Introduction to Geometric Integration.

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This course is meant to provide the students with a basic theory of geometric mechanics and its application to the construction of structure-preserving numerical integrators. In the first half of the course key results of classical mechanics on manifolds will be presented. The second half of the course will be focused on the development of a number of numerical schemes able to preserve the underlying differential structure of the problem. Of most interest will be Lie-Poisson integrators and their application to two-dimensional hydrodynamics.

At the end of the course an **assignment** will be given, which will consist of an actual implementation and numerical simulation of an ODE system where structure-preserving is key.

Prerequisites: basic knowledge of Lagrangian and Hamiltonian mechanics.

Total hours: 12.

Table of contents:

1. Smooth manifolds: construction of the tangent and cotangent space.
2. Differential forms and symplectic manifolds.
3. Basic introduction to Lie theory: Lie groups, their algebras and left invariant vector fields.
4. Actions of a Lie group: adjoint and coadjoint actions.
5. Hamiltonian mechanics on Poisson manifolds: Poisson brackets and Casimirs.
6. Symplectic Runge-Kutta methods for Hamilton systems.
7. Non-canonical Poisson systems, Lie-Poisson systems.
8. Integration on manifolds: the Munthe-Kaas method.
9. Application to 2d ideal fluid dynamics.

References

[1] Hairer, E., Hochbruck, M., Iserles, A. and Lubich, C., 2006. Geometric numerical integration. Oberwolfach Reports, 3(1), pp.805-882.

[2] Arnold, V.I., 2013. Mathematical methods of classical mechanics (Vol. 60). Springer Science & Business Media.

[3] Holm, D.D., Schmah, T. and Stoica, C., 2009. Geometric mechanics and symmetry: from finite to infinite dimensions (Vol. 12). Oxford University Press.