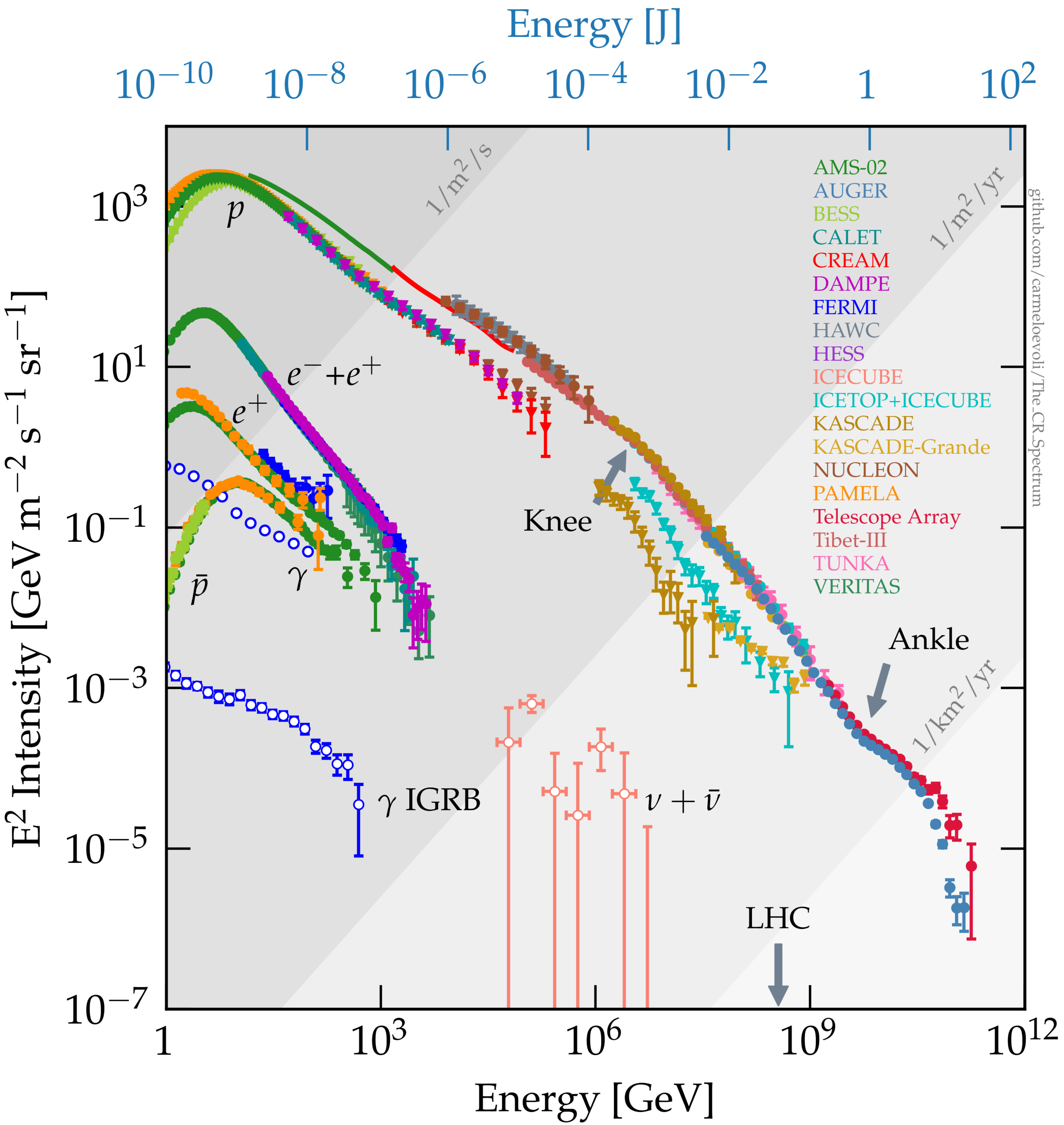
GRAN SASSO SCIENCE INSTITUTE

The role of turbulence in CR phenomenology

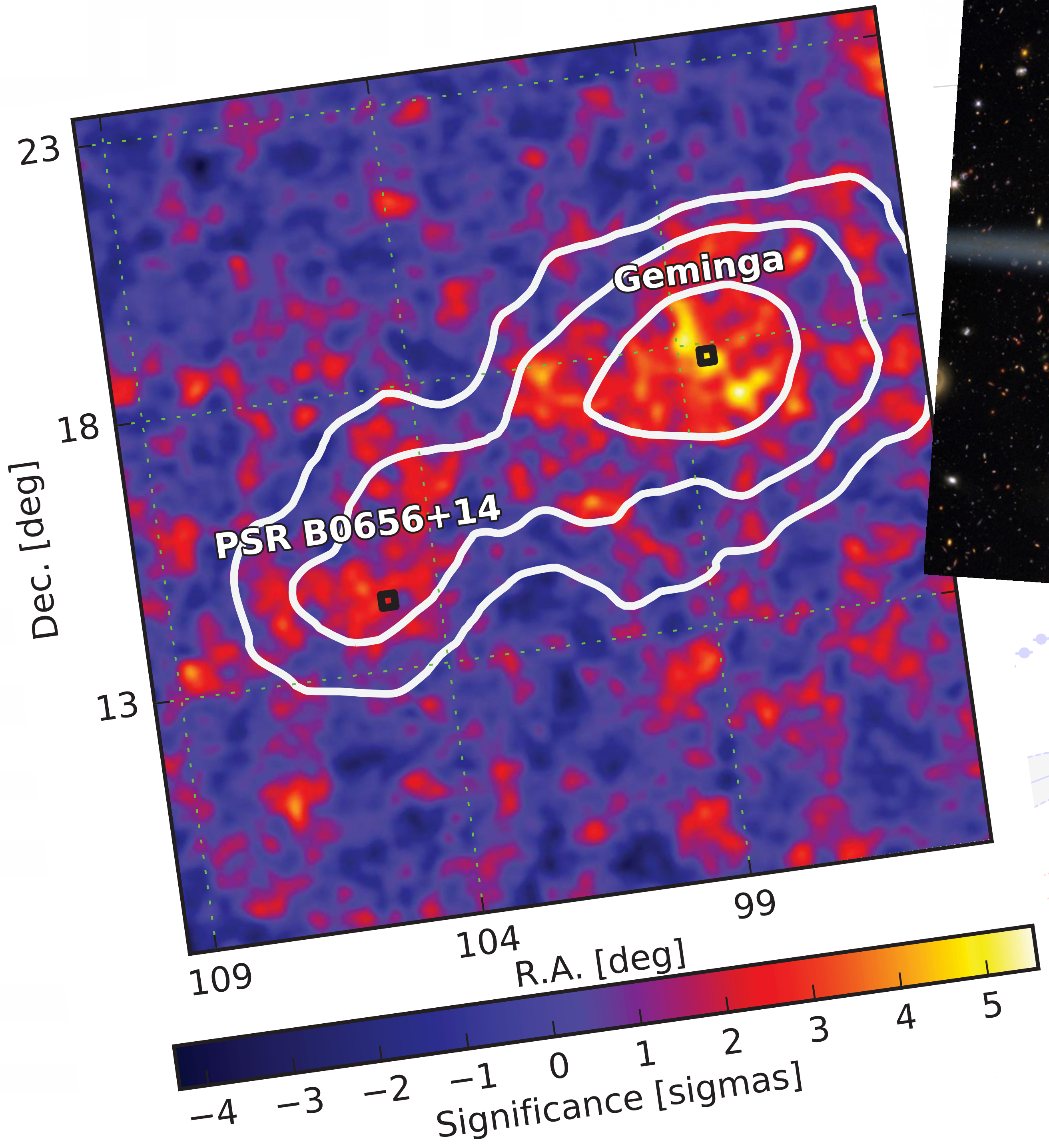
Presented by: Ottavio Fornieri

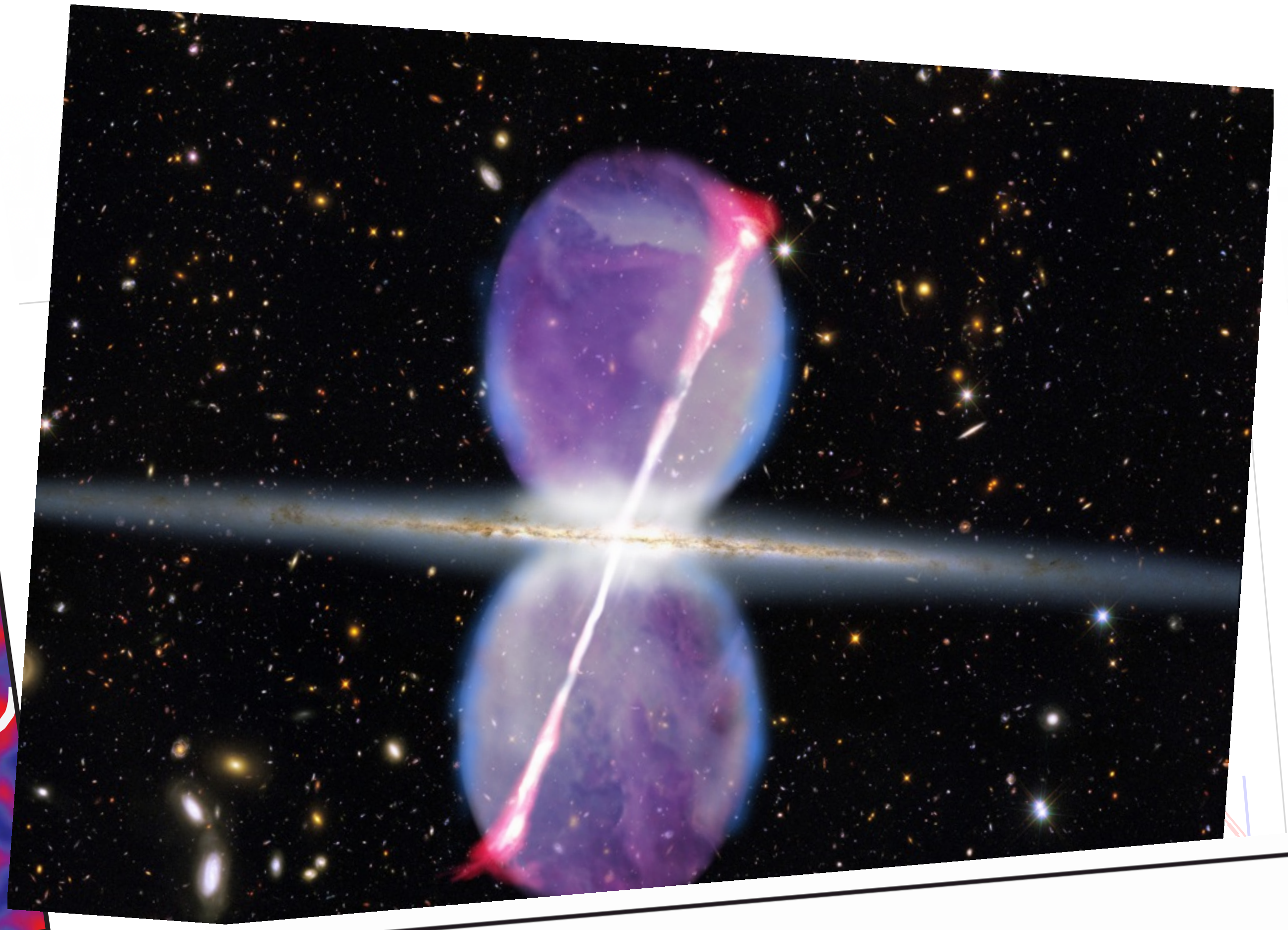
The general picture

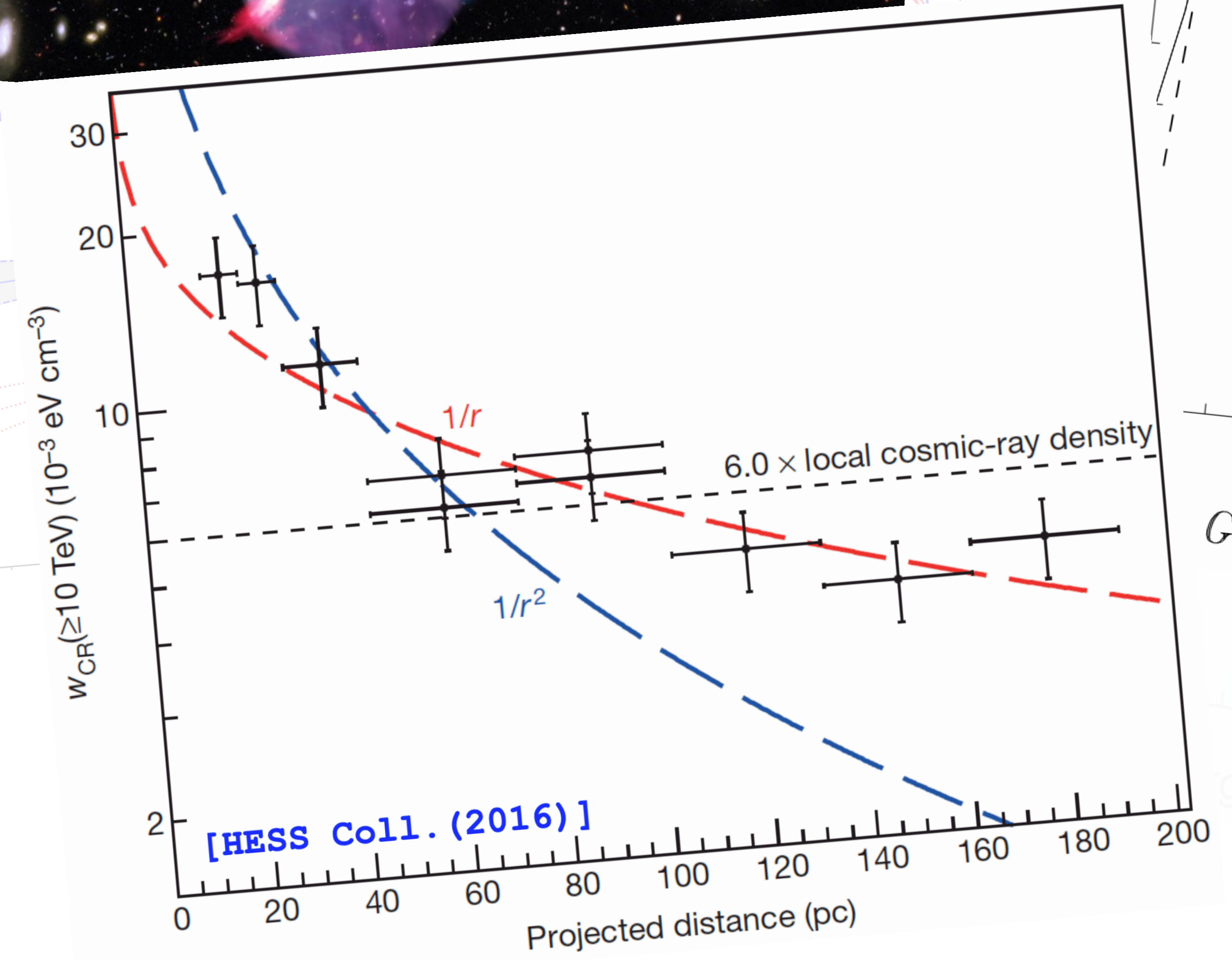


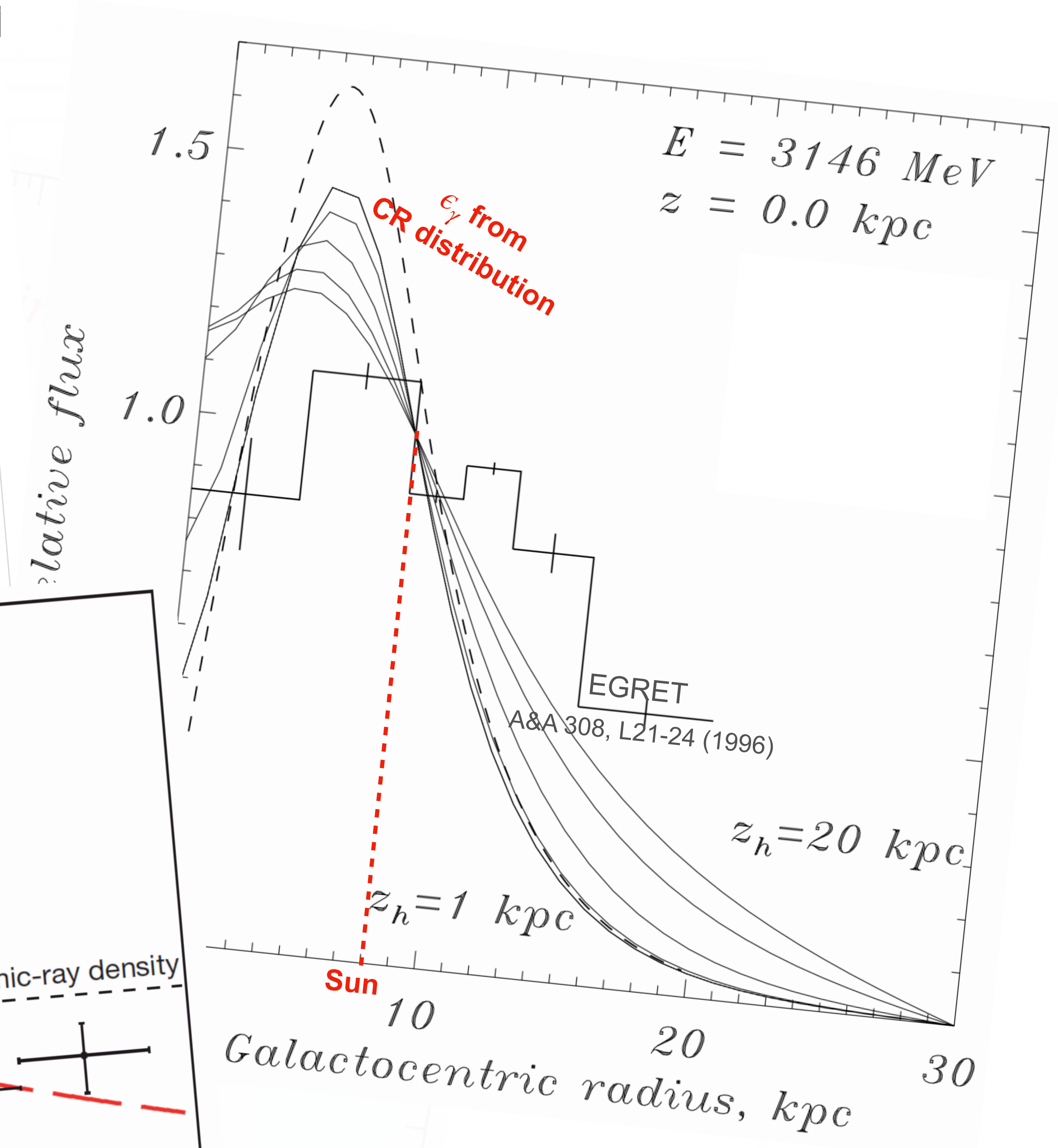
GSI

Anomalies in the details









