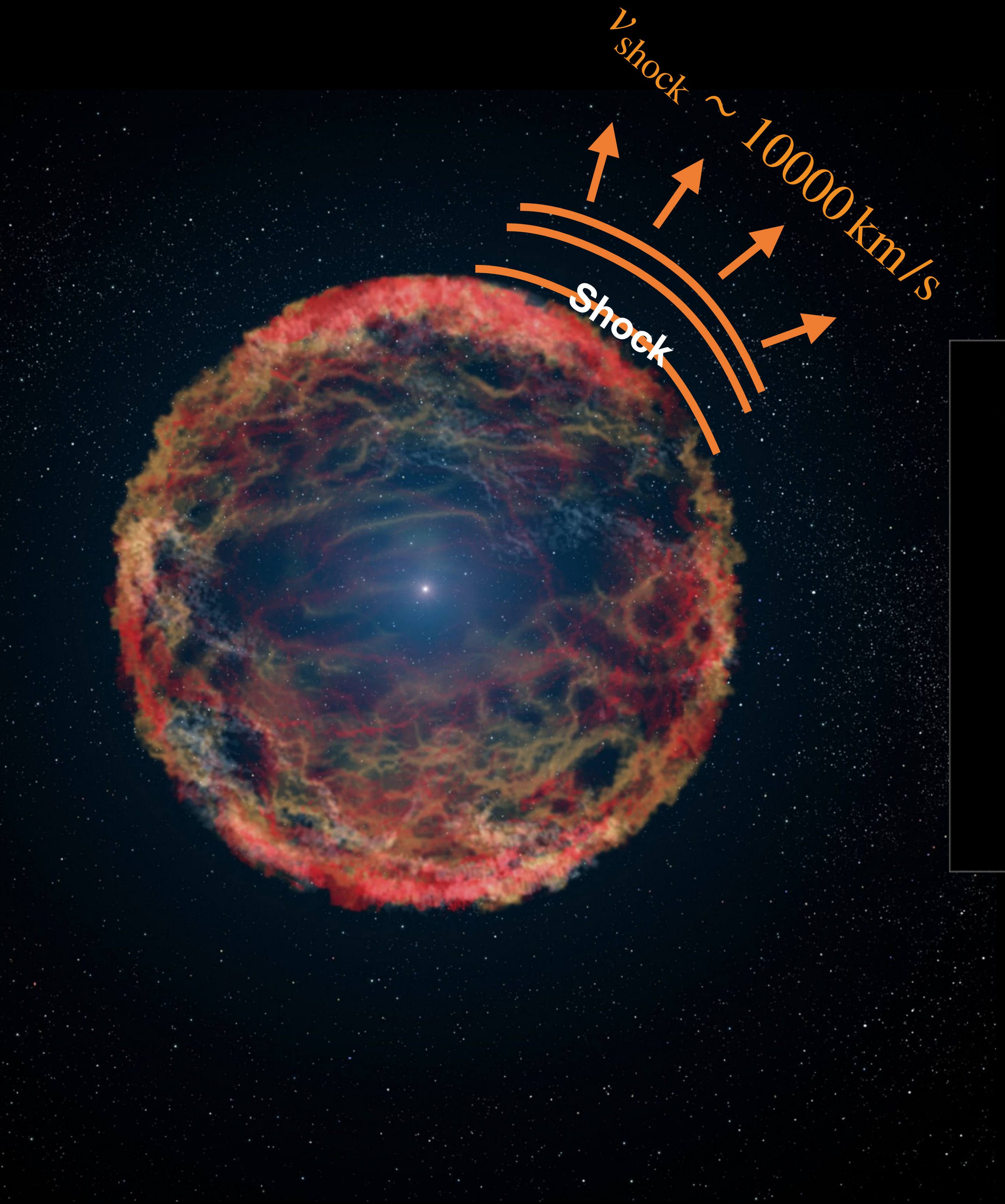
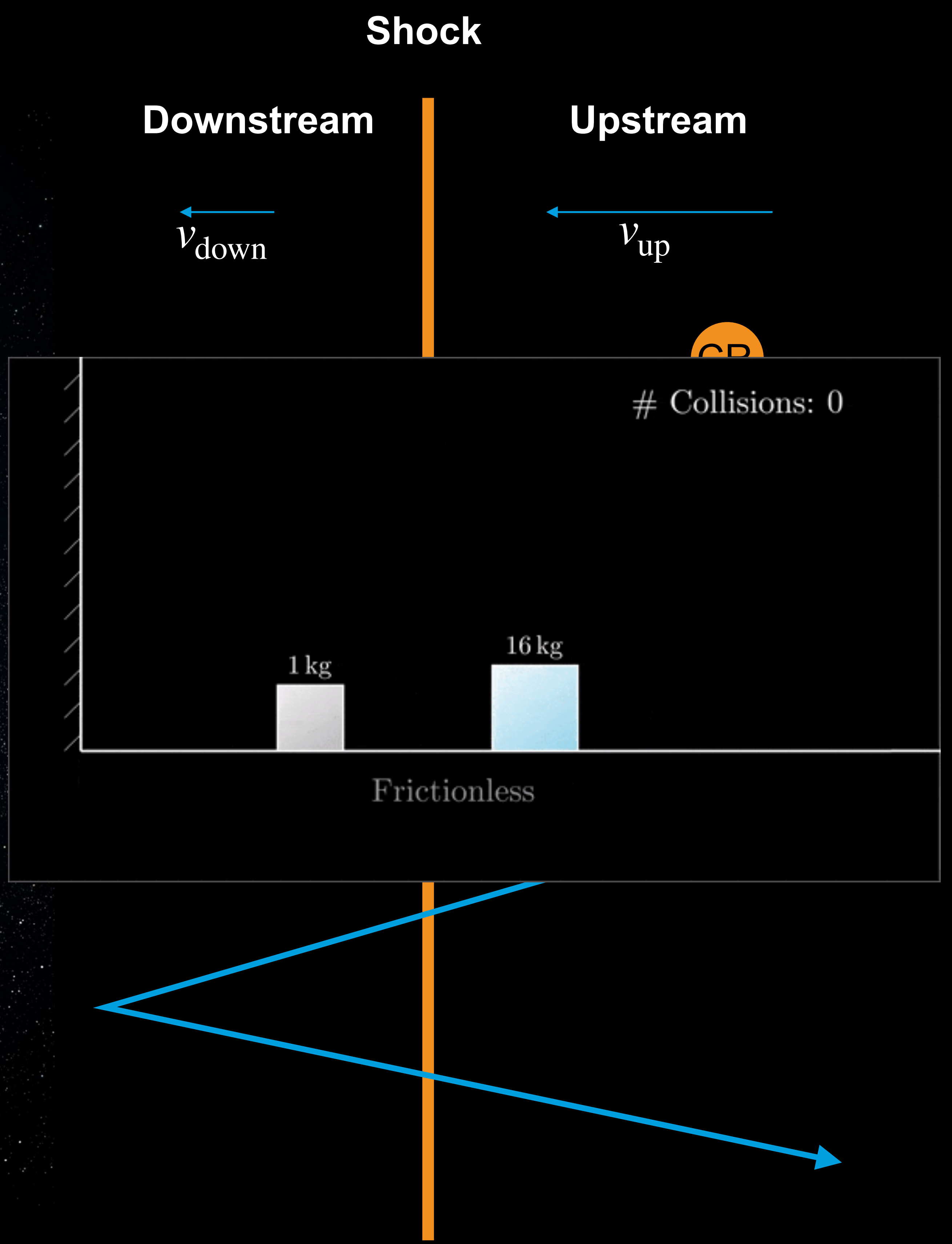
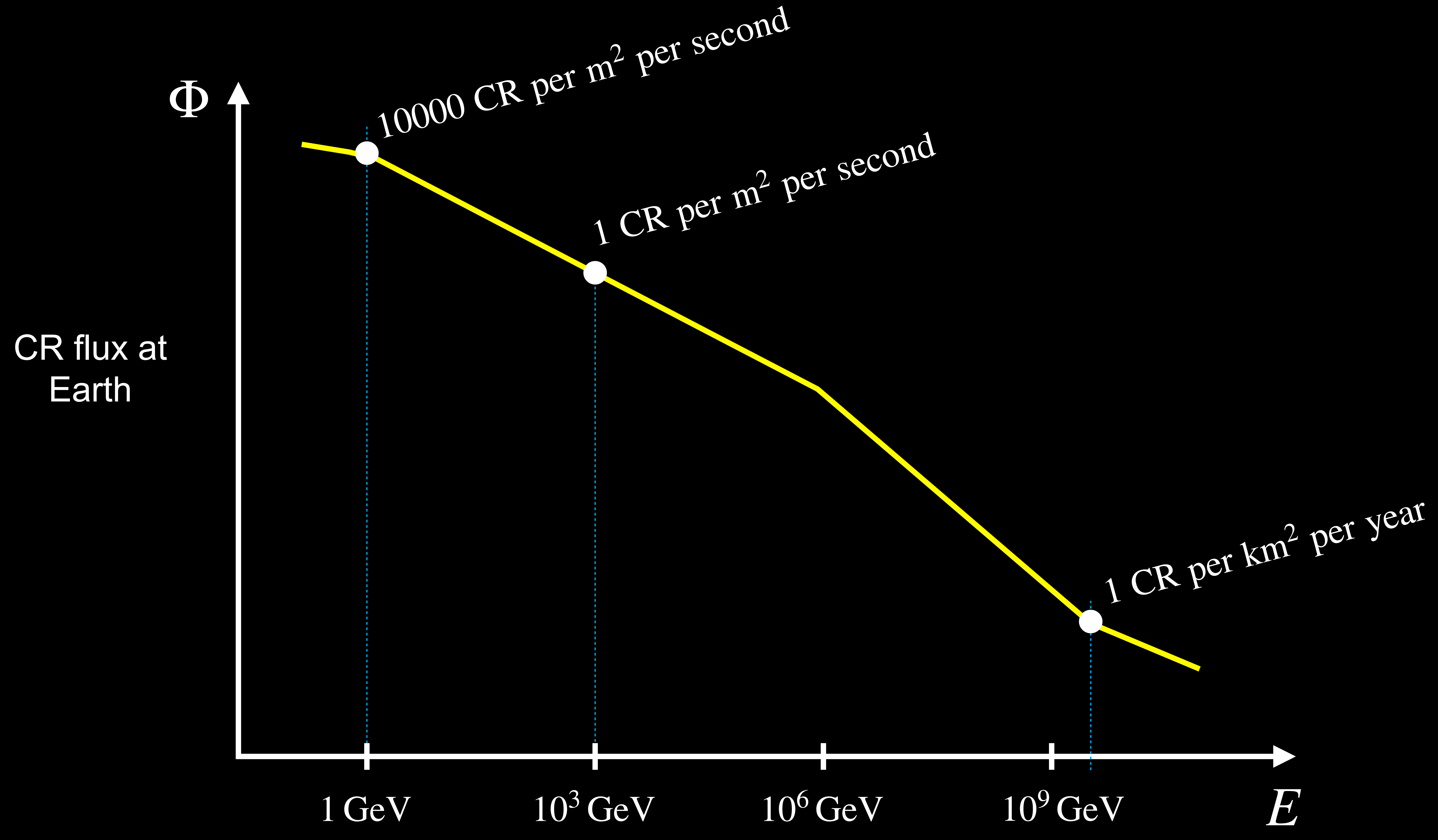


