Contribution ID: **18** Type: **Talk**

A low rank ODE for spectral clustering stability

Spectral clustering is a well-known technique which identifies k clusters in an undirected graph with weight matrix $W \in \mathbb{R}^{n\times n}$ by exploiting its graph Laplacian
L(W)=diag(W**1**)-W, **1**=(1,...,1)$^T \in \mathbb{R}^n, whoseeigenvalues$0=$\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n$ and eigenvectors are related to the k clusters. Since the computation of λ_{k+1} and λ_k affects the reliability of this method, the k-th spectral gap $\lambda_{k+1} - \lambda_k$ is often considered as a stability indicator. This difference can be seen as an unstructured distance between $L(W)$ and an arbitrary symmetric matrix $L_\star$ with vanishing k-th spectral gap.

A more appropriate structured distance to ambiguity such that $L_\star$ represents the Laplacian of a graph has been proposed in [1]. Slightly differently, we consider the objective functional F$(\Delta) = \lambda_{k+1}\left(L(W+\Delta)\right) - \lambda_k\left(L(W+\Delta)\right), where\Delta$ is a perturbation such that $W+\Delta$ has non-negative entries and the same pattern of W. We look for an admissible perturbation $\Delta_\star$ of smallest Frobenius norm such that $F(\Delta_\star) = 0$.

In order to solve this optimization problem, we exploit its low rank underlying structure. Similarly to [2], we formulate a rank-4 symmetric matrix ODE whose stationary points are the optimizers sought. The integration of this equation benefits from the low rank structure with a moderate computational effort and memory requirement, as it is shown in some illustrative numerical examples.

[1] E. Andreotti, D. Edelmann, N. Guglielmi, C. Lubich, Measuring the stability of spectral clustering, Linear Algebra and its Applications, 2021

[2] N. Guglielmi, C. Lubich, S. Sicilia, Rank-1 matrix differential equations for structured eigenvalue optimization, arXiv preprint arXiv:2206.09338, 2022

Primary authors: Prof. GUGLIELMI, Nicola (Gran Sasso Science Institute); SICILIA, Stefano (Gran Sasso Science Institute)

Presenter: SICILIA, Stefano (Gran Sasso Science Institute)