GRAN SASSO SCIENCE INSTITUTE

Gravitational wave signals from star clusters

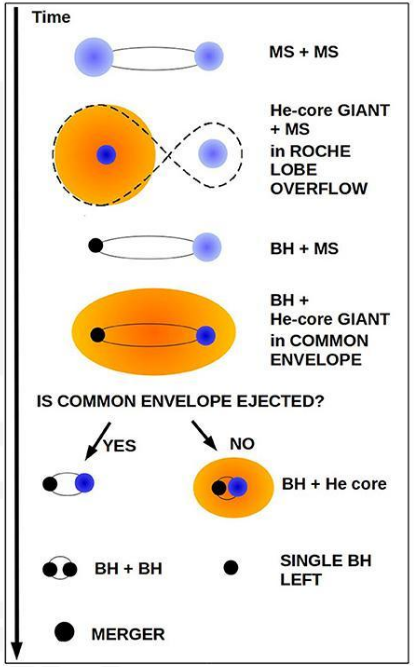
Elena Codazzo, PhD student
Advisor: Jan Harms

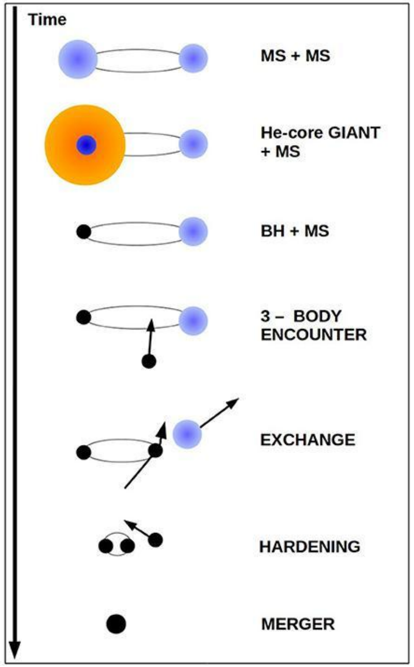
Report 2nd year, 18/10/2021

Background

Isolated binary

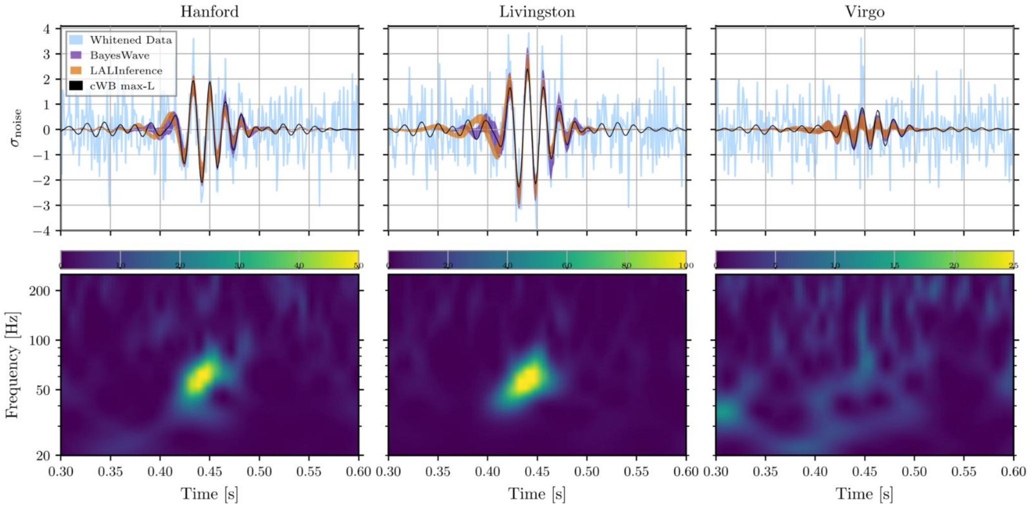
Star clusters





Credits: M.Mapelli

GW190521

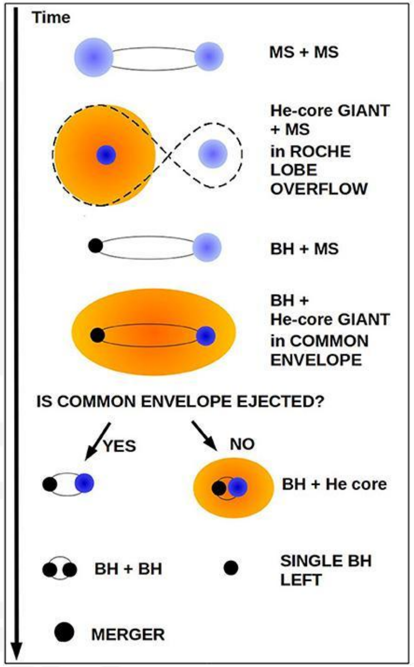


Def HARD BINARY:

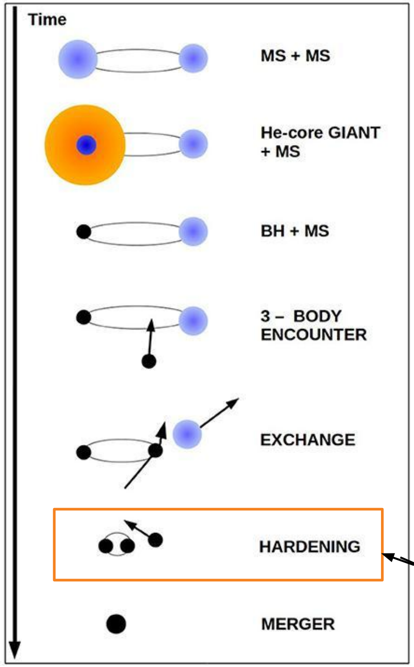
$$E_b = \frac{G\, m_1\, m_2}{2\, a} \geq \frac{1}{2} m_* \, \sigma^2$$

Background

Isolated binary



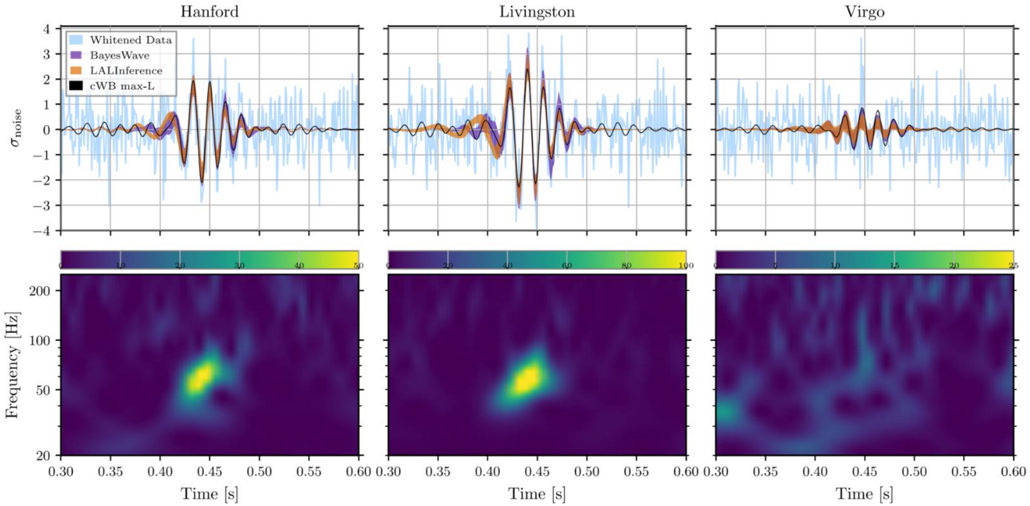
Star clusters



Credits: M.Mapelli

I am interested in these Single-Binary encounters

GW190521



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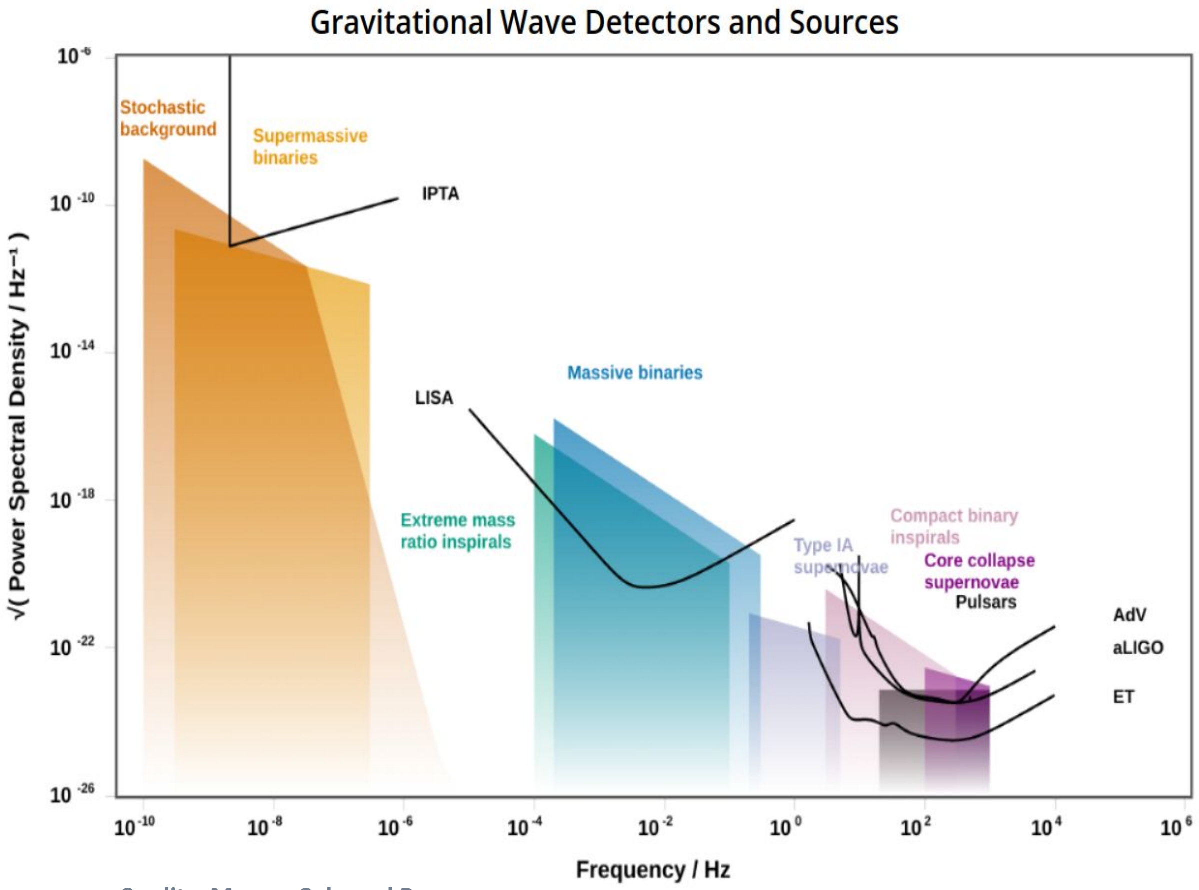
GSSI