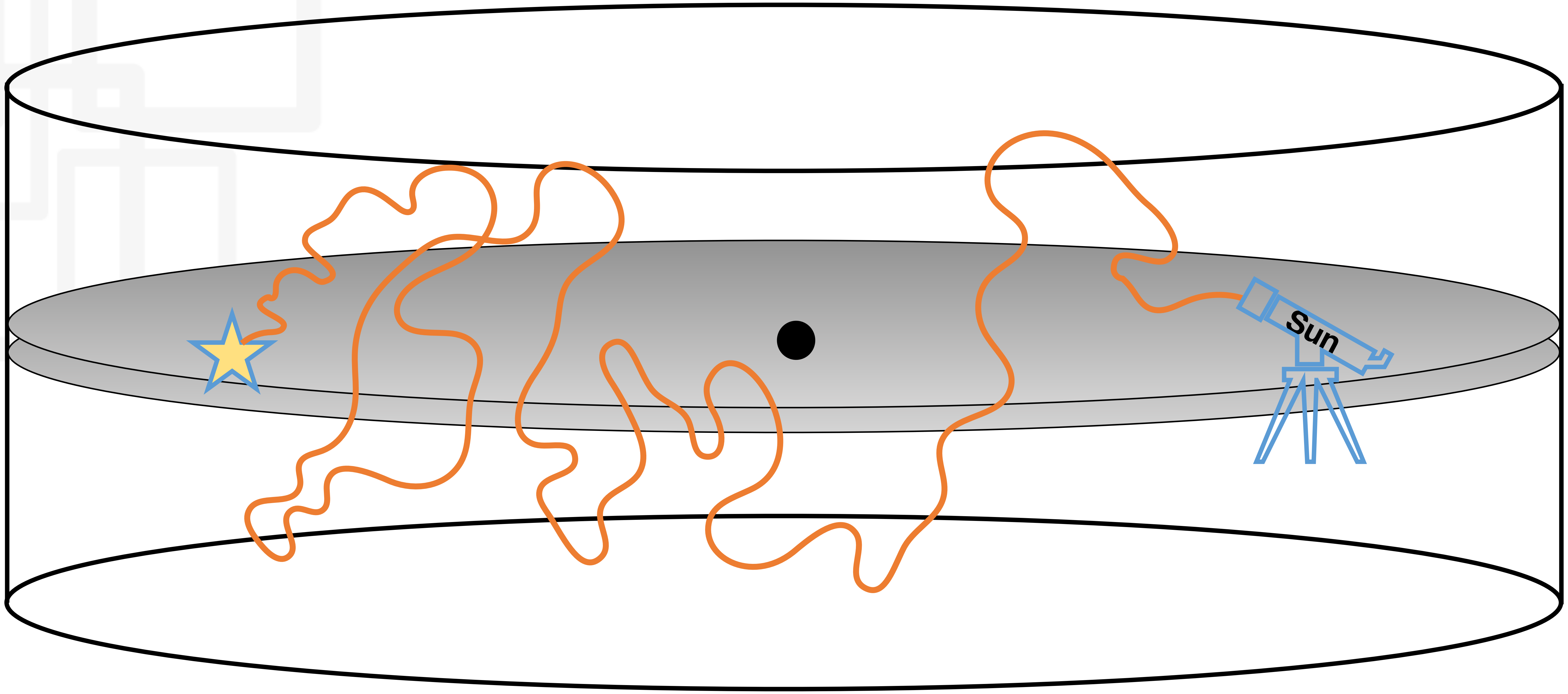
Why studying CR physics?

1

Acceleration at the highest energies

Why studying CR physics?

Propagation through ISM plasma

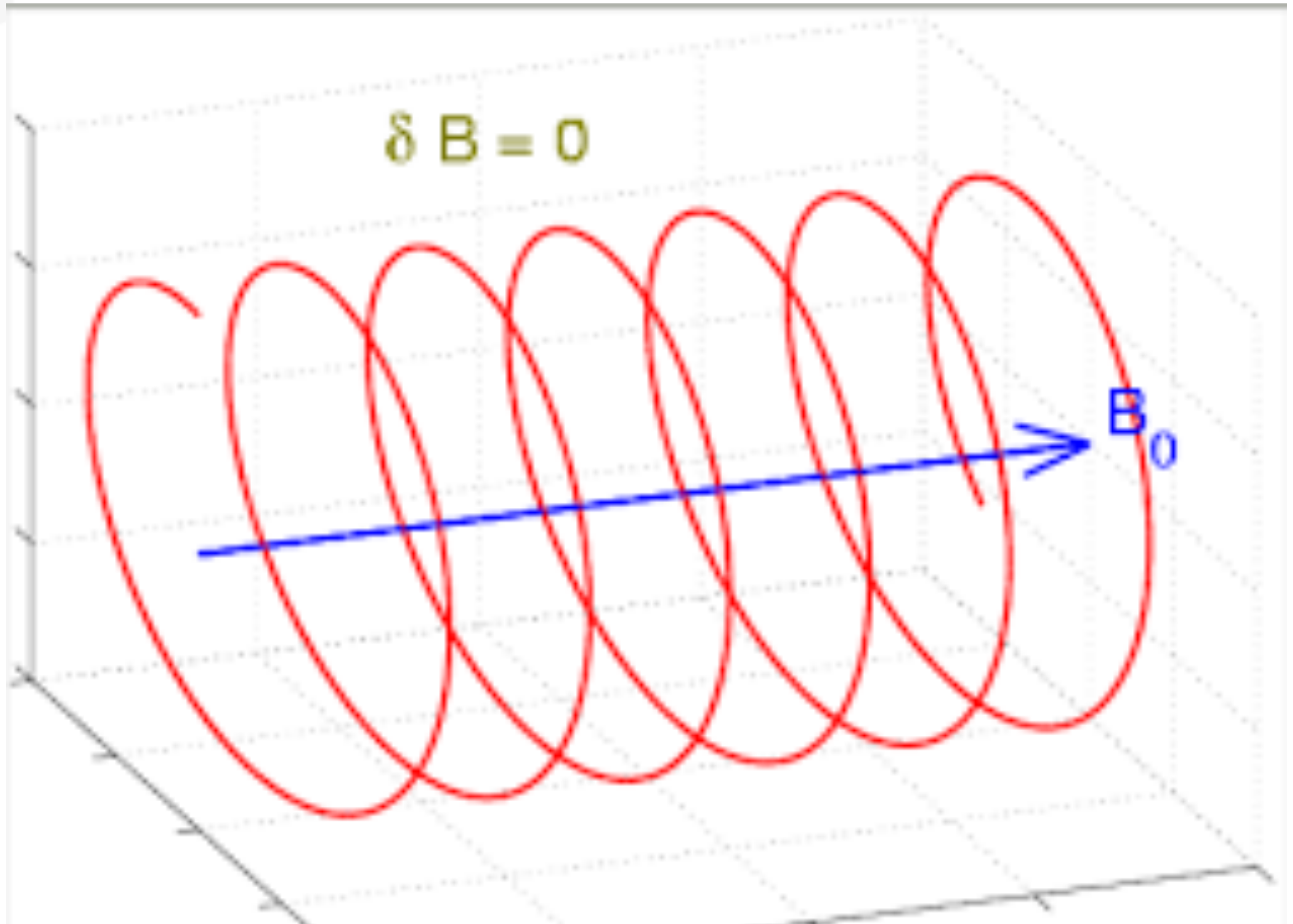


GSSI

Particle motion in magnetic fields

Gyro-motion of charged particles

$$\boxed{m\gamma \frac{d\mathbf{v}}{dt} = \frac{q}{c}\left(\mathbf{v} \wedge \mathbf{B}_{\rm tot}\right)}$$



