APPLIED NONLINEAR PERRON–FROBENIUS THEORY

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The Perron-Frobenius theory (1907-1912) provides fundamental properties about the eigensystems of positive and nonnegative matrices. This theory has impacted many areas of mathematics, including graph theory, Markov chains, and matrix computation, and it forms a fundamental component in the analysis of a range of models in areas such as demography, economics, wireless networking, and search engine optimization.

Nonnegative matrices are also pervasive in a data mining applications. For example, distance and similarity matrices are fundamental tools for data classification, affinity matrices are key instruments for graph matching, adjacency matrices are at the basis of almost every graph mining algorithm, transition matrices are the main tool for studying stochastic processes on data. The Perron-Frobenius theory makes the algorithms based on these matrices very attractive from a linear algebra point of view.

At the same time, most recent machine learning and data mining methods rely on nonlinear mappings rather than just matrices, which however still have some notion of nonnegativity.

The nonlinear Perron-Frobenius theory allows us to transfer most of the theoretical and computational niceties of nonnegative matrices to the much broader class of nonlinear multihomogeneous operators. These types of operators include for example commonly used maps associated with tensors and are tightly connected to the formulation of nonlinear eigenvalue problems with eigenvector nonlinearities.

In this minitutorial we will introduce the concept of multihomogeneous operators and we will present the state-of-the-art version of the nonlinear Perron-Frobenius theorem for nonnegative nonlinear mappings. We will discuss several numerical optimization implications connected to nonlinear and higher-order versions of the Power and the Sinkhorn methods and several open challenges, both from the theoretical and the computational viewpoints. We will also discuss some example problems in data mining, machine learning and network science which can be cast in terms of nonlinear eigenvector problems and we will show how the nonlinear Perron-Frobenius theory can help solve them.

References

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