Goal



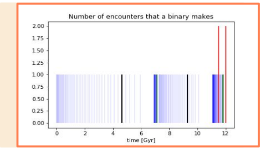
Credits: Moore, Cole and Berry

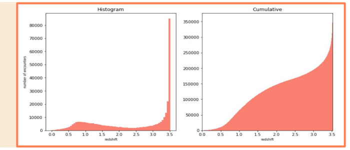
- ➔ What is the frequency range that such GW bursts occupy?
- ➔ What is the number of events that will be detectable by the upcoming generation of interferometers?

GSSI

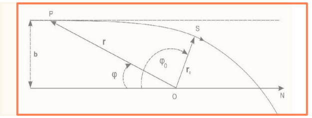
Overview

- Initial conditions to set the problem
- Number of encounters in the last 12Gyr

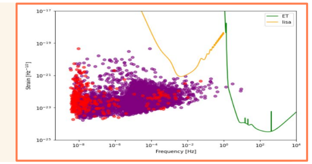




- The Nbody code ARWV $\longrightarrow$ discrimination between hyperbolic and resonant encounters
- 2body encounter with the intruder



- Check on the angle for the 2body encounter $\longrightarrow$ 2body hyperbolic
- Characteristic frequency and strain amplitude



- Preliminary results and future work

Initial conditions

Types of clusters:

- Young star clusters
- Globular clusters
- Nuclear Star Cluster (NSC)

  $\longrightarrow$ higher escape velocity (hundreds km s^{-1})

  $\longrightarrow$ easier to estimate, long lived, same age

$$N^{\mathrm{gal}}(D) = \begin{cases} N_1(D/1\ \mathrm{Mpc})^{0.9}, & D \leq 3\ \mathrm{Mpc}, \\ N_2(D/3\ \mathrm{Mpc})^{1.5}, & 3\ \mathrm{Mpc} < D < 16\ \mathrm{Mpc}, \\ N_3(D/16\ \mathrm{Mpc})^{2.4}, & 16\ \mathrm{Mpc} < D < 60\ \mathrm{Mpc}, \\ N_4(D/60\ \mathrm{Mpc})^{3}, & D > 60\ \mathrm{Mpc}, \end{cases}$$

Kocsis et al 2006. N4= 26000

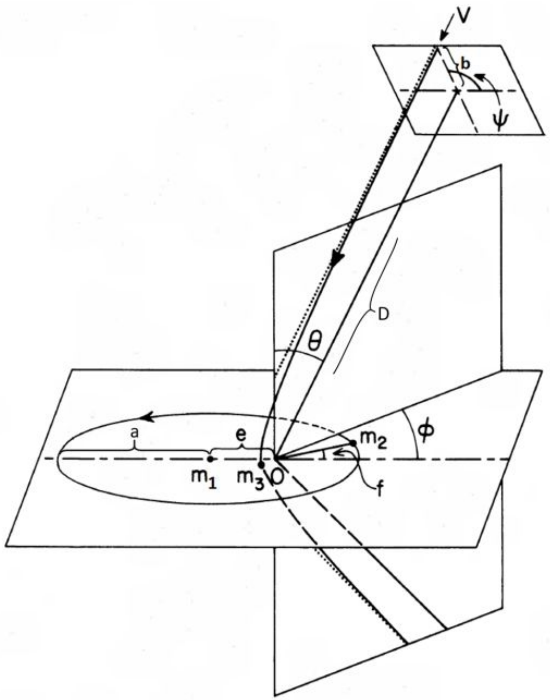
At D=160Mpc Ngal=5 10^5

✗ we don't know the number of BBHs in NSC as a function of redshift

✓ we can estimate it at the formation of the cluster $\longrightarrow N_{BBH} \approx 7$

To be conservative:
we assume that there are 10^6 evolving binaries in a 160 Mpc radius
-> 10^6 first generation binaries

Initial conditions



Credit: Hut & Bahcall 1983

- Single (m_3) & Binary (m_1, m_2) BHs masses from Di Carlo et al. 2019 (https://arxiv.org/abs/1901.00863)

masses computed from astrophysical evolution of stars in a dynamical environment

- The semi-major axis are generated with a uniform distribution from 0 to 1000 AU and then rejected if the binary is soft or if the binary is too hard.
- other parameters: spins, eccentricity, phase of the binary, angles, impact parameters, relative velocity.