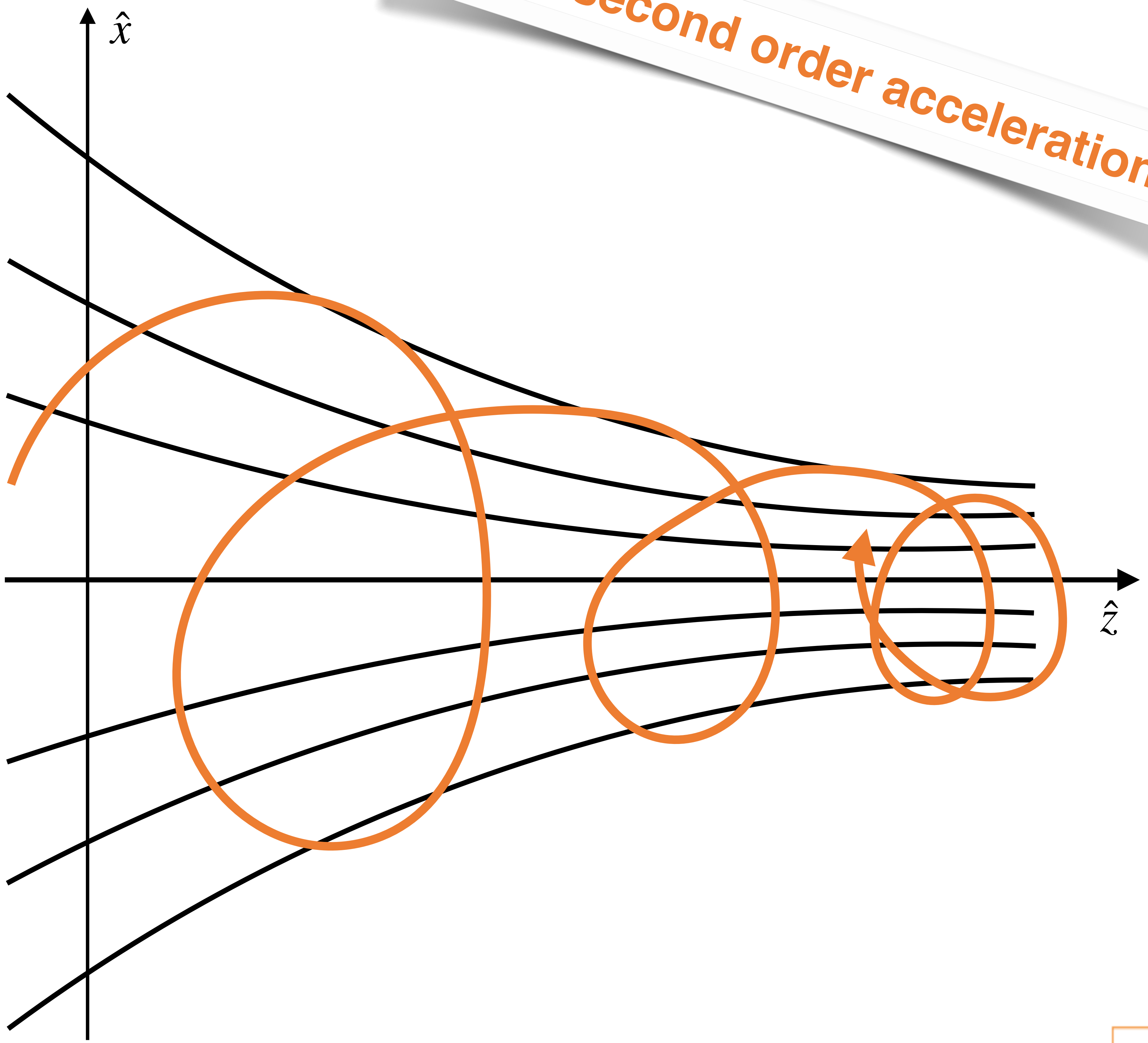
$$\mathbf{B}_{\rm tot} = \mathbf{B}_0 + \delta\mathbf{B}$$

Magnetic bottles

$$v_0^2 = v_{\parallel,0}^2 + v_{\perp,0}^2$$
$$\underset{\forall z}{=} v_\parallel^2 + v_\perp^2$$

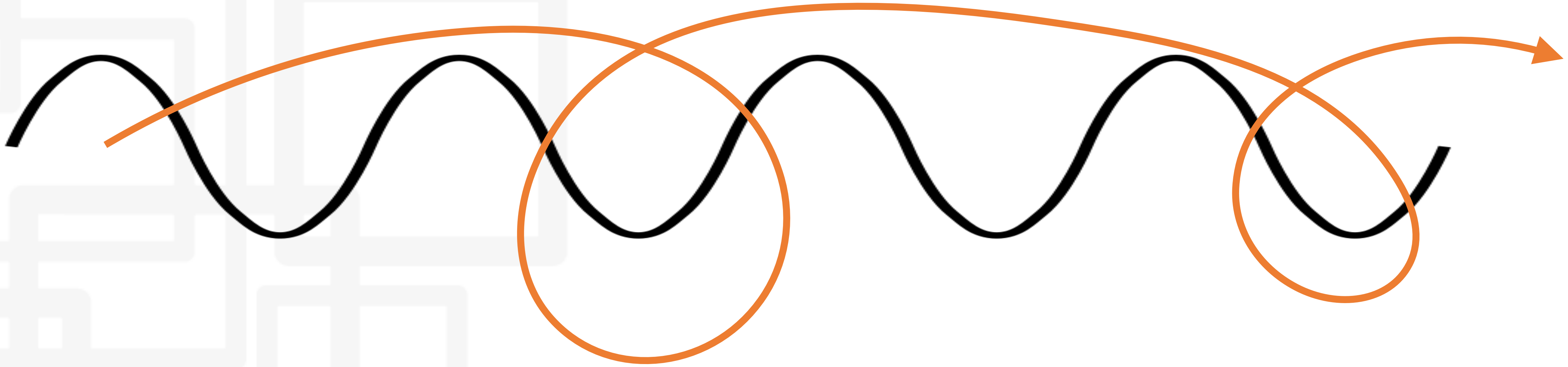
$$\frac{v_{\perp,0}^2}{B} \underset{\forall z}{=} \frac{v_\perp^2}{B(z)}$$

$$\Delta(v_\parallel^2) = v_{\perp,0}^2 \left(B_0 - B(z)\right) \cdot \frac{1}{B_0}$$

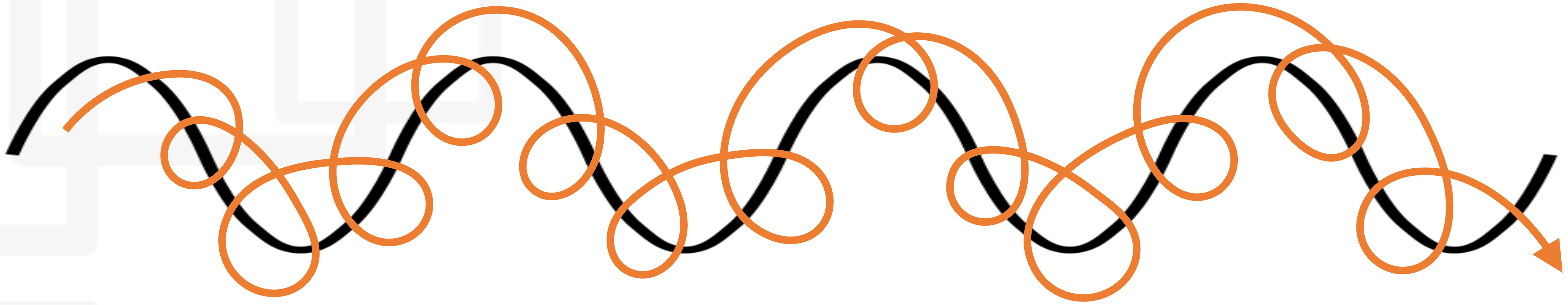


Fermi second order acceleration

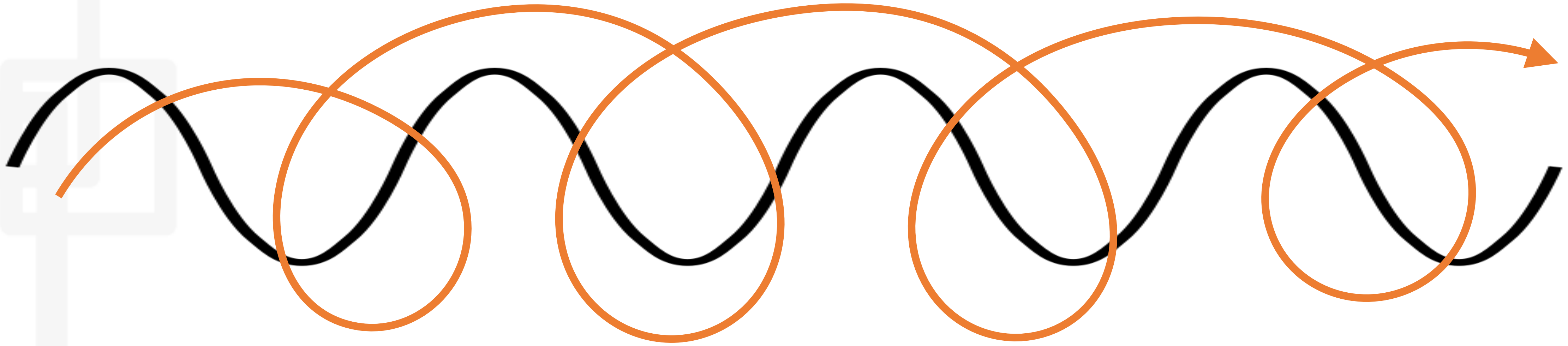
Pitch-angle scattering on $\mathbf{B}$-fluctuactions



$$r_L \gg \frac{1}{k_{\text{fluctuation}}} \quad \rightarrow \quad \mathbf{B}_0$$



$$r_L \ll \frac{1}{k_{\text{fluctuation}}} \quad \rightarrow \quad \delta\mathbf{B}$$



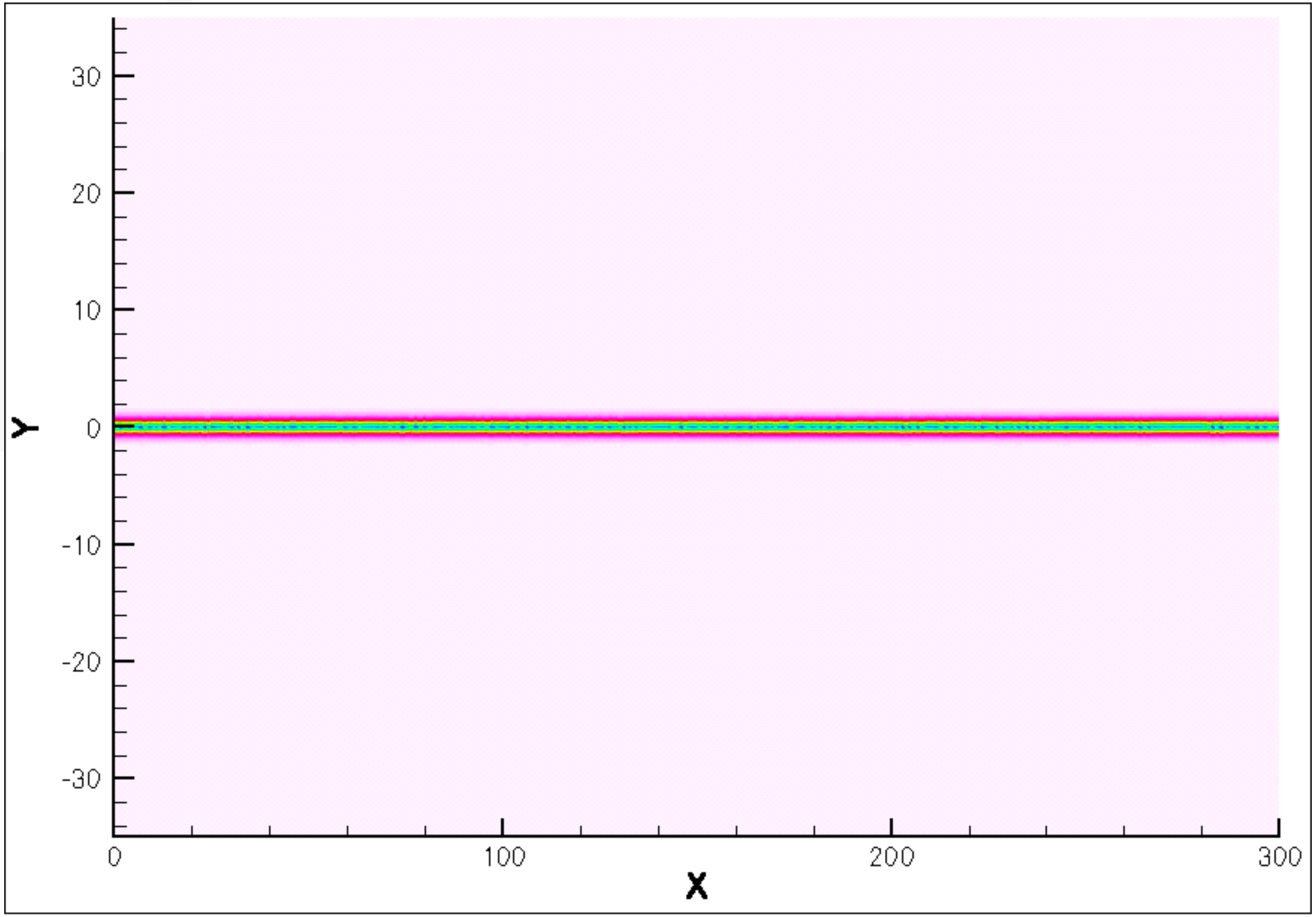
$$r_L \sim \frac{1}{k_{\text{fluctuation}}} \quad \rightarrow \quad \mathbf{B}_0 + \delta\mathbf{B}$$

