INFN
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Analog models of physical phenomena

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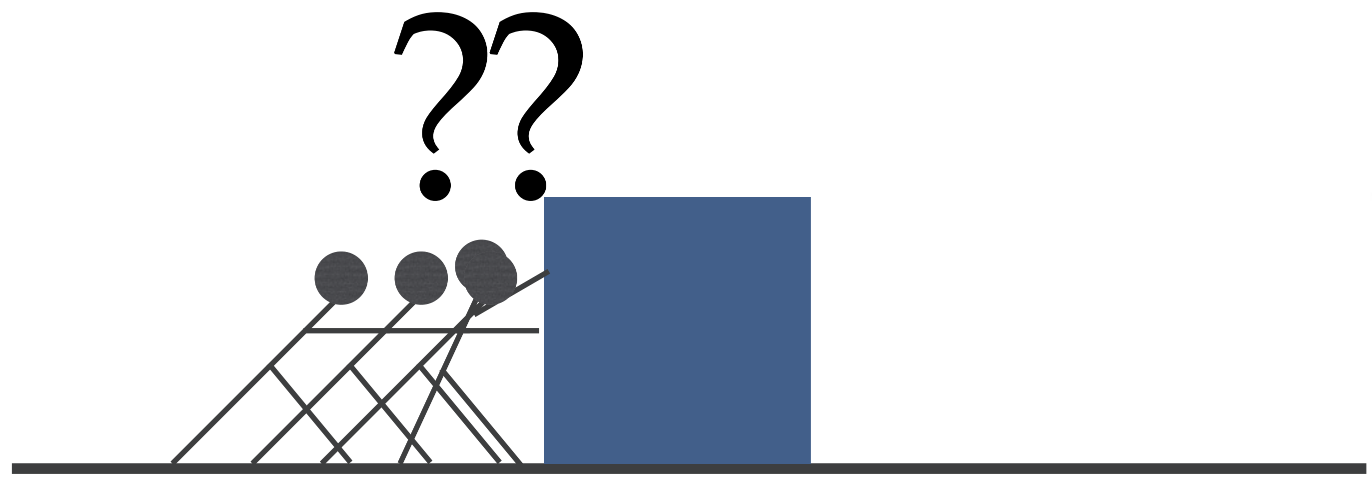
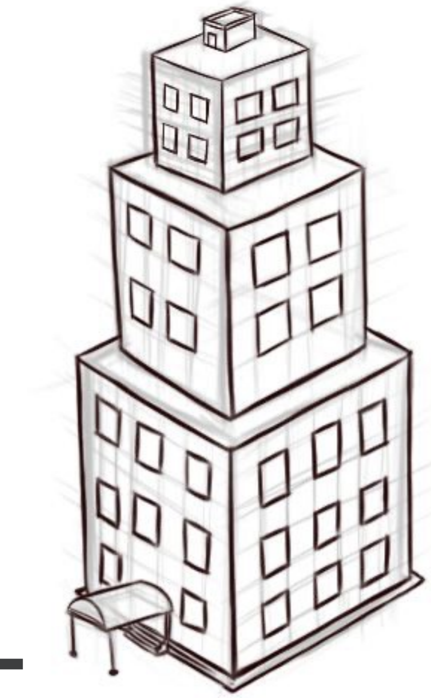
Outline

- **Introduction to analog models**
- **Gravity analogs**
- **Barotropic fluid**
- **Hawking radiation**
- **Conclusions**

References
MM and C. Manuel, Phys.Rev.D 77 (2008) 103014
MM, C. Manuel and B. A. Sa'd, Phys.Rev.Lett. 101 (2008) 241101
M. A. Escobedo, MM and C. Manuel, Phys.Rev.A 79 (2009) 063623
MM and C. Manuel, Phys.Rev.D 81 (2010) 043002
MM, C. Manuel and L. Tolos, Annals Phys. 336 (2013) 12-35
MM, D. Grasso, S. Trabucco and L. Chiofalo Phys.Rev.D 103 (2021) 7, 076001

How to solve problems…





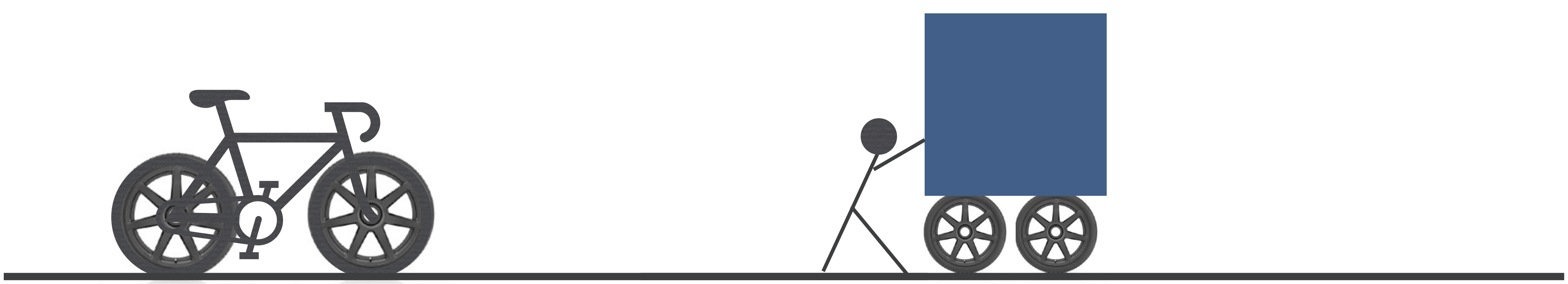
Try harder!

Use approximations!

Use numerical methods!

Ask others to join!

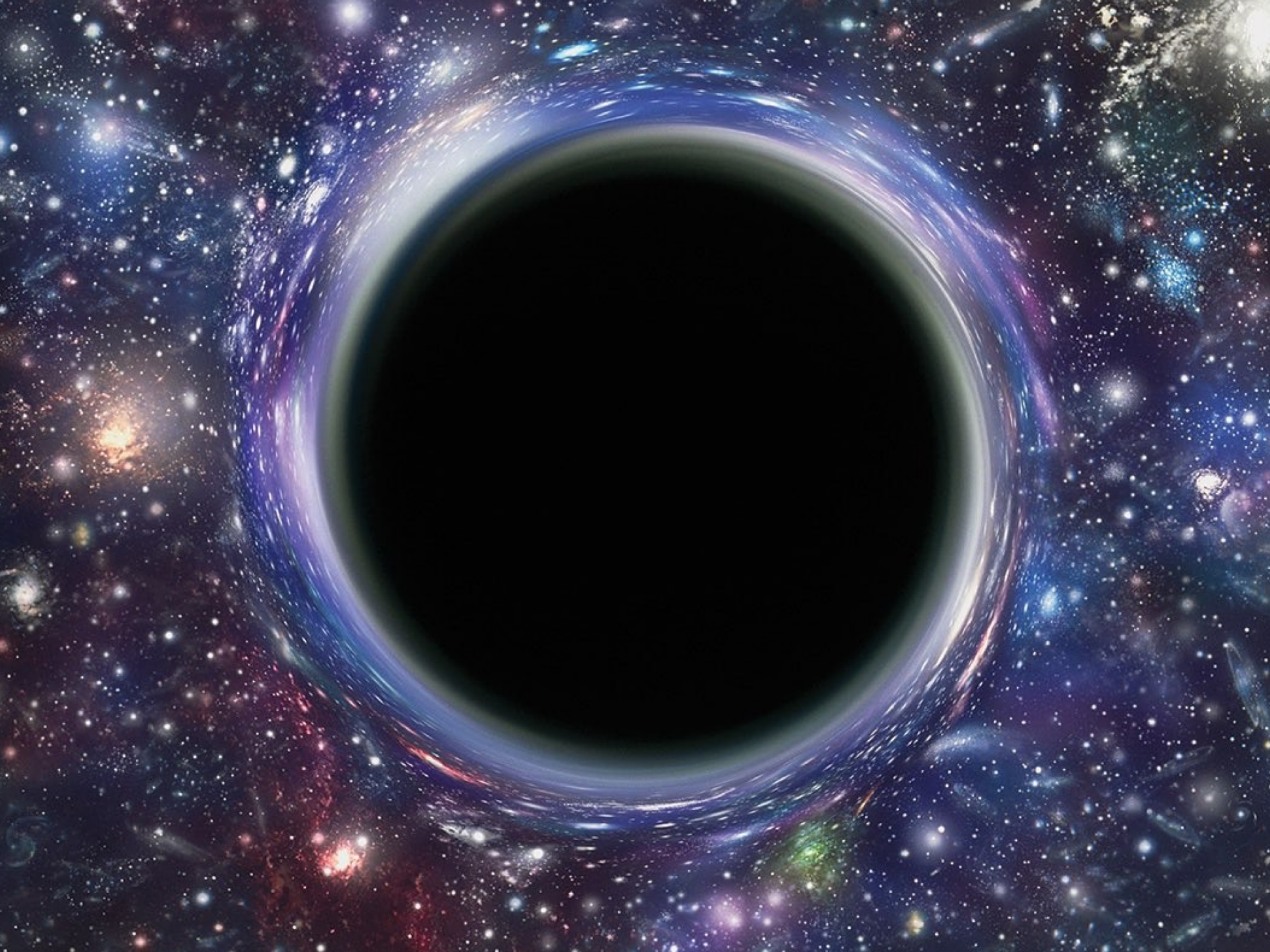
Take a different perspective: use analogies



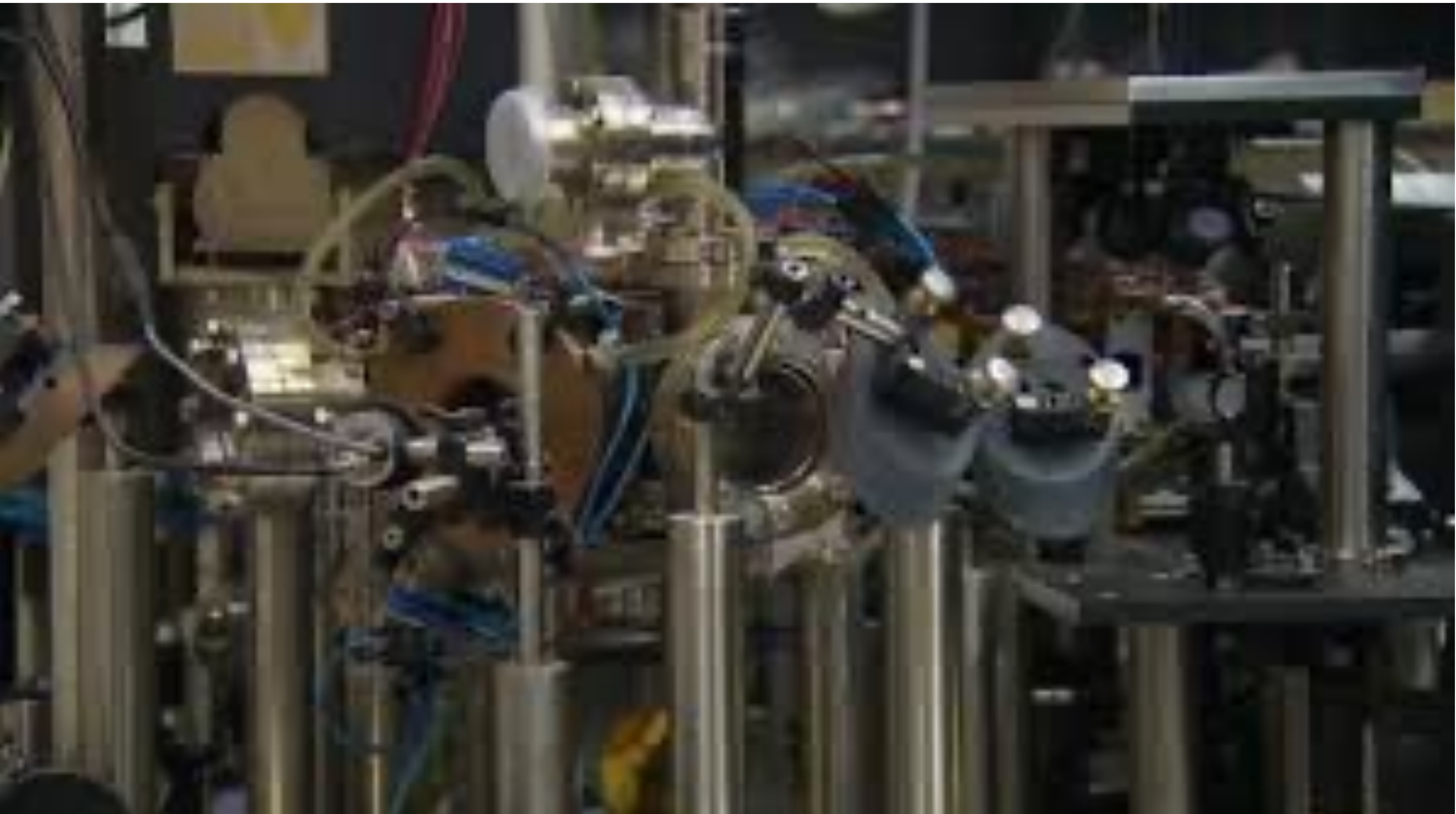
Maybe your problem can be rephrased (mapped) as a different solvable problem

In some cases, we have not direct access to the physical system.