# Resulting CR spectrum



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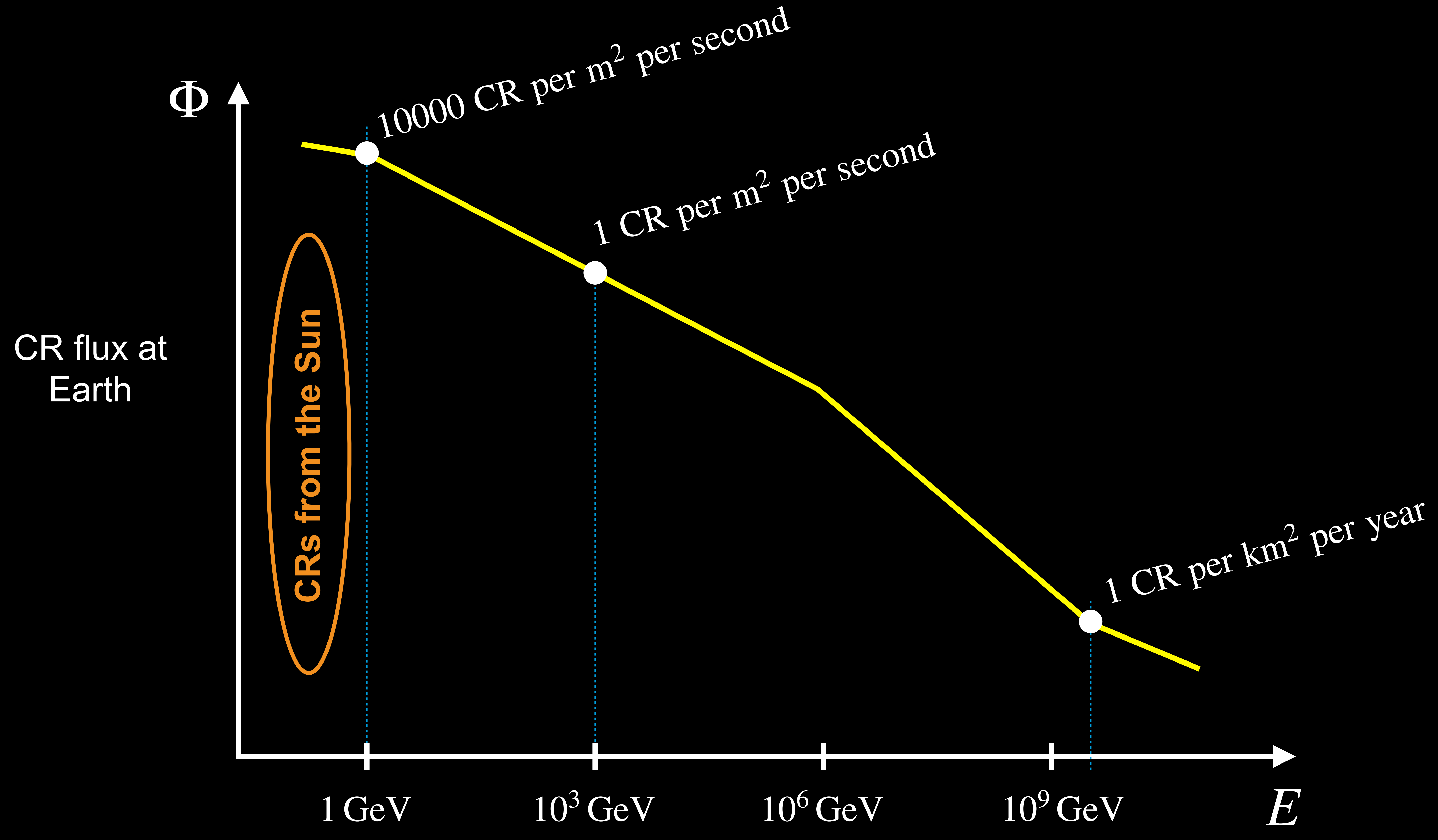