GSI

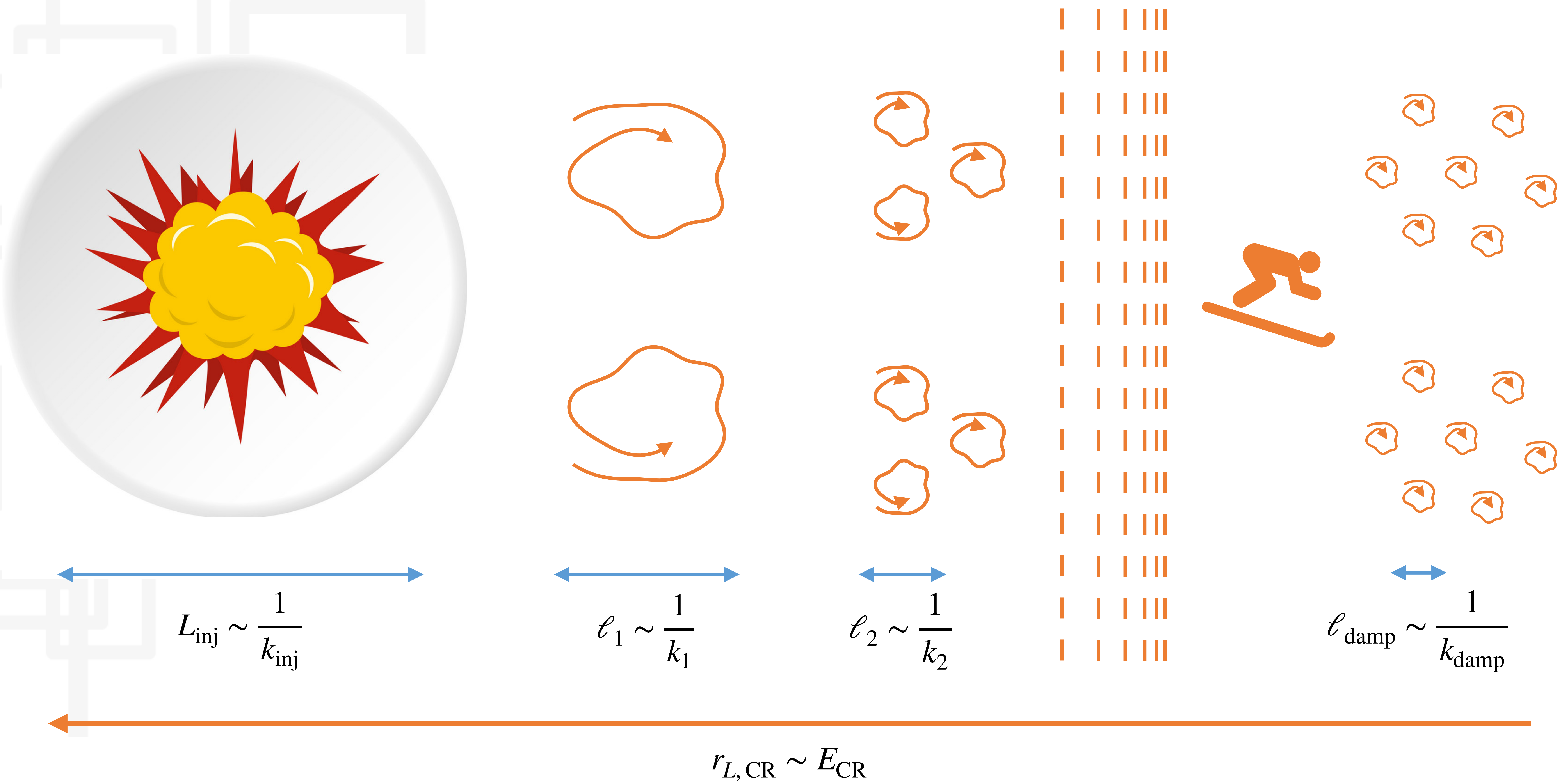
Properties of turbulence

GS
SI

Turbulent cascade in the inertial range



Turbulent cascade in the inertial range



Turbulent cascade in the inertial range

Kolmogorov's approach

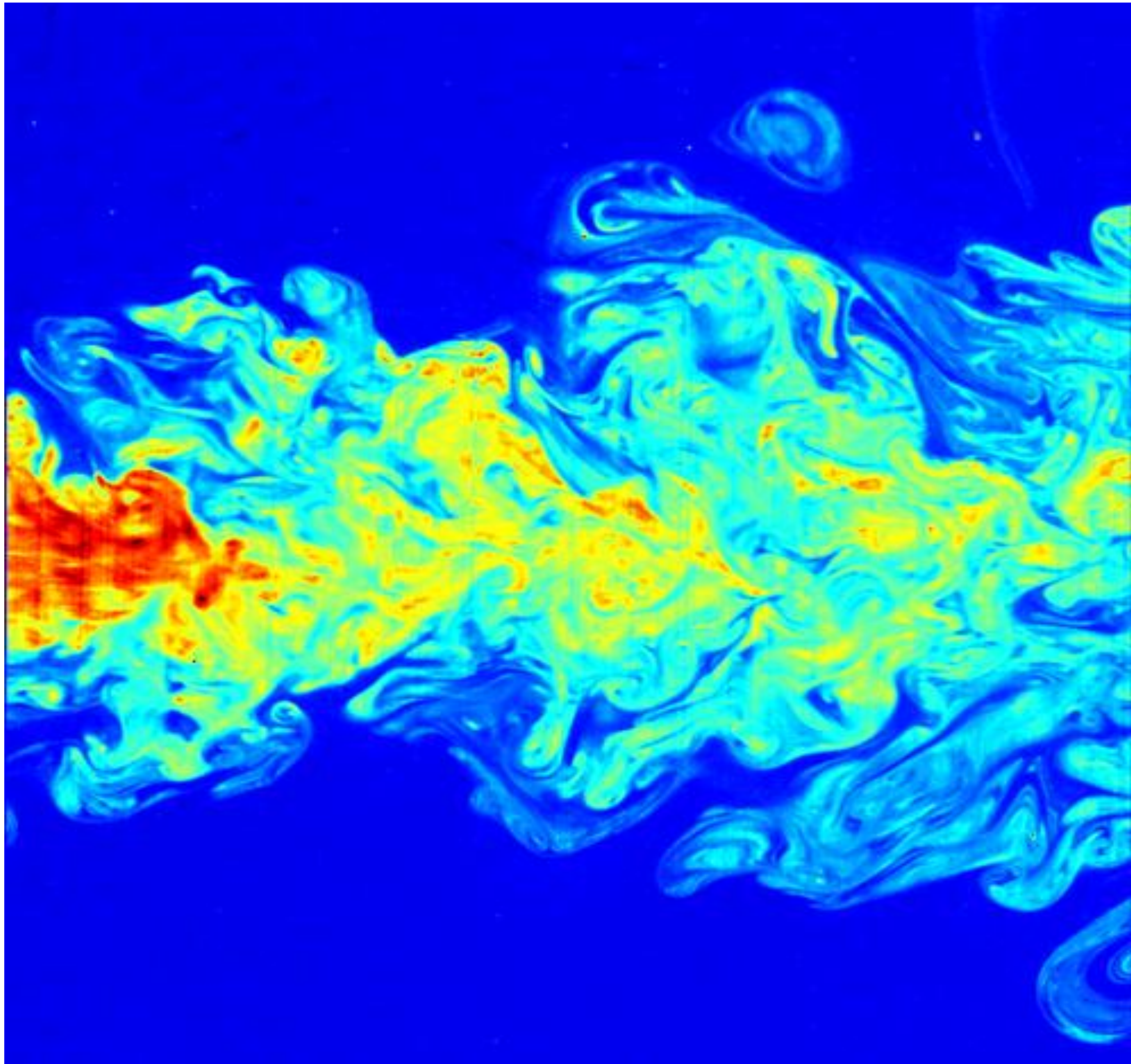
$$\boxed{\frac{E_{\mathrm{K}}/V}{\tau_{\mathrm{turn}}} \sim \frac{\rho v_\ell^2}{\ell/v_\ell} = \mathrm{const}}$$

$$\Downarrow$$

$$v_\ell^3 \sim \ell \quad \Rightarrow \quad v_\ell \sim \ell^{1/3} \quad \Rightarrow \quad v_k \sim k^{-1/3}$$

$$\Downarrow$$

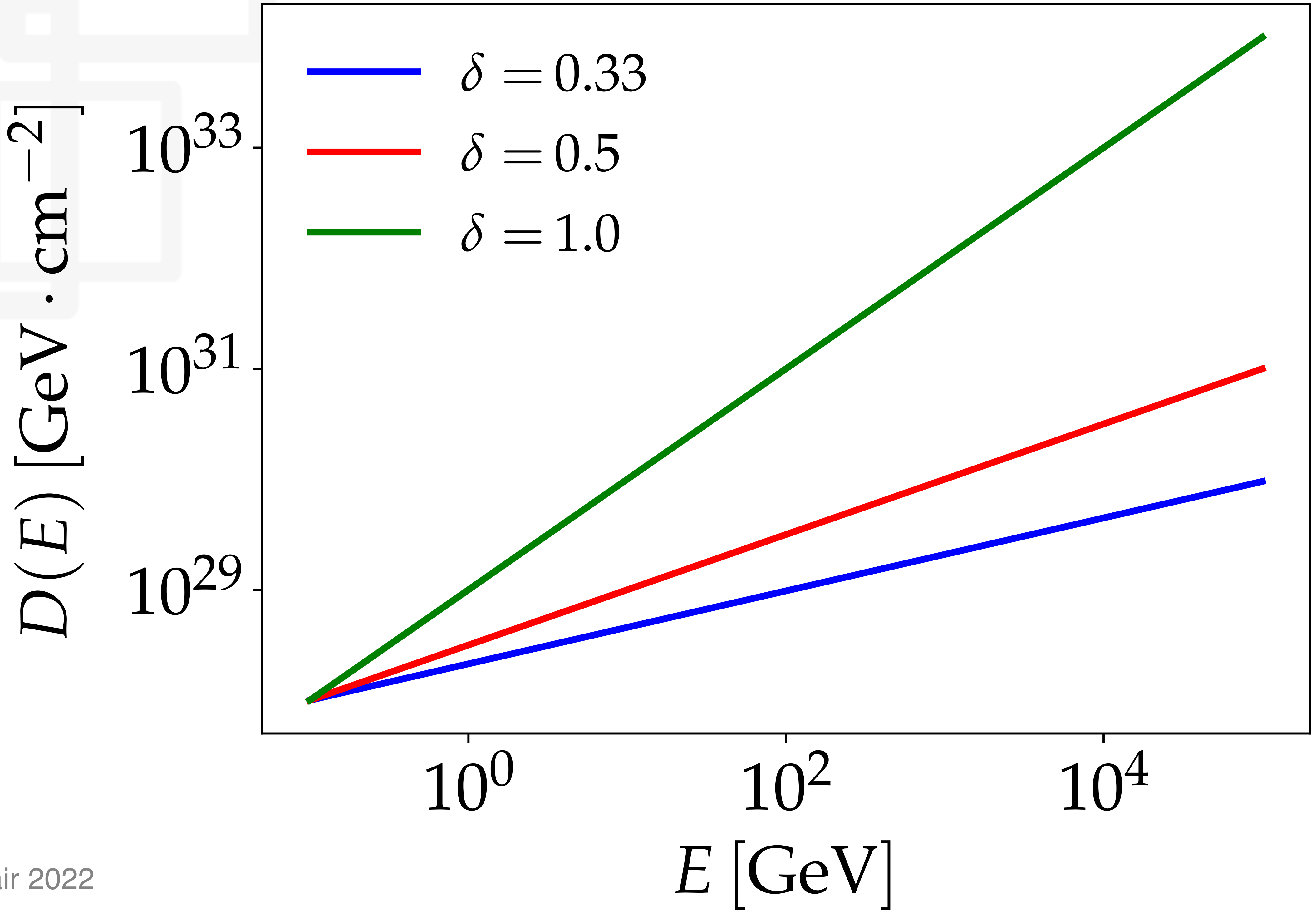
$$k \cdot E(k) \sim \rho v_k^2 \quad \Rightarrow \quad \boxed{E(k) \sim k^{-5/3}}$$