Number of encounters

evolution of the semi major axis of the binary

$$\frac{\mathrm{d}a}{\mathrm{d}t} = -2\pi\xi \frac{G\rho_c}{\sigma} a^2 - \frac{64}{5} \frac{G^3 m_1 m_2 (m_1+m_2)}{c^5 a^3 (1-e^2)^{7/2}} f_1(e)$$

(1) hardening because of single-binary encounters

$E_{new} = E_{prev} [1+ \xi m_3/(m_1+m_2)]$

(2) hardening because of GW emission

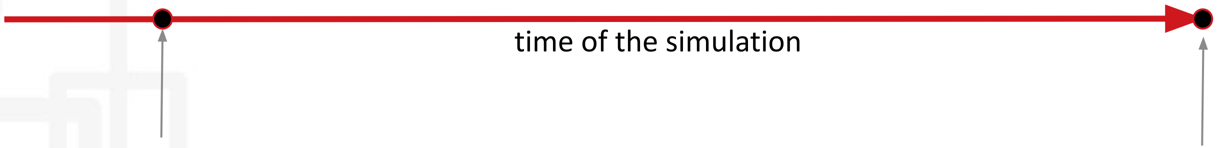
Def: $a_{critical} = \max \{ \; a_{GW} = \left[\frac{32 G^2}{5\pi\xi c^5} \frac{\sigma m_1 m_2 (m_1+m_2)}{\rho_c (1-e^2)^{7/2}} f_1(e) \right]^{1/5}, \quad a_{ej} = \frac{2\xi {m_*}^2}{(m_1+m_2)^3} \frac{G m_1 m_2}{v_{esc}^2} \; \}$

- if $a_{critical} = a_{ej}$ the binary is ejected from the cluster
- if $a_{critical} = a_{GW}$ the binary merge inside the cluster

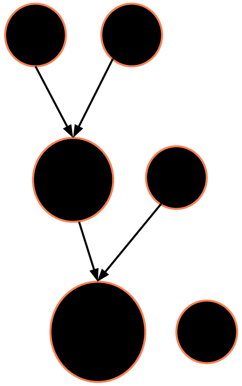
GSSI

Number of encounters

- Age of a NSC today ≈ 13.6 Gyr
- Time to form hard and massive BBH = $t_{form} + t_{DF} + \min(t_{3bb}, t_{12}, t_{cap}) + t_{hard} \approx 1.6$ Gyr
  - t_{form} = time to form a BH
  - t_{DF} = time that takes a BH to drift in the core
  - t_{3bb}, t_{12}, t_{cap} = time to form BBHs
  - t_{hard} = time to form hard and massive BBH
- Time of the simulation: 12 Gyr



Scheme of hierarchical mergers



Number of encounters

Conditions :

- t must be < 12Gyr & v must be $<v_{esc}$
- until $a > a_{critical}$ → evolution due to SB encounters
- after $a_{critical} = \max\{a_{GW}, a_{ej}\}$: change of regime
  - if $a_{critical} = a_{ej}$ → stop
  - if $a_{critical} = a_{GW}$ →evolution due to SB+GW
- after the merger $v_{kick}<v_{esc}$
  - $m_{1,\,new} = 0.95(m_1+m_2)$
  - $m_{2,\,new}$ from intruder list >10 Msun
  - spin $m_{1,\,new}$ random from a uniform distribution between [0.6-0.9]
  - t + delay time to form a new binary

$$v_{kick} = (v_m^2 + v_\perp^2 + 2 v_m v_\perp cos\phi + v_\parallel^2)^{1/2},$$

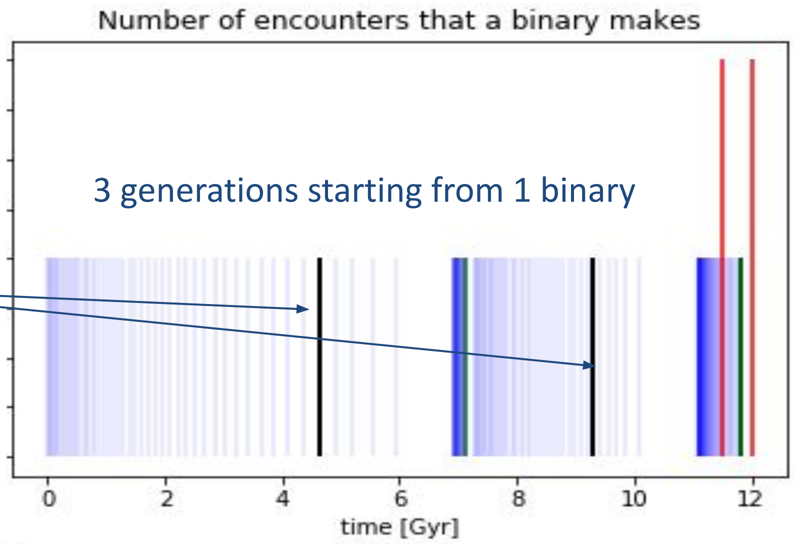
where

$$v_m = A\eta^2 \frac{1-q}{1+q}(1+B\eta)$$

$$v_\perp = H\frac{\eta^2}{1+q}\left|\chi_{1\parallel} - q\chi_{2\parallel}\right|$$

$$v_\parallel = \frac{16\eta^2}{1+q}\left[V_{1,1} + V_A S_\parallel + V_B S_\parallel^2 + V_C S_\parallel^3\right] \left|\chi_{1\perp} - q\chi_{2\perp}\right| cos(\phi_\Delta - \phi)$$

Lousto et al. (2012)



Legend:

