

Parallel-in-time solver for the all-at-once Runge–Kutta discretization

Time-dependent PDEs arise very often in many scientific areas, such as mechanics, biology, economics, or chemistry, just to name a few. Of late, many researchers have devoted their effort to devising parallel-in-time methods for the numerical solution of time-dependent PDEs. As opposed to the classical approach, in which an approximation of the solution at a time t is computed by a sequential time-stepping, parallel-in-time methods approximate the solution of the problem for all times concurrently. This in turns adds a new dimension of parallelism and allows to speed-up the numerical solution on modern supercomputers.

In this talk, we present a fully parallelizable preconditioner for the all-at-once linear system obtained when employing a Runge–Kutta method in time. The resulting system is solved iteratively for the numerical solution and for the stages of the method. By employing classical theory of block matrices, one is able to derive an optimal preconditioner for the system considered. This results in a block-diagonal solve for all the stages at all the time-steps, and a Schur complement whose inverse can be applied by solving again for the systems for the stages of the method. Since at each linear iteration one has to solve for the latter system, we introduce a new block-preconditioner based on the SVD of the (real) Runge–Kutta coefficient matrix $A_{RK} = U\Sigma V^T$.

A range of numerical experiments validate the robustness of the preconditioner with respect to the discretization parameters and to the number of stages, as well as very promising scalability and parallel efficiency indices.

Primary author: LEVEQUE, Santolo (Scuola Normale Superiore)

Co-authors: BERGAMASCHI, Luca (University of Padova); MARTINEZ, Angeles (University of Trieste); PEARSON, John (University of Edinburgh)

Presenter: LEVEQUE, Santolo (Scuola Normale Superiore)