Quantum Turbulence

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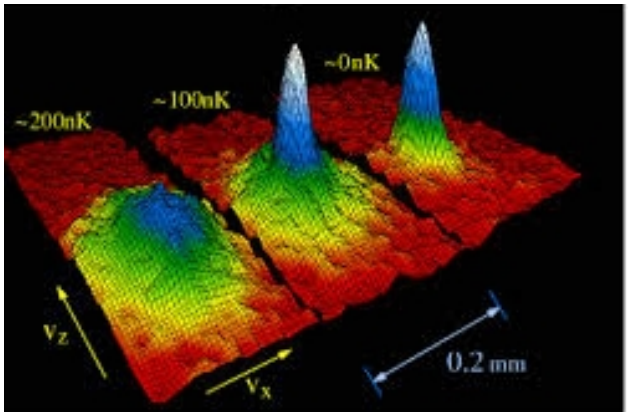
Outline

- **Context:**
  Superfluid helium, atomic Bose-Einstein condensates
- **Turbulence:**
  with and without quantum mechanical constraints

Quantum fluids

Context:

- Liquid helium, cold atomic gases
- Macroscopic occupation of the ground state (BEC)



Key properties:

- Superfluidity (no viscosity)
- Quantised vorticity

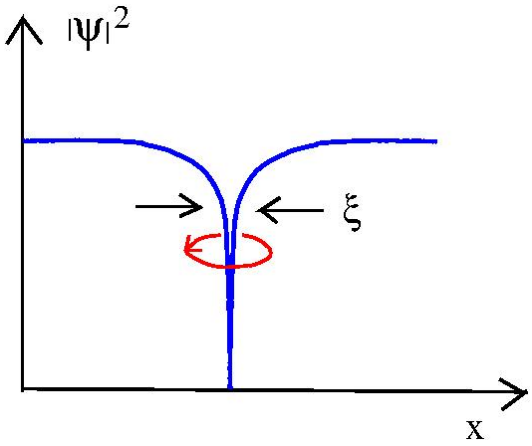
Quantum vortex line

Macroscopic wavefunction $\Psi(\mathbf{x}, t) = \sqrt{n(\mathbf{x}, t)} e^{i\phi(\mathbf{x},t)}$

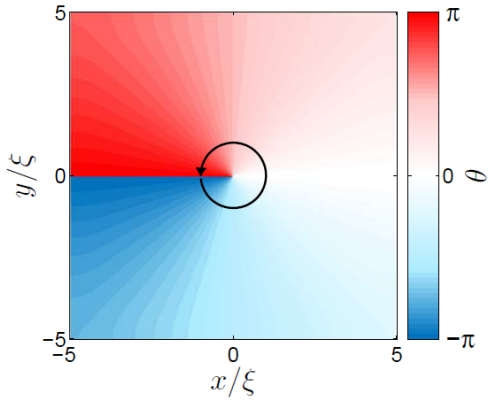
Velocity $\mathbf{v}(\mathbf{x}, t) = (\hbar/m)\nabla\phi(\mathbf{x}, t)$, Density $n(\mathbf{x}, t) = |\Psi(\mathbf{x}, t)|^2$

$$\text{Circulation is quantised} \quad \oint_C \mathbf{v} \cdot \mathbf{dr} = \frac{h}{m} = \kappa$$

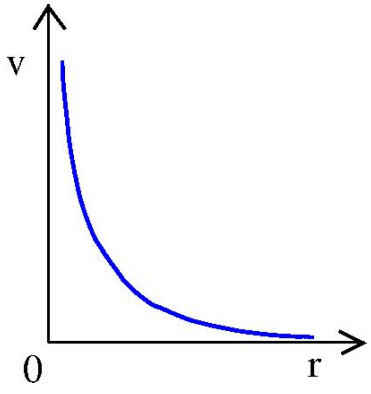
A vortex is a hole around which the phase ϕ changes by 2π
Core size ξ and velocity $v = \kappa/(2\pi r)$ are fixed