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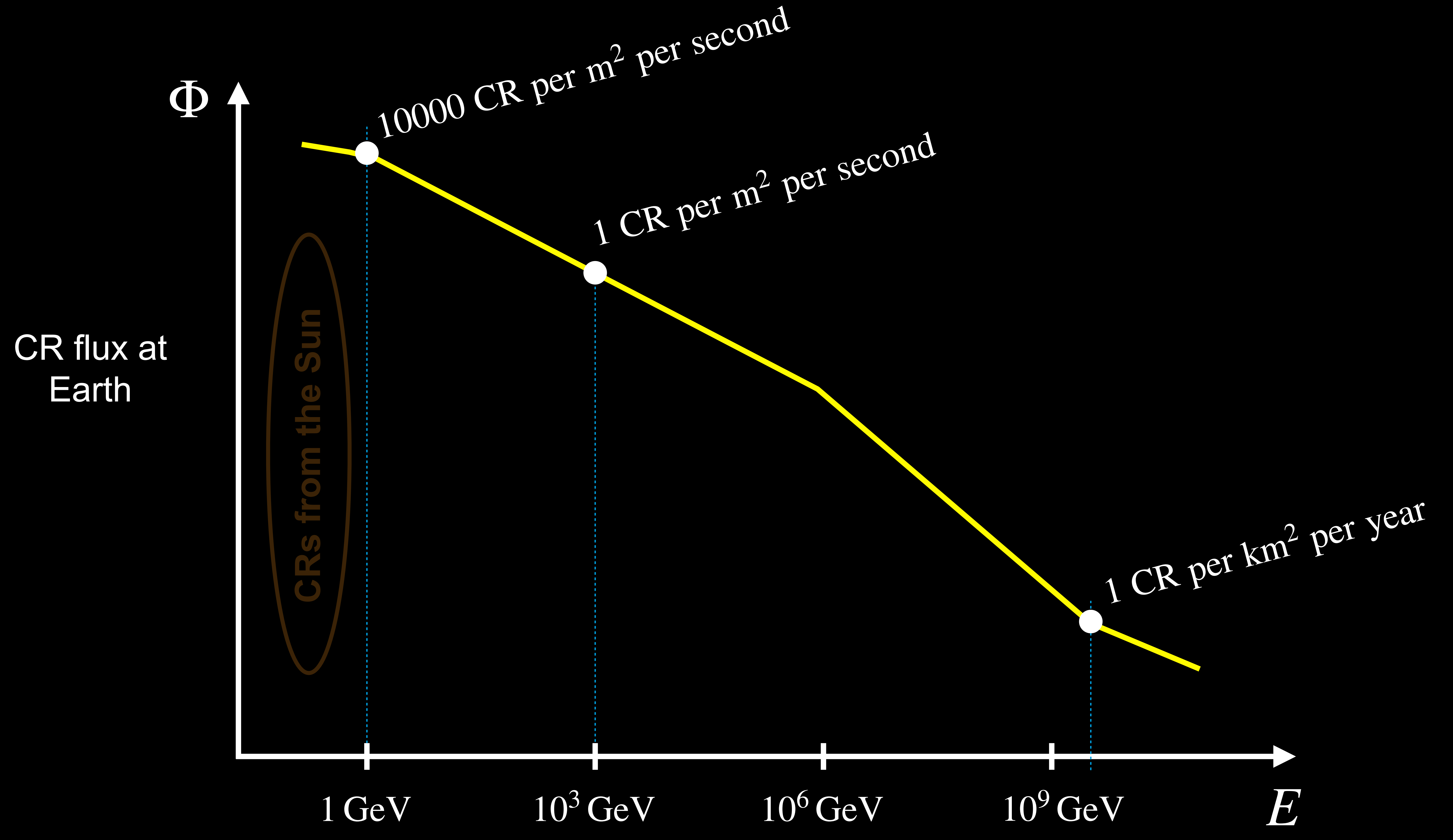
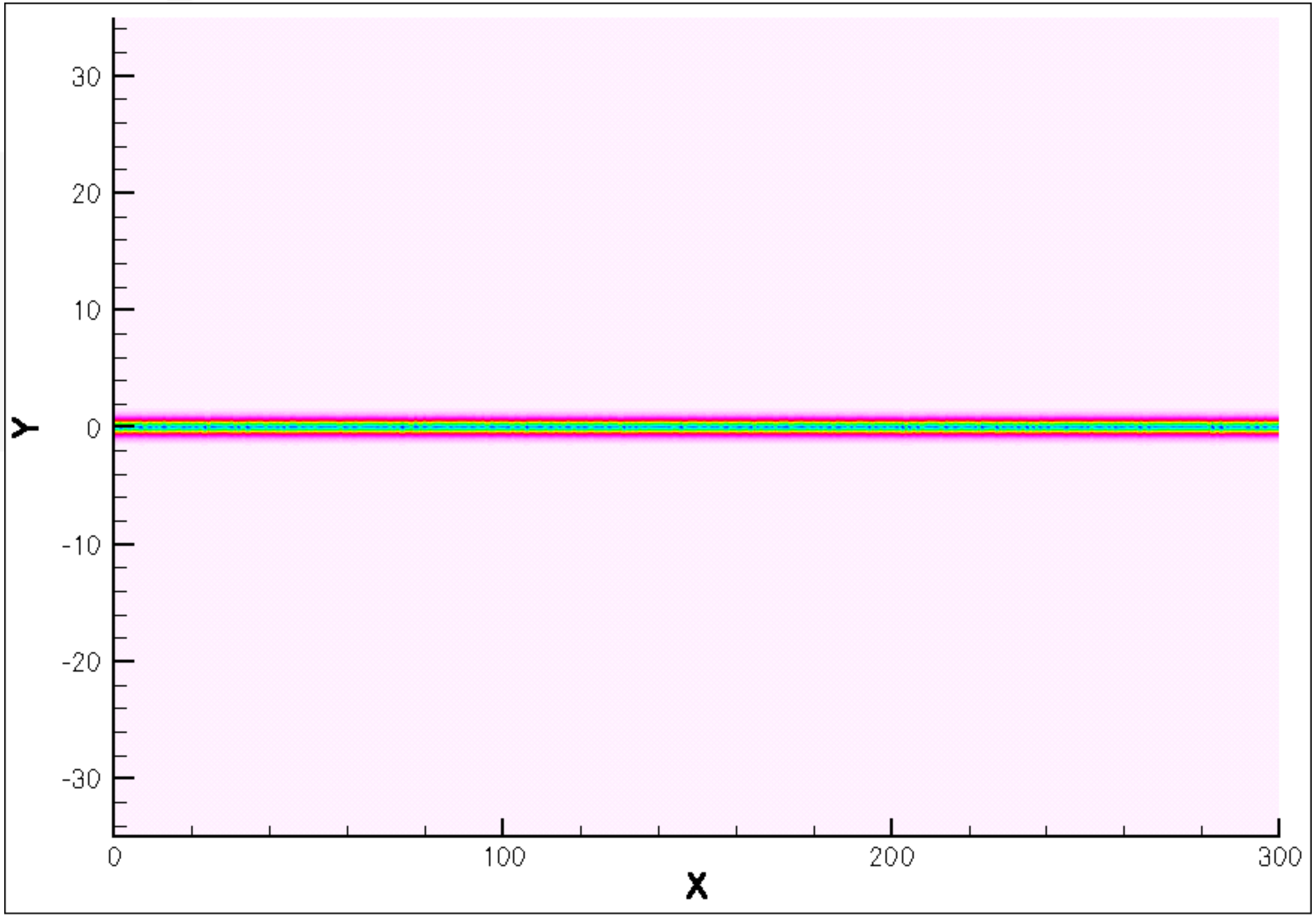