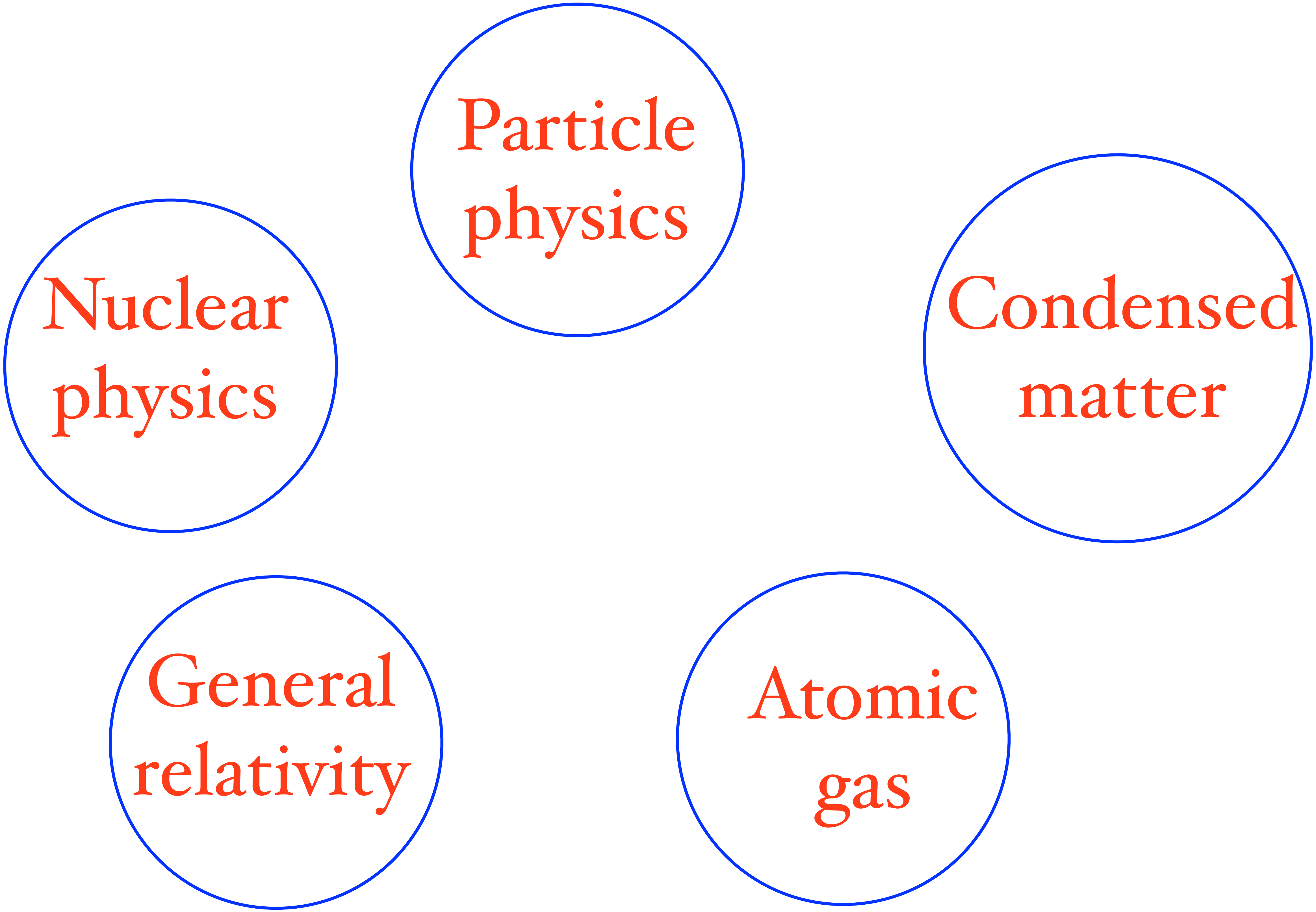
But we can realize the analog one in a lab



BH analog in a lab



The richness of physics



Some analogies

Particle propagation ⟷ Wave propagation

Higgs mechanism ⟷ Meissner effect

Kaon oscillations

Neutrino oscillations

B-meson oscillations

Light polarization

Phonon in a fluid ⟷ Photons in GR

The realm of the analogy I

In most cases the analogy is about **kinematics not dynamics**

The analogy holds in **restricted energy regions**, for restricted processes

- **Higgs-Anderson mechanism**

Sponantenous breaking of a local symmetry $\Rightarrow$ Gauge field acquire mass M

Range of the gauge field propagation $\sim 1/M$

Higgs mechanism
or W and Z bosons of the

analogous

Anderson effect
magnetic field screening in superconductors

The interactions of photons, Z and W bosons are completely different

The realm of the analogy II

- ***Particle-wave duality (analogy)***

Propagation of **massless bosons**

analogous

wave propagation in hydodynamics

$$\frac{\partial^2\psi}{\partial t^2} - c^2\nabla^2\psi = 0$$

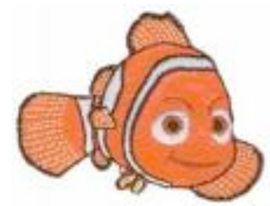
This analogy is valid in the absence of interactions.

Including interactions the particle behavior is different: scattering, quantum corrections etc.

Gravity analogs

If we can rephrase a given problem as a geometrical problem we can look for a solution using the analogy with general relativity (GR)

A particle in a moving medium

Our particle: the phonon 

Phonon velocity in the comoving frame $\mathbf{c}_\mathrm{s} = c_\mathrm{s}\,\hat{\mathbf{n}}$ with $|\hat{\mathbf{n}}|^2 = 1$

Velocity of the fluid $\mathbf{v}$

Phonon velocity in the lab frame $$\frac{d\mathbf{x}}{dt} = c_s\hat{\mathbf{n}} + \mathbf{v}$$

A simple geometrical picture

Rewrite $\dfrac{d\mathbf{x}}{dt} = c_s \hat{\mathbf{n}} + \mathbf{v}$ as $c_s \hat{\mathbf{n}}\, dt = d\mathbf{x} - \mathbf{v}\, dt$

Square it $c_s^2 dt^2 - (d\mathbf{x} - \mathbf{v} dt)^2 = 0$

$$\Updownarrow$$

Null geodesic $g_{\mu\nu} dx^\mu dx^\nu = 0$

acoustic metric $g_{\mu\nu} = \left(\begin{array}{c|c} c_s^2 - v^2 & \mathbf{v}^t \\ \hline \mathbf{v} & -I \end{array} \right)$

Note that $\sqrt{-g} = \sqrt{-\det g} = c_s$

Acoustic metric

Promoting to special relativity we have that

$$g_{\mu\nu} = \eta_{\mu\nu} + \left(c_s^2 - 1\right) v_\mu v_\nu \qquad \text{where} \qquad v_\mu = \gamma(1, -\mathbf{v})$$

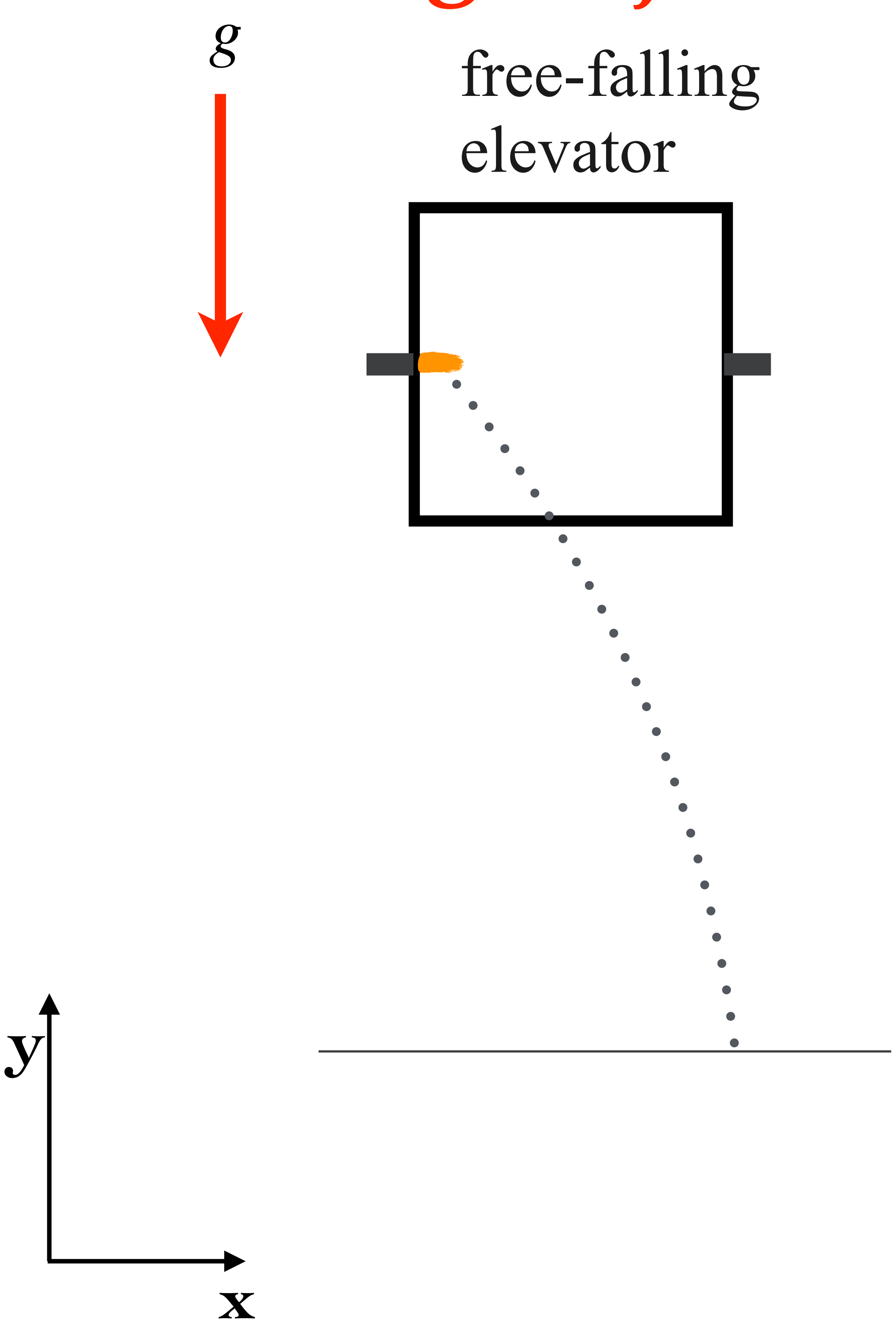
flat spacetime

in-medium effects

Particle in a moving medium $\overset{?}{\longleftrightarrow}$ Particle in gravity