Density



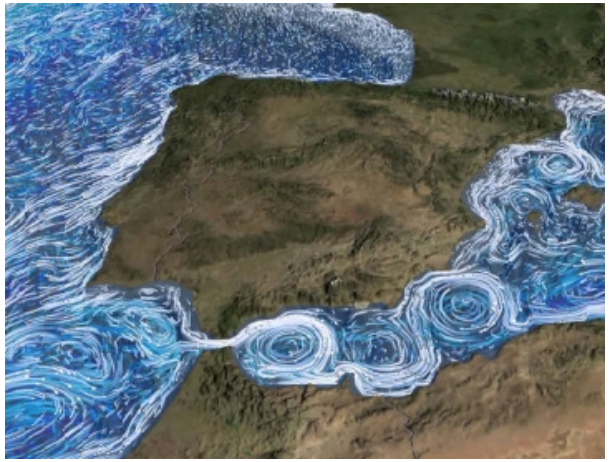
Phase



Velocity

Quantum vortices vs ordinary (classical) vortices

- **Quantum vortices** are stable topological defects; circulation and core radius are fixed
- **Classical vorticity:** $\boldsymbol{\omega} = \nabla \times \mathbf{v}$ ($\mathbf{v}$ = velocity)
  Circulation and core size are arbitrary.
  Vorticity decays (unless it is forced)



$$\frac{D\boldsymbol{\omega}}{Dt} = (\boldsymbol{\omega} \cdot \nabla)\mathbf{v} + \nu \nabla^2 \boldsymbol{\omega}$$

Turbulence

Turbulence consists of vortices
(Leonardo da Vinci 1510)

Turbulence

Applications:

-Life sciences (blood flow in the aorta)

-Engineering (airflow around wings, air-fuel mixture in engines)

-Geophysics (oceans, atmosphere)

-Astrophysics (Earth's and Sun interiors, interstellar gas)

Challenges:

-**Mathematics:** the equations, written long ago, are deceptively simple (singularity problem: the $ 1,000,000 Clay prize)

-**Physics:** Turbulence is a problem of nonlinear statistical physics: a huge number of coupled degrees of freedom/length scales excited simultaneously

Kolmogorov's 1941 theory (K41)

The energy spectrum $E(k)$ is the distribution of kinetic energy E_{kin} over the length scales $2\pi/k$ (k= wavenumber)

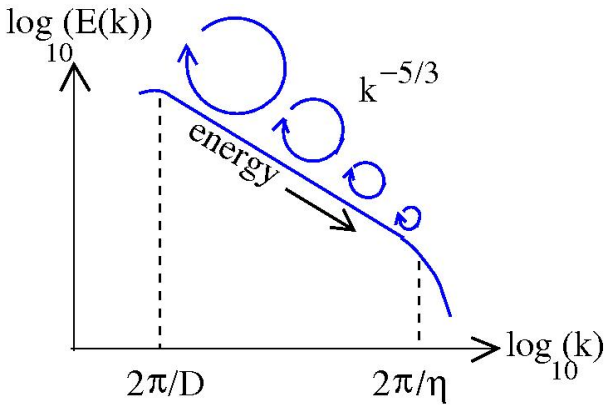
$$E_{kin} = \frac{1}{V}\int_V \frac{\mathbf{v}^2}{2} dV = \int_0^\infty E(k)dk$$

$$E(k) = C\epsilon^{2/3}k^{-5/3}$$

$\epsilon = -dE_{kin}/dt$ (dissipation rate)

$C \approx 1$

$\mathrm{Re} = UD/\nu$ (Reynolds number)