From turbulence to CR diffusion

$$D(E) = \frac{1}{3} \cdot \frac{c\, r_L}{k_{\rm res} \cdot E(k_{\rm res})} \quad \Rightarrow \quad D(E) \sim \frac{r_L}{r_L^{-1} \cdot r_L^{\alpha}}$$

$$\boxed{D(E) \sim E^{2-\alpha} \equiv E^{\delta}}$$

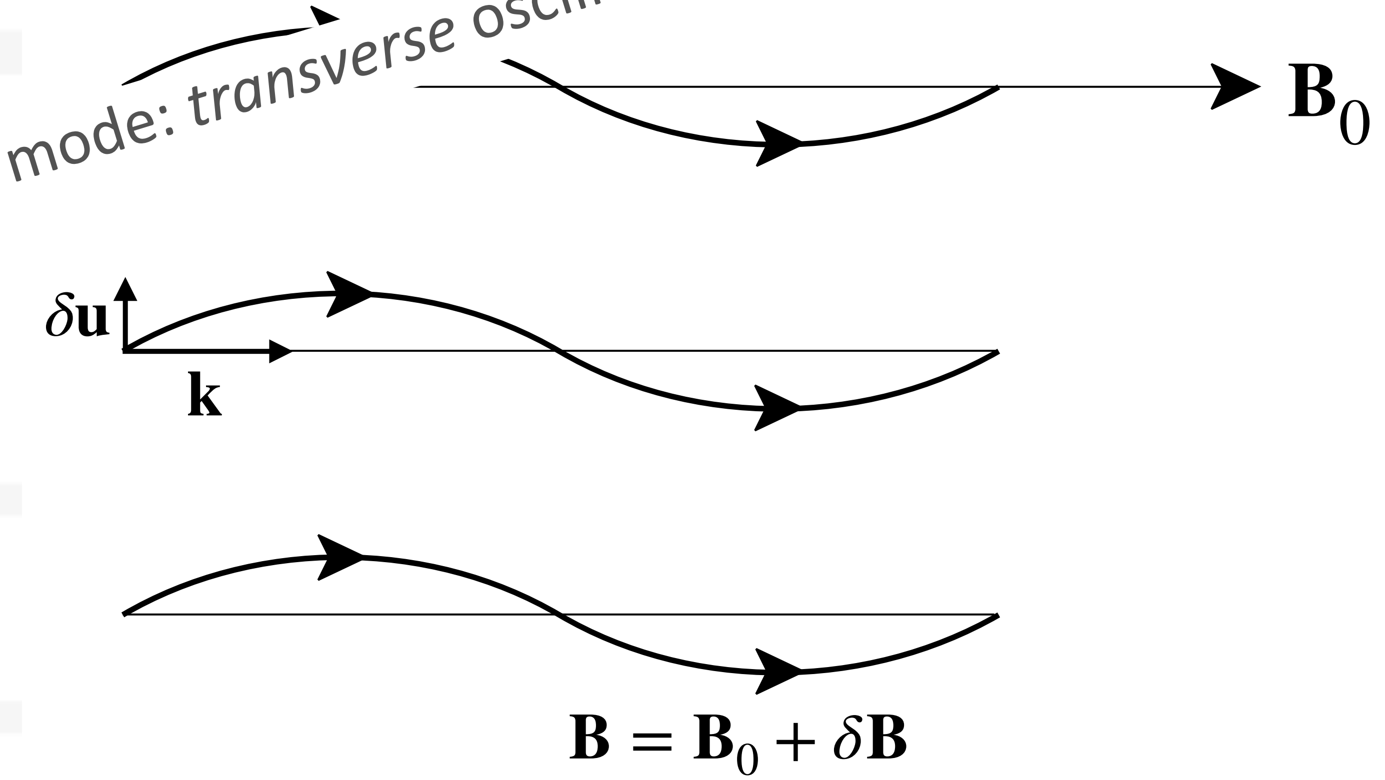


MHD decomposition

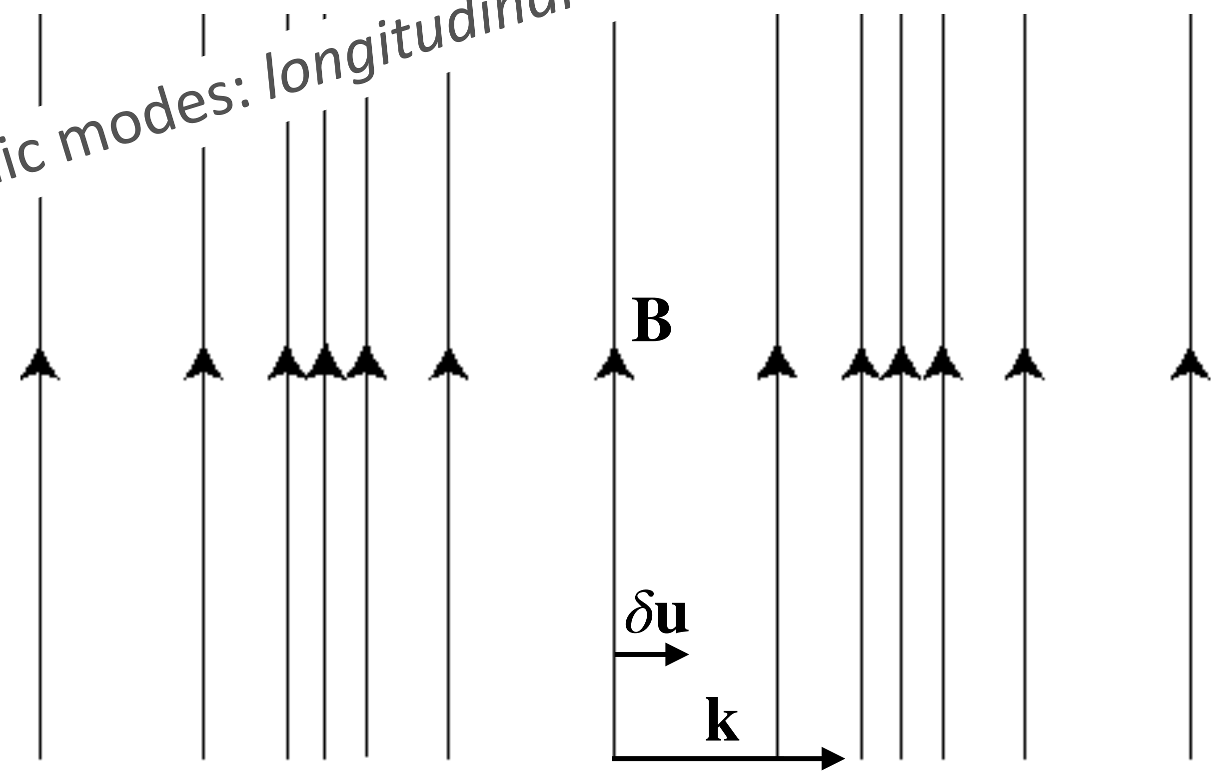
Magnetized medium ⇨ MHD equations ⇨

$$\begin{pmatrix} \omega^2 - k^2 v_A^2 - k_\perp^2 c_s^2 & 0 & -k_\perp k_\parallel c_s^2 \\ 0 & \omega^2 - k_\parallel^2 v_A^2 & 0 \\ -k_\perp k_\parallel c_s^2 & 0 & \omega^2 - k_\parallel^2 c_s^2 \end{pmatrix} \begin{pmatrix} \delta u_x \\ \delta u_y \\ \delta u_z \end{pmatrix} = 0$$

Alfvén mode: *transverse* oscillation



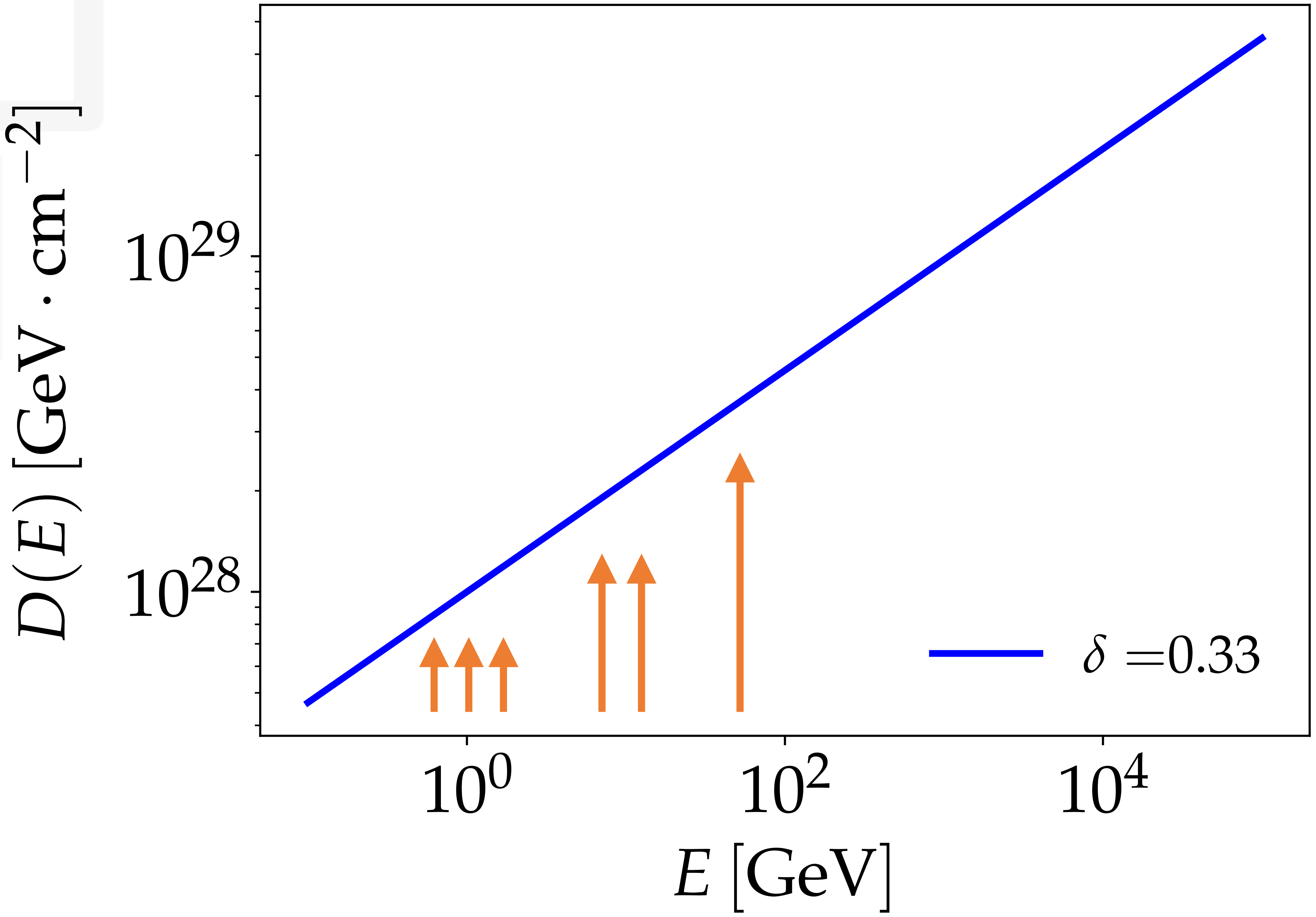
Magnetosonic modes: *longitudinal* oscillations



