

ON COSMIC RAY TRANSPORT IN THE MAGNETIZED INTERSTELLAR MEDIUM

A BIAS TOWARDS MICROPHYSICS

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OUTLINES

- A basic question : Why doing microphysics ?
- **Turbulence > Cosmic Rays** : Effect of turbulence on Cosmic Rays : transport coefficients.
- **Cosmic rays > Turbulence** : Effect of Cosmic Rays on turbulence : CR driven instabilities and their fate.
- **Turbulence < > Cosmic rays** : Numerical studies.
- Impact over anisotropy studies.
- Perspectives.

Nota bene

CR = Cosmic Rays

MF = Magnetic field

MS = Magnetosonic, FMS = fast MS

MFLW = Magnetic field line wandering

b = Magnetic turbulence level = $\delta B/B$

L = Turbulence injection scale

WHY DOING MICROPHYSICS ?

Well ... it is more satisfactory to have a full understanding of the CR transport (Zweibel 2013)

- Turbulence ISM studies:
 - At CR sources we can have $\delta B > B \Rightarrow$ highly non-linear transport.
 - Around sources, we may have some transition towards $\delta B < B$, to which extends \Rightarrow need for microphysics.
 - In the ISM transport by magnetic chaos is important and intrinsically non-linear.
- Space plasma studies: determination of diffusion tensor is of prime importance to understand (Bieber 2003)
 - The anomalous CR component
 - Solar modulation effects
 - Solar energetic particle transport
- this requires a fine understanding of processes at play in particle-turbulence interaction.
- Advantage of space plasmas: observational constraints on the EP mean free path !
- But the description of the microphysics of the transport is a quite complex task...involved in analytical and numerical approaches.

DEVELOPMENTS IN NUMERICAL METHODS

- Two basic approaches:
 - 1) Particle transport in synthetic turbulence: a model of turbulence is implemented in the simulation cube.
 - a) Turbulence modelled with plane wave developments (Giacalone & Jokipii 1999, Candia & Roulet 2004, Qin & Shalchi 2008, Ivascenko et al 2016 ...).
 - b) Turbulence modelled with Fourier transform (FT) (Casse et al 2002).
 - 2) Particle transport in code-generated turbulence (usually in the magnetohydrodynamic approximation).
 - a) Test-wave approximation (Xu & Yan 2013, Cohet & A.M. 2016): No particle backreaction over turbulence, usually use snapshots of magnetic realizations.
 - b) Backreaction included (Bai et al 2015, van Marle, Casse, A.M. 2018, Lebiga et al 2018, Bai et al 2019) : Particle-in-cell-MHD simulations.
- => Then need analytical calculations (eg Shalchi'09) for a good interpretation.

TURBULENCE > COSMIC RAYS

- Derivation of transport coefficients, e.g. diffusion coefficients.
- See talks by [W. Matthaeus](#), [G. Giacinti](#).

QUASI-LINEAR THEORY: MAIN ASSUMPTIONS

- Pioneering works: Jokipii 1966, Hall & Sturrock 1968, Goldstein 1976, Urch 1977, see Schlickeiser 2002. I
- In field perturbation theory, Quasi-Linear theory (QLT) corresponds to the first perturbation level (Born approximation).
- In practice: particle motion is approximated to be the (unperturbed) Larmor gyromotion in the calculation of FT of turbulent fields.

Two basic assumptions are mandatory for the QLT to apply

1. The level of perturbations need to be small : eg $b^2 = \delta B^2 / B_0^2$ is a small parameter.
 2. The theory is applicable at times $t_c < t < t_s$
 - t_c = correlation time of stochastic forces,
 - t_s = scattering time, timescale of the evolution of the mean distribution function.
- This requires to have fully developed turbulent spectrum $k W(k) \sim \delta B^2(k)$, if particle undergo resonant interaction with modes k then $t_c \sim \Omega^{-1} k / \Delta k$, Δk is the spectrum width in the parallel direction n (see §A in Casse et al 2002).
- Hence $t_c \ll t_s$ means $b^2 \ll \Delta k / k$.

QLT DIFFUSION COEFFICIENTS MAIN RESULTS

- Derived in several textbooks (eg Schlickeiser 02, Shalchi 09). An important quantity: the angular scattering frequency $\nu \leftrightarrow$ the rate of change of the particle pitch-angle cosine μ given by $D_{\mu\mu}$; $\nu = 2 \frac{D_{\mu\mu}}{(1-\mu^2)}$
 - Controls the propagation along the magnetic field, and the // diffusion coefficient, so CRA dipole if the CR gradient and the local MF are aligned.
- E.G.: In case of Alfvén waves: $\omega_j = jk_{\parallel}V_a$, $j=+1$ forward waves, $j=-1$ backward waves.

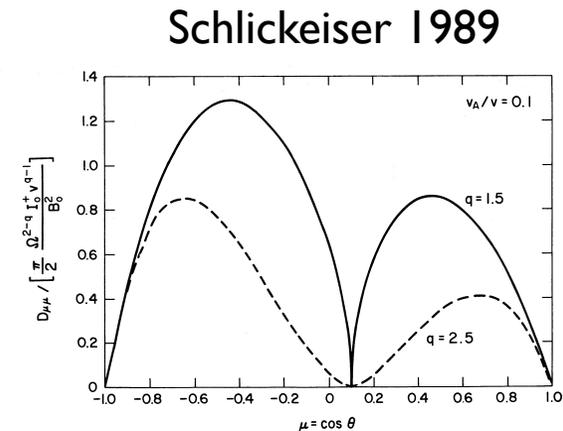
$$D_{\mu\mu} = \frac{2\Omega^2}{B_m^2} (1 - \mu^2) \sum_{j=\pm 1} \sum_n \int d\vec{k} \left(1 - \frac{\mu\omega_j}{k_{\parallel}v}\right)^2 \left(\frac{nJ_n(z)}{z}\right)^2 \left\langle \vec{B}_R^j(k) \vec{B}_R^j(k) \right\rangle \mathfrak{R}_j(\vec{k}, \omega_j)$$

two main unknowns



Magnetic
turbulent
tensor: M^B

Resonance
function



In the QLT, magnetostatic turbulence $\mathfrak{R}_j(\vec{k}, \omega_j) \sim \pi(vk_{\parallel}\mu + n\Omega) \Rightarrow D_{\mu\mu} \rightarrow 0$ around $90^\circ \Rightarrow$ infinite parallel mean free path

CHOICE FOR M^B AND R, NON-LINEAR EXTENSIONS

- Choice for the resonance function R and M^B (Giacinti & Kirk'17, GK17) to enlarge the resonance condition
- 1. Dynamical turbulence model: $M^B(\mathbf{k},t) = M^B(\mathbf{k}) \Gamma(\mathbf{k},t)$, Γ is the correlation function, eg to account for a finite correlation time $\Gamma(\vec{k},t)=\exp(-t/t_c)$ then $\mathfrak{R} = \frac{t_c^{-1}}{t_c^{-2} + (v\mu k_{\parallel} + n\Omega)^2}$, model R₁ in GK17

- 2. Resonance broadening due to a non-linear particle trapping effect (Völk'75) $\mathfrak{R} = \frac{\sqrt{\pi}}{vb k_{\parallel}} e^{\left[-\left(\frac{k_{\parallel}v\mu + n\Omega}{vb k_{\parallel}}\right)^2\right]}$, model R₂ in GK17

