

The construction of orthogonal Cauchy-like matrices

A matrix $K \in \mathbb{R}^{n \times n}$ is Cauchy-like if its entries have the form

$$K_{ij} = \frac{a_i b_j}{x_i - y_j}, \quad i, j = 1, \dots, n,$$

where x_i, y_j for $i, j = 1, \dots, n$ are pairwise distinct real numbers. Besides their pervasive occurrence in computations with rational functions, Cauchy-like matrices play an important role in deriving algebraic and computational properties of many displacement-structured matrix families and occur as fundamental blocks (together with trigonometric transforms) in decomposition formulas and fast solvers for, e.g., Toeplitz, Hankel, and related matrices.

This contribution provides a complete description of orthogonal Cauchy-like matrices [1]. Interest in these matrices stems from the paper [2], where they are needed to design all-pass filters for signal processing, and a novel characterization of Cauchy matrices as transition matrices between eigenbases of particular matrix pairs [3]. We illustrate their relationships with secular equations, the diagonalization of symmetric quasi-separable matrices and the construction of orthogonal rational functions with free poles. Moreover, we characterize matrix algebras that are simultaneously diagonalized by orthogonal Cauchy-like matrices.

[1] D. Fasino, Orthogonal Cauchy-like matrices, *Numerical Algorithms*, 92 (2023), 619–637.

[2] S. J. Schlecht, Allpass feedback delay networks, *IEEE Trans. Signal Process.*, 69 (2021), 1028–1038.

[3] A. G. Lynch, Cauchy pairs and Cauchy matrices, *Linear Algebra Appl.*, 471 (2015), 320–345.

Primary author: FASINO, Dario (University of Udine)

Presenter: FASINO, Dario (University of Udine)