- Blue: the intruder is a star
- Green: the intruder is a BH
- Black: $a = a_{critical}$
- Red: sphere of 160Mpc radius

Number of encounters

Output of the simulation:

```
Estimate of the number of encounters expected close to us in NSC

 How many events there are for each generation:

Gen1: 1000000
Gen2: 440491
Gen3: 18731
Gen4: 2616
Gen5: 917
Gen6: 346
Gen7: 69
Gen8: 5

Encounters with black holes: 359543 !!!

All encounters: 240787822

Binaries: 287323
NB: each binary is responsible for multiple encounters
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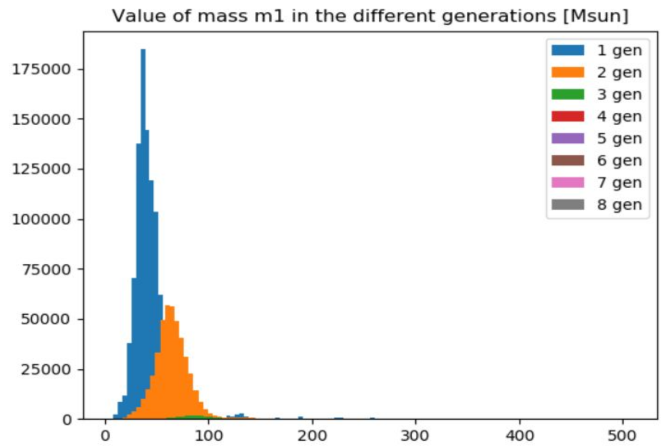
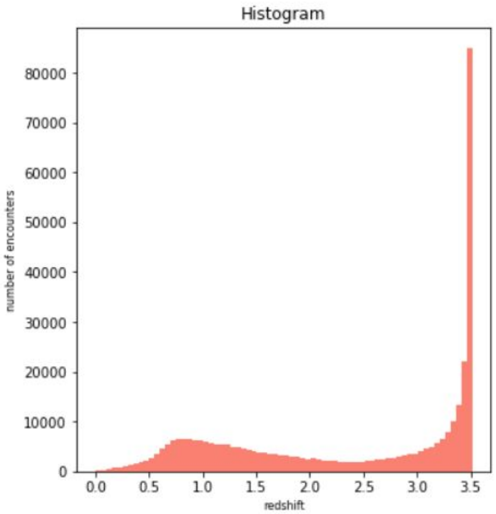
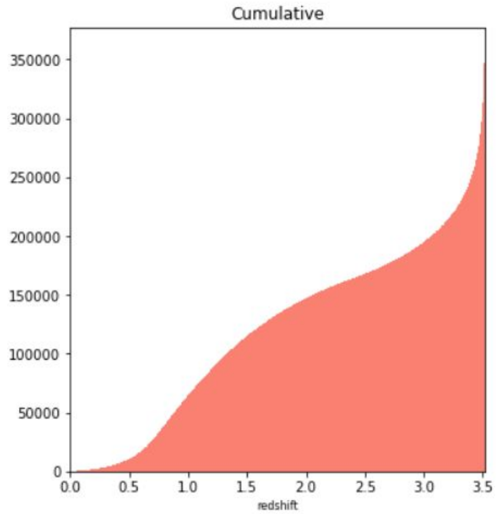
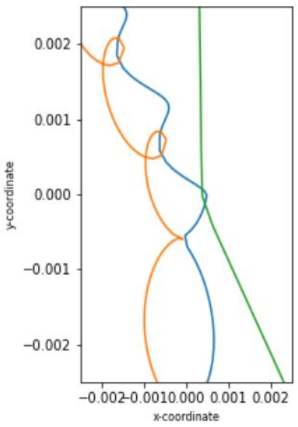
- 196 encounters within 160Mpc

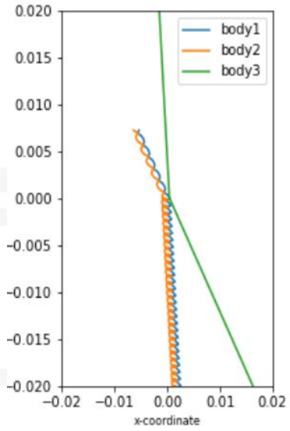






Three-body encounters with ARWV code

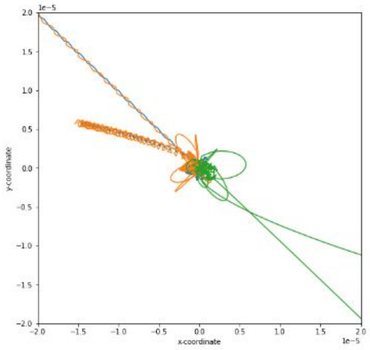


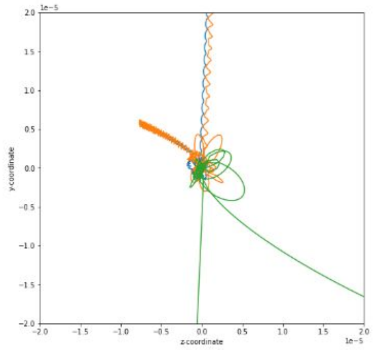


From the *Number of encounters* code:

★ 359543 encounters BBH-BH from the evolution of 10^6 first generation binaries

