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Spectral analysis of weights-based finite element methods

A finite element method is defined by specifying three objects: a mesh, a finite dimensional functional space of shape functions and a collection of degrees of freedom, namely linear functionals on shape functions. It is rather this third aspect that characterises the method.

In the last twenty years a large amount of research has been devoted to generalising existing methods. These extensions moved towards several directions: flexible frameworks, more abstract formulations involving differential forms and, of course, several choices of alternative degrees of freedom have been consequently proposed. Among them we recall, for instance, *moments* and *weights*. A weight is, in its greatest generality, the integral of a multilinear k-form (tipically a differential form) on a k-simplex. As a consequence, this latter choice naturally generalises Lagrangian finite elements, allowing for considering physical measurements as degrees of freedom.

In practice, changing degrees of freedom consists in choosing a different basis for the bilinear form of the problem on the discrete level, from any given basis to that in (a fixed) duality with the specified degrees of freedom. Any different basis thus yields a different stiffness matrix and hence a different spectral symbol, which is, in the case of finite elements, a $r \times r$ Hermitian matrix-valued function, being $r+1=R$ the dimension of the local space of shape functions. For a d-variate problem, the resulting matrices have essentially the structure of d-level $R \times R$ positive definite Toepliz matrices.

Although the dimension of the stiffness matrix of the problem grows linearly with the number of elements, the rank of the symbol does not change and such a function perfectly captures the spectral behaviour of the whole stiffness matrix. As a consequence, this tool has been used to analyse and optimise several approximation methods, such as Lagrangian finite elements. We rely on this function for a spectral analysis of finite elements based on non-standard degrees of freedom. In particular, we focus on the Laplacian operator and offer a spectral analysis of weights, seeking for optimal distributions of supports in terms of the conditioning of the corresponding stiffness matrix. Results are compared with the well established counterpart for classical Lagrangian elements.

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