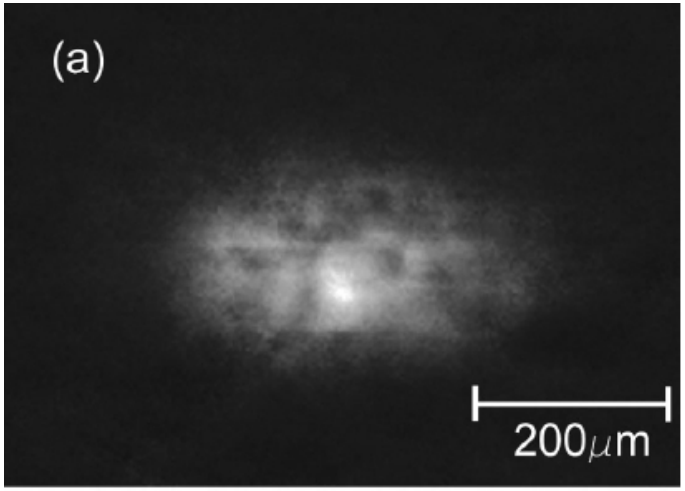


Energy is injected at large scale D, **cascades** to larger and larger k in the inertial range and becomes heat at the Kolmogorov scale η.

$$\eta/D \approx \mathrm{Re}^{-3/4}$$

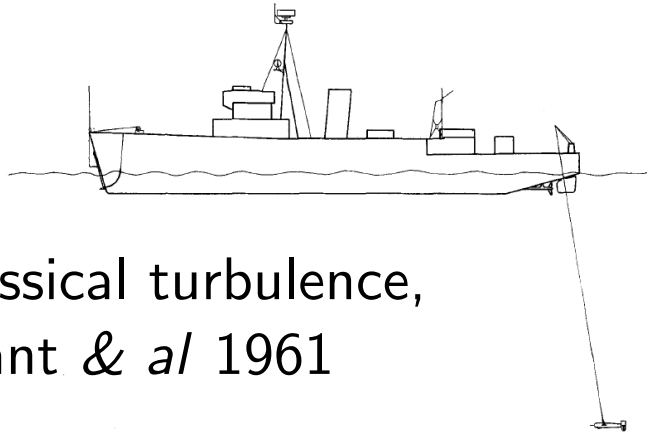
Turbulence: helium vs atomic vs classical

Helium (SHREK, Grenoble)



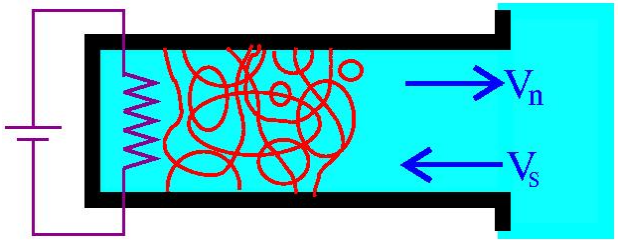
Atomic BECs
(Henn & Bagnato PRL 2009)

