On the existence of N-junctions for a symmetric nonnegative potential with $N+1$ zeros.

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Abstract

We consider a nonnegative potential $W:\mathbb{R}^2\to\mathbb{R}$ which is invariant under C_N, the rotation group of the regular polygon with N sides:

$$W(\omega z)=W(z),\ \ z\in\mathbb{R}^2,$$

where ω is the rotation of angle $\frac{2\pi}{N}$.
We assume that W has $N+1$ zeros:

$$\{W=0\}=\{0,a,\omega a,\ldots,\omega^{N-1}a\},$$

for some $a\in\mathbb{R}^2\setminus\{0\}$.
We prove that, if the ratio between the surface tension for the interface $0\leftrightarrow a$ and the surface tension for the interface $a\leftrightarrow\omega a$ is sufficiently large, then there exists a N-junction, that is a solution $U:\mathbb{R}^2\to\mathbb{R}^2$ of

$$\Delta u=W_u(u)$$

which remains away from 0 (in spite of the fact that $W(0)=0$) and divides the plane in N equal sectors where $U\sim a,\ldots,U\sim\omega^{N-1}a$.