- Model for M^B based on Yan & Lazarian'04, Cho+02, Chandran'00

➤ Alfvénic turbulence : Goldreich-Sridhar model

Anisotropic MHD turbulence

➤ Fast magnetosonic turbulence : Kraichnan model

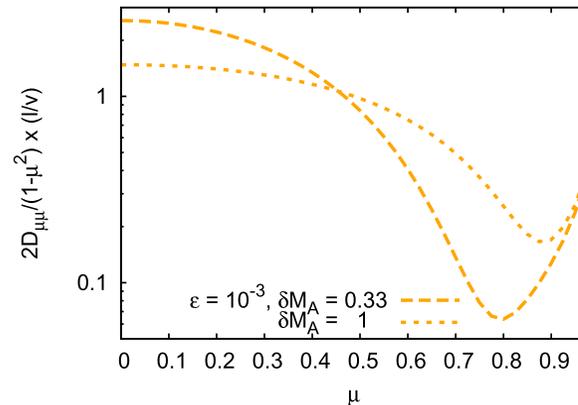
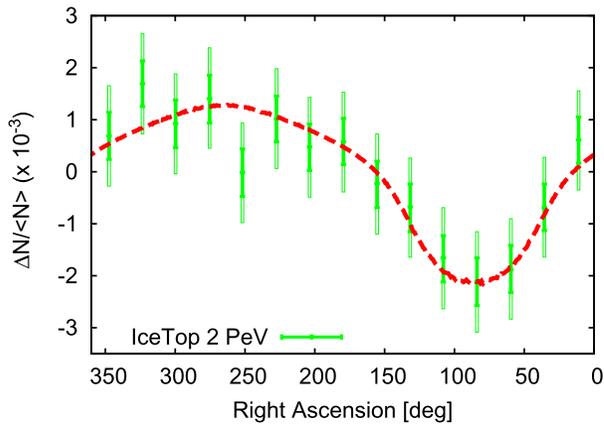
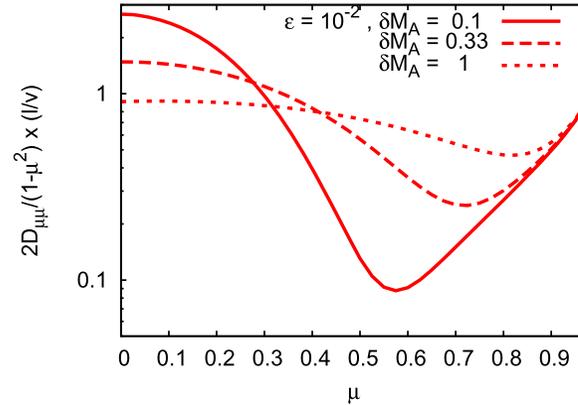
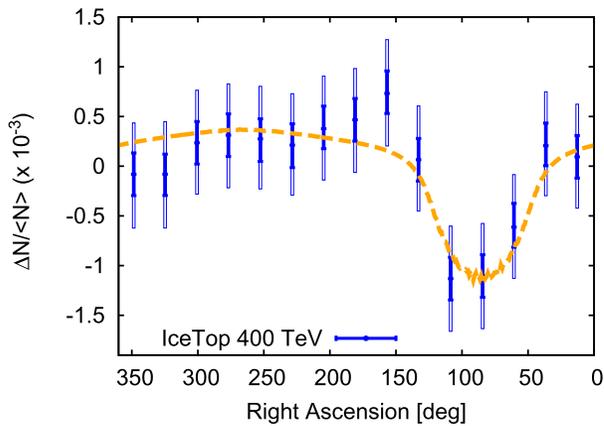
Isotropic MHD turbulence

Properties of the Turbulence Models

Model	Type		Spectrum	Resonance Function
A	GS (incompressible)	Anisotropic	Heaviside ($\mathcal{I}_{A,S,1}$)	Narrow – $R_{n,1}(\delta = v_A/v)$
B	GS (incompressible)	Anisotropic	Exponential ($\mathcal{I}_{A,S,2}$)	Narrow – $R_{n,1}(\delta = v_A/v)$
C	GS (incompressible)	Anisotropic	Heaviside ($\mathcal{I}_{A,S,1}$)	Broad – $R_{n,2}(\delta\mathcal{M}_A)$
D	GS (incompressible)	Anisotropic	Exponential ($\mathcal{I}_{A,S,2}$)	Broad – $R_{n,2}(\delta\mathcal{M}_A)$
E	Fast modes (compressible)	Isotropic	$\mathcal{I}_F \propto k^{-3/2}$	Narrow – $R_{n,1}(\delta = v_A/v)$
F	Fast modes (compressible)	Isotropic	$\mathcal{I}_F \propto k^{-3/2}$	Broad – $R_{n,2}(\delta\mathcal{M}_A)$

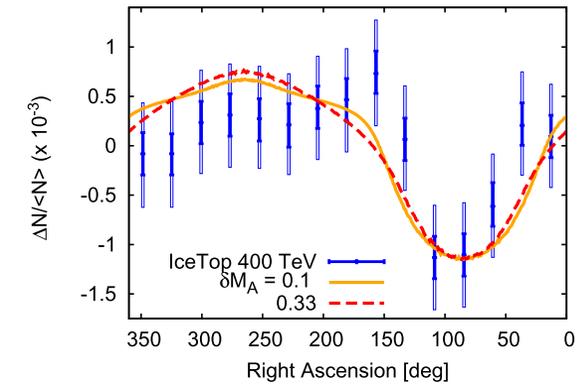
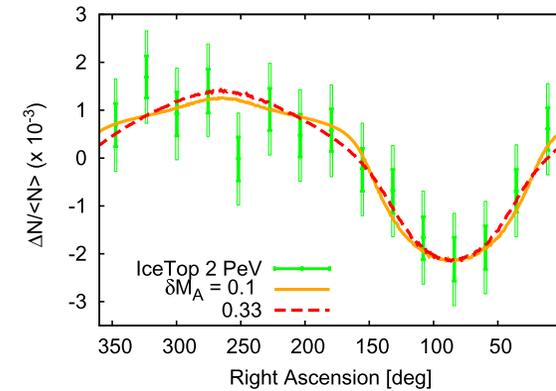
CR ANISOTROPY DIPOLE CALCULATIONS

Alfvén turbulence (R_2)



$b \sim \delta M_A / 2 = 0.16$

FastMS turbulence (R_2)



Giacinti & Kirk'17 main assumptions: 1) CR mfp $< L$ 2) Anisotropy // regular local MF 3) Homogeneous turbulence (see G.Giacinti talk).
CRA starts to put constraints on M^B and R

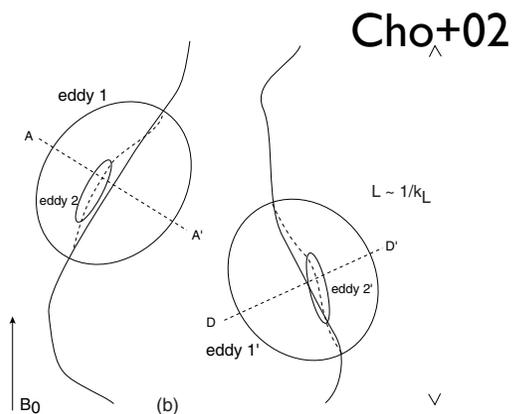
- Alfvénic/FMS turbulence with resonance broadening (R_2) leads to a good fit of the dipole amplitude.
- Models with dynamical turbulence (R_1) have difficulties to fit the data.