- ○ I already have: <u>m1, m2, sma of the binary, m3, redshift</u>
- ○ I need: <u>ecc of the binary, v, b and angles</u>





Simulated with ARWV:

I selected a sample in order to find some useful events for ET and LISA:

- R < 160Mpc (196)
- v/b $>10^{-6}$ Hz (194)
- v > 190 km/s (805)
- M1> 450 Msun (233)
- Sma < 0.2 AU (9890)

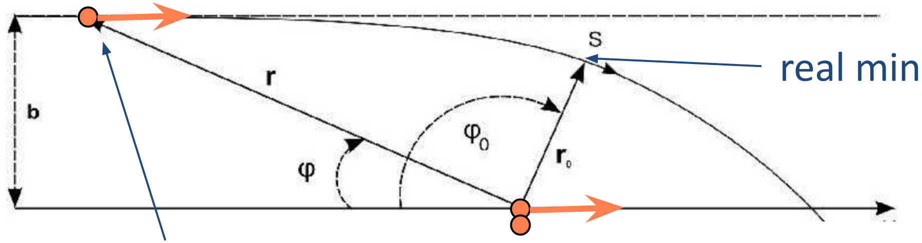
Total =11318 ⟶ 3%

Two-body hyperbolic encounters

1st selection → I consider the single binary encounters in which the intruder makes a flyby around the binary
I rejected resonant encounters

2nd selection	*the binary is narrow*	*the binary is wide*
Intruder INSIDE the binary	interesting but not hyperbolic	I consider only the closest body
Intruder OUTSIDE the binary	I consider the binary as a single object	I consider only the closest body

3rd selection

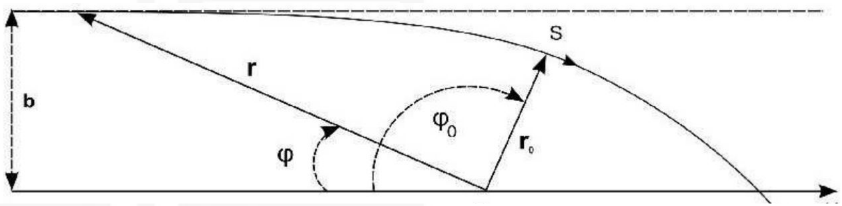


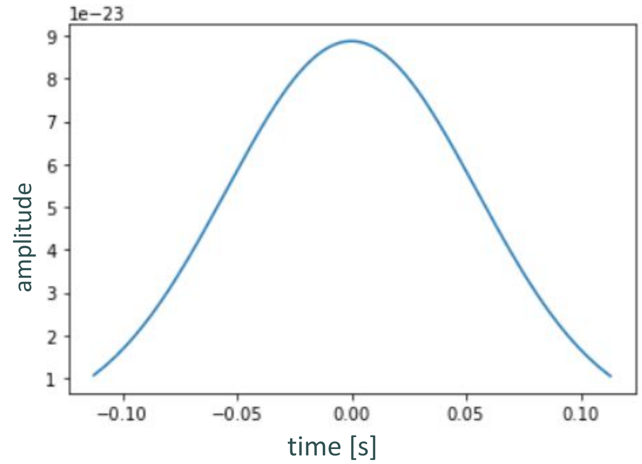
real minimum

Closest distance for arwv

- check on the ϕ angle:
  good: $\phi < 40°$ or $\phi > 150°$
  reject: $40° < \phi < 150°$