# Collisions: 0
1 kg
16 kg
Frictionless

# Turbulent cascade in the inertial range



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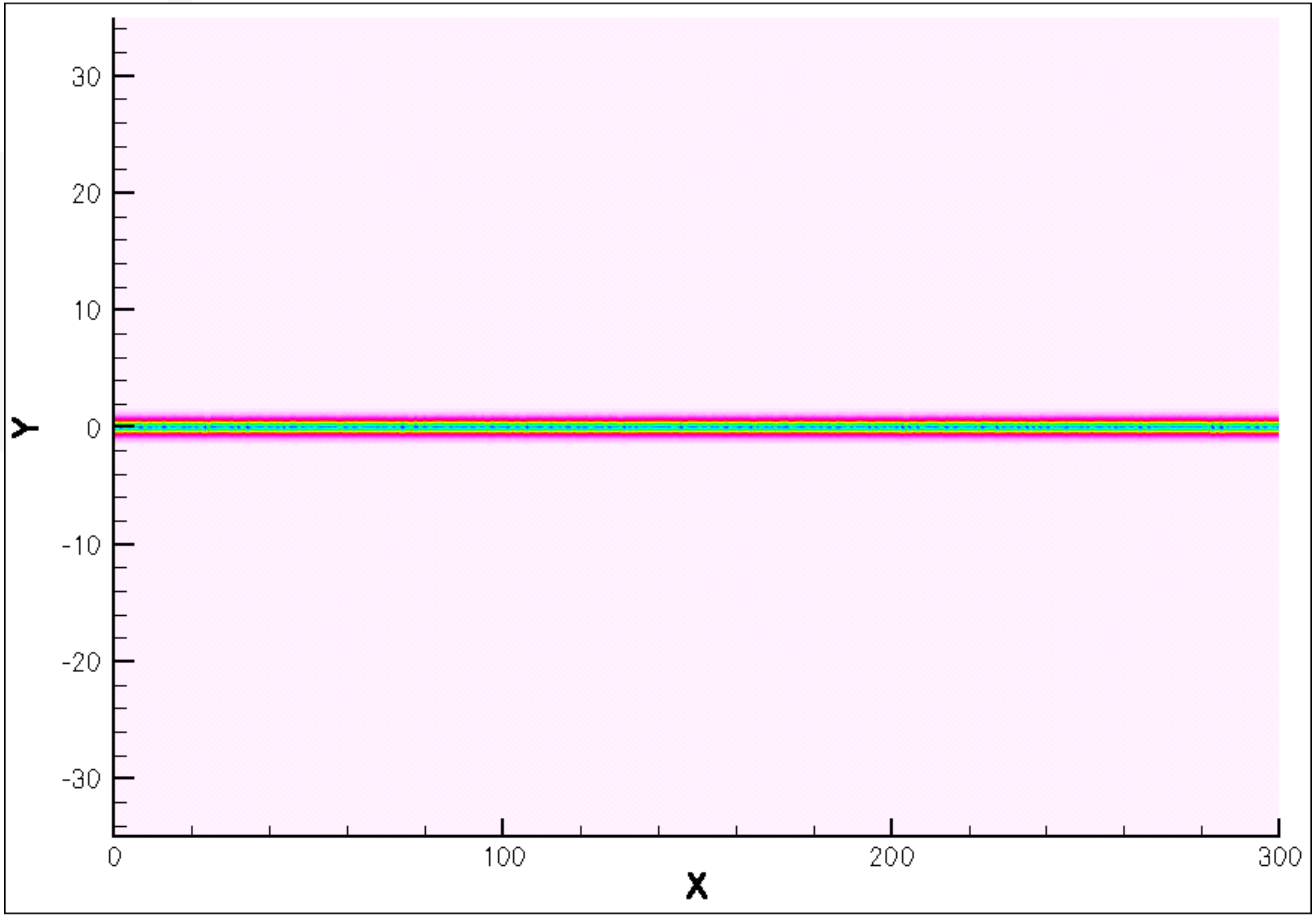


# Turbulent cascade in the inertial range

Kolmogorov's approach

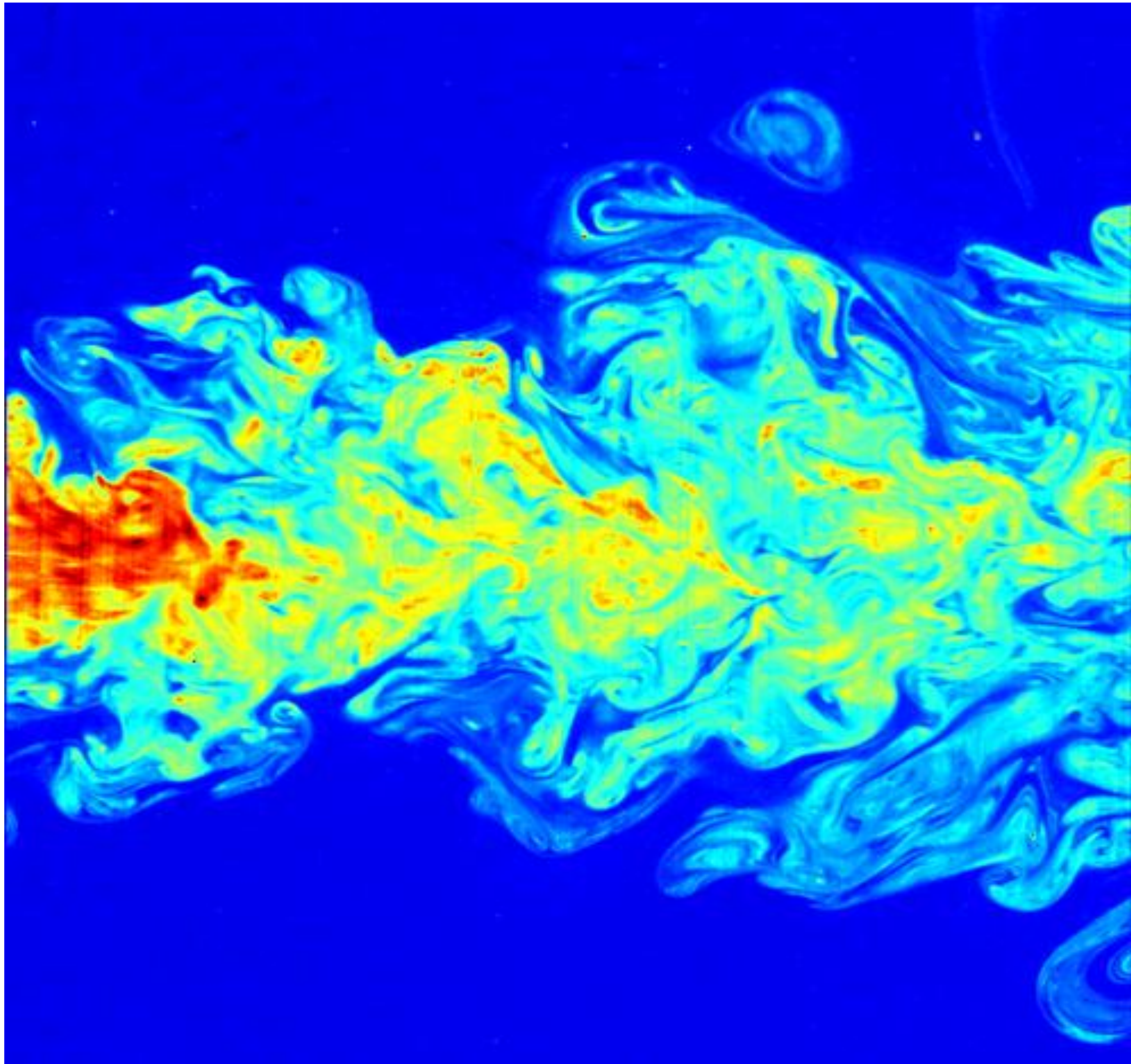
$$\frac{E_K/V}{\tau_{turn}} \sim \frac{\rho v_\ell^2}{\ell/v_\ell} = \text{const}$$

