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Nodal domain count for the graph p-Laplacian

The nodal domain count of the graph p-Laplacian eigenfunctions allows to relate the p-Laplacian eigenvalues to different topological invariants of the graph, i.e. the Cheeger constants and the packing radii of the graph. We prove that the nodal domain count of any eigenfunction can be bounded, both from above and below, in terms of the position of the corresponding eigevalue in the variational spectrum. To this end, we prove that the variational spectrum of the p-Laplacian operator on forests exhaust the spectrum, we transfer the Weyl's inequalities for the Laplacian matrix to the nonlinear p-Laplacian and we provide a Perron-Frobenious-like characterization of the first eigenpair of the p-Laplacian. Our new results show that the variational p-Laplacian eigenvalues on trees equal the Cheeger constants and packing radii of the graph, respectively for $p = 1$ and $p = \infty$. Moreover, when applied to the linear case $p = 2$, the new results imply well-known properties of the linear Laplacian matrix as well as novel ones.

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