TESTING TRANSPORT THEORIES WITH NUMERICS

- Are these estimations for $D_{\mu\mu}$ or $\kappa_{//}$ recovered by simulations ? CR propagation in synthetic or MHD-code-generated turbulent snapshots.

SYNTHETIC TURBULENCE SIMULATIONS

- Scattering studies in Goldreich-Sridhar turbulence is very challenging as the mean MF direction changes with the scale.

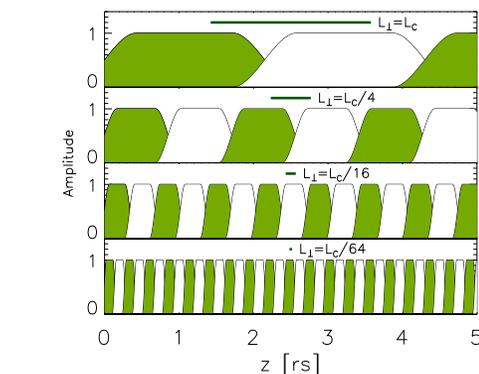


If $k_{\parallel} r_L = 1$
resonant
condition

$$k_{\parallel} L = (k_{\perp} L)^{2/3}$$

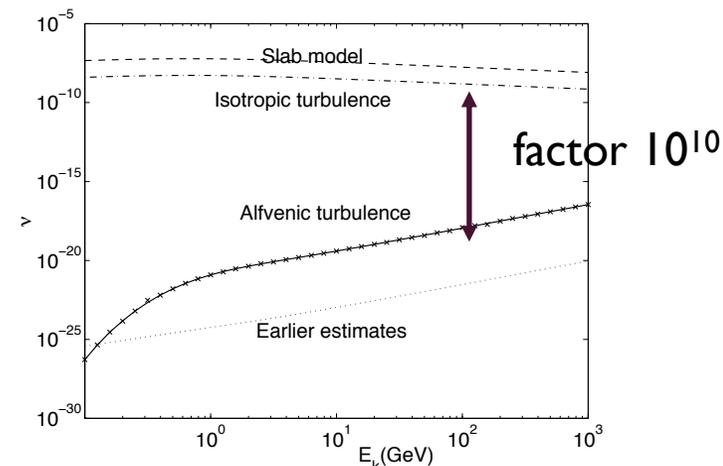
$$k_{\perp} r_L = \left(\frac{L}{r_L}\right)^{1/2} > 1$$

A CR with r_L encompasses several Eddies if these are uncorrelated



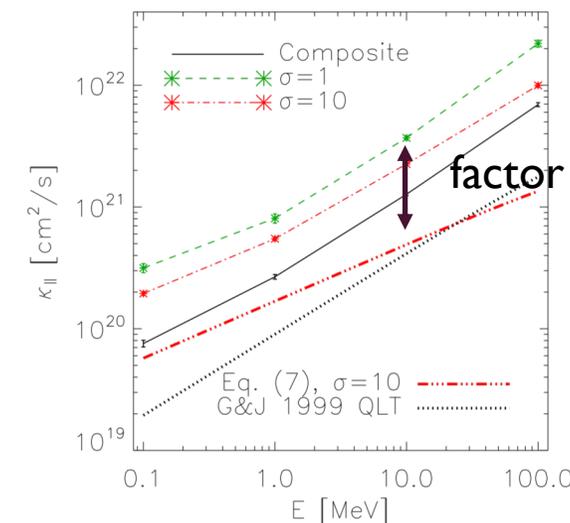
One way is to use envelopes of the plane waves following Goldreich-Sridhar scaling (Laitinen+13). (here 4 envelopes in the perpendicular direction are shown).

Yan & Lazarian '02



rigidity $\rho = 2\pi r_L / L \sim 2 \cdot 10^{-4} E_{100\text{GeV}} B_{3.5\mu\text{G}}$

AM+06: in GS turbulence resonance
 $k_{\perp} r_L = 1$ may control the // transport



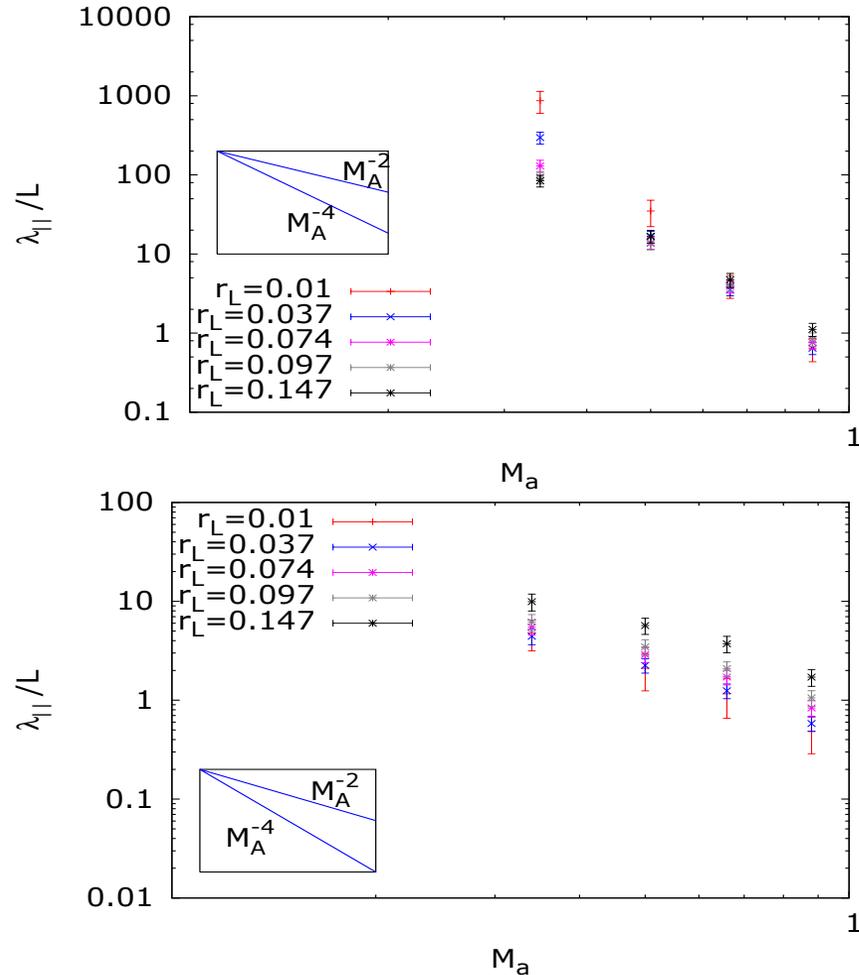
rigidity $\rho = 2\pi r_L / L \sim 2 \cdot 10^{-4} E_{10\text{MeV}} B_G$

$b=1$

factor 10-30

factor 10¹⁰

TURBULENCE GENERATED BY MHD CODES



Cohet & AM'16 : 512^3 resolution simulations. Turbulence is forced using a function \mathbf{F} with different geometries.