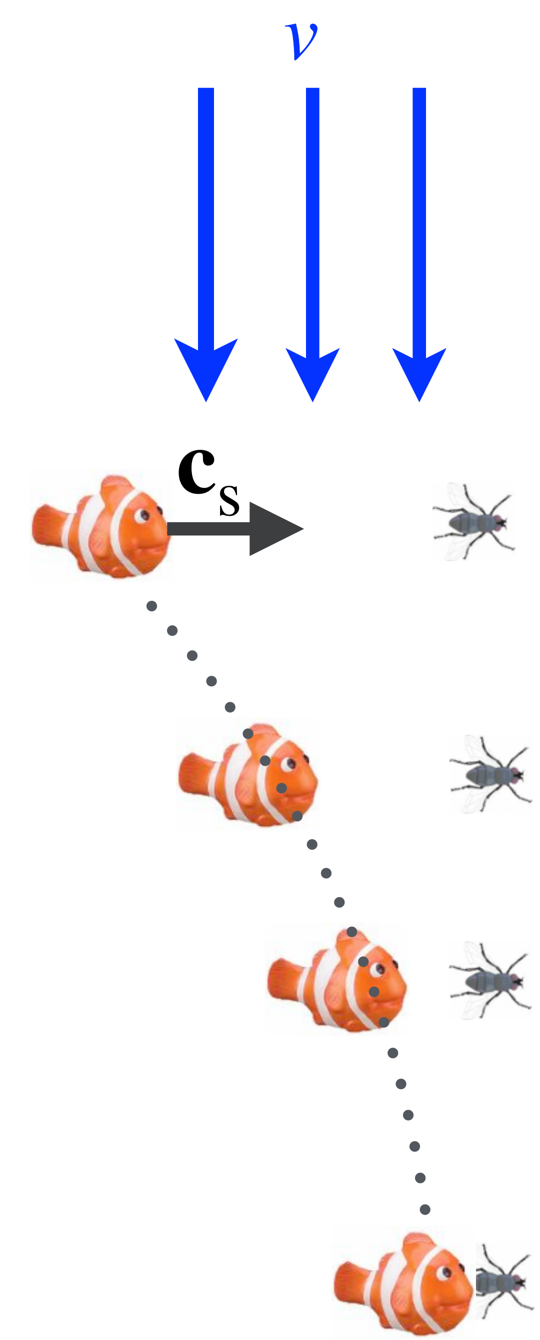
To which extent does it hold?

Bending trajectories



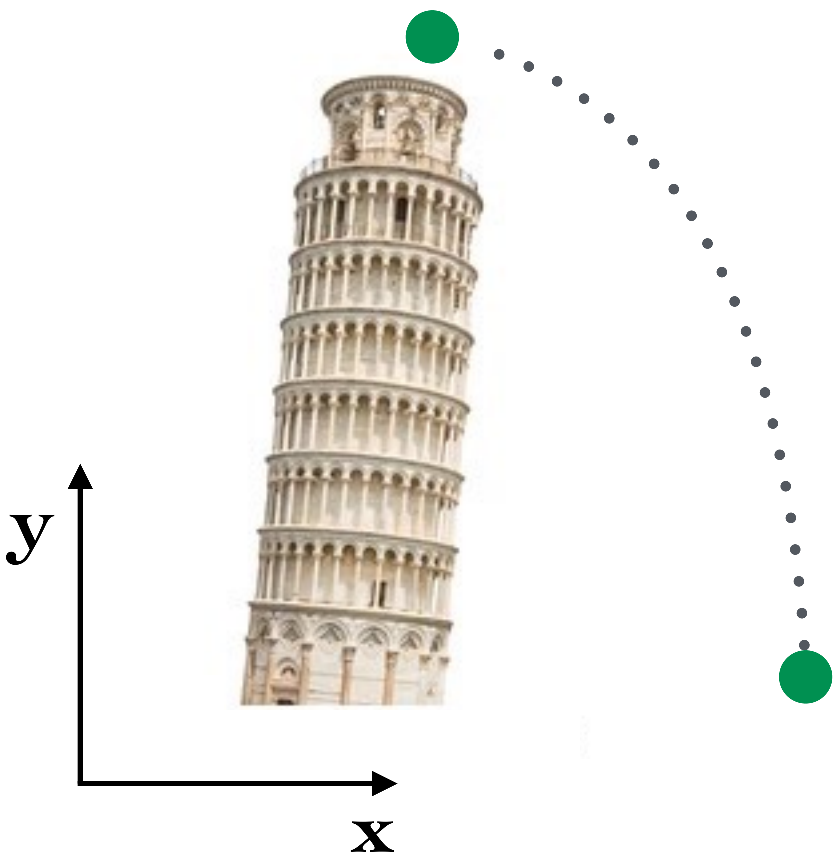
A bent trajectory

$$\frac{dy}{dx} = \frac{v}{c_s}$$



A **velocity space gradient** produces the analog of light bending

Space gradients to emulate gravity

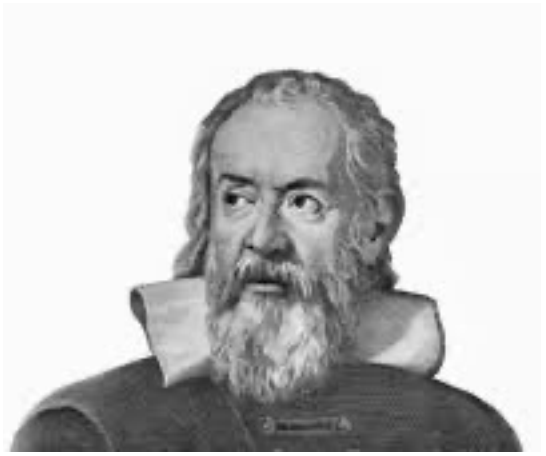


Gravity

$$v_x = c_s \qquad dx = c_s dt \qquad x = c_s t$$

$$v_y = gt \qquad dy = g\,t\,dt \qquad y = \frac{1}{2} g t^2$$

trajectory $y = \dfrac{g}{2c_s^2} x^2$



Galileo

An analog model

Unrhu

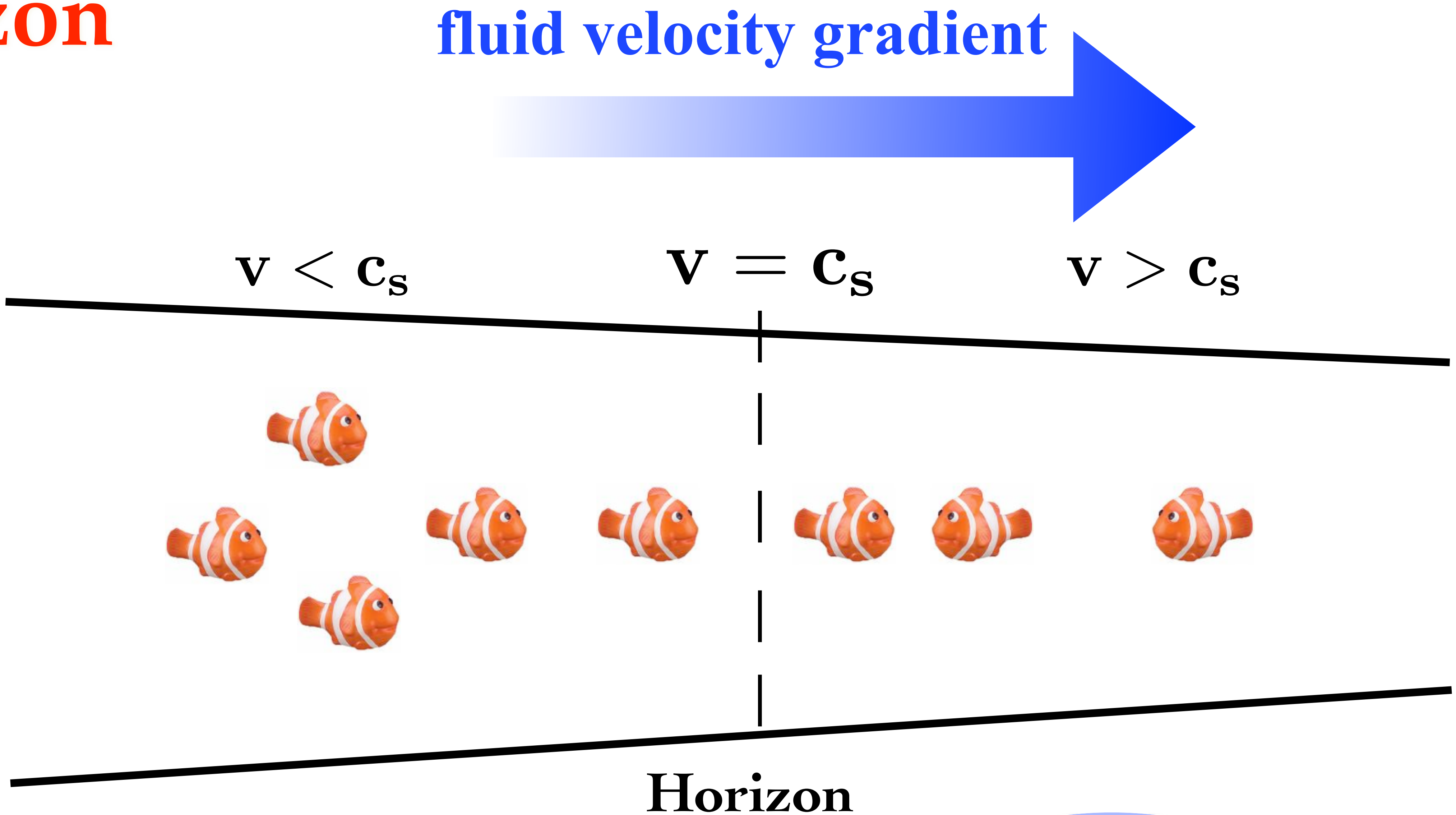
$$v_x = c_s \qquad dx = c_s dt$$

$$v_y = k\frac{x}{c_s} \qquad dy = k\frac{x}{c_s} dt \qquad dy = k\frac{x}{c_s^2} dx$$

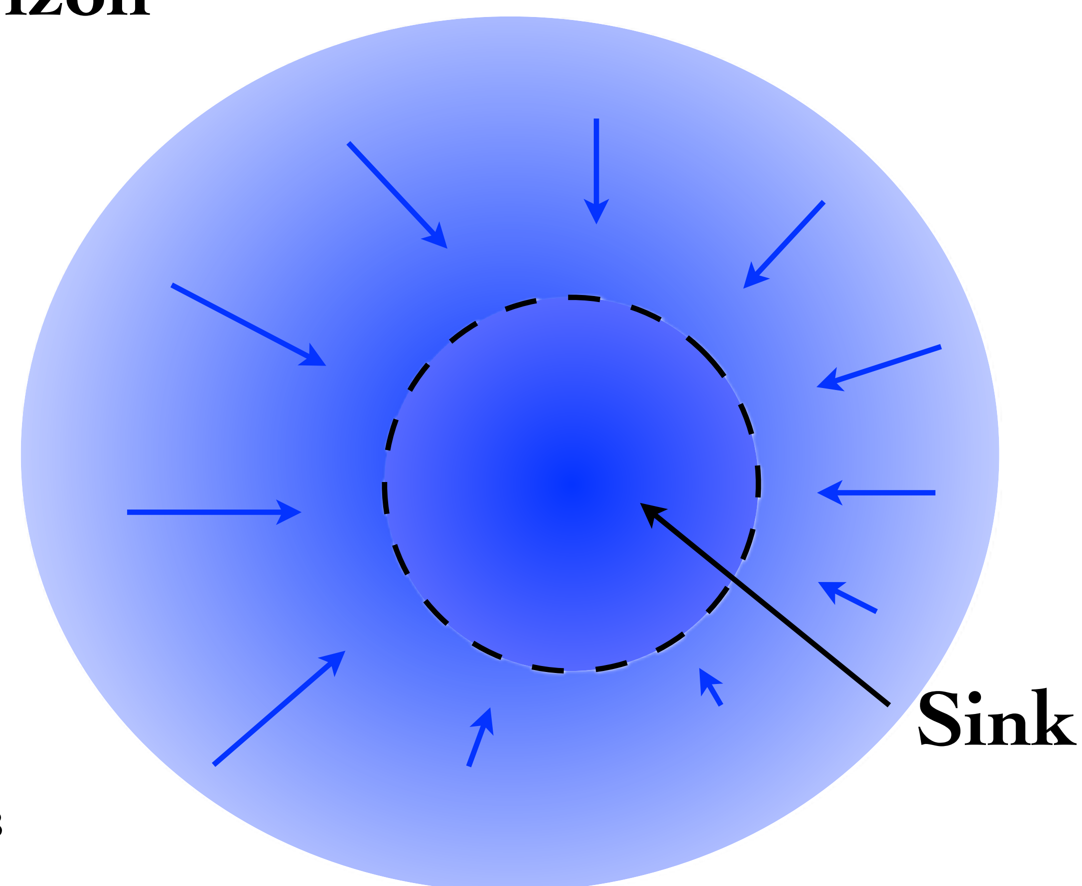
trajectory $y = \dfrac{k}{2c_s^2} x^2$

***k* is related to the "surface" acceleration**

Horizon



Trapped surface



Is gravity an emerging phenomenon?