Seymour Narrows, Vancouver

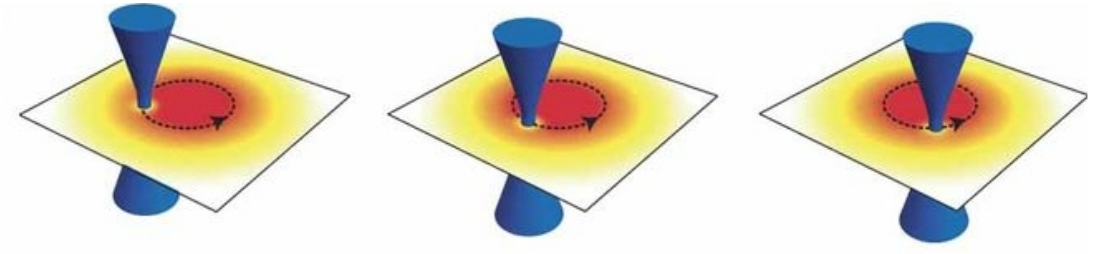


Classical turbulence,
Grant *& al* 1961

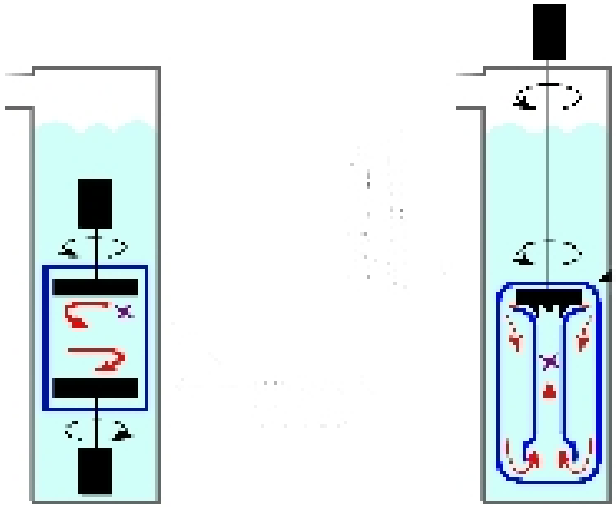
Quantum turbulence: how is it generated ?



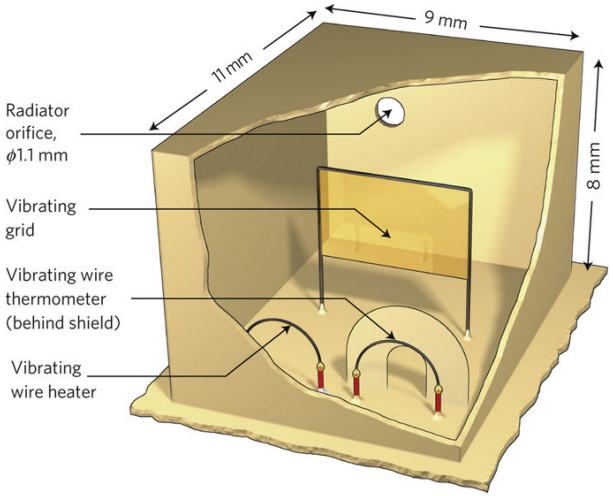
(Prague)



(Paris)

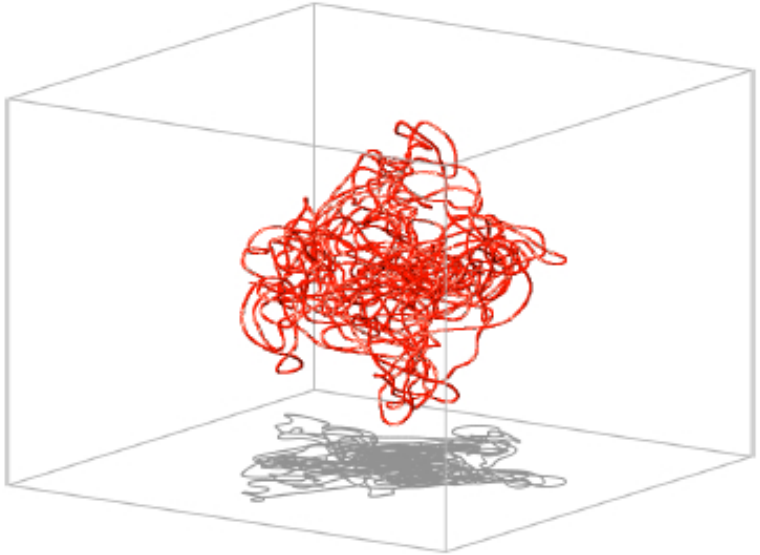


(Grenoble)