- if rejected, check previous and next step.

Frequency range and amplitude



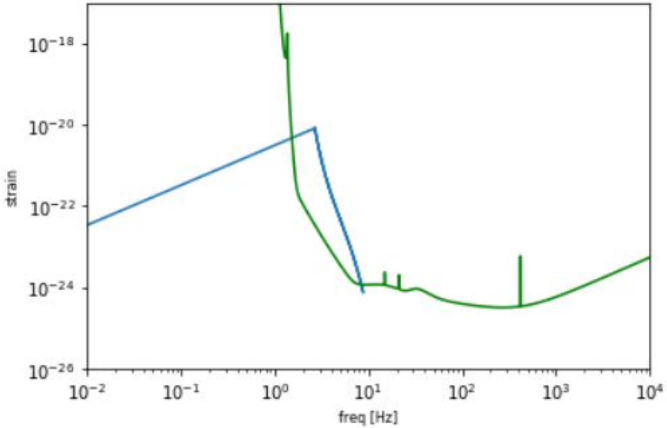


- Analytical procedure to find the minimum
  - Amplitude [Capozziello et al 2008]:

$$h = \frac{2G}{Rc^4}\mu v_0^2 \csc^2\phi_0 \left\{2\left[59\cos 2(\phi_0-\phi) - \cos\phi(54\cos(2\phi_0)+101)\right]\cos^2\phi_0 - 9\cos(3\phi-4\phi_0) - 9\cos(3\phi-2\phi_0) + 95\cos 2\phi_0 + 9\cos 4\phi_0 - \sin\phi\left[101\sin 2\phi_0 + 27\sin 4\phi_0\right] + 106\right\}^{1/2},$$

  - Time [Garcia Bellido et al 2018]:

$$t = \frac{b}{v_0}\int \frac{\sin^2\varphi_0\, d\varphi}{(\cos(\varphi-\varphi_0)-\cos\varphi_0)^2} = \frac{b}{v_0\, e^2}\left[\frac{e\,\sin(\varphi-\varphi_0)}{1+e\,\cos(\varphi-\varphi_0)} - \frac{2}{e+1}\tan\frac{\varphi-\varphi_0}{2}\right]$$



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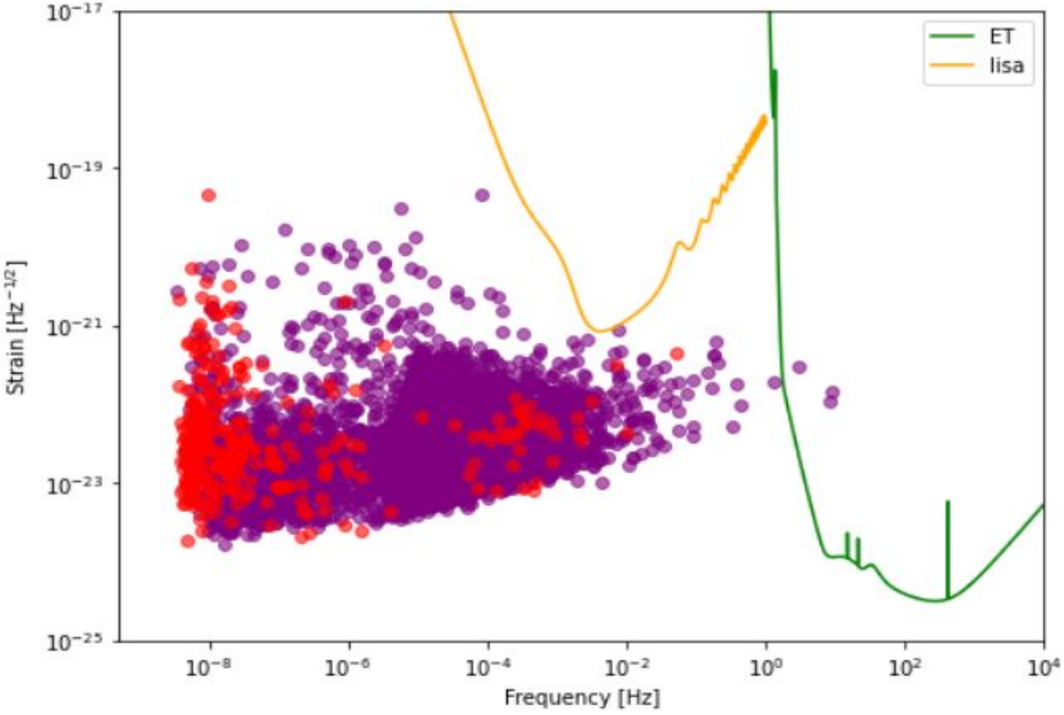
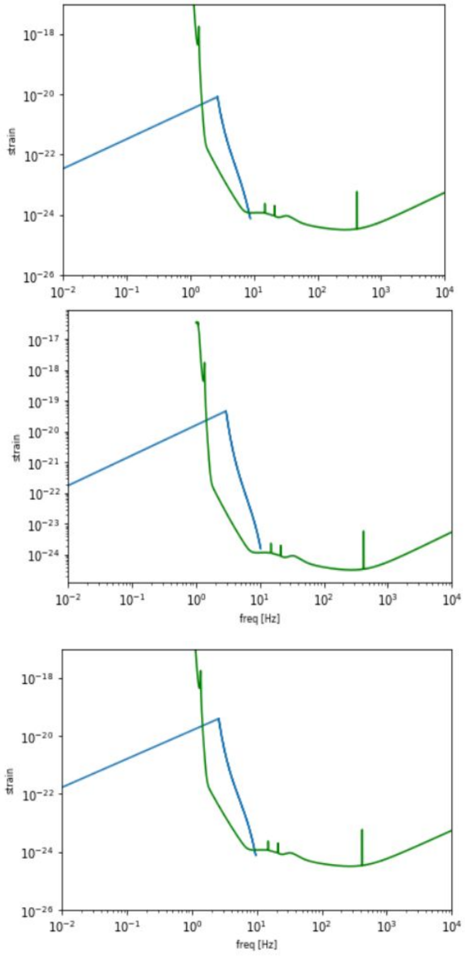
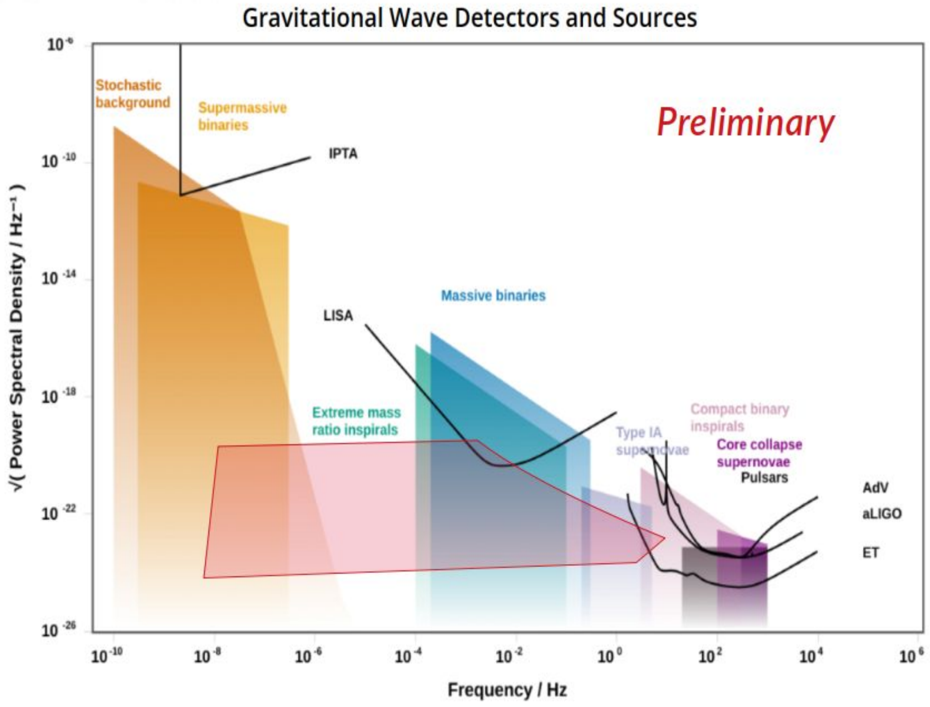




Legend:

- Red: the 2 bodies are the intruder and the closest component of the binary