Gravity as emerging from a more fundamental theory has been proposed by many, including Sakharov

In Einstein's theory of gravitation one postulates that the action of space-time depends on the curvature (R is the invariant of the Ricci tensor):

$$S(R) = -\frac{1}{16\pi G}\int (dx)\sqrt{-g}R. \tag{1}$$

The presence of the action (1) leads to a "metrical elasticity" of space, i.e., to generalized forces which oppose the curving of space.

Here we consider the hypothesis which identifies the action (1) with the change in the action of quantum fluctuations of the vacuum if space is curved. Thus, we consider the metrical elasticity of space as a sort of level displacement effect (cf. also Ref. 1).[1)]

An interesting one page reading

Vacuum quantum fluctuations in curved space and the theory of gravitation

A. D. Sakharov

Dokl. Akad. Nauk SSSR **177**, 70–71 (1967) [Sov. Phys. Dokl. **12**, 1040–1041 (1968). Also S14, pp. 167–169]

Usp. Fiz. Nauk **161**, 64–66 (May 1991)

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In present-day quantum field theory it is assumed that the energy-momentum tensor of the quantum fluctuations of the vacuum $T^i_k(0)$ and the corresponding action $S(0)$, formally proportional to a divergent integral of the fourth power over the momenta of the virtual particles of the form $\int k^3 dk$, are actually equal to zero.

Recently Ya. B. Zel'dovich[3] suggested that gravitational interactions could lead to a "small" disturbance of this equilibrium and thus to a finite value of Einstein's cosmological constant, in agreement with the recent interpretation of the astrophysical data. Here we are interested in the dependence of the action of the quantum fluctuations on the curvature of space. Expanding the density of the Lagrange function in a series in powers of the curvature, we have (A and $B \sim 1$)

$$(R) = \mathcal{L}(0) + A\int k dk\cdot R + B\int \frac{dk}{k} R^2 + \dots \tag{2}$$

The first term corresponds to Einstein's cosmological constant.

The second term, according to our hypothesis, corresponds to the action (1), i.e.,

$$G = -\frac{1}{16\pi A\int k dk}, \quad A \sim 1. \tag{3}$$

The third term in the expansion, written here in a provisional form, leads to corrections, nonlinear in R, to Einstein's equations.[2)]

The divergent integrals over the momenta of the virtual particles in (2) and (3) are constructed from dimensional considerations. Knowing the numerical value of the gravitational constant G, we find that the effective integration limit in (3) is

$$k_0 \sim 10^{28}\ \text{eV} \sim 10^{+33}\,\text{cm}^{-1}.$$

In a gravitational system of units, $G = \hbar = c = 1$. In this case $k_0 \sim 1$. According to the suggestion of M. A. Markov, the quantity k_0 determines the mass of the heaviest particles existing in nature, and which he calls "maximons." It is natural to suppose also that the quantity k_0 determines the limit of applicability of present-day notions of space and causality.

Consideration of the density of the vacuum Lagrange function in a simplified "model" of the theory for noninteracting free fields with particles $M \sim k_0$ shows that for fixed ratios of the masses of real particles and "ghost" particles (i.e., hypothetical particles which give an opposite contribution from that of the real particles to the R-dependent action), a finite change of action arises that is proportional to M^2R and which we identify with R/G. Thus, the magnitude of the gravitational interaction is determined by the masses and equations of motion of free particles, and also, probably, by the "momentum cutoff."

This approach to the theory of gravitation is analogous to the discussion of quantum electrodynamics in Refs. 4 to 6, where the possibility is mentioned of neglecting the Lagrangian of the free electromagnetic field for the calculation of the renormalization of the elementary electric charge. In the paper of L. D. Landau and I. Ya. Pomeranchuk the magnitude of the elementary charge is expressed in terms of the masses of the particles and the momentum cutoff. For a further development of these ideas see Ref. 7, in which the possibility is established of formulating the equations of quantum electrodynamics without the "bare" Lagrangian of the free electromagnetic field.

The author expresses his gratitude to Ya. B. Zel'dovich for the discussion which acted as a spur for the present paper, for acquainting him with Refs. 3 and 7 before their publication, and for helpful advice.

1) Here the molecular attraction of condensed bodies is calculated as the result of changes in the spectrum of electromagnetic fluctuations. As was pointed out by the author, the particular case of the attraction of metallic bodies was studied earlier by Casimir.[2]

2) A more accurate form of this term is $\int (dk/k)(BR^2 + CR^{ik}R_{ik} + DR^{iklm}R_{iklm} + ER^{iklm}R_{iklm})$ where $A, B, C, D, E \sim 1$. According to Refs. 4 to 7, $\int dk/k \sim 137$, so that the third term is important for $R \gtrsim 1/137$ (in gravitational units), i.e., in the neighborhood of the singular point in Friedman's model of the universe.

1. E. M. Lifshits, ZhETF 29:94 (1954); Sov. Phys. JETP 2:73 (1954), trans.
2. H. B. G. Casimir, Proc. Nederl. Akad. Wetensch. 51:793 (1948).
3. Ya. B. Zel'dovich, ZhETF Pis'ma 6:922 (1967); JETP Lett. 6:345 (1967), trans.
4. E. S. Fradkin, Dokl. Akad. Nauk SSSR 98:47 (1954).
5. E. S. Fradkin, Dokl. Akad. Nauk SSSR 100:897 (1955).
6. L. D. Landau and I. Ya. Pomeranchuk, Dokl. Akad. Nauk SSSR 102:489 (1955), trans. in Landau's Collected Papers (D. ter Haar, ed.), Pergamon Press, 1965.
7. Ya. B. Zel'dovich, ZhETF Pis'ma 6:1233 (1967).

 0038-5670/91/050394-01$01.00

Gravity analogs
using a barotropic fluid

Starting from Euler equations

Description of the fluid

Continuity equation $$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

Euler equation $$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right) = \mathbf{f}$$

Characteristics of the fluid

- barotropic $p \equiv p(\rho)$
- inviscid $\mathbf{f} = -\nabla p$
- irrotational $\mathbf{v} = \nabla \phi$

Small perturbations

Fluctuations around a background configuration

$$
\begin{aligned}
\rho &= \rho_0 + \epsilon\rho_1 + \mathcal{O}(\epsilon^2) \\
p &= p_0 + \epsilon p_1 + \mathcal{O}(\epsilon^2) \\
\phi &= \phi_0 + \epsilon\phi_1 + \mathcal{O}(\epsilon^2)
\end{aligned}
\qquad \mathbf{v}_0 = \nabla\phi_0 \qquad \mathbf{v}_1 = \nabla\phi_1
$$

Bulk **Acoustic perturbation**

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0$$ general

$$\frac{\partial\rho_0}{\partial t} + \nabla\cdot(\rho_0\mathbf{v_0}) = 0$$ bulk

$$\frac{\partial\rho_1}{\partial t} + \nabla\cdot(\rho_0\mathbf{v_1}) + \nabla\cdot(\rho_1\mathbf{v_0}) = 0$$ perturbation

Small perturbations

Combining linearized Euler and continuity equations:

$$\frac{\partial}{\partial t}\left(c_s^{-2}\rho_0\left(\frac{\partial \phi_1}{\partial t}+\mathbf{v}_0\cdot\nabla\phi_1\right)\right)-\nabla\cdot\left(\rho_0\nabla\phi_1-c_s^{-2}\rho_0\mathbf{v}_0\left(\frac{\partial \phi_1}{\partial t}+\mathbf{v}_0\cdot\nabla\phi_1\right)\right)=0$$

where $\quad c_s^2=\dfrac{\partial p}{\partial \rho}$

$$\boxed{\mathbf{v_0}=0\,,\quad \rho_0=\mathrm{const},\quad c_s=\mathrm{const}\qquad\qquad \frac{\partial^2\phi_1}{\partial t^2}-c_s^2\nabla^2\phi_1=0}$$

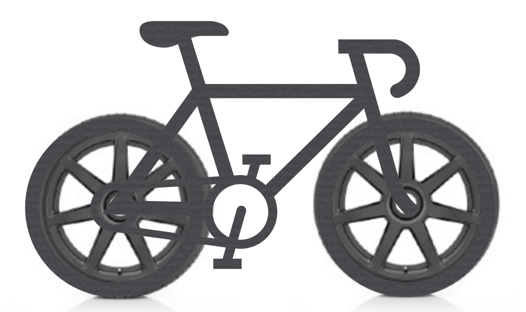
The non uniform medium changes the propagation

The gravity analog emerges

$$\frac{\partial}{\partial t}\left(c_s^{-2}\rho_0\left(\frac{\partial\phi_1}{\partial t}+\mathbf{v}_0\cdot\nabla\phi_1\right)\right)-\nabla\cdot\left(\rho_0\nabla\phi_1-c_s^{-2}\rho_0\mathbf{v}_0\left(\frac{\partial\phi_1}{\partial t}+\mathbf{v}_0\cdot\nabla\phi_1\right)\right)=0$$

Solving this equation…

GR bike



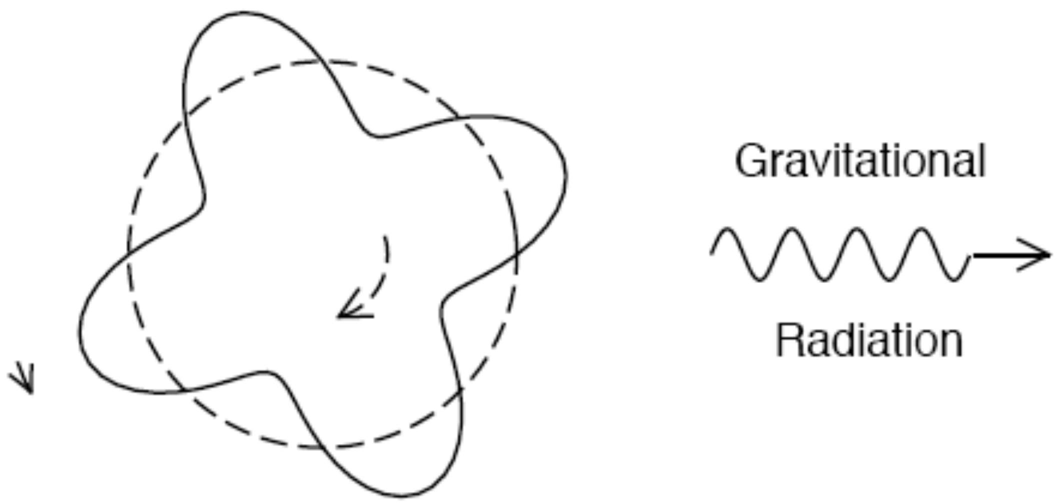
Rewrite the above equation as

$$\boxed{\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi_1\right)=0}$$

where $g_{\mu\nu}=\Omega\begin{pmatrix} c_s^2-v^2 & \mathbf{v}^t \\ \mathbf{v} & -I \end{pmatrix}$