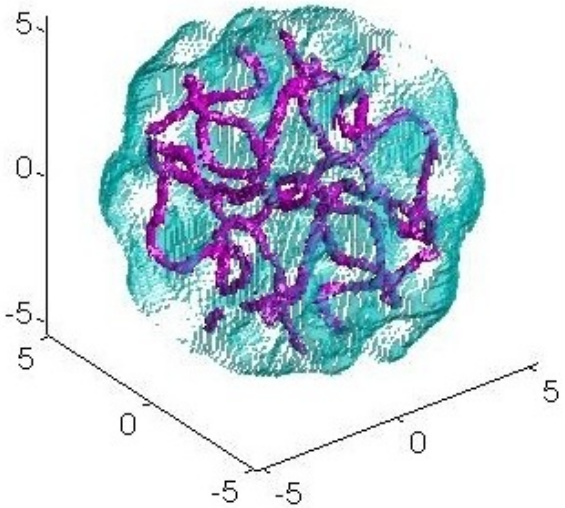


(Lancaster)

Quantum turbulence: what does it look like ?



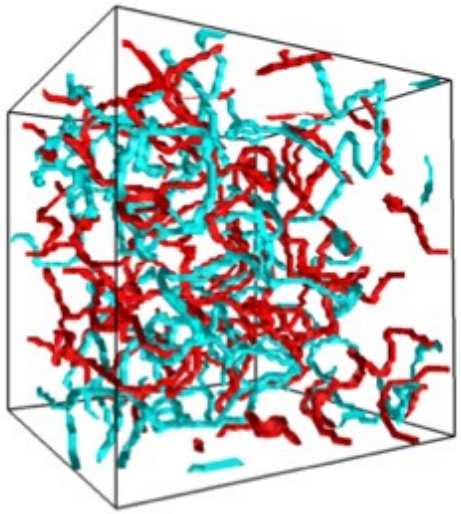
A small patch of quantum turbulence in superfluid helium
Cooper, CFB & al



Quantum turbulence in an atomic BEC
White, CFB & al, PRL 2010

Quantum turbulence: what does it look like ?

Quantum turbulence in the quench of a Bose gas
Stagg, Parker & CFB, PRA 2016



Quantum turbulence in a two-component Bose gas
Takeuchi & al, PRL 2010

Turbulence: helium vs atomic BECs vs classical

Compare length scales:

D = system's size, η = Kolmogorov length,

ℓ = average inter-vortex distance, ξ = vortex core size

Atomic BECs:

$D/\xi \approx 10$ to 100

$D/\ell \approx 1$ to 10

Liquid helium (SHREK, Grenoble):

$D/\xi \approx 8 \times 10^9$

$D/\ell \approx 10^4$ to 10^5

Classical turbulence (Grant & *al* 1961):

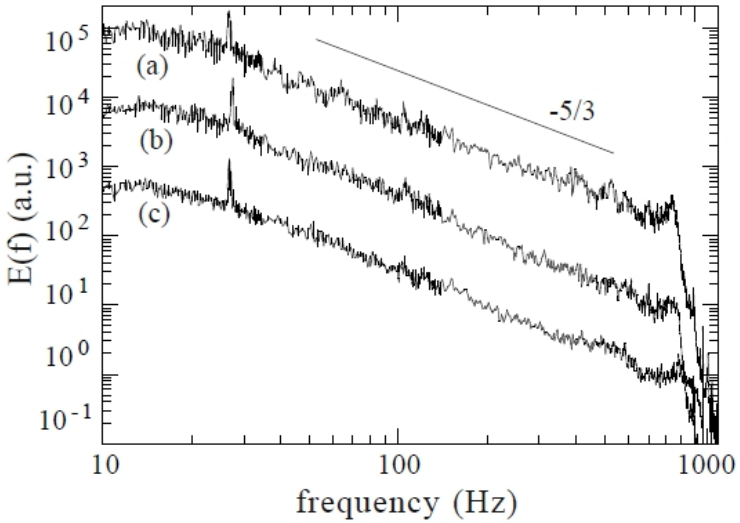
$D/\eta \approx 2.5 \times 10^6$

Opportunities:

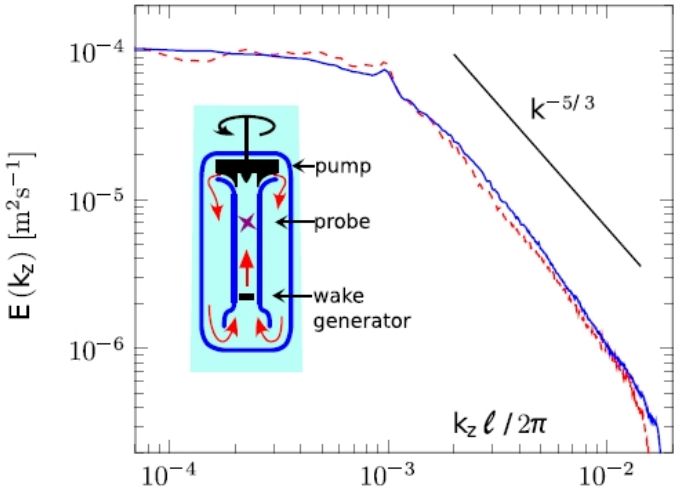
- Liquid helium: more intense than classical
- Atomic BECs: crossover chaos-turbulence

Kolmogorov spectrum in quantum turbulence

The $k^{-5/3}$ law has been observed in superfluid helium
at length scales larger than the inter-vortex distance ℓ



Maurer & Tabeling, EPL 1998
at $T = 2.3K$, $2.08K$ and $1.4K$

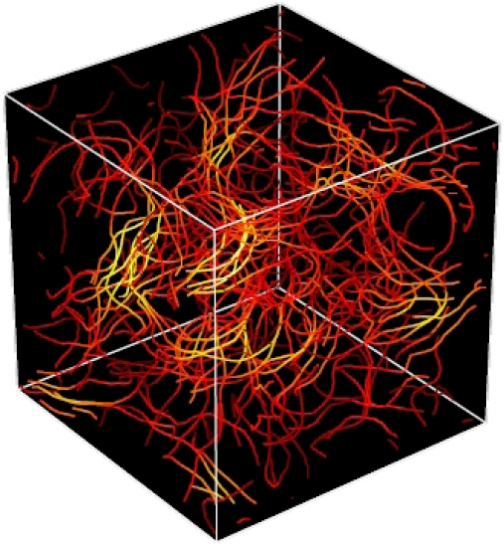


Salort & al, EPL 2012

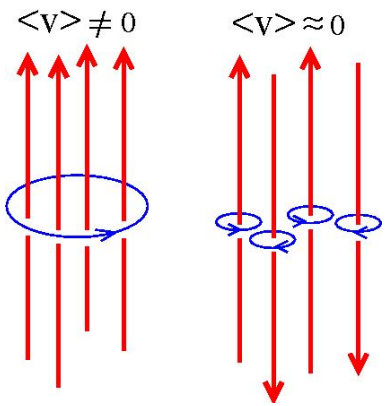
Additional evidence from decay measurements

Kolmogorov spectrum in quantum turbulence

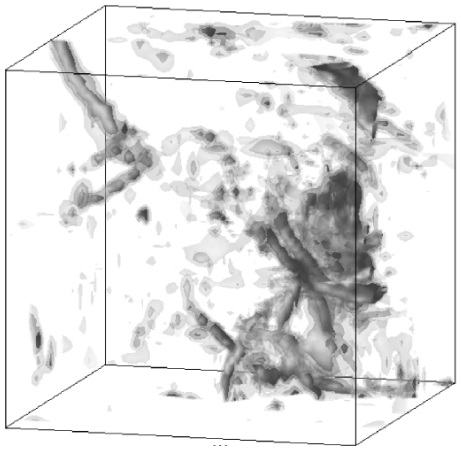
$E(k) \sim k^{-5/3}$ arises from polarisation of vortex lines for $k < 2\pi/\ell$ (Baggaley, CFB & al, PRL 2012)

