Simulating the results obtained by LGWA with GWFish

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What we want to know

- Will LGWA be able to detect *source X*?
- Will it be able to measure its parameters?

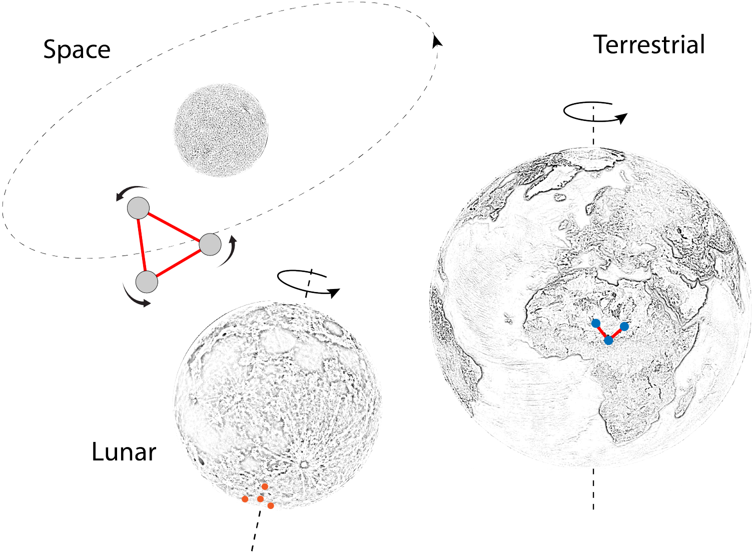
The Moon's response

In the long-wavelength, quasi-monochromatic approximation:

$$h(f) = \mathcal{A}_{ij}(t(f))h_{ij}(f)$$
$$\mathcal{A}_{ij}(t) = \vec{e}_{\text{displacement}}(t) \otimes \vec{e}_{\text{surface normal}}(t)$$

where h_{ij} (with only two independent components, h_+ and $h_\times$) is the strain tensor computed according to some theoretical signal model.

Moon motion



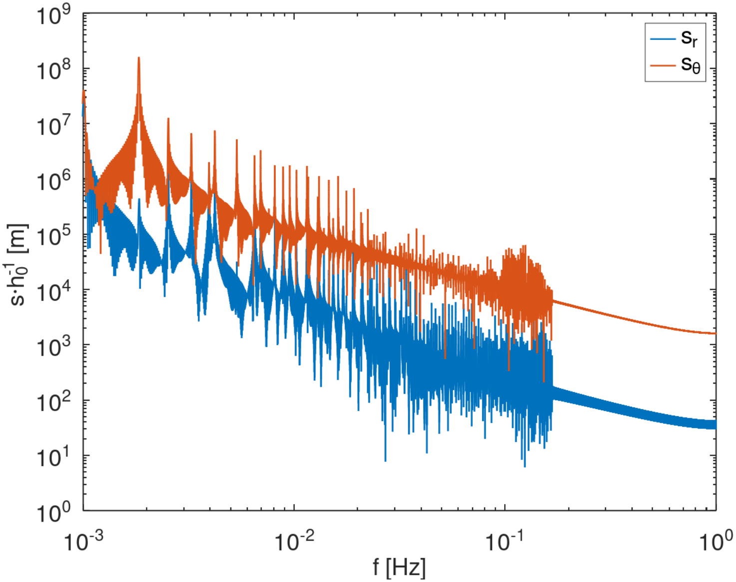
The detector's response

$h(f)$ is strain-at-seismometer, then we need:

- a map from "monochromatic" strain to measured displacement, $d(f)/h(f)$ — difficult, $\lesssim R_{\rm moon} \sim 1.7 \times 10^6$ m;
- a displacement noise PSD $\langle d^*(f) d(f') \rangle = S_d(f)\delta(f - f')/2$.

Map the displacement PSD S_d [m^2/Hz] to a strain PSD S_h [$1/\mathrm{Hz}$].

A tentative strain-to-displacement function



SNR and source detection

With a matched-filtering method like the one LIGO-Virgo-Kagra use today:

$$\mathrm{SNR} = \sqrt{4\int_0^\infty \frac{|h(f)|^2}{S_h(f)}\mathrm{d}f} = \sqrt{\int_0^\infty \frac{h_c^2(f)}{h_n^2(f)}\mathrm{dlog}\, f}$$