The gravity analog at work

R-mode instability of rotating stars



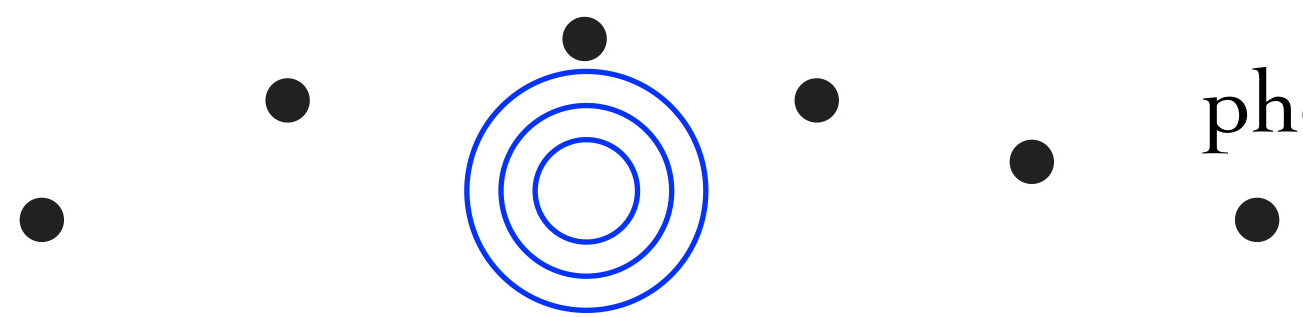
Quick spin down of pulsars

Lindblom, astro-ph/0101136
Andersson, Kokkotas
Int.J.Mod.Phys.D10:381-442,2001

Dissipative processes damp this mode

elastic phonon-vortex scattering

phonon



superfluid vortex

Analytic cross section

$$\frac{d\sigma}{d\theta} = \frac{c_s}{2\pi E}\frac{\cos^2\theta}{\tan^2\frac{\theta}{2}}\sin^2\frac{\pi E}{\Lambda}$$

MM, C. Manuel and B. A. Sa'd, Phys.Rev.Lett. 101 (2008) 241101

Acoustic vs GR

- The sound waves propagation can be described as that of a **scalar field** in an **emerging GR background**

- **The background however does not obey the Einstein equations, it indeed obeys the Euler equations!**

One can certainly calculate the **Ricci and Einstein tensors** of the fluid using the acoustic metric.
However, they **do not satisfy the Hilbert-Einstein equation**.

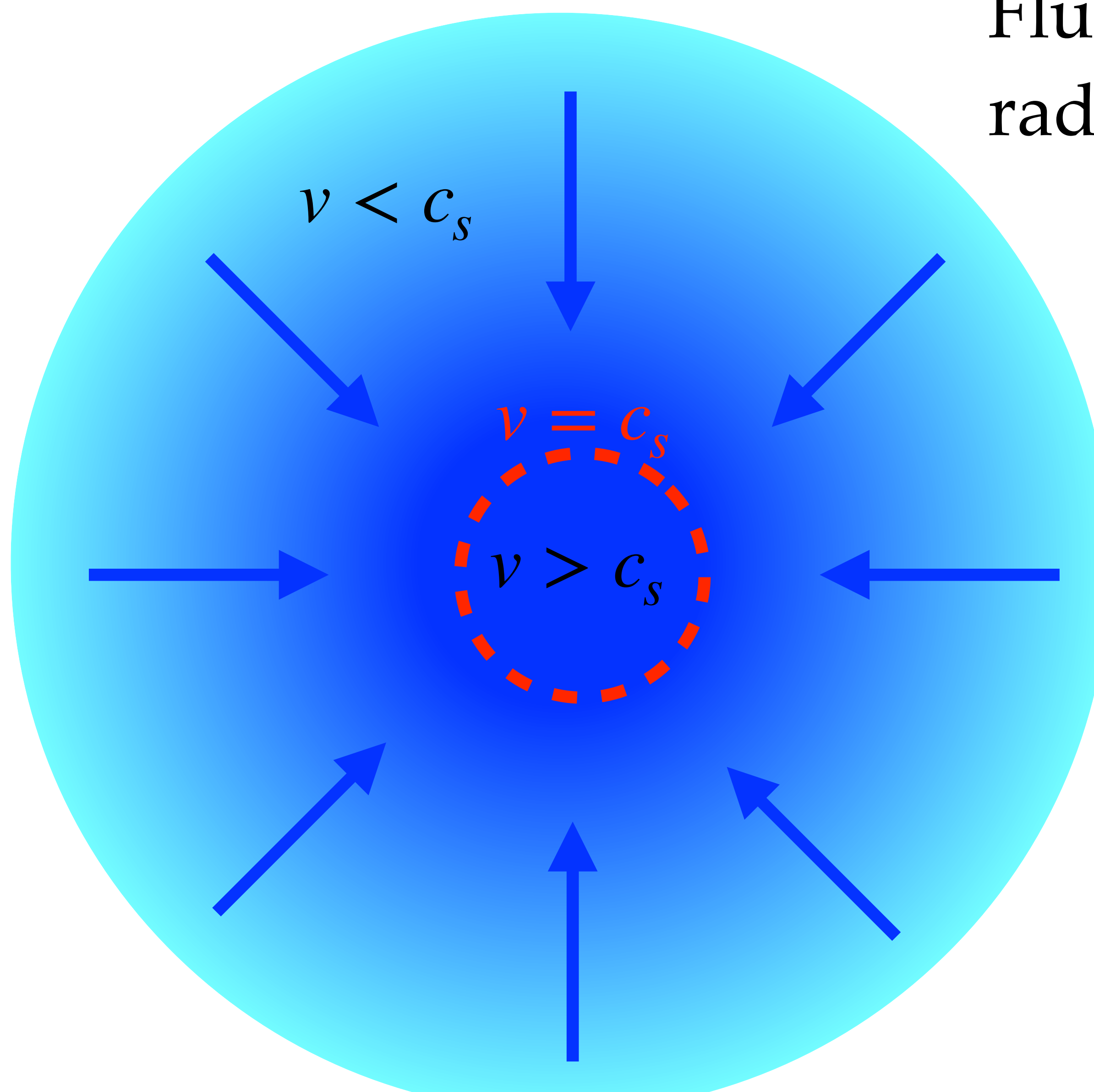
Schwarzschild acoustic metric?

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2(d\theta^2 \sin^2\theta d\phi^2)$$

Schwarzschild radius $R_s = 2M$

Fluid with radial flow

Does the fluid analog exist?



Acoustic metric

$$\boxed{ds^2 = \frac{\rho}{c_s}\Big(-(c_s^2 - v^2)dt^2 + 2\mathbf{v}\cdot\mathbf{dx}\,dt + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\Big)}$$

Painleve'–Gullstrand representation of Schwarzschid metric

$$\boxed{ds^2 = -\left(1 - \frac{2GM}{r}\right)dt^2 \pm \sqrt{\frac{2GM}{r}}drdt + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)}$$

$$v \propto \frac{1}{\sqrt{r}}$$ divergent flow at the origin

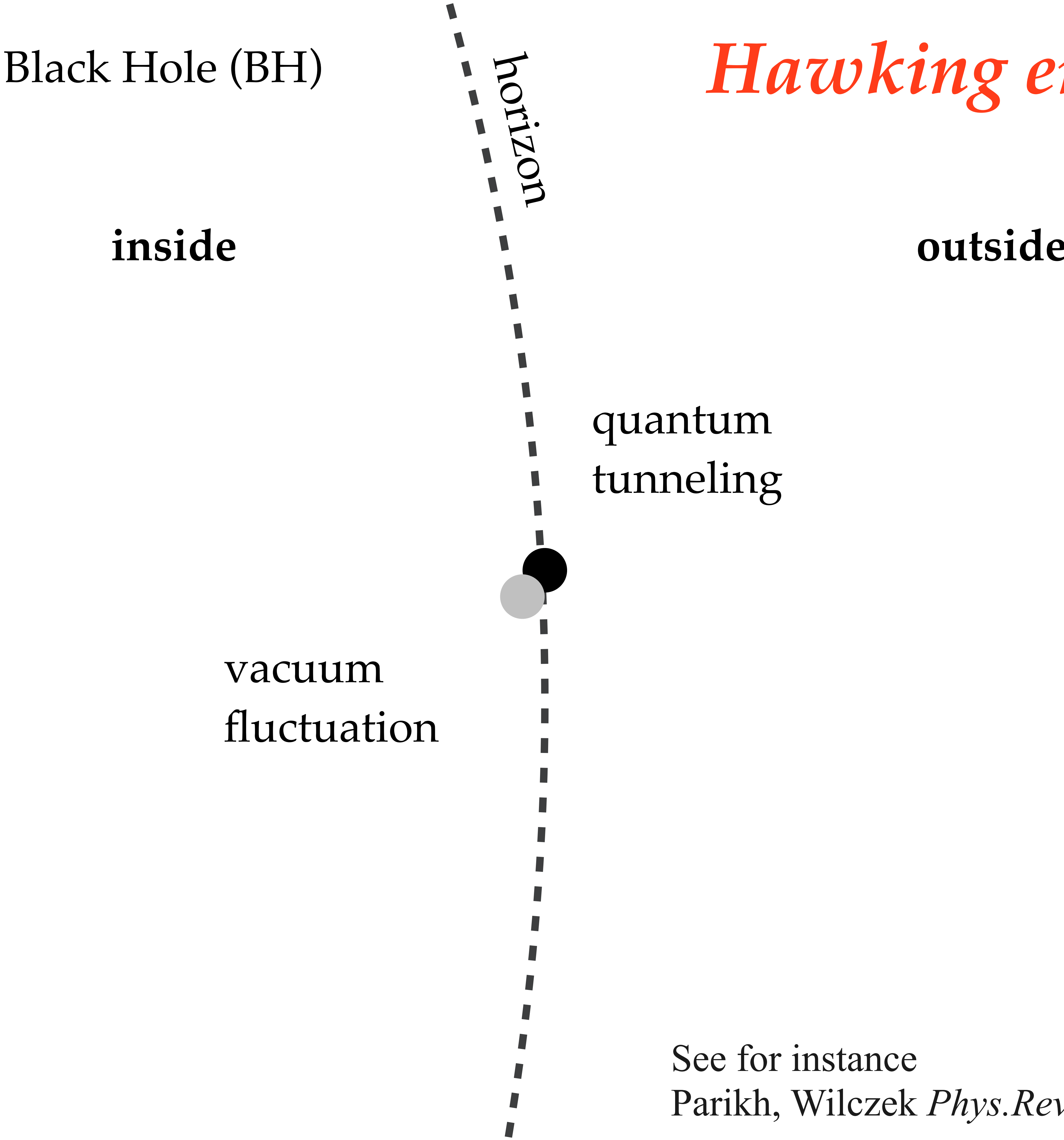
Abandon the 3D spherical geometry

Hawking radiation

S. W. Hawking, Particle creation by black holes, *Commun. Math. Phys. 43, 199 (1975)*

W. Unruh, Experimental black hole evaporation, *Phys.Rev.Lett. 46, 1351 (1981)*.

Hawking emission



See for instance
Parikh, Wilczek *Phys.Rev.Lett.* 85 (2000) 5042

BH thermodynamics

A particle/nuclear physics perspective

WKB tunneling amplitude $\Gamma \sim e^{-2\,\mathrm{Im}\,\mathrm{S}}$

where $\mathrm{S} = \int_{r_{in}}^{r_{out}} p_r\, dr$ using the geodesic equation $\mathrm{Im}\,\mathrm{S} = 4\pi\omega M$

$$\boxed{\Gamma \sim e^{-8\pi M\omega} = e^{-\omega/T} \qquad T = \frac{1}{8\pi M}}$$

BH entropy variation $dS = \frac{dQ}{T} = \frac{dM}{T} = 8\pi M dM$

Area law $S = 4\pi M^2 = \frac{4\pi R_s^2}{4} = \frac{A}{4}$

A dim emission

$$T = \frac{\hbar c^3}{8\pi G k_B M} \simeq 6 \times 10^{-8} \left(\frac{M_\odot}{M}\right) K$$

$$T = \frac{g}{2\pi} \qquad g = \frac{M}{R_s^2} = \frac{M}{4M^2} = \frac{1}{4M} \qquad g = \left(1 - \frac{2M}{r}\right)'\bigg|_H$$

If an acoustic hole is realizable and if it emits the Hawking radiation is it detectable?