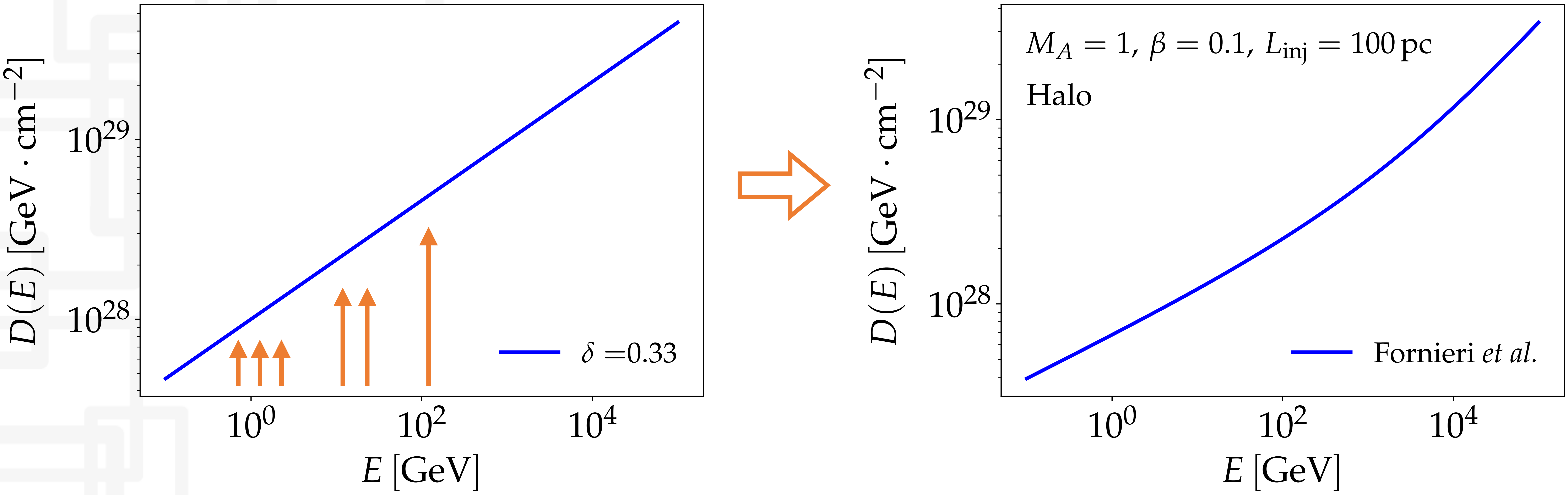
$\delta\mathbf{u}$ = plasma displacement