⇩

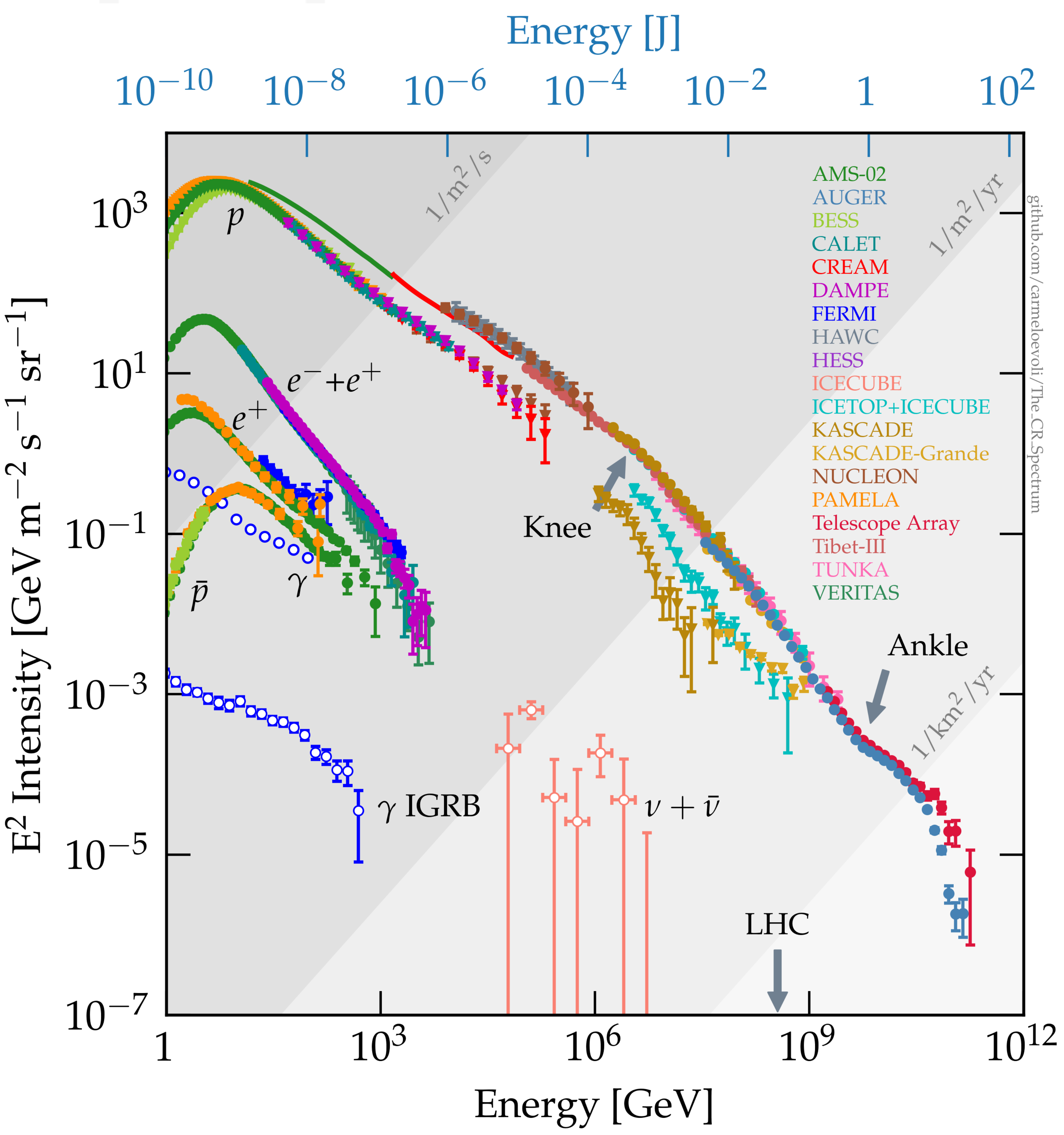
$$v_\ell^3 \sim \ell \quad \Rightarrow \quad v_\ell \sim \ell^{1/3} \quad \Rightarrow \quad v_k \sim k^{-1/3}$$

⇩

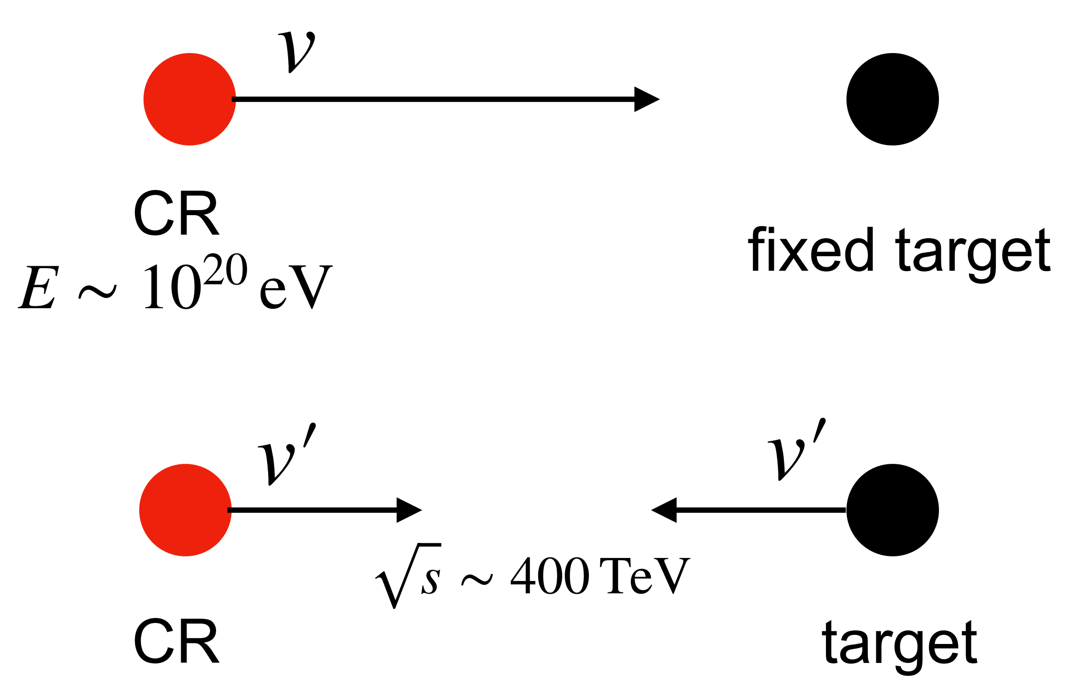
$$k \cdot E(k) \sim \rho v_k^2 \quad \Rightarrow \quad E(k) \sim k^{-5/3}$$



# Why studying CR physics?




- $\frac{dN}{dE} \propto E^{-\gamma} \qquad 2.7 \lesssim \gamma \lesssim 3.1$
- Very energetic particles

