From microscopic Hamiltonian dynamics to collisional kinetic equations. The case of the Boltzmann equation.

January 13, 2022

Synopsis

The problem of deriving effective macroscopic equations from the microscopic description based on the fundamental laws of mechanics, through suitable scaling limits, is a central problem of non-equilibrium statistical mechanics. The resulting equations are essential to describe the relevant physical properties of the system and their time evolution. In this course we will address the aforementioned question in the framework of kinetic theory of gases. One of the most famous example of kinetic equation is the Boltzmann equation which describes the evolution of a rarefied gas, namely

$$\partial_t f + v \cdot \nabla_x f = Q(f,f) \tag{1}$$

$$Q(f,f)(t,x,v) = \int_{S^2} d\omega \int_{\mathbb{R}^3} dv_* B(v - v_*, \omega) \left\{ f(t,x,v') f(t,x,v'_*) - f(t,x,v) f(t,x,v_*) \right\}. \tag{2}$$

The goal of this course is to present some fundamental results on the mathematical aspects of the Boltzmann equation and to outline the limiting procedure which leads from the microscopic description based on the Hamiltonian dynamics of N interacting particles to the Boltzmann equation.

The course is divided into two parts. In the first part I will present the main properties of the Boltzmann equation and the heuristic derivation. Furthermore, I will review the celebrated result by O. Lanford from 1975 on the rigorous derivation of the Boltzmann equation from a system of N hard spheres in the low-density limit, for short times and discuss the main steps of the proof. In the second part the focus will shift towards a simpler microscopic model, the Lorentz gas, which is a gas of non-interacting particles in a random configuration of scatterers. This model is paradigmatic since it provides a rare source of exact results in kinetic theo ry. We will see in detail the proof of the rigorous validation of the linear Boltzmann equation from the Lorentz Gas, in the low-density limit.