$\mathbf{k}$ = wave vector of the fluctuation

Damping of the fast modes

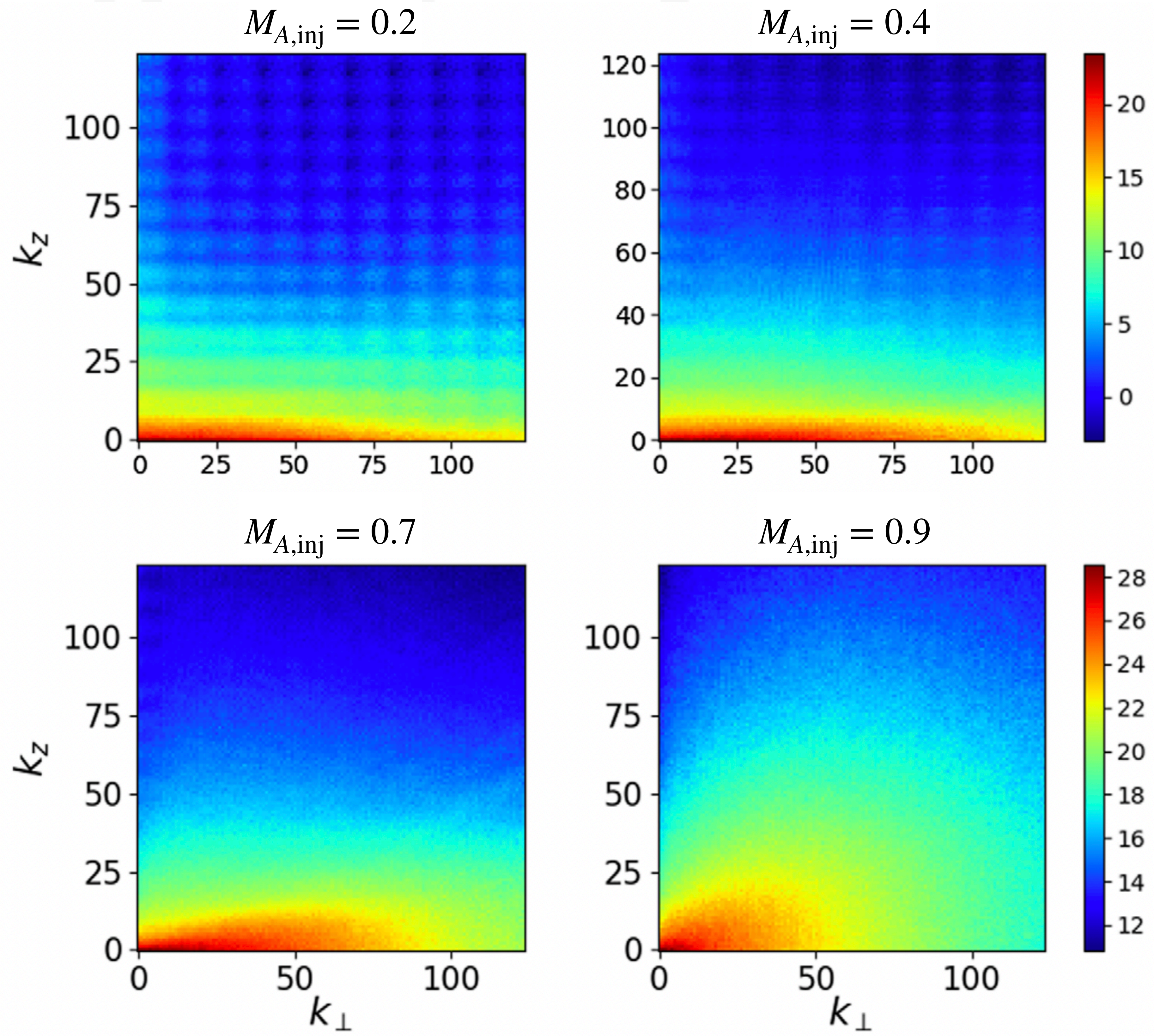


Damping of the fast modes

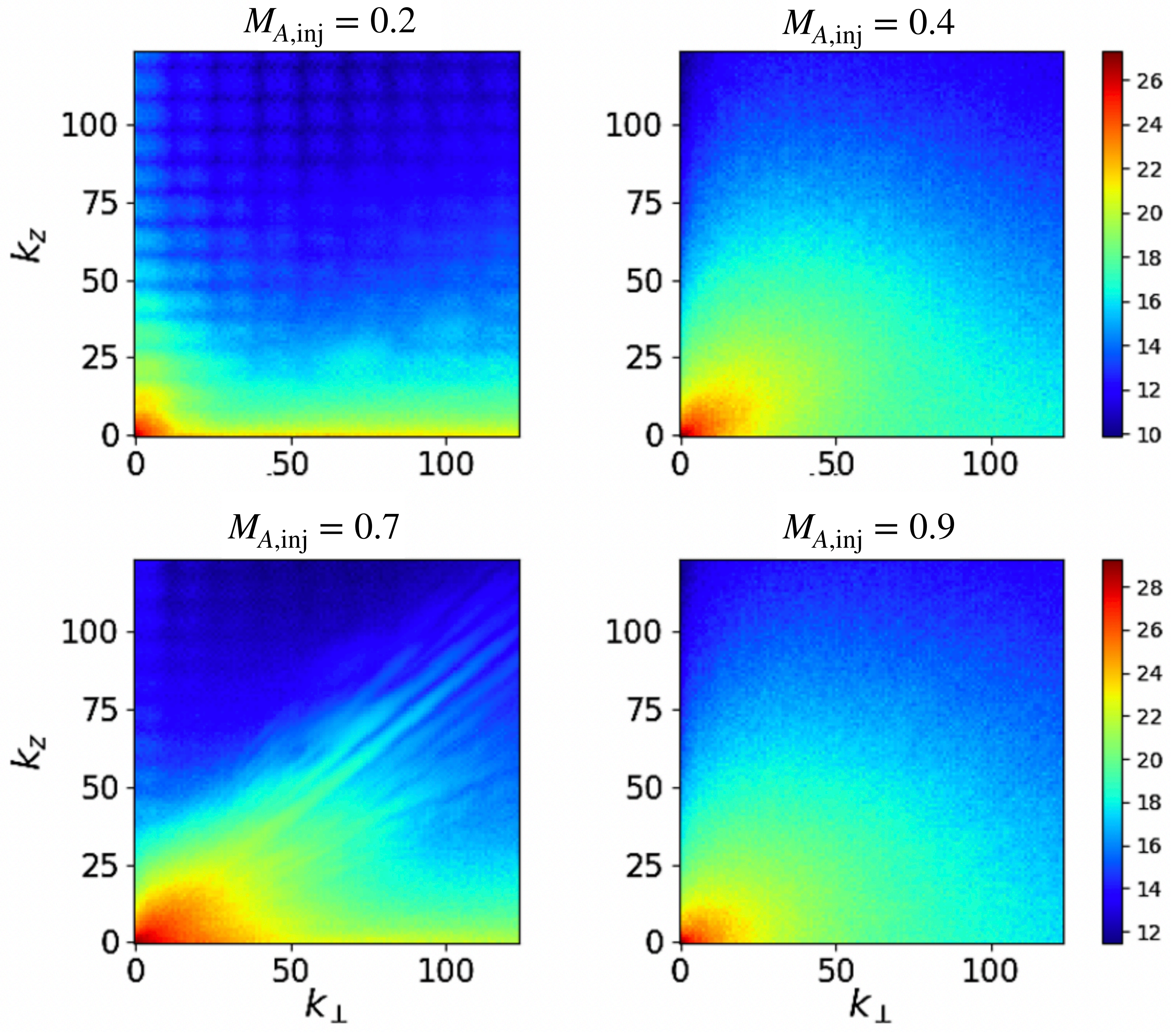


Distribution of the turbulent power

Alfvén modes



Fast modes

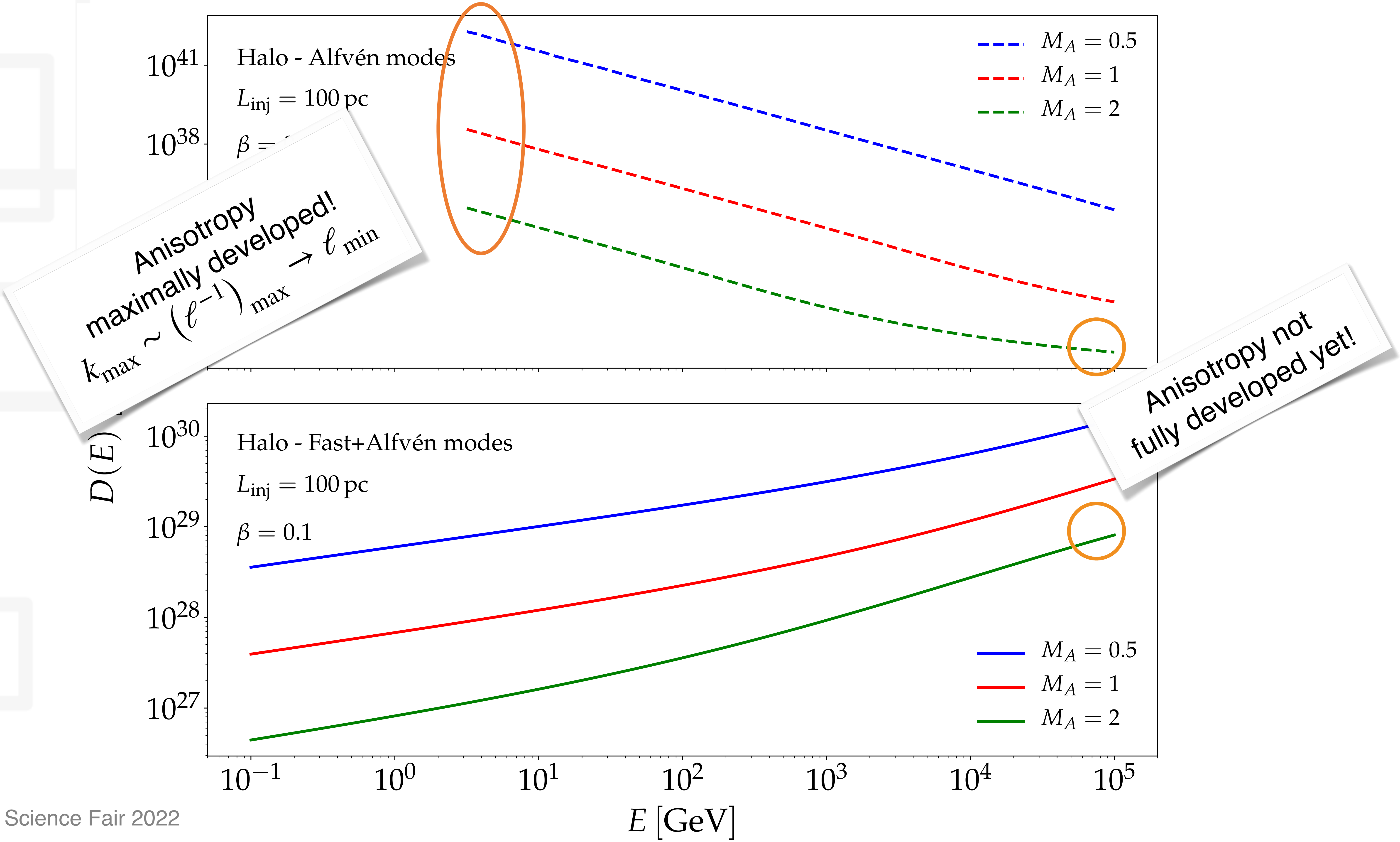


Conclusion…

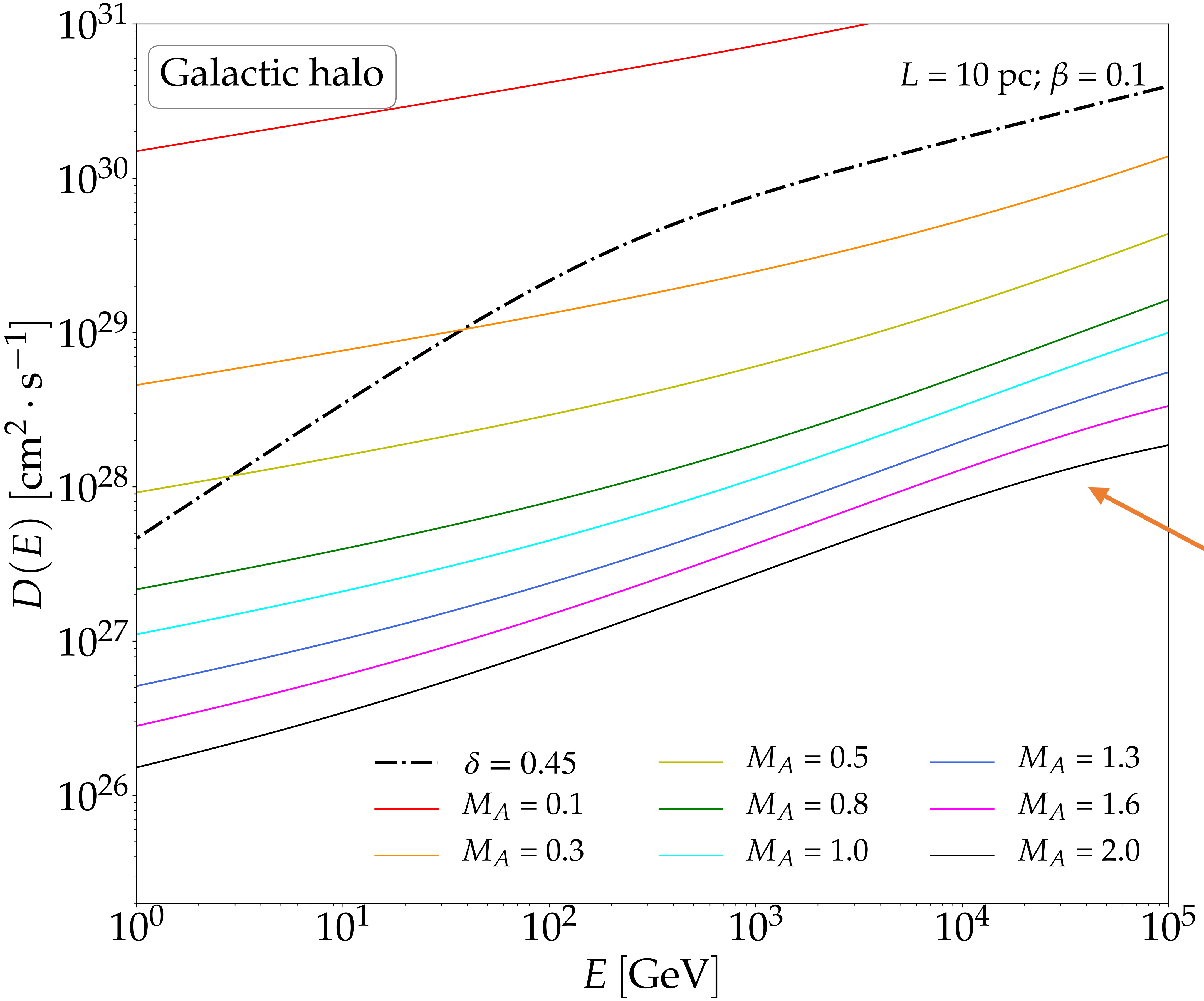
Cosmic-ray phenomenology

GSSI

Resulting CR diffusivity



Resulting CR diffusivity



Contribution from Alfvén modes

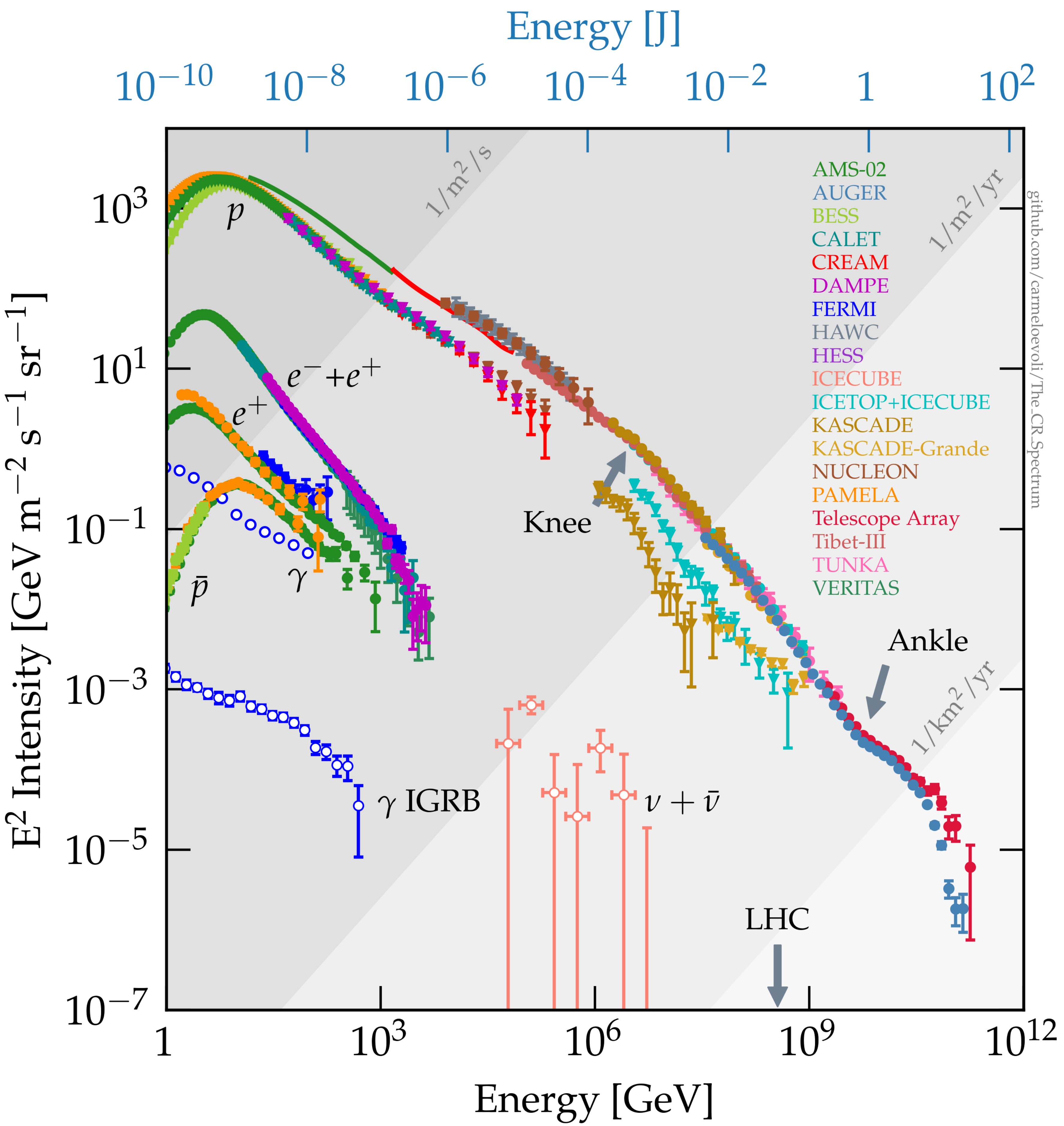
Take-home message

Accurate *measurements require detailed knowledge of the* ***microphysics of CR transport*** *in our Galaxy.*

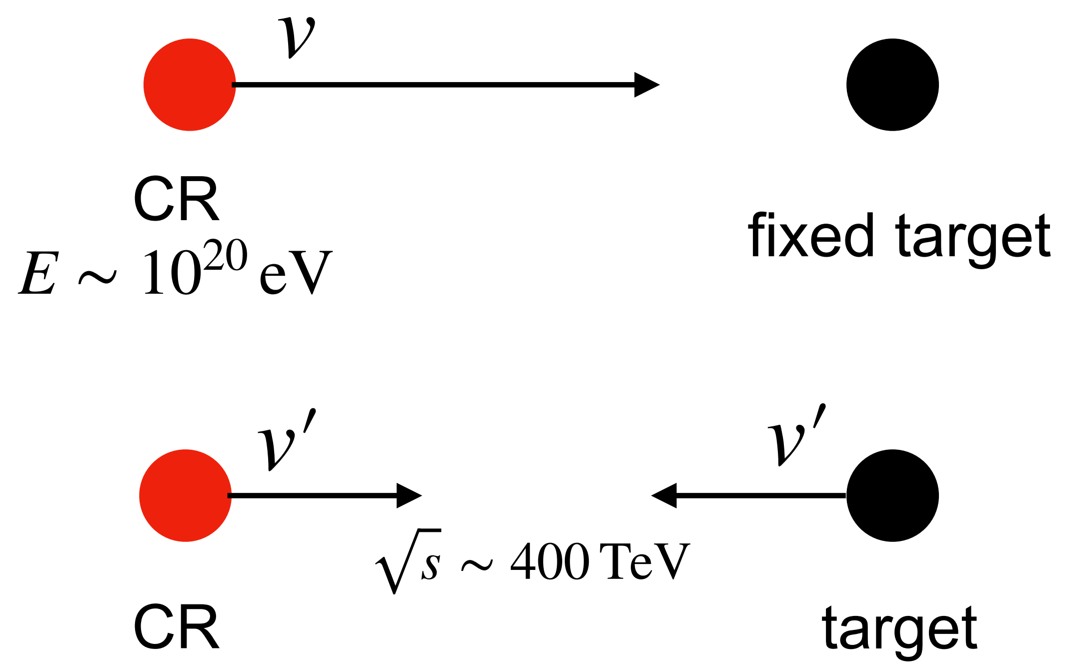
GSSI

Backup slides

Why studying CR physics?



- $\dfrac{dN}{dE} \propto E^{-\gamma} \qquad 2.7 \lesssim \gamma \lesssim 3.1$
- Very energetic particles

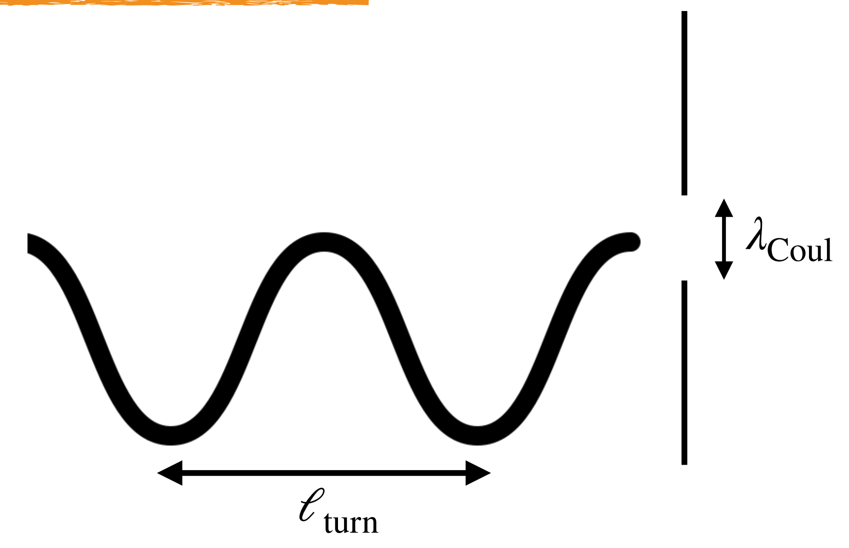


- Unique probe of extreme astrophysical phenomena

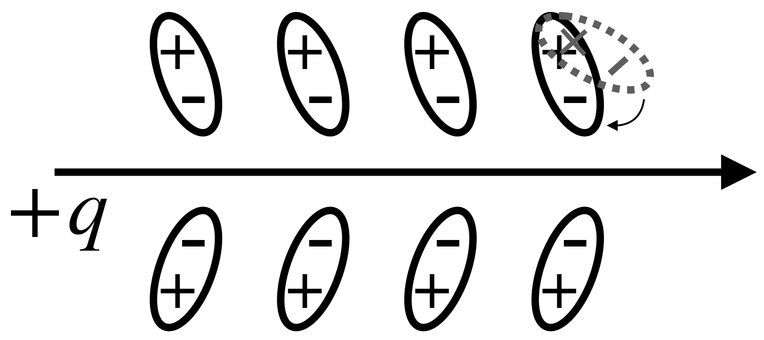
Damping of the fast modes

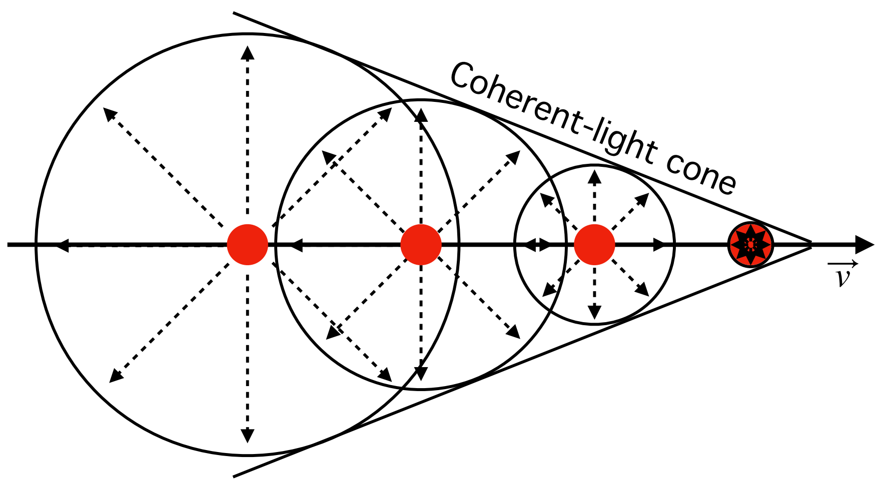
$$\boxed{\lambda_{\text{Coul}} \approx 1.3 \cdot 10^{-5} \left(\frac{\text{cm}^{-3}}{n_{\text{ISM}}} \right) \cdot \left(\frac{T}{10^4\,\text{K}} \right)^2 \text{pc}}$$