(up) solenoidal forcing (div $\mathbf{F}=0$, eg shearing flows): λ scales as $\sim M_a^{-4.91 \pm 0.35}$

➤ Weak sub-Alfvénic turbulence produces cascades almost in the perpendicular direction so scattering is expected to be weak.

(down) compressible forcing (rot $\mathbf{F}=0$, eg supernova): λ scales as $\sim M_a^{-1.82 \pm 0.35}$

➤ One can expect to have more FMS modes close to the QLT expectation (Jokipii'66)

! all MHD modes contribute, so one has to filter out each of them to isolate their effect (Kowal & Lazarian'10)

NON-DIFFUSIVE TRANSPORT: CONDITIONS

- Usually diffusive transport is assumed in anisotropic cosmic rays studies. But sub- or super-diffusion regimes may occur depending on the properties of the magnetic turbulence.

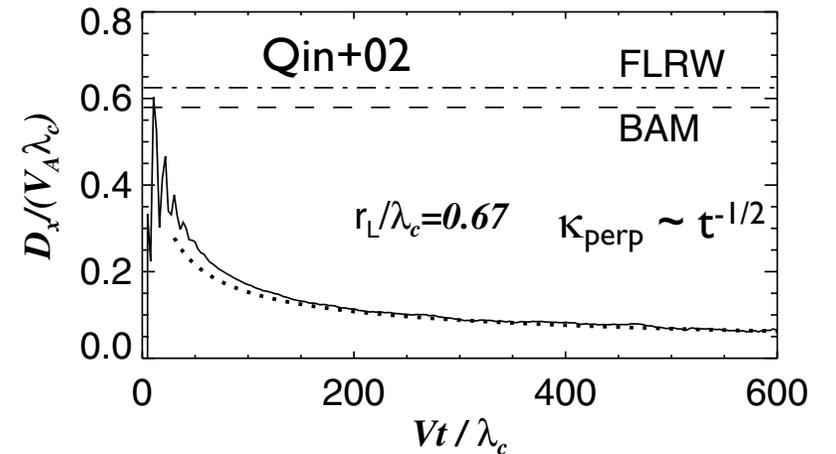
Subdiffusion : specific conditions CR need to stay on a MF patch for a time where it starts a parallel diffusion but before escaping the patch. The regime is found in simulations with slab-type turbulence (Qin+02)

- reinforces the role of κ_{\parallel} in CR anisotropy.

Superdiffusion : MF line wandering can lead to superdiffusion. If CR scattering is inefficient MFLW controls the perpendicular transport (Lazarian & Yan'16).

Superdiffusion may occur in the Goldreich-Sridhar regime at scales $<$ injection scale L

- CR anisotropy underestimates the effect of perpendicular diffusion.



Type of MHD turbulence	Injection velocity	Range of scales	Magnetic diffusion
Weak	$V_L < V_A$	$[l_{trans}, L]$	diffusion
Strong subAlfvénic	$V_L < V_A$	$[l_{min}, l_{trans}]$	Richardson
Strong superAlfvénic	$V_L > V_A$	$[l_A, L]$	diffusion
Strong superAlfvénic	$V_L > V_A$	$[l_{min}, l_A]$	Richardson

l_{min} = damping scale, $l_{trans} = LM_a^2$, $l_A = L/M_a^3$

COSMIC RAYS > TURBULENCE

- Cosmic Rays carry a lot of power.
 - In sources up to 10-30% of shock ram pressure, almost carried by TeV (and beyond) CRs.
 - In the ISM the mean CR component is in pressure equipartition with the gas, the magnetic field, carried by GeV CRs
 - In between (sources and ISM) CR pressure can dominate gas and magnetic pressures ! \Leftrightarrow source vicinity effects
- Cosmic Rays can trigger their own turbulence via different instabilities. [See talk by G. Morlino.](#)
 - Kinetic : Resonant streaming instability (Skilling'75, Nava+16+19, Brahim+19); Gyroresonant ($p_{\text{perp}} > p_{\parallel}$) instability (Lazarian & Berezhnyak'06 , Lebiga+18)
 - Hydrodynamic: Firehose/mirror type instability (Achterberg'13, Bykov'13), Non-resonant streaming instability (Bell'04, Blasi & Amato'19, Inoue 19).

SELF-GENERATED TURBULENCE AND COSMIC RAY TRANSPORT

- CR streaming induces the growth of 3 modes (2 resonant, 1 non-resonant, Gary'90, Bell & Lucek'01, Bell'04).

Resonant modes: 2 polarizations (left/right-handed)

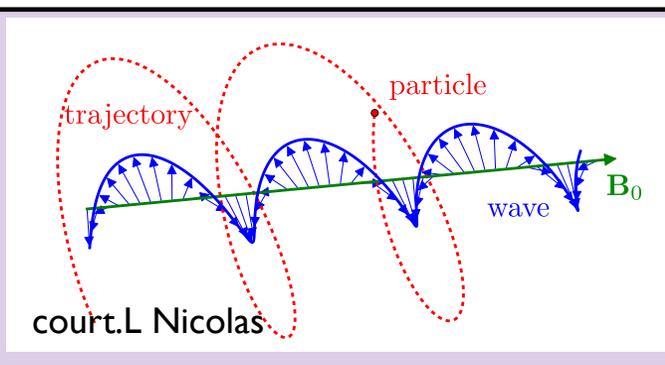
$$\omega - k_{\parallel} v_{\parallel} + n\Omega = 0$$

$n=-1$ Left H mode, $n=1$ Right H mode

Linear growth rate

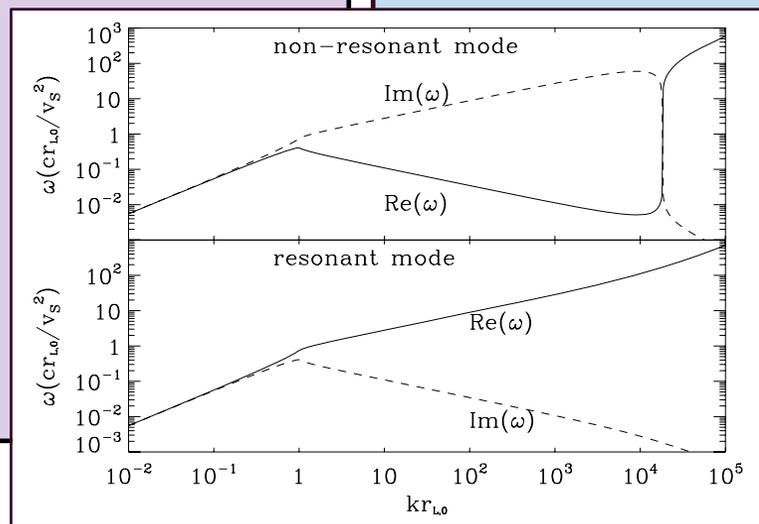
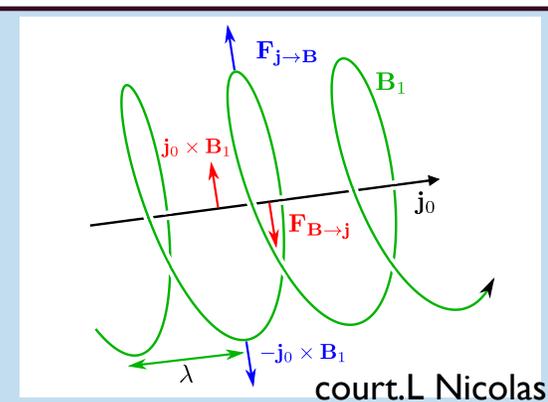
$$\Gamma_{growth} = -\frac{V_a}{I(k)} \nabla_{\parallel} P_{CR}$$