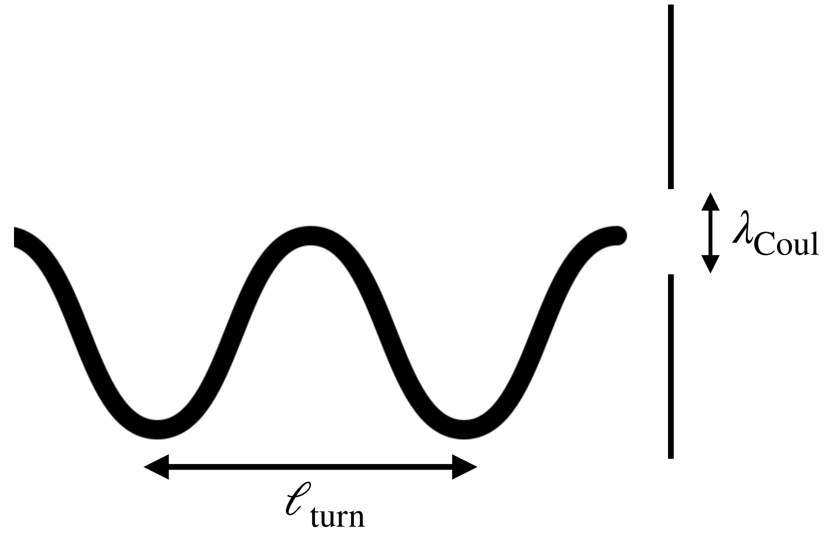



- Unique probe of extreme astrophysical phenomena

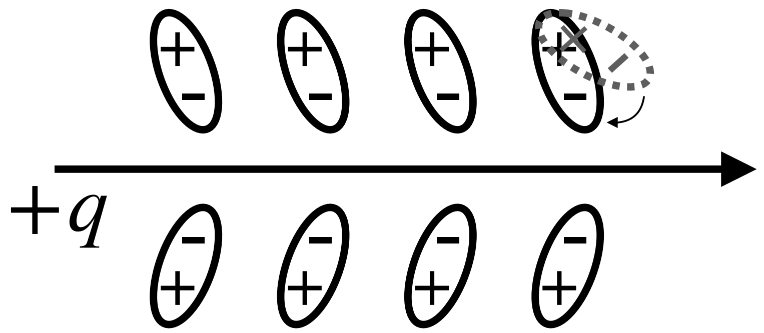
# Damping of the fast modes

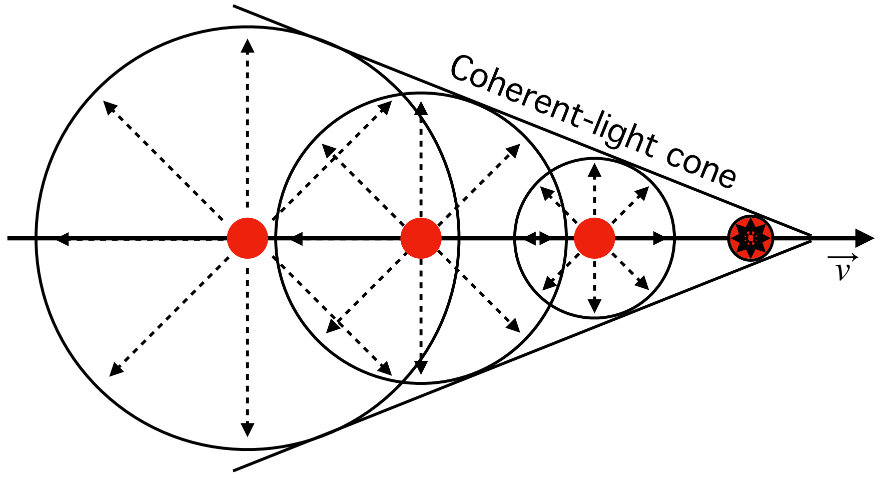
$$\lambda_{\rm Coul} \approx 1.3 \cdot 10^{-5} \left( \frac{\rm cm^{-3}}{n_{\rm ISM}} \right) \cdot \left( \frac{T}{10^4\,\rm K} \right)^2 \rm pc$$

- **Collisional** damping



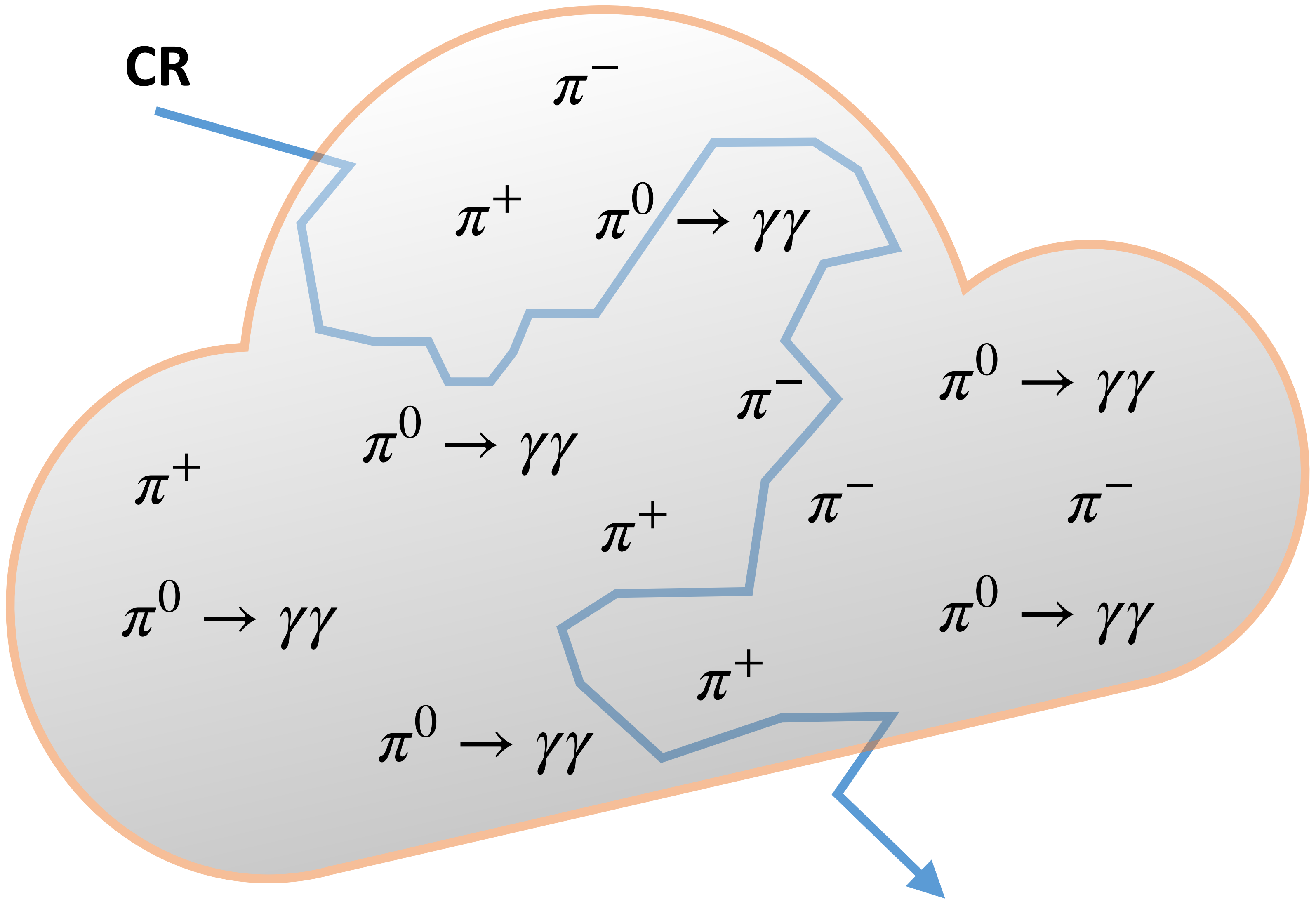

- **Collisionless** damping




$$\lambda_{\rm Coul}^{\rm disk} \underset{\substack{n_{\rm disk} = 1\,\rm cm^{-3} \\ T = 10^4\,\rm K}}{\approx} 1.3 \cdot 10^{-5}\,\rm pc, \qquad \lambda_{\rm Coul}^{\rm halo} \underset{\substack{n_{\rm Halo} = 10^{-3}\,\rm cm^{-3} \\ T = 10^6\,\rm K}}{\approx} 1.3 \cdot 10^{2}\,\rm pc \simeq L_{\rm inj}$$