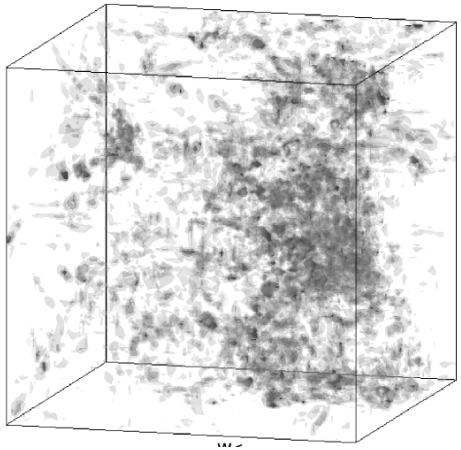


Total lines: $E(k) \sim k^{-5/3}$
Polarised: $E(k) \sim k^{-5/3}$
Unpolarised: $E(k) \sim k^{-1}$



Classical turbulence:
Total: $E(k) \sim k^{-5/3}$
Coherent: $E(k) \sim k^{-5/3}$
Incoherent: $E(k) \sim k^{2}$
(Farge & al, PRL 2001)

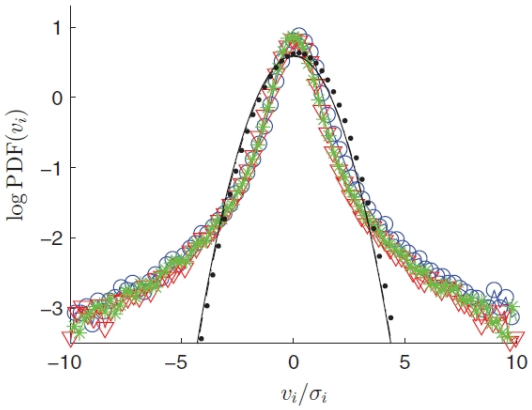


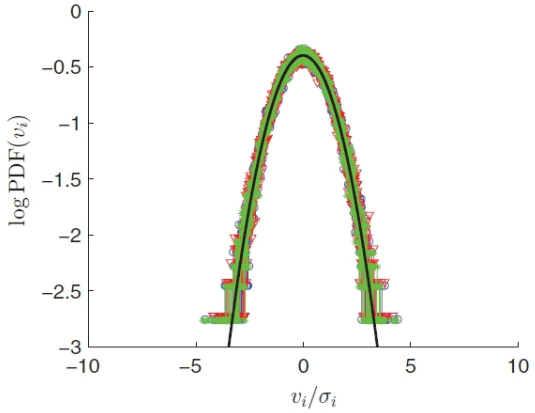


Turbulent velocity statistics

- Classical turbulence: Gaussian
- Quantum turbulence: power-laws
  (Paoletti & al PRL 2008, White & al PRL 2010)
- Cross-over from Gaussian to power-law:
  Δ = measurement region, ℓ = average inter-vortex distance

$\Delta < \ell$ (left): power-laws $\Delta > \ell$ (right): Gaussians





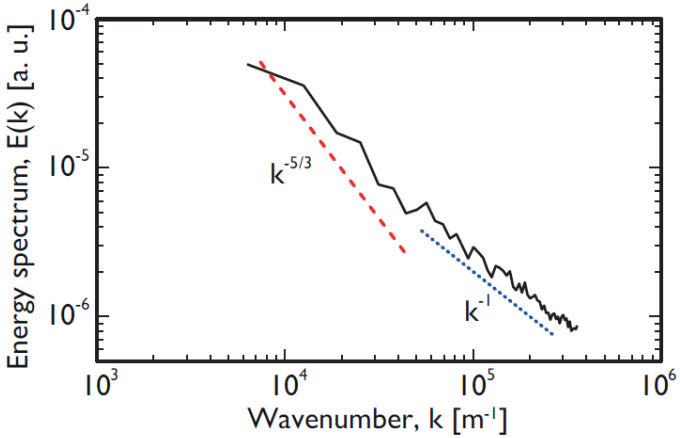
(Baggaley & CFB, PRE 2011, LaMantia & Skrbek, EPL 2014)