By analogy, the temperature of an acoustic hole $T = \frac{1}{2\pi} \frac{\partial |c_s - v|}{\partial n}\bigg|_H$

$$T \simeq mc_s^2 \simeq 10^{-9} K$$

Boson isotope with a large mass: ^{87}Rb

The speed of sound is small $c_s \sim \mathrm{mm\,s^{-1}}$

Quick recap

To have an horizon we need a transonic flow

$$v < c_s \qquad v = c_s \qquad v > c_s$$

It cannot be 3D

- We need to embed quantum effects

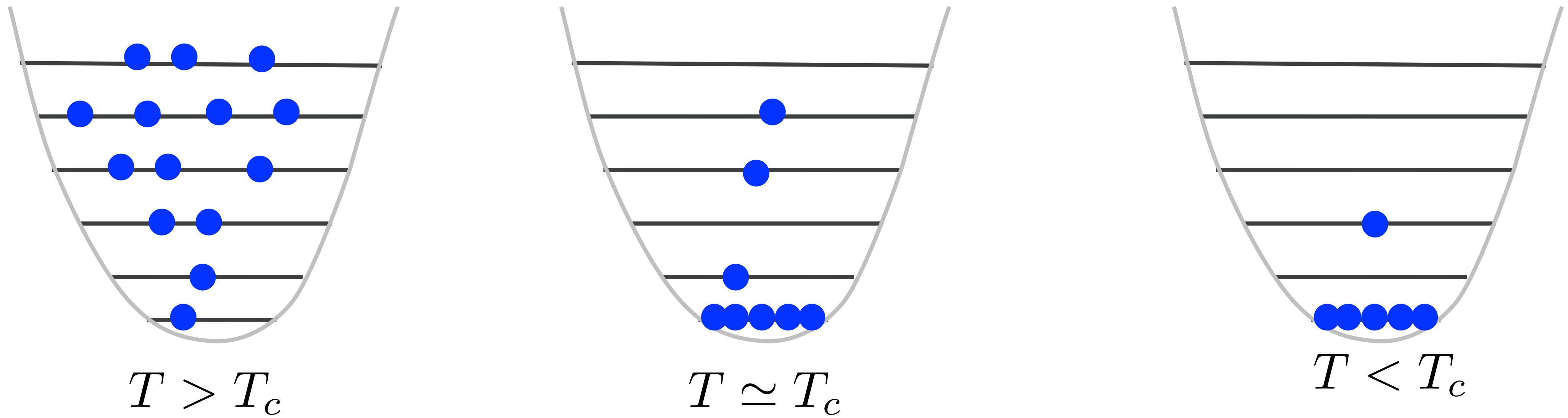
- Measure a dim phonon emission

- **How to avoid turbulence?** Use a Bose-Einstein condensate!

Bose-Einstein condensate (BEC)

It is a **coherent state of matter** with a “thermodynamically” large number of particles in the same quantum state

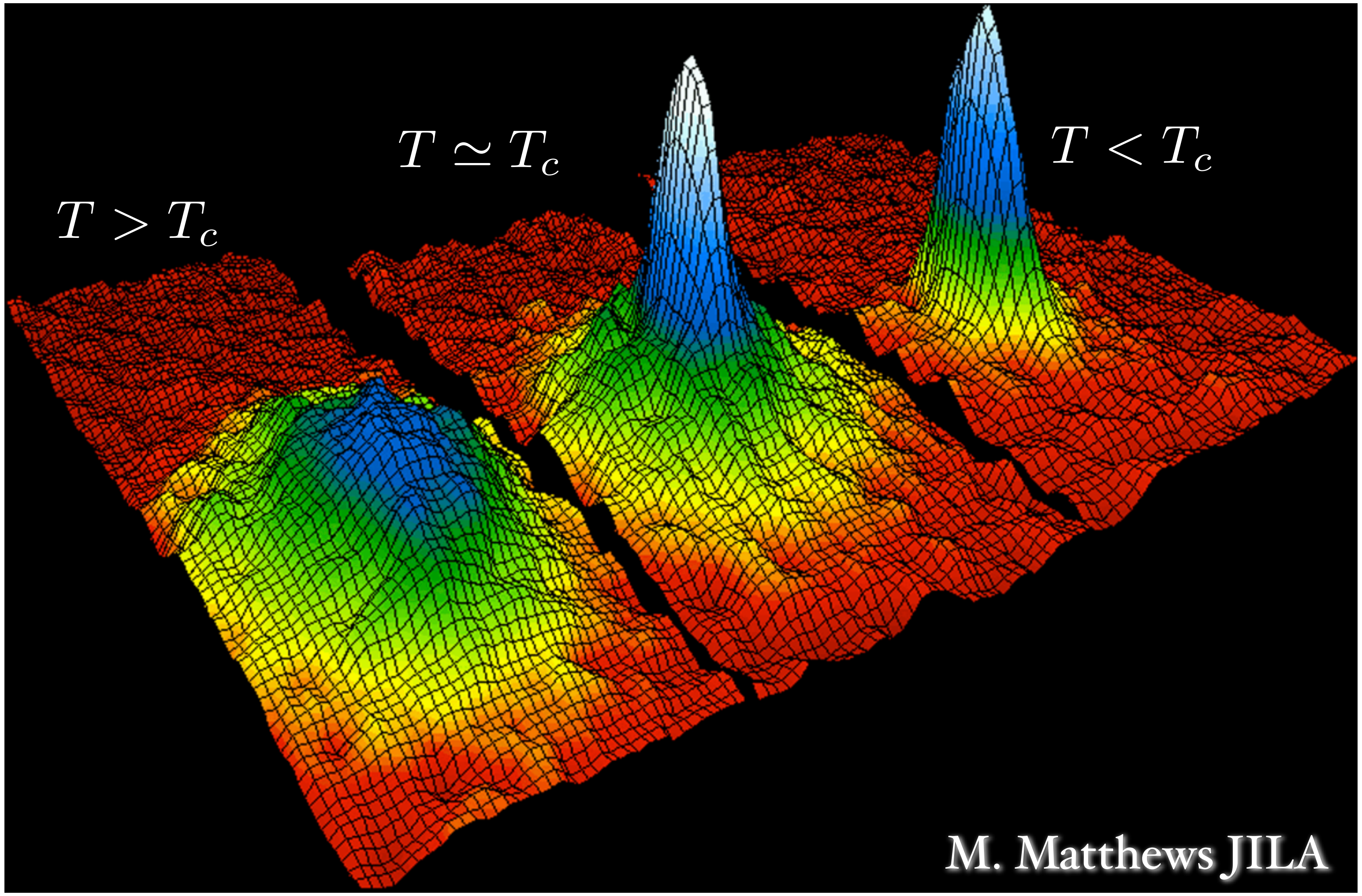
BOSONS@ low temperature in a potential well



Requirements:

1. Particles must be **bosons**
2. **Cold system**: A fight between thermal disorder and quantum coherence
3. Particles must be **stable**

Ultracold atoms in an optical trap

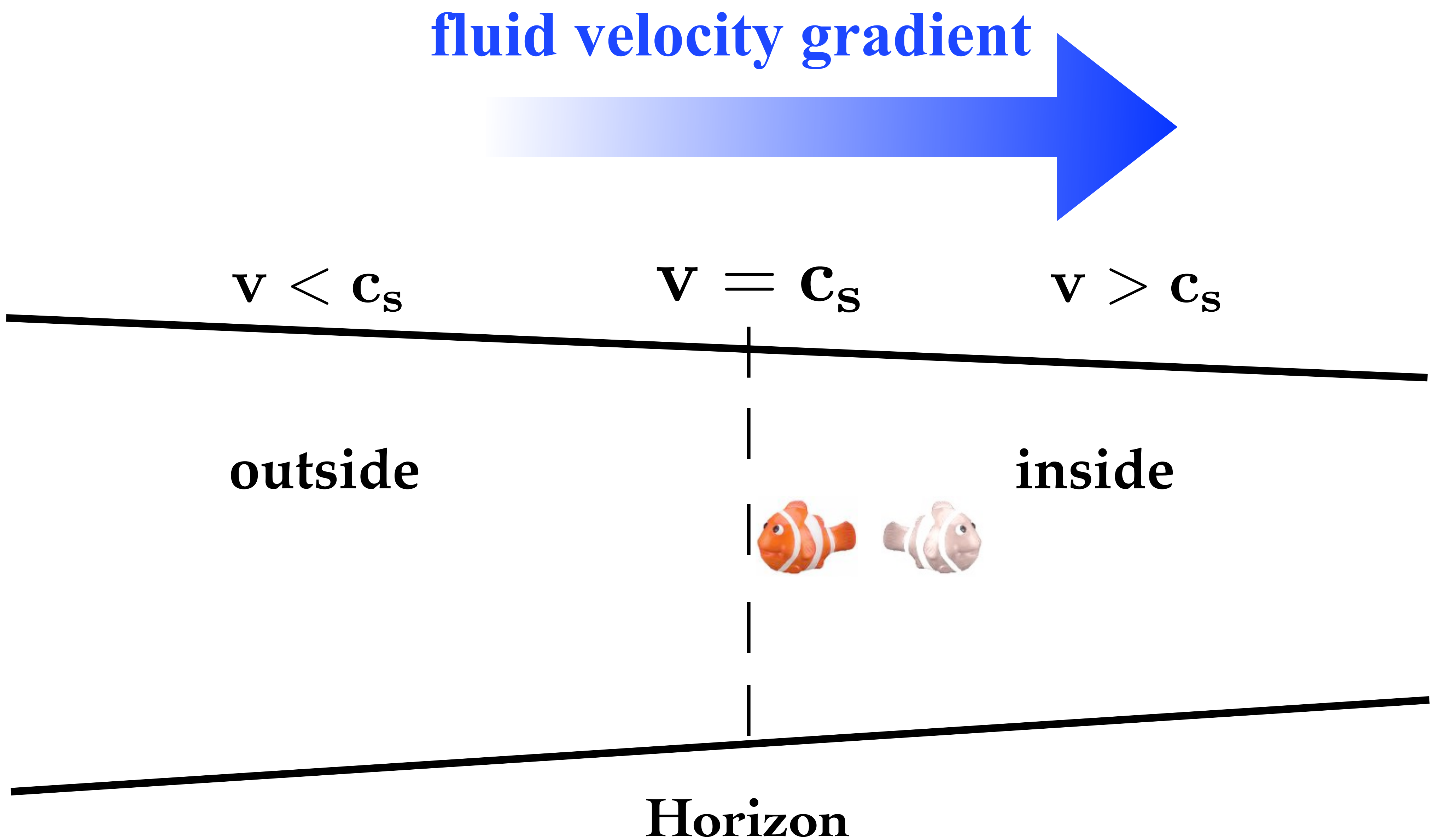


Velocity distribution of ^{87}Rb atoms $\quad T_c \simeq 200$ nK

1. ^{87}Rb is **bosonic**
2. can be **cooled**
3. has a lifetime of about $\mathbf{10^{10}}$ **years** (the experiment lasts $\sim 10^3$s)