- Purple: the 2 bodies are the intruder and the CoM of the binary

Summary



- Number of single binary encounters as a function of redshift
- ARWV simulation with 1yr timestep
- Analysis of hyperbolic encounters
  - we are now working at decreasing the timestep of the simulation to avoid any resonances between steps.
- Region occupied in the log-log plot

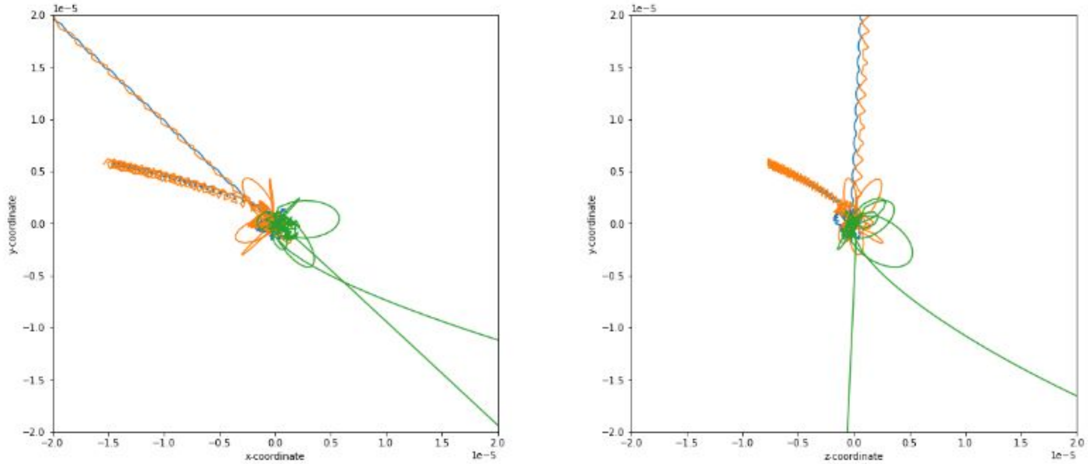
Demographics of gravitational-wave signals from star clusters

Elena Codazzo,[1,2] Matteo Di Giovanni,[1,2] Jan Harms,[1,2] Marco Dall'Amico,[3,4] and Michela Mapelli[3,4]

Future work

Near future:

❖ find the GW bursts emitted during resonant encounters by looking at the quadrupole emission at the points of minimum distance.



Long term:

❖ obtain the contribution of GCs and YSCs.

❖ inspect the waveform signal in the data of the interferometers.

GS
SI

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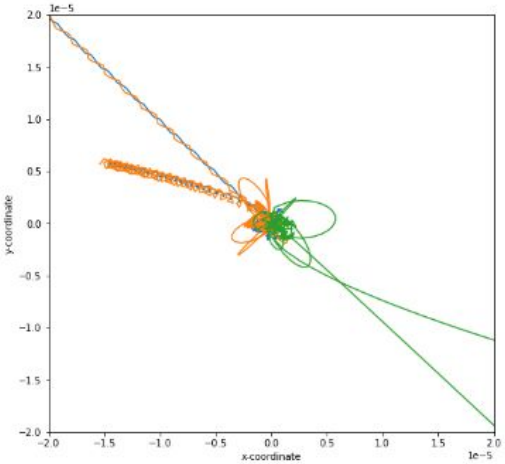

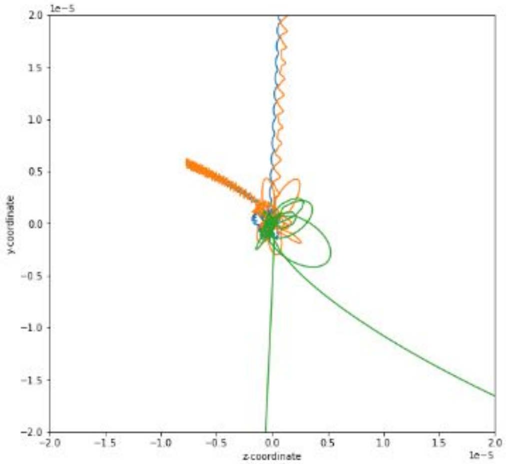

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Thank you.