# Inferred CR density from pion decay



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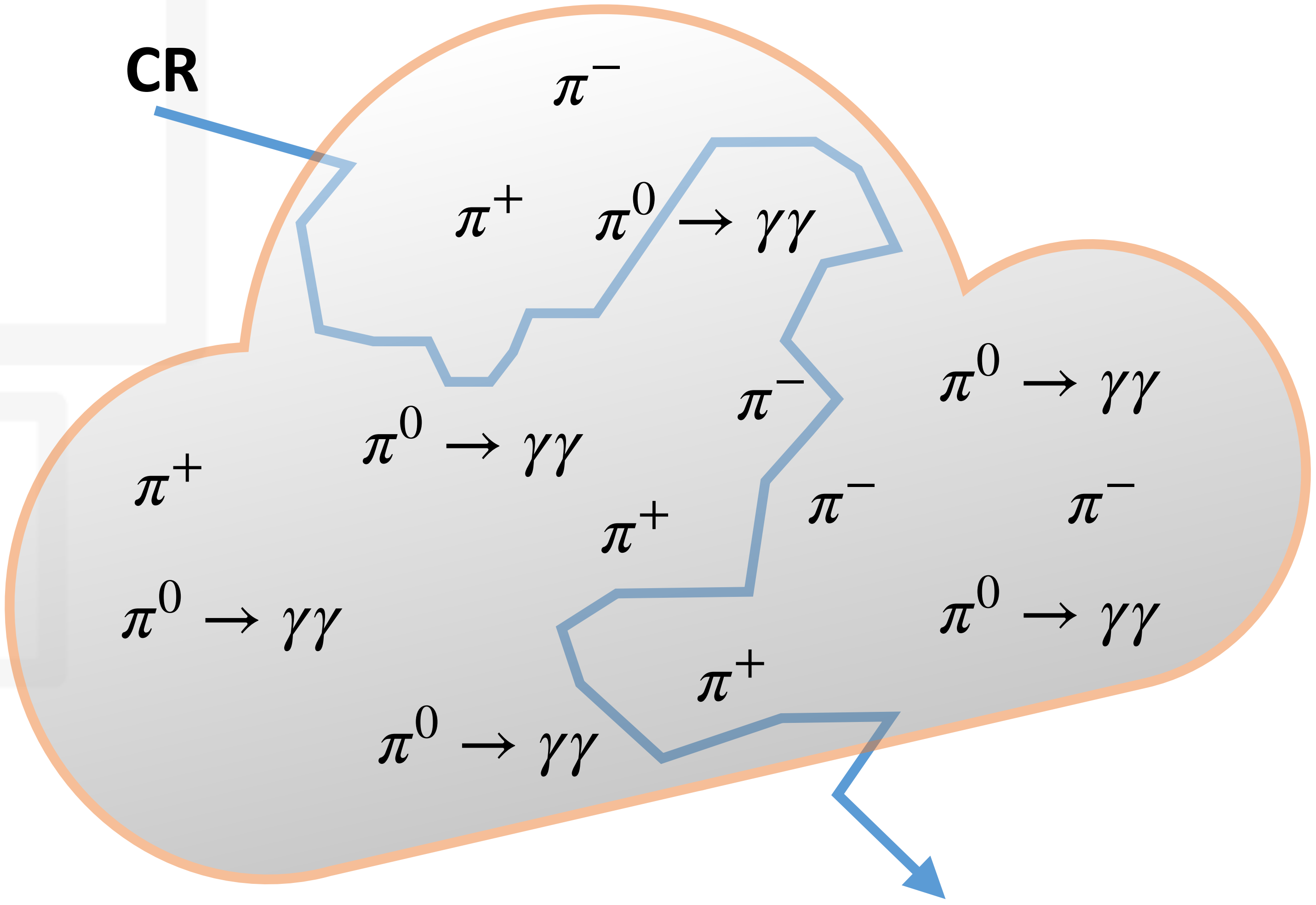


$$\langle E_\gamma \rangle \simeq 0.1\, E_{\rm CR}$$

$$E_{\rm flux} = \int_{E_{\rm min}}^{E_{\rm max}} dE\, E \cdot \overbrace{\left( \frac{dN_\gamma}{dE} \frac{c}{4\pi} \right)}^{\Phi_\gamma} \quad \left[ \frac{E}{L^2 \cdot T} \right]$$

$$\Rightarrow \quad L_\gamma = E_{\rm flux} \cdot 4\pi d^2 \quad \left[ \frac{E}{T} \right]$$



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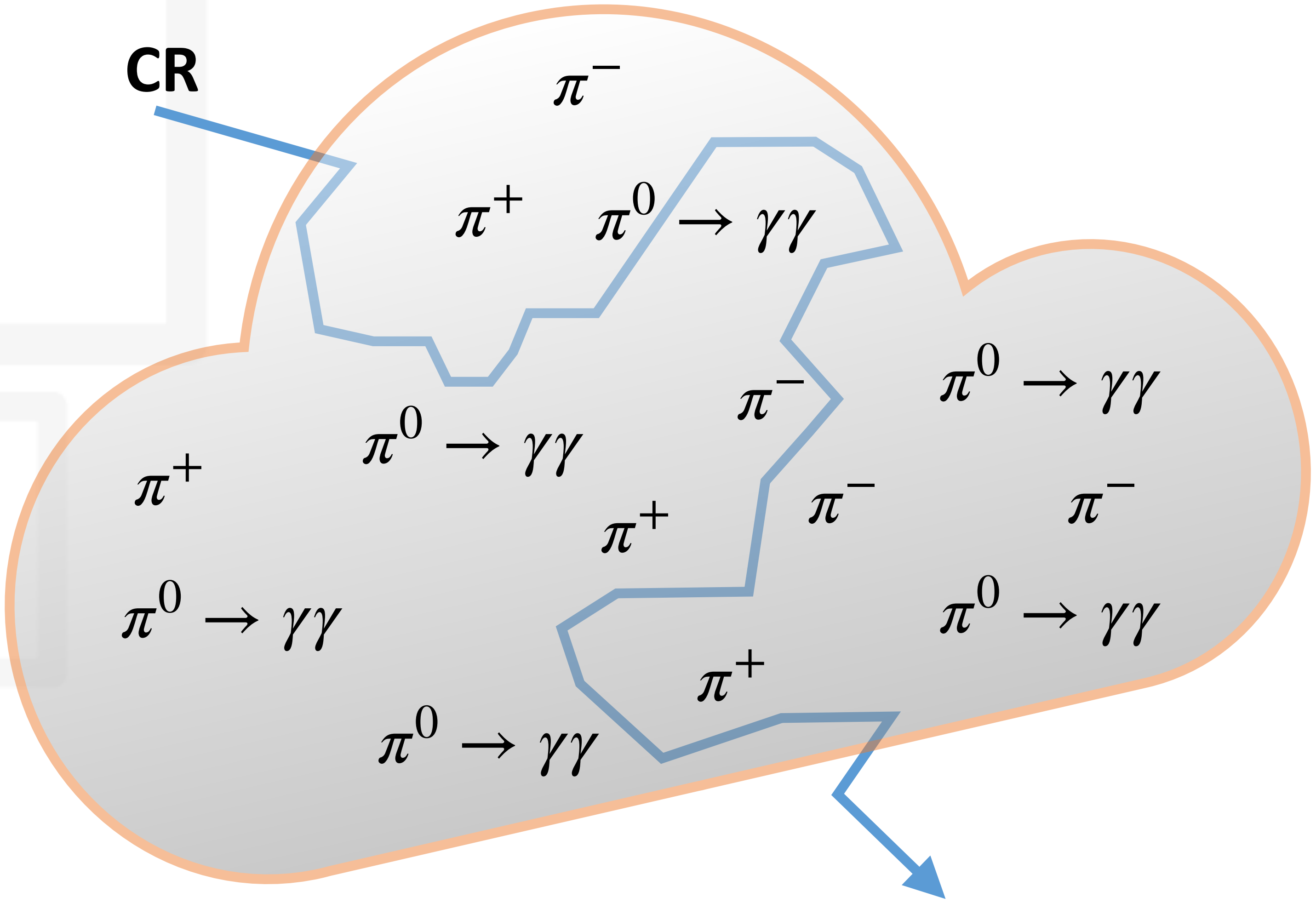
# Inferred CR density from pion decay

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$$\Rightarrow \quad L_\gamma = E_{\rm flux} \cdot 4\pi d^2 \quad \left[ \frac{E}{T} \right]$$

$$L_\gamma(\geq E_\gamma) \approx \eta_N \cdot \frac{W_p(\geq 10\, E_\gamma)}{\tau_{pp\to\pi^0}} = \eta_N \cdot \frac{W_p(\geq E_\gamma)}{1.6 \cdot 10^8\,{\rm yr} \left( \frac{\rm cm^{-3}}{n_H} \right)}$$

$$\omega_{\rm CR} \equiv \frac{W_p(\geq 10\, E_\gamma)}{V_{\rm crossed}} = \frac{W_p(\geq 10\, E_\gamma)}{M_{\rm tot}} \cdot n_H \approx 1.8 \cdot 10^{-2} \left( \frac{\eta_N}{1.5} \right)^{-1} \left( \frac{L_\gamma(\geq E_\gamma)}{10^{34}\,{\rm erg \cdot s^{-1}}} \right) \left( \frac{M_{\rm tot}}{10^6 M_\odot} \right)^{-1} {\rm erg \cdot cm^{-3}}$$