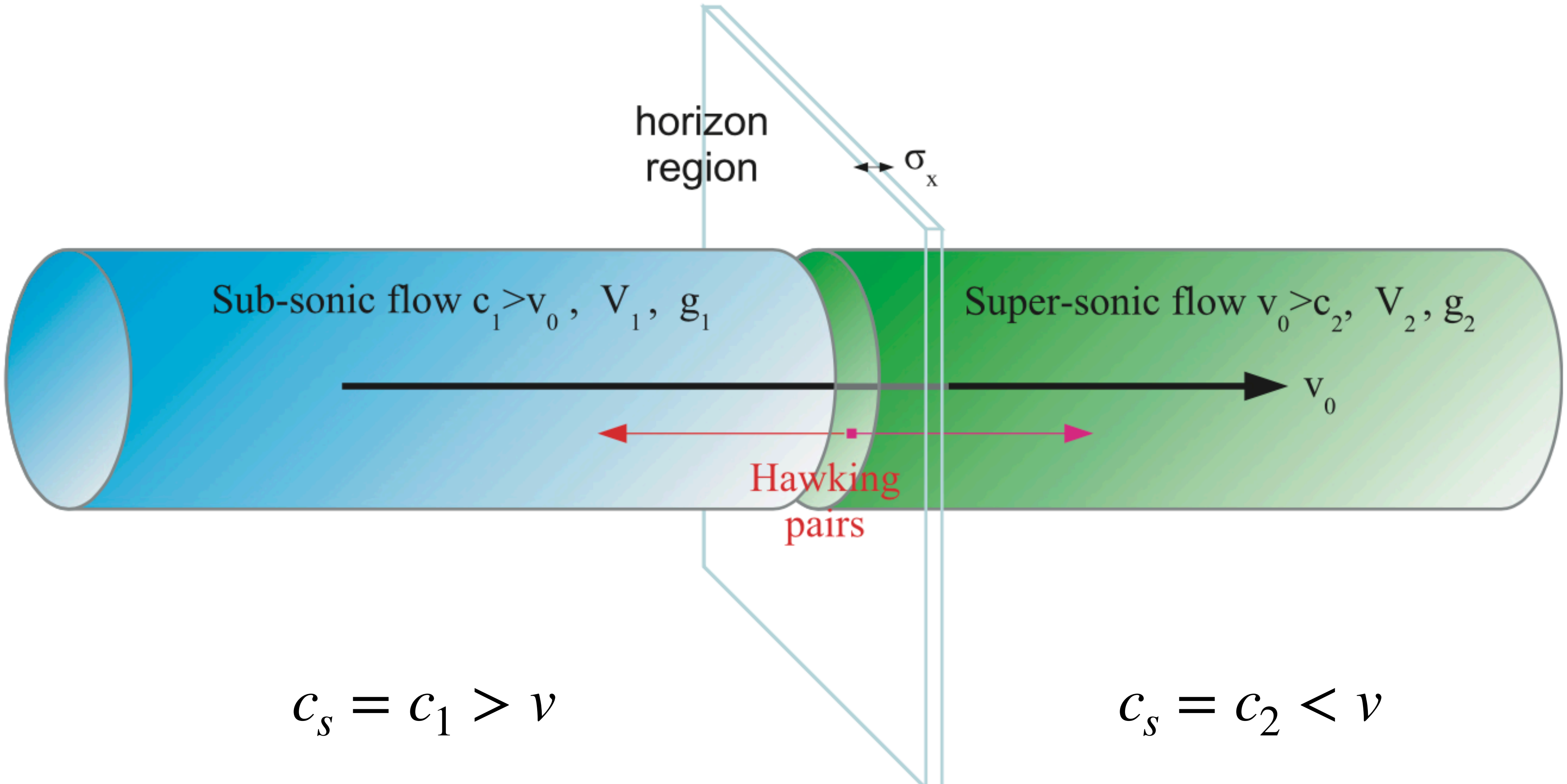
Quantum effects

Quantum effects in the analog picture



The phonon to escape has to do quantum tunneling

Setup: trapped BEC condensate



$c_s = c_1 > v$

$c_s = c_2 < v$

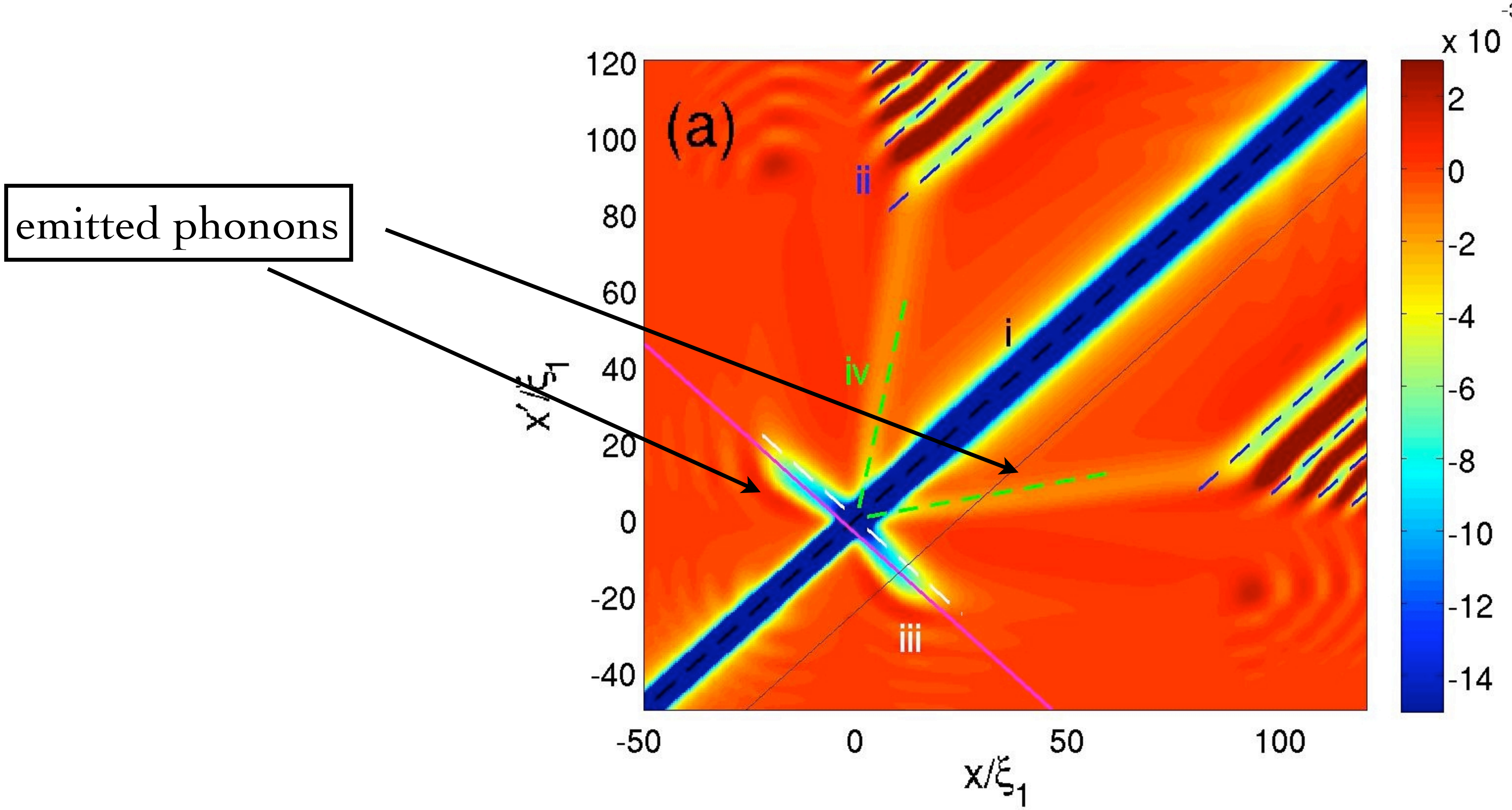
Carusotto *et al New J. Phys.* 10 103001 (2008)

Instead of changing the velocity, change the speed of sound

Detection strategy

The phonon emission perturbs the system producing long-range density correlations

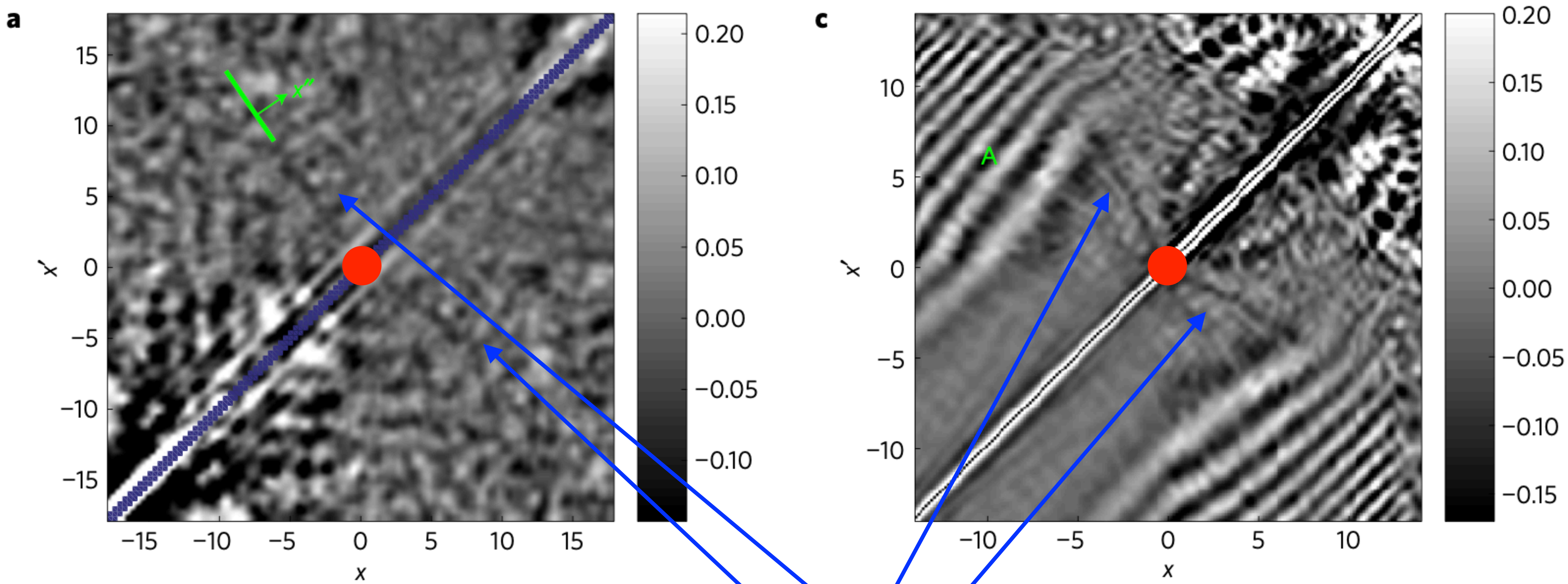
Parametric plot of the density-density correlation function



Carusotto *et al New J. Phys.* 10 103001 (2008)

Experimental observation

experiment numeric



black hole position

effect of phonon emission on the density

Image obtained by 4600 repetitions of the experiment

Fitted Hawking temperature $\sim 10^{-9}K$

Steinhauer, Nature Phys. 12 (2016) 959

Kinetic theory

From GR

R. W. Lindquist, Annals of Physics 37, 487 (1966).
J. Stewart, Lecture Notes in Physics, Lecture Notes in Physics No. v. 10 (Springer-Verlag, 1969).