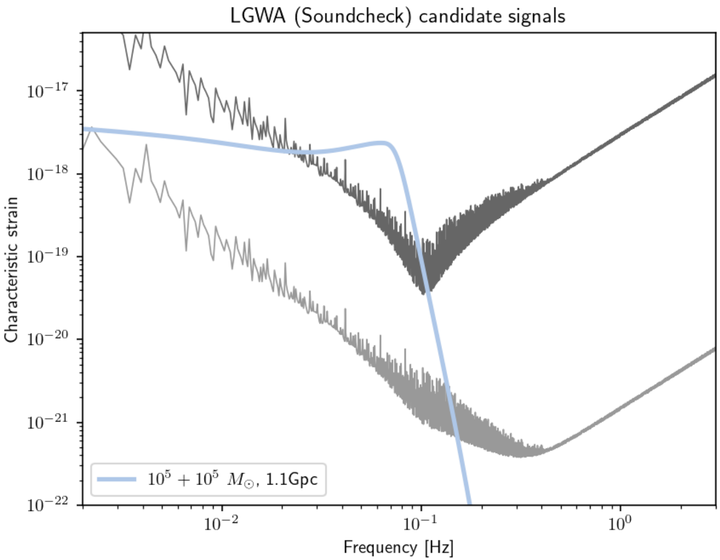
$$h_c(f) = 2f\, h(f)$$

$$h_n(f) = \sqrt{f S_h(f)}$$

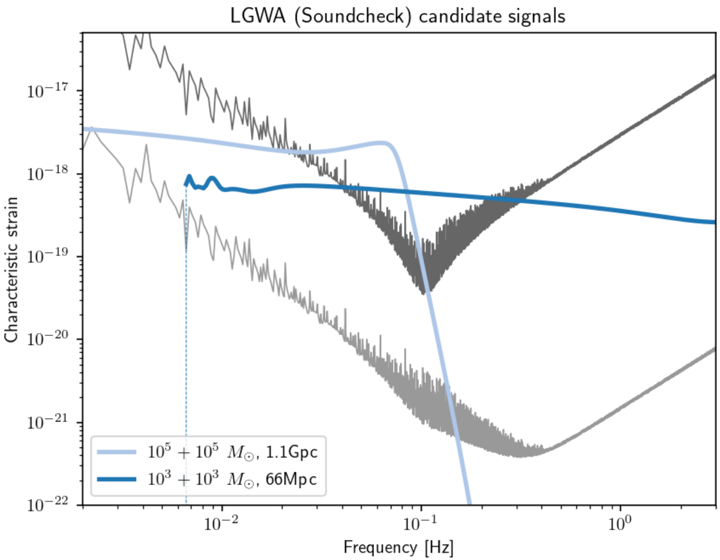
Computing the horizon distance - 1

- Fix all the parameters θ of a signal (sky position, polarization, arrival time, object masses...) except for distance;
- change the distance until the SNR is computed to be equal to some threshold value (here, 9).