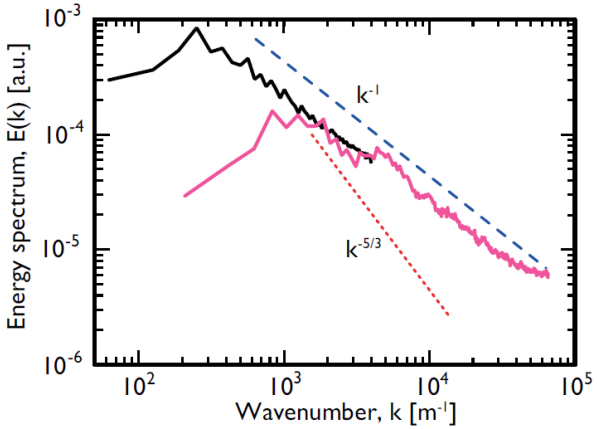
A second kind of quantum turbulence: Vinen turbulence

Another form of turbulence, called **Vinen turbulence**, has also been observed in:

- ^{4}He experiment (Walmsley & Golov, PRL 2008)
- ^{3}He experiments (Bradley & al, PRL 2006)
- simulations of Walmsley & Golov's experiment (Baggaley, CFB & Sergeeev, PRB 2012)
- simulations of counterflow (T1) turbulence in ^{4}He (Baggaley, Sherwin, CFB & Sergeev, PRB 2012)
- simulations of thermal quench of a Bose gas (Stagg, Parker & CFB, PRA 2016)
- simulations of turbulence in atomic BECs (Cidrim, CFB & Bagnato, PRA 2017)
- simulations of dark matter (Mocz & al, MNRAS 2017)

Vinen vs Kolmogorov





- **Kolmogorov:** $E(k)$ peaks at low k, $E(k) \sim k^{-5/3}$ for $k < \delta$, coherent structures, $L \sim t^{-3/2}$, $E_{kin} \sim t^{-2}$
- **Vinen:** $E(k)$ peaks at intermediate k, $E(k) \sim k^{-1}$ at larger k, $L \sim t^{-1}$, $E_{kin} \sim t^{-1}$, velocity correlation decays rapidly with r

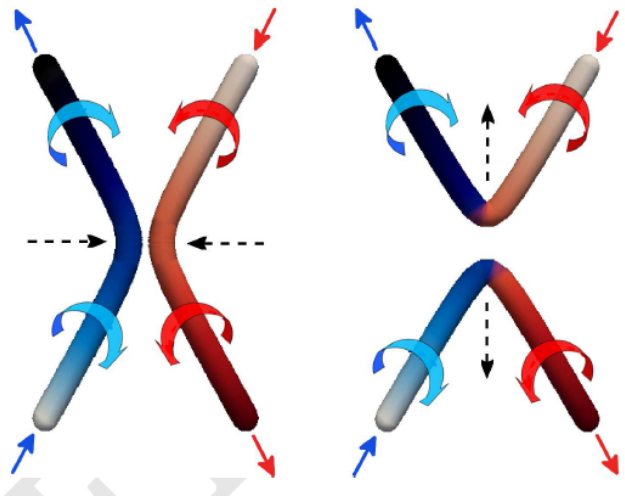
Interpretation of Vinen turbulence:
turbulence without a cascade, random-like flow
(CFB, Sergeeev & Baggaley, Sci Rep 2016)

Vortex reconnections

Without reconnections there would be no turbulence

In quantum fluids vorticity consists of **individual vortex lines**, thus reconnections are **isolated events**

Are there **universal** laws of reconnections ?