- **Collisional** damping



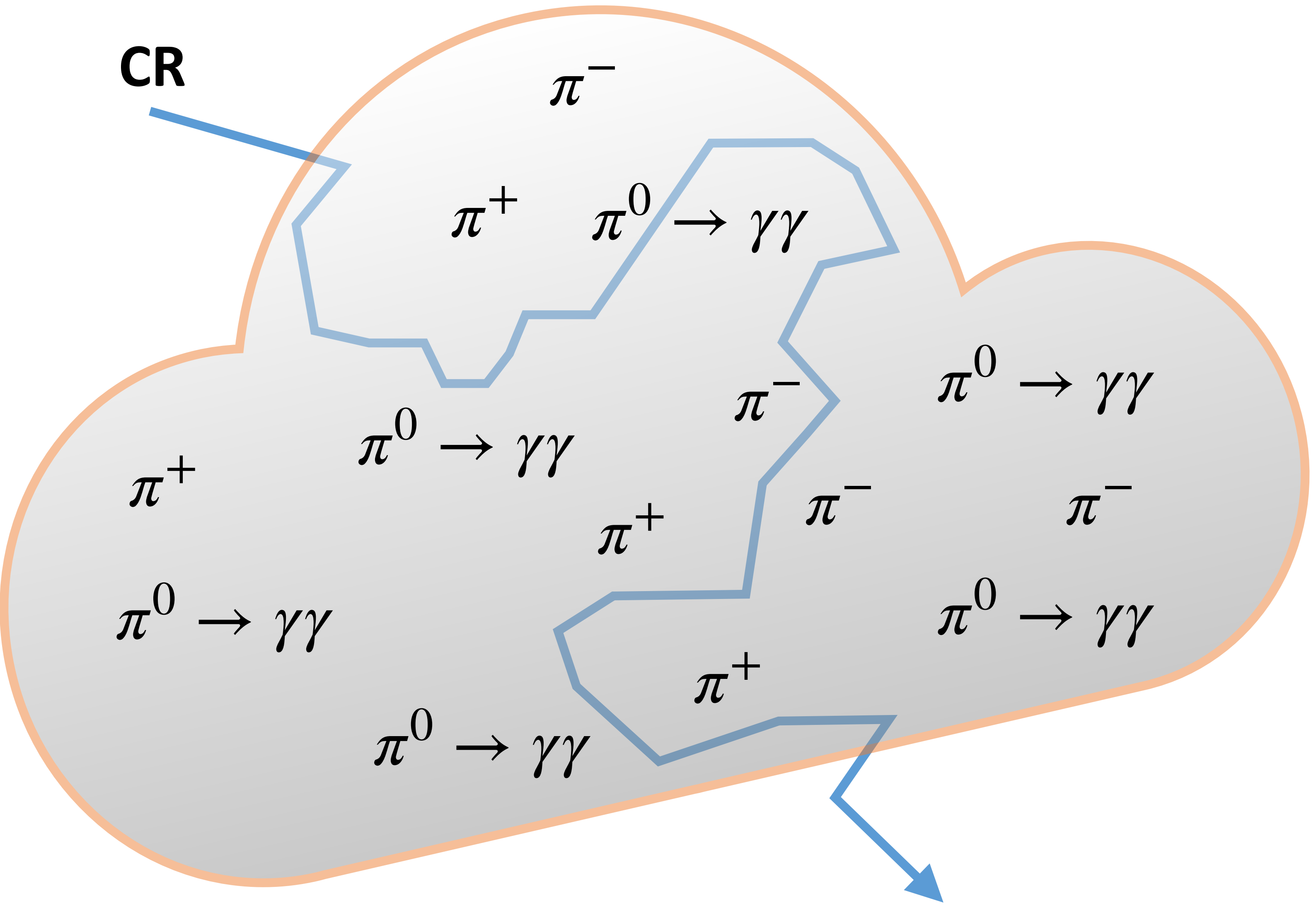
- **Collisionless** damping



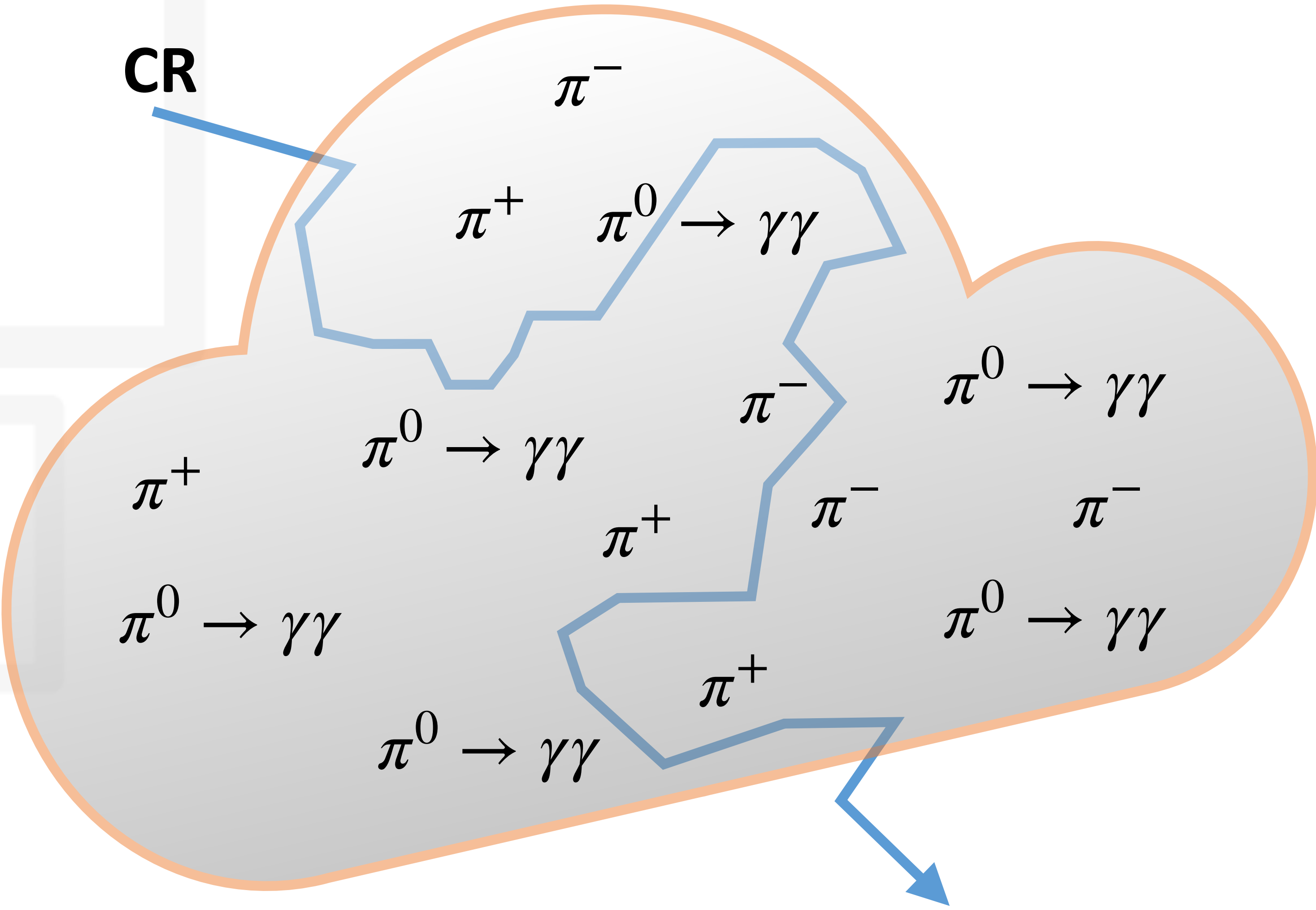


$$\lambda^{\text{disk}}_{\text{Coul}} \underset{\substack{n_{\text{disk}} = 1\,\text{cm}^{-3} \\ T = 10^4\,\text{K}}}{\approx} 1.3 \cdot 10^{-5}\,\text{pc}, \qquad \lambda^{\text{halo}}_{\text{Coul}} \underset{\substack{n_{\text{Halo}} = 10^{-3}\,\text{cm}^{-3} \\ T = 10^6\,\text{K}}}{\approx} 1.3 \cdot 10^{2}\,\text{pc} \simeq L_{\text{inj}}$$

Inferred CR density from pion decay



Inferred CR density from pion decay



$$\langle E_\gamma \rangle \simeq 0.1\, E_{\mathrm{CR}}$$

$$E_{\mathrm{flux}} = \int_{E_{\mathrm{min}}}^{E_{\mathrm{max}}} dE\, E \cdot \left(\overbrace{\frac{dN_\gamma}{dE}\, \frac{c}{4\pi}}^{\Phi_\gamma} \right) \quad \left[\frac{E}{L^2 \cdot T} \right]$$

$$\Rightarrow \quad L_\gamma = E_{\mathrm{flux}} \cdot 4\pi d^2 \quad \left[\frac{E}{T} \right]$$

$$L_\gamma(\geq E_\gamma) \approx \eta_N \cdot \frac{W_p(\geq 10\, E_\gamma)}{\tau_{pp\to\pi^0}} = \eta_N \cdot \frac{W_p(\geq E_\gamma)}{1.6 \cdot 10^8\, \mathrm{yr} \left(\frac{\mathrm{cm}^{-3}}{n_H} \right)}$$

$$\omega_{\mathrm{CR}} \equiv \frac{W_p(\geq 10\, E_\gamma)}{V_{\mathrm{crossed}}} = \frac{W_p(\geq 10\, E_\gamma)}{M_{\mathrm{tot}}} \cdot n_H \approx 1.8 \cdot 10^{-2} \left(\frac{\eta_N}{1.5} \right)^{-1} \left(\frac{L_\gamma(\geq E_\gamma)}{10^{34}\, \mathrm{erg} \cdot \mathrm{s}^{-1}} \right) \left(\frac{M_{\mathrm{tot}}}{10^6 M_\odot} \right)^{-1} \mathrm{erg} \cdot \mathrm{cm}^{-3}$$

Inefficiency of Alfvén modes

$$\boxed{\int dk_\parallel \, E(k_\parallel) = \int dk_\perp \, E(k_\perp)}$$

$$E^{\mathrm{GS}}(k_\perp) \sim k_\perp^{-5/3} = k_\perp^{-1.67}$$

$$k_\parallel \sim k_\perp^{2/3} \;\Rightarrow\; k_\parallel^{3/2} \sim k_\perp$$

$$\rightarrow \; \frac{dk_\perp}{dk_\parallel} = \frac{3}{2} k_\parallel^{3/2 - 1} = \frac{3}{2} k_\parallel^{1/2}$$

$$\Rightarrow$$

$$\int dk_\perp \, E(k_\perp) = \int \frac{3}{2} k_\parallel^{1/2} \, dk_\parallel \, E(k_\perp) = $$

$$= \frac{3}{2} \int k_\parallel^{1/2} \, dk_\parallel \, k_\perp^{-5/3} = \frac{3}{2} \int dk_\parallel \, k_\parallel^{1/2} \left(k_\parallel^{3/2}\right)^{-5/3}$$

$$\Rightarrow \quad \boxed{E^{\mathrm{GS}}(k_\parallel) \sim k_\parallel^{-2}}$$

But $D_{\mu\mu} \propto R_n(k_\parallel v_\parallel - \omega + n\Omega) \ldots$