To the analog model

MM and C. Manuel, Phys.Rev.D 77 (2008) 103014
MM, D. Grasso, S. Trabucco and L. Chiofalo Phys.Rev.D 103 (2021) 7, 076001

Lagrangian formulation

Consider the Lagrangian for a scalar field $\mathscr{L} \equiv \mathscr{L}(\phi, \partial_\mu \phi)$

Scale separation

$$\phi(x) = \underbrace{\phi_0(x)}_{\text{background / long-wavelength}} + \epsilon \underbrace{\phi_1(x)}_{\text{“phonon” / “short-wavelength”}}$$

Expand the action

$$S[\phi] = S[\phi_0] + \frac{\epsilon^2}{2} \int d^4x \left[\underbrace{\frac{\partial^2 \mathcal{L}}{\partial(\partial_\mu \phi_0)\partial(\partial_\nu \phi_0)} \partial_\mu \phi_1 \partial_\nu \phi_1 + \left(\frac{\partial^2 \mathcal{L}}{\partial \phi_0 \partial \phi_0} - \partial_\mu \frac{\partial^2 \mathcal{L}}{\partial(\partial_\mu \phi_0)\partial \phi_0} \right) \phi_1 \phi_1}_{} \right]$$

Phonon's action $S[\phi_1] = \dfrac{1}{2} \displaystyle\int d^4x \sqrt{-g} \left(g^{\mu\nu} \partial_\mu \phi_1 \partial_\nu \phi_1 - M^2_{\phi_0} \phi_1 \phi_1 \right)$

Phonon distribution

Since phonons are bosons and are emitted at a temperature T we expect that they have a Bose-Einstein distribution f

Solution of $$L[f] \equiv p^{\alpha}\frac{\partial f}{\partial x^{\alpha}} - \Gamma^{\alpha}_{\beta\gamma}p^{\beta}p^{\gamma}\frac{\partial f}{\partial p^{\alpha}} = C[f]$$

for $C[f] = 0$

Assuming $$f(x,p) = \frac{1}{\exp(p^{\mu}\beta_{\mu}) - 1}$$

$$\beta_{\lambda;\rho} + \beta_{\rho;\lambda} = 0$$ solution $$\beta^{\mu} = (\beta, \mathbf{0})$$

Thermodynamics

Knowing the distribution function we can obtain the thermodynamics

distribution function

Phonon number $n^{\mu}_{\text{ph}} = \int p^{\mu} f(x,p)\, d\mathscr{P}$ **integral measure**

Energy momentum tensor $T^{\mu\nu}_{\text{ph}} = \int p^{\mu} p^{\nu} f(x,p)\, d\mathscr{P}$

Entropy $s^{\alpha}_{\text{ph}} = -\int p^{\alpha} \left[f \ln f - (1+f) \ln(1+f) \right] d\mathscr{P}$

Transport of "phonon" number

Covariant conservation $$\partial_\nu n^\nu_{\rm ph} + \Gamma^\mu_{\mu\nu} n^\nu_{\rm ph} = \int C[f] d\mathcal{P}$$

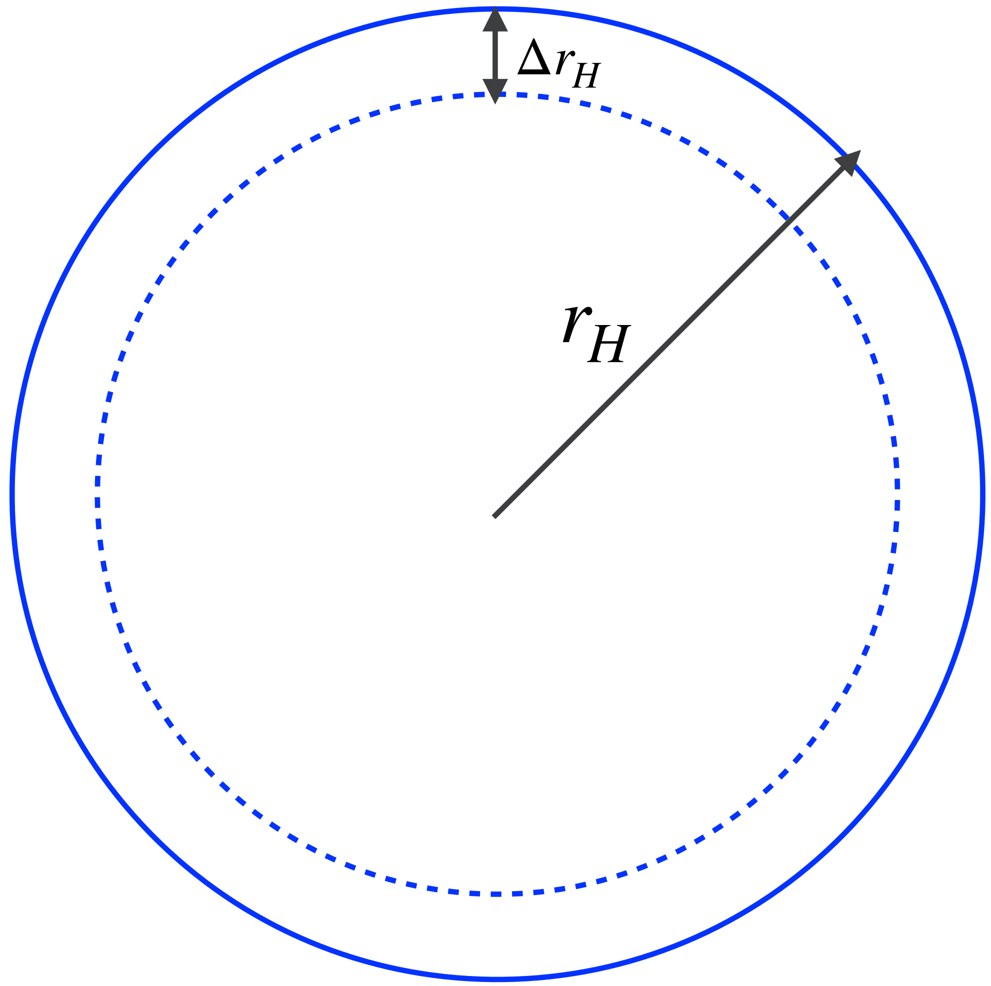
collision integral

Where $$\Gamma^\mu_{\mu\nu} = \frac{1}{\sqrt{-g}} \partial_\nu \sqrt{-g} = \frac{1}{c_s} \frac{\partial c_s}{\partial x^\nu}$$

We keep $C[f] = 0$

Change in the number of phonons due to the background non uniformity!

Hawking temperature



radius variation due to phonon emission

Associate an entropy to the sonic hole $S_H = \dfrac{A}{4L_c^2}$

Entropy variation due to horizon shrinking $\Delta S_H = 2\pi \dfrac{r_H}{L_c^2} \Delta r_H$

The phonon emission results in an entropy loss of the horizon

The entropy flux

The entropy lost by the horizon is gained by the phonon gas

$$\Delta S_{\mathrm{ph}} = -\,\Delta S_{\mathrm{H}}$$

The actual entropy flux

From the Fluid $\longrightarrow$ To the Phonon gas

by means of the horizon

There are dissipative processes localized at the horizon

Entropy balance

Entropy loss of the fluid $$\Delta S_H = 2\pi \frac{r_H}{L_c^2} \Delta r_H$$

Entropy gain of the phonon gas $$\Delta S_{\rm ph} = 4\pi r_H^2 \, d_g \, s_{\rm ph} \Delta r_H$$

number of degrees of freedom

$$s_{\rm ph} = \frac{\pi T}{6 L_c^2 C x}$$

with $C = (v + c_s)'|_H$

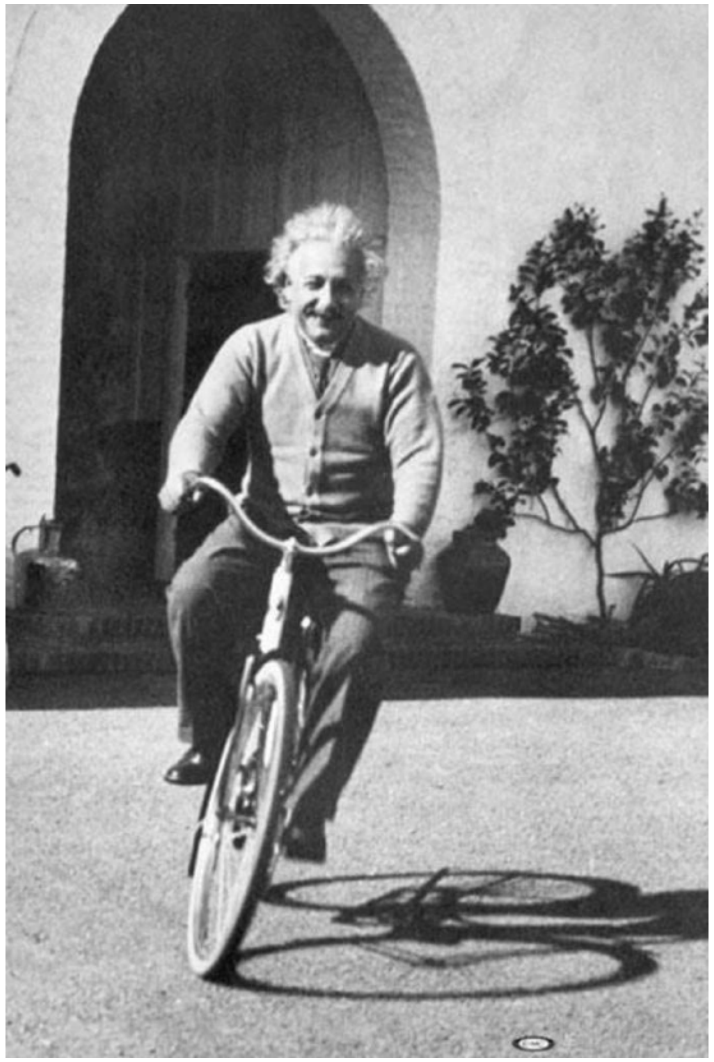
$$\Delta S_{\rm ph} = -\Delta S_{\rm H} \quad \Longrightarrow \quad T = \frac{1}{2\pi} \left(\frac{c_s - |v|}{1 - c_s |v|} \right)' \Bigg|_H$$

Conclusion

- **There is a large number of physical systems linked by analogies**

- **We can use them to solve some problems or to reproduce phenomena of unreachable systems**

- **We focused on Hawking radiation in superfluids, but this was just an example**



Thanks for your attention!

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Backup slide

Not only fluids

1. Dielectric media: A refractive index can be reinterpreted as an effective metric, the Gordon metric. (Gordon [2], Skrotskii [3], Balazs [4], Plebanski [5], de Felice [6], and many others.)
2. Acoustics in flowing fluids: Acoustic black holes, *aka* “dumb holes”. (Unruh [7], Jacobson [8], Visser [9], Liberati *et al* [10], and many others.)
3. Phase perturbations in Bose–Einstein condensates: Formally similar to acoustic perturbations, and analyzed using the nonlinear Schrodinger equation (Gross–Pitaevskii equation) and Landau–Ginzburg Lagrangian; typical sound speeds are centimetres per second to millimetres per second. (Garay *et al* [11], Barceló [12] *et al.*)
4. High-refractive-index dielectric fluids (“slow light”): In dielectric fluids with an extremely high group refractive index it is experimentally possible to slow lightspeed to centimetres per second or less. (Leonhardt–Piwnicki [13], Hau *et al* [14], Visser [15], and others.)
5. Quasi-particle excitations: Fermionic or bosonic quasi-particles in a heterogeneous superfluid environment. (Volovik [16], Kopnin–Volovik [17], Jacobson–Volovik [18], and Fischer [19].)
6. Nonlinear electrodynamics: If the permittivity and permeability themselves depend on the background electromagnetic field, photon propagation can often be recast in terms of an effective metric. (Plebanski [20], Dittrich–Gies [21], Novello *et al* [22].)
7. Linear electrodynamics: If you do not take the spacetime metric itself as being primitive, but instead view the linear constitutive relationships of electromagnetism as the fundamental objects, one can nevertheless reconstruct the metric from first principles. (Hehl, Obukhov, and Rubilar [23, 24, 25].)
8. Scharnhorst effect: Anomalous photon propagation in the Casimir vacuum can be interpreted in terms of an effective metric. (Scharnhorst [26], Barton [27], Liberati *et al* [28], and many others.)
9. Thermal vacuum: Anomalous photon propagation in QED at nonzero temperature can be interpreted in terms of an effective metric. (Gies [29].)
10. “Solid state” black holes. (Reznik [30], Corley and Jacobson [31], and others.)
11. Astrophysical fluid flows: Bondi–Hoyle accretion and the Parker wind [coronal outflow] both provide physical examples where an effective acoustic metric is useful, and where there is good observational evidence that acoustic horizons form in nature. (Bondi [32], Parker [33], Moncrief [34], Matarrese [35], and many others.)
12. Other condensed-matter approaches that don’t quite fit into the above classification [36, 37].