# Inefficiency of Alfvén modes

$$\int dk_{\parallel}\, E(k_{\parallel}) = \int dk_{\perp}\, E(k_{\perp})$$

$$E^{\mathrm{GS}}(k_{\perp}) \sim k_{\perp}^{-5/3}$$

$$k_{\parallel} \sim k_{\perp}^{2/3} \;\Rightarrow\; k_{\parallel}^{3/2} \sim k_{\perp}$$

$$\rightarrow \frac{dk_{\perp}}{dk_{\parallel}} = \frac{3}{2} k_{\parallel}^{3/2-1} = \frac{3}{2} k_{\parallel}^{1/2}$$

$\Rightarrow$

$$\int dk_{\perp}\, E(k_{\perp}) = \int \frac{3}{2} k_{\parallel}^{1/2}\, dk_{\parallel}\, E(k_{\perp}) =$$

$$= \frac{3}{2} \int k_{\parallel}^{1/2}\, dk_{\parallel}\, k_{\perp}^{-5/3} = \frac{3}{2} \int dk_{\parallel}\, k_{\parallel}^{1/2} \left(k_{\parallel}^{3/2}\right)^{-5/3}$$



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# Inefficiency of Alfvén modes

$$\int dk_{\parallel}\, E(k_{\parallel}) = \int dk_{\perp}\, E(k_{\perp})$$

$$E^{\rm GS}(k_{\perp}) \sim k_{\perp}^{-5/3} = k_{\perp}^{-1.67}$$

$$k_{\parallel} \sim k_{\perp}^{2/3} \Rightarrow k_{\parallel}^{3/2} \sim k_{\perp}$$

$$\rightarrow \frac{dk_{\perp}}{dk_{\parallel}} = \frac{3}{2} k_{\parallel}^{3/2-1} = \frac{3}{2} k_{\parallel}^{1/2}$$

$$\int dk_{\perp}\, E(k_{\perp}) = \int \frac{3}{2} k_{\parallel}^{1/2}\, dk_{\parallel}\, E(k_{\perp}) =$$

$$= \frac{3}{2} \int k_{\parallel}^{1/2}\, dk_{\parallel}\, k_{\perp}^{-5/3} = \frac{3}{2} \int dk_{\parallel}\, k_{\parallel}^{1/2} \left(k_{\parallel}^{3/2}\right)^{-5/3}$$

$$E^{\rm GS}(k_{\parallel}) \sim k_{\parallel}^{-2}$$

# Inefficiency of Alfvén modes

$$\int dk_{\parallel}\, E(k_{\parallel}) = \int dk_{\perp}\, E(k_{\perp})$$

$$E^{\rm GS}(k_{\perp}) \sim k_{\perp}^{-5/3} = k_{\perp}^{-1.67}$$

$$k_{\parallel} \sim k_{\perp}^{2/3} \;\Rightarrow\; k_{\parallel}^{3/2} \sim k_{\perp}$$

$$\rightarrow \frac{dk_{\perp}}{dk_{\parallel}} = \frac{3}{2} k_{\parallel}^{3/2-1} = \frac{3}{2} k_{\parallel}^{1/2}$$

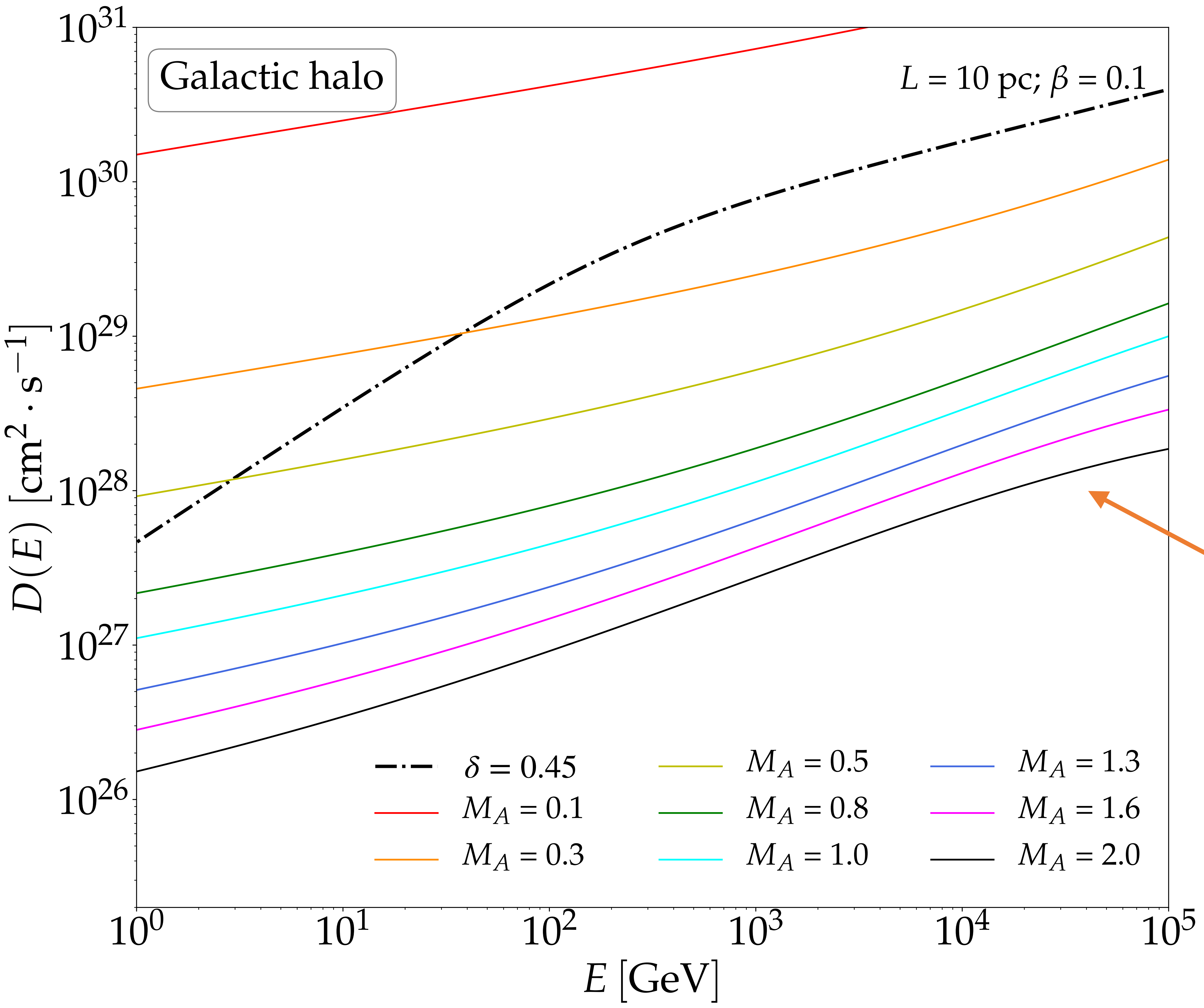
$$\int dk_{\perp}\, E(k_{\perp}) = \int \frac{3}{2} k_{\parallel}^{1/2}\, dk_{\parallel}\, E(k_{\perp}) = \frac{3}{2}\int k_{\parallel}^{1/2}\, dk_{\parallel}\, k_{\perp}^{-5/3} = \frac{3}{2}\int dk_{\parallel}\, k_{\parallel}^{1/2} \left(k_{\parallel}^{3/2}\right)^{-5/3}$$

$$E^{\rm GS}(k_{\parallel}) \sim k_{\parallel}^{-2}$$

But $D_{\mu\mu} \propto R_n(k_{\parallel} v_{\parallel} - \omega + n\Omega) \ldots$

GSSI

# Resulting CR diffusivity




Contribution from Alfvén modes