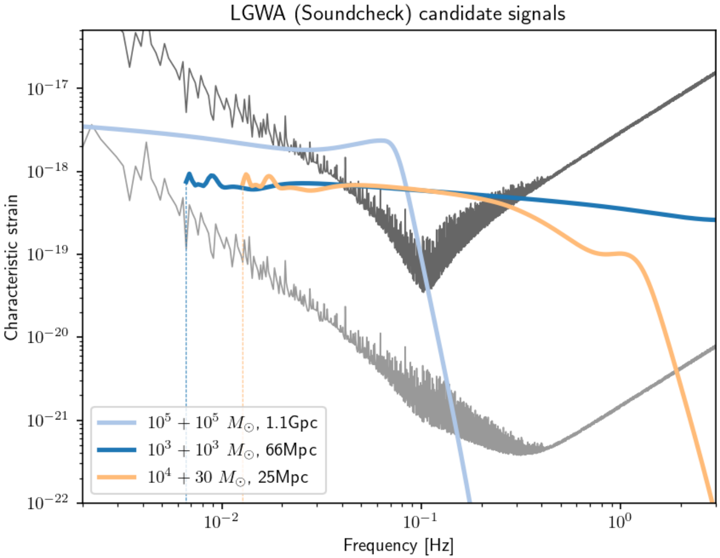
Computing the horizon distance - 2



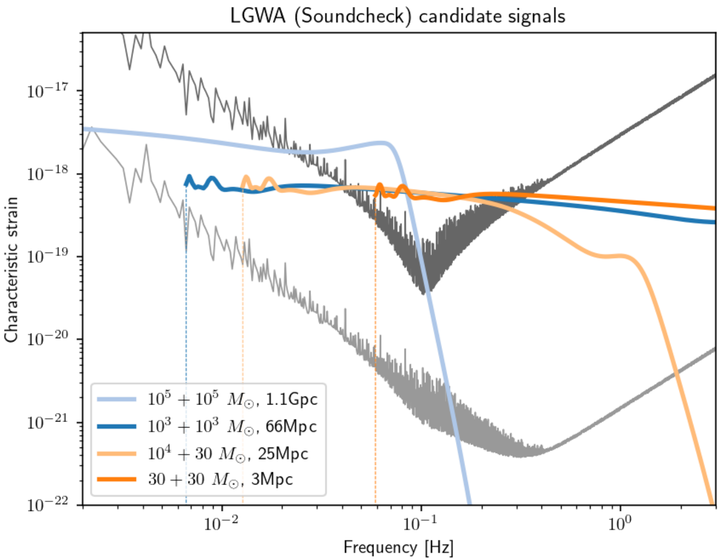
Effect of the Moon's rotation



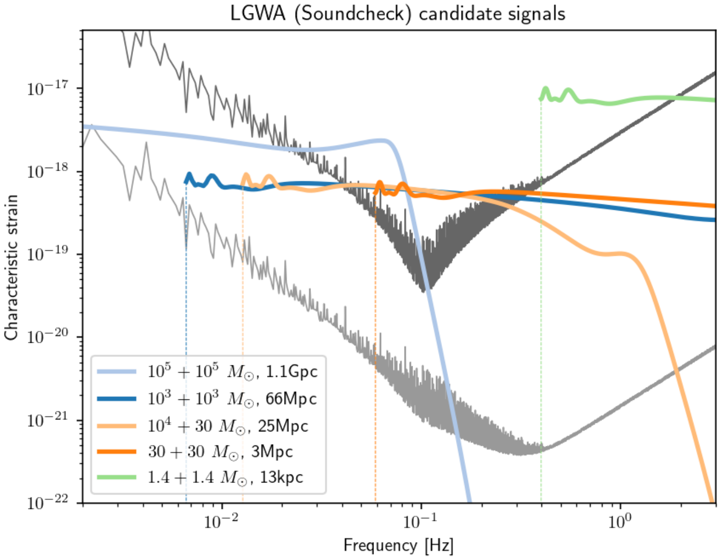
Effect of higher order modes



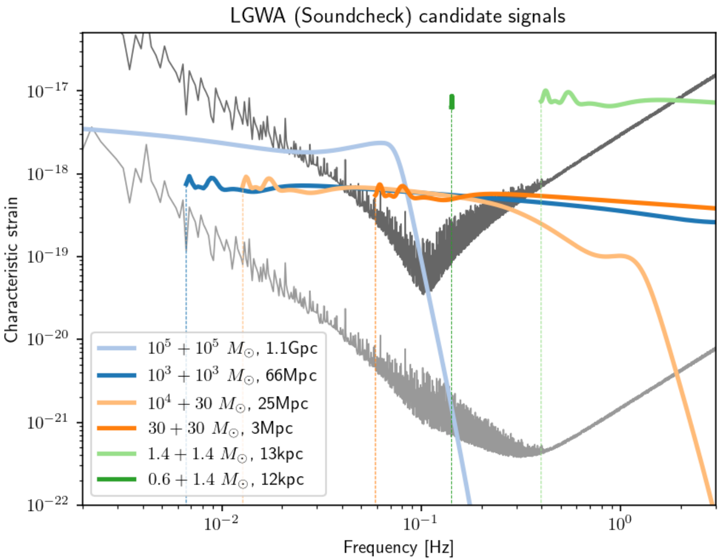
Effect of changing the mass



Effect of truncating the detector lifetime



Quasi-monochromatic sources



GWFish

Publically available at
`https://github.com/janosch314/GWFish`.
Documentation and tutorials at
`https://gwfish.readthedocs.io`.
It is being developed here at GSSI (Ulyana Dupletsa, Jan Harms, Biswajit Banerjee,Boris Goncharov, me).

Error estimation

In terms of the signal parameters θ, approximate the likelihood as a Gaussian

$$\mathcal{L} \approx \mathcal{N} \exp\left(-\frac{1}{2}\mathcal{F}_{ij}\Delta\theta^i\Delta\theta^j\right) \quad (1)$$

$$\Delta\theta^i = \theta^i - \theta^i_0 \quad (2)$$

$$\mathcal{F}_{ij} = 4\int_0^\infty \frac{1}{S_h(f)}\frac{\partial h^*(f)}{\partial\theta^i}\frac{\partial h(f)}{\partial\theta^j}\mathrm{d}f\bigg|_{\theta_0} \quad (3)$$

This converges to the true likelihood in the high-SNR limit, and we can compute $\operatorname{cov}\theta_i = (\mathcal{F})^{-1}_{ii}$.