P_{CR} , $I(k)$ CR and wave pressures



Non-resonant mode (right-handed)

Lorentz force of the particle current over perturbed background MF



Develop at scales $kr_L > 1$ with a linear growth rate $\Gamma_{growth} = \frac{B_0 J_0}{2\rho V_a}$

if $v_d/c > U_B/U_{CR}$

U_B U_{CR} MF and CR energy densities

Amato & Blasi'09: linear growth rates

SELF-GENERATED TURBULENCE AND CR MEAN FREE PATH

- The problem is non-linear but based on a linear theory: the diffusion coefficients are derived in the QLT framework.
- Need to solve a couple system of Eqs on CR pressure $P_{\text{CR}}(E)$, and wave pressure $I(k)$ (Ptuskin+08, Malkov+13, d'Angelo+16+18, Nava+16,'19, Evoli+18, Brahim+19, Blasi'19)

$$\frac{\partial P_{\text{CR}}}{\partial t} + V_A \frac{\partial P_{\text{CR}}}{\partial z} = \frac{\partial}{\partial z} \left(D \frac{\partial P_{\text{CR}}}{\partial z} \right)$$

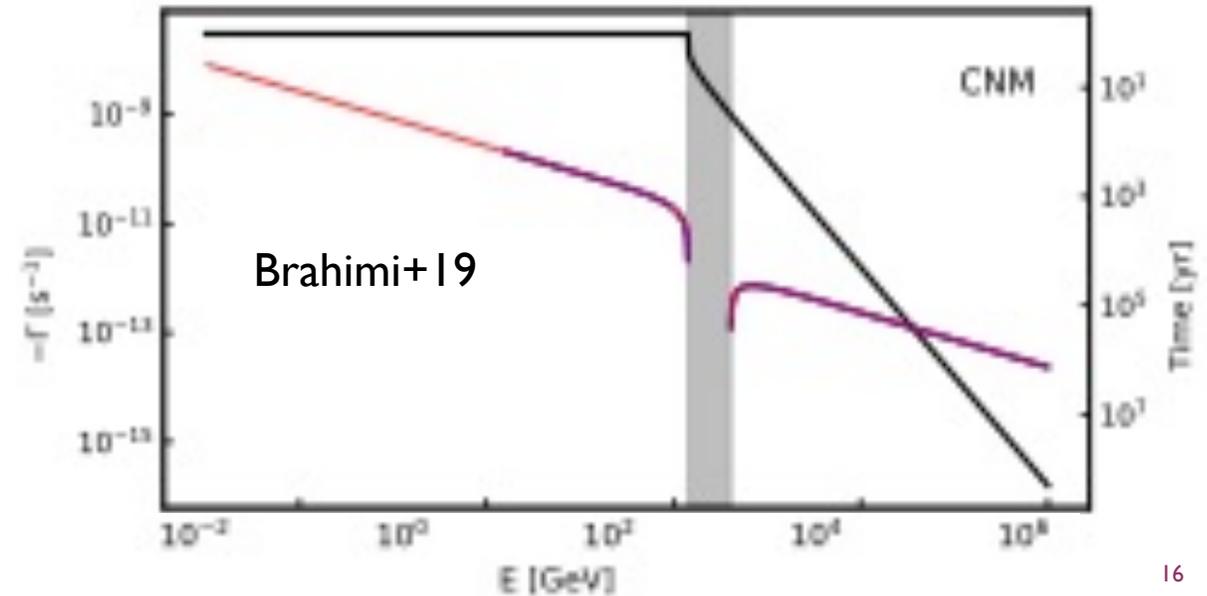
$$\frac{\partial I}{\partial t} + V_A \frac{\partial I}{\partial z} = 2(\Gamma_{\text{growth}} - \Gamma_{\text{d}})I + Q$$

$D = D_B / I$, $D_B =$ Bohm diffusion coefficient $r_L v / 3$

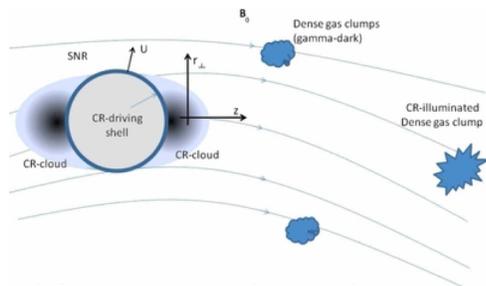
Q : turbulence injection term

V_a drift speed of the scattering centres and CRs

The damping process Γ_{d} dependent on the interstellar medium properties:
(eg Cold neutral medium)

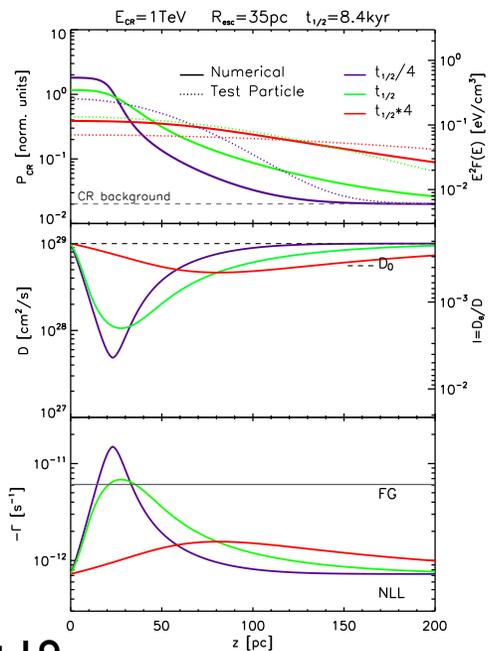


PROPAGATION CLOSE TO SOURCES @ 1 TEV



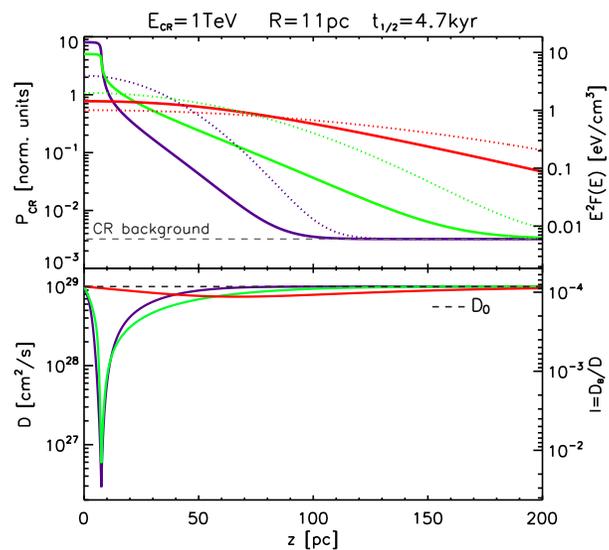
Flux tube approximation = 1D propagation along background MF up to a distance $z=L$, at one coherence length (Malkov+13)
 ➤ effects over less than a few times 10^4 years

Hot ionised medium



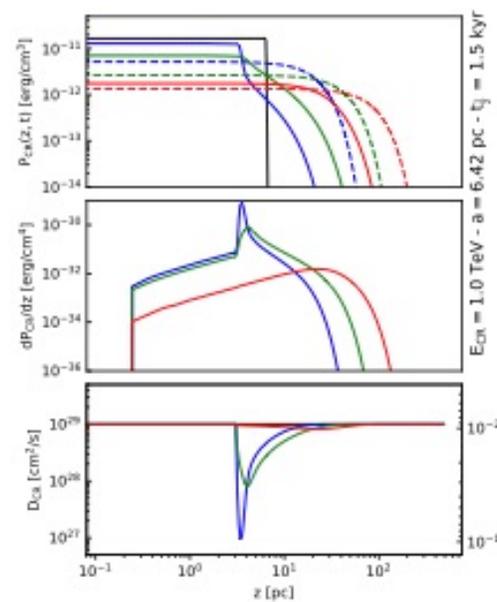
Nava+19

Warm ionised medium



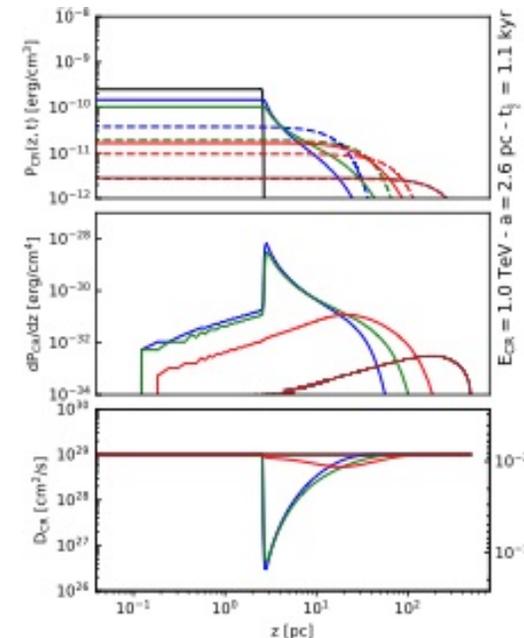
Nava+16

Warm neutral medium



Brahimi+19

Cold neutral medium



PROPAGATION ABOVE 10 TEV

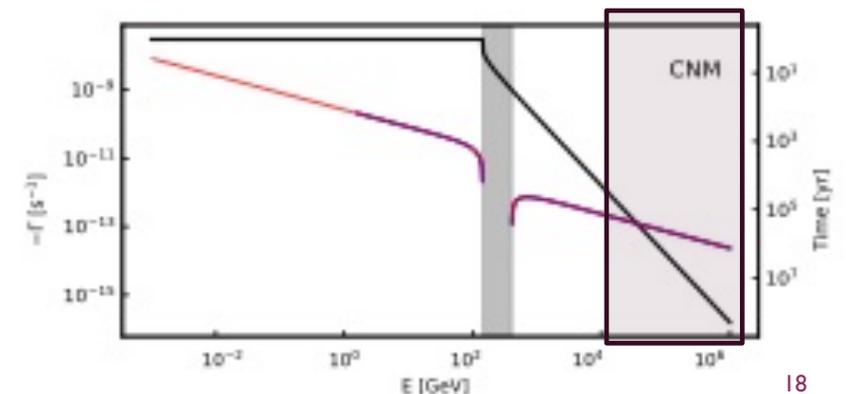
Damping

1. Below ion-neutral collision frequency partially ionized phases ion-neutral damping rate drops as k^2 (Mc Ivor'77), so drops as E^{-2} if we deal with resonant wave-particle interaction.
2. Interaction rate of self-generated waves with background turbulence (Lazarian or Farmer-Goldreich damping) also drops as $E^{-1/2}$.
3. Non-linear Landau damping varies as E^{a-1} where the self generated turbulence spectrum scales as k^{-a}

Growth rate

1. Resonant instability growth rate scales as E^{1-b} , where the energy CR spectrum scales as E^{-b}
2. The CR can trigger many other instabilities, especially the non-resonant streaming instability, efficient in producing MF (Inoue'19).

- Not so easy to anticipate over the final result...
- Importance of local source effects for CRA (Ahlers'16).



TURBULENCE < > COSMIC RAYS

- CR-Turbulence constitutes a complex system. Its dynamics is difficult to handle unless using numerics.

PARTICLE-IN-CELL-MAGNETOHYDRODYNAMICS

- Combine a MHD solver with a PIC module to handle CR dynamics. (alternative di-hybrid method; Gargaté+07, Caprioli & Spitkovsky'14).
- Methodology 1) Solve MHD Eqs [Bai+15 ATHENA, van Marle+18 MPI-AMR-VAC, Mignone+18 PLUTO]

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot \left(\rho \mathbf{v} \otimes \mathbf{v} - \frac{\mathbf{B} \otimes \mathbf{B}}{4\pi} + P_{tot} \mathbb{1} \right) = - \mathbf{F}_{part}$$

$$\frac{\partial e}{\partial t} + \nabla \cdot \left((e + P_{tot}) \mathbf{v} + (\mathbf{E} - \mathbf{E}_0) \times \frac{\mathbf{B}}{4\pi} \right) = - \mathbf{u}_{part} \cdot \mathbf{F}_{part}$$

MHD Eqs + source terms due to the non-thermal Lorentz force and its work over the magnetized thermal fluid, $\mathbf{E}_0 = -\mathbf{v} \times \mathbf{B}$

2) Modified Ohm's law by the NT component = $R = \rho_{NT}/\rho_c$

$$c \mathbf{E} = -((1 - R) \mathbf{v} + R \mathbf{u}_{part}) \times \mathbf{B}$$

$$\mathbf{u}_{part} = \frac{\sum_{\alpha} n_{\alpha} q_{\alpha} \mathbf{u}_{\alpha}}{\sum_{\alpha} n_{\alpha} q_{\alpha}} = \frac{\mathbf{J}_{part}}{n_{part} e} \quad \text{averaged NT velocity}$$

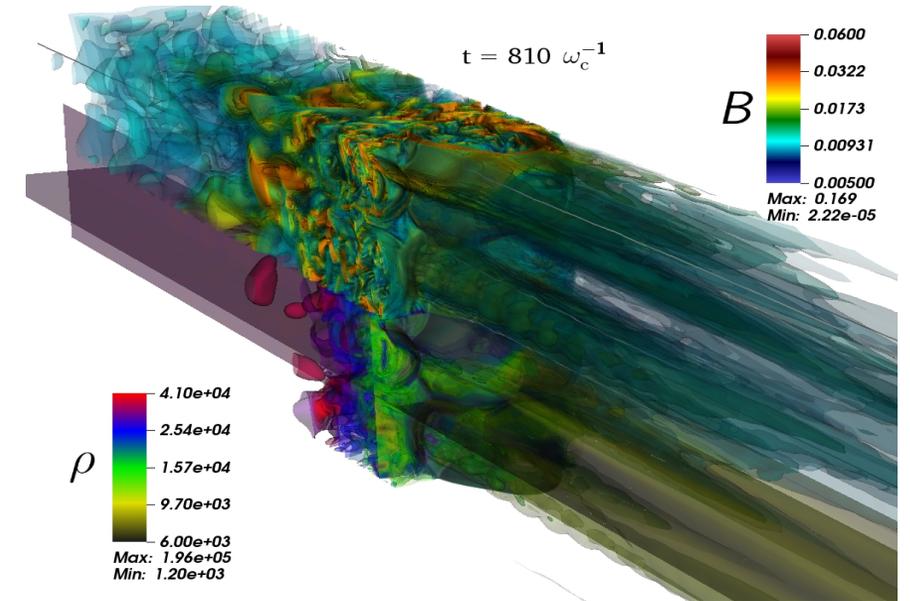
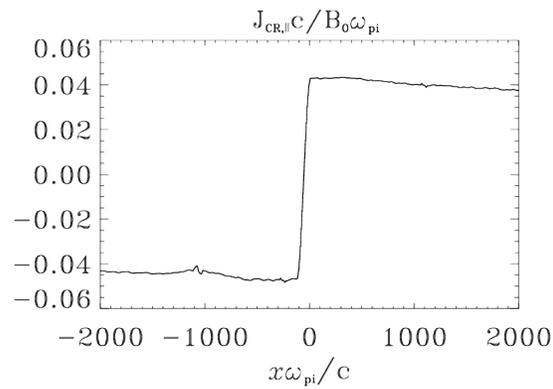
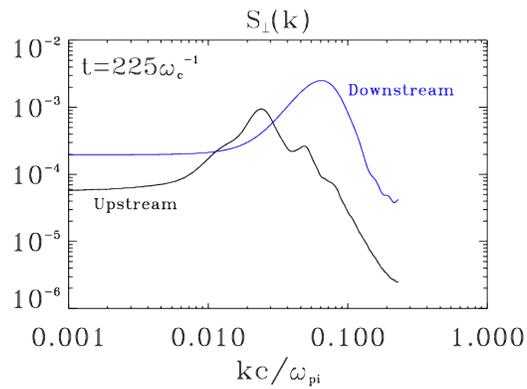
3) Lorentz Eq for NT particles

$$\frac{\partial \mathbf{p}_{\alpha,j}}{\partial t} = \frac{q_{\alpha}}{c} (\mathbf{u}_{\alpha,j} - (1 - R) \mathbf{v} - R \mathbf{u}_{part}) \times \mathbf{B}$$

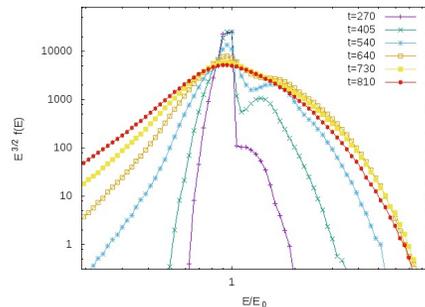
NON-RESONANT STREAMING INSTABILITY

- In the context of shock acceleration physics (van Marle+18, 19)

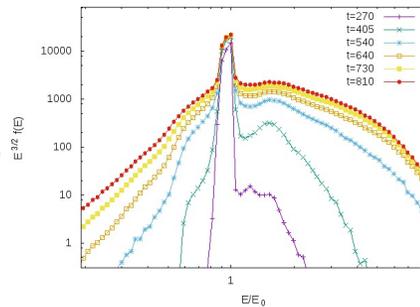
MF Fourier spectra = init of the NR streaming instability



Particle SED in 3D



Particle SED in 2D



gas density and MF in 3D simulations of non-relativistic shock

particle acceleration efficiency a bit reduced in 3D due to the strong shock front corrugation (HE particle feedback over injection)

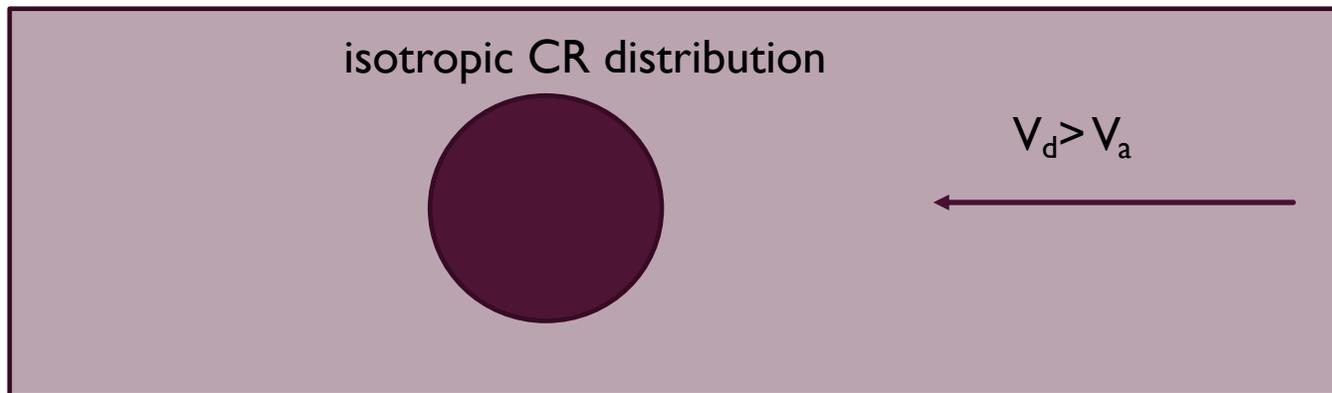
RESONANT STREAMING INSTABILITY

- Anisotropic CR-driven turbulence (Bai+19:): Here two major difficulties 1) the ratio $n_{\text{CR}}/n_{\text{ISM}} \ll 1$, 2) among these n_{CR} CRs very few are in resonance with a wave with a wave number k .
- Bai+19 adopted a δF method where each particle represents a Lagrangian marker of the difference between the full solution F and the equilibrium solution $F_0 \Rightarrow$ reduced noise level.

$$n_{\text{CR}}(t, \mathbf{x}) = n_{\text{CR},0} + \int \delta f(t, \mathbf{x}, \mathbf{p}) d^3 \mathbf{p}$$

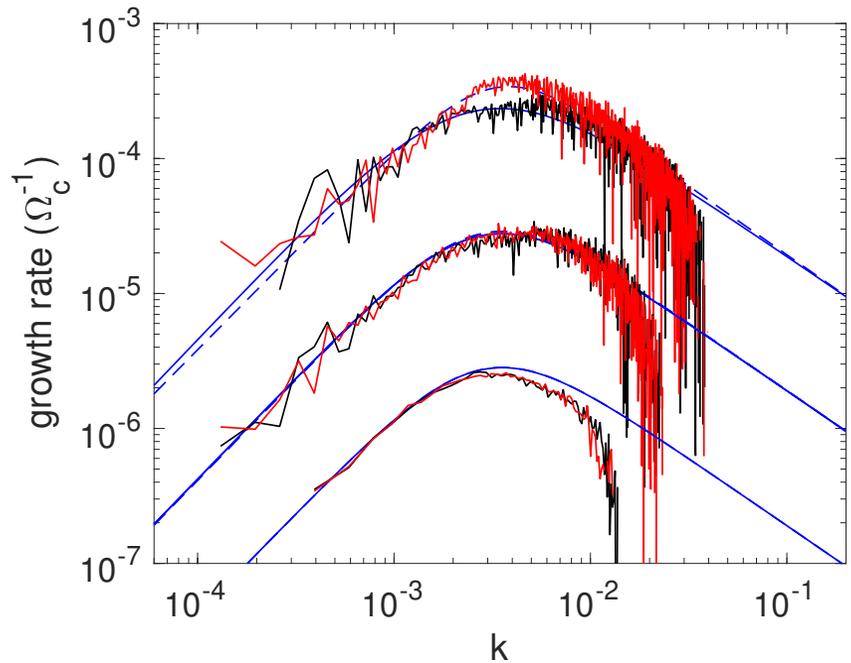
$$\simeq n_{\text{CR},0} + \sum_{j=1}^{N_p} w_j S(\mathbf{x} - \mathbf{x}_j)$$

CR density expressed in terms of the equilibrium solution.

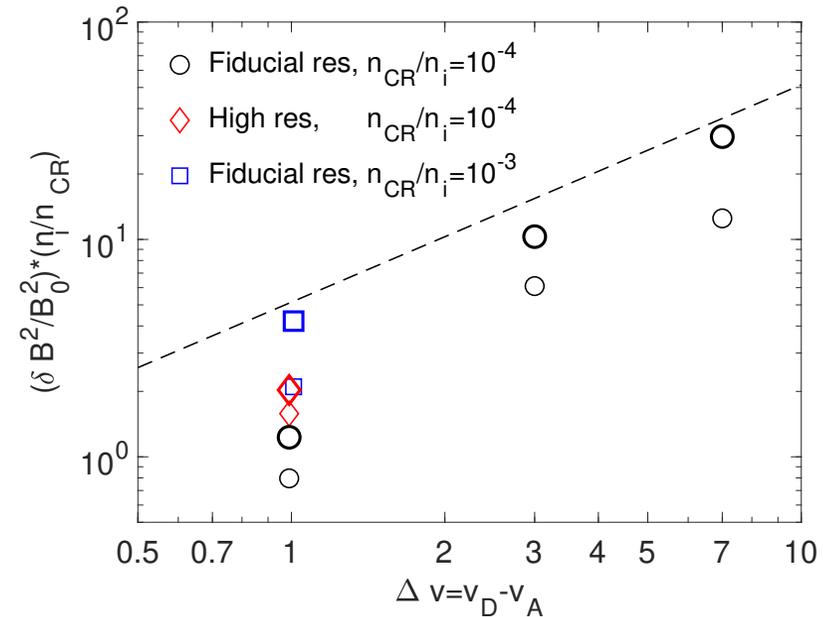


set-up in the frame where CR distribution is isotropic – the background plasma drifts with a speed V_d

GROWTH RATE AND SATURATION



Linear growth rate for left (black) and right (red) handed polarized modes. Blue analytical calculation for three different levels of n_{CR}/n_{ISM}



Saturation magnetic energy density for three different runs varying the drift speed. Dashed line: analytical saturation level

$$\frac{\delta B^2}{B_0^2} \approx \frac{4}{3} \frac{\langle p \rangle}{mc} \frac{n_{CR}}{n_i} \frac{\Delta v}{v_A}$$

PERSPECTIVES

- CR propagation in background turbulence (high-energy CRs, $E > \text{TeV}$ in the ISM,)
 - Need to a better exploration of CR transport properties in different type of MHD turbulence : influence of the forcing, wave-particle interaction especially in Goldreich-Sridhar turbulence.
- CR-driven turbulence
 - Explore the model of CR escape from, sources more deeply.
 - Explore more set-ups with PIC-MHD or PIC techniques to study the non-resonant streaming instability (galactic escape Blasi & Amato'19, SNR/molecular cloud interaction Inoue'19, ...) => to study impact of sources over CR anisotropy.
 - Explore the physics of different resonant instabilities, retrace particle trajectories to derive the transport properties (eg Lebiga + 18).

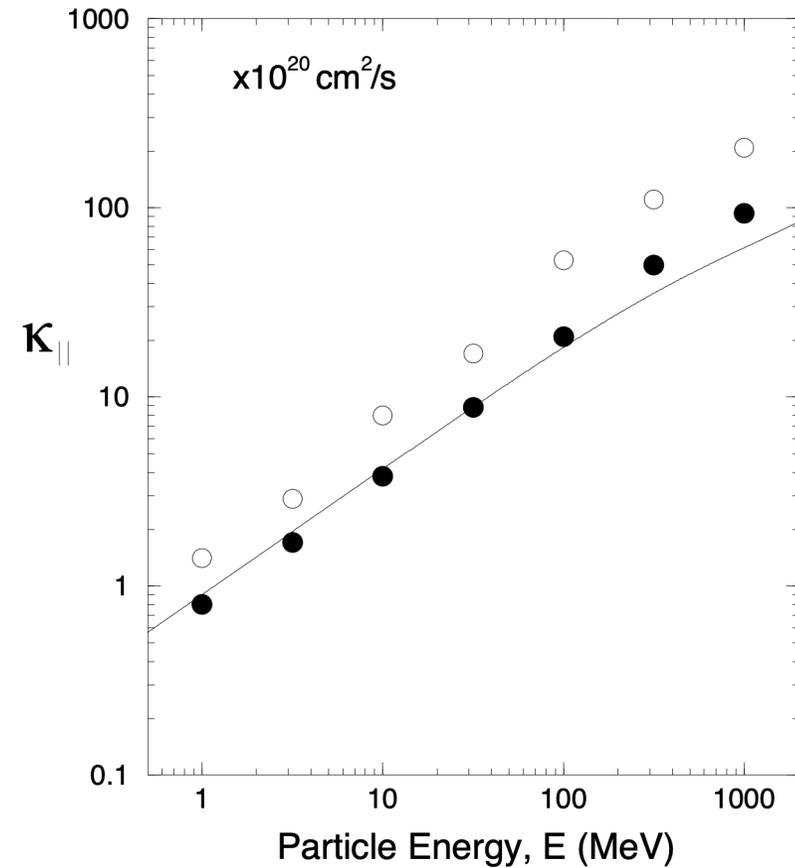


Thank you for your attention

BACK-UP

GIACALONE & JOKIPII 1999

- Results on parallel mfp for composite (slab+2D) [open circles] and isotropic turbulence [filled circles] with a Kolmogorov spectrum.



LAITINEN ET AL 2013

Methodology:

Plane wave development modulated by an envelope (a kind of wavelet)

$$\delta \mathbf{B}(x, y, z) = \sum_{n=1}^N \sum_{i=1}^{M_n} \mathcal{A}_{n,i}(z) A(k_n) \hat{\xi}_{n,i} e^{i(k_{\perp n} z'_{n,i} + \beta_{n,i})}.$$

Envelope function

$$\mathcal{A}_{n,i}(z) = \frac{1}{2} \left[F \left\{ \frac{z - z_i}{2S_n} \right\} - F \left\{ \frac{z - L_n - z_i}{2S_n} \right\} \right]$$

$$F\{z\} = \begin{cases} -1 & z < -1 \\ \frac{3}{2}z - \frac{1}{2}z^3 & -1 \leq z \leq 1 \\ 1 & z > 1. \end{cases} \quad \begin{aligned} S_n &= L_n / \sigma \\ L_n &= L_C^{1/3} \lambda_{\perp n}^{2/3} \end{aligned}$$

impact of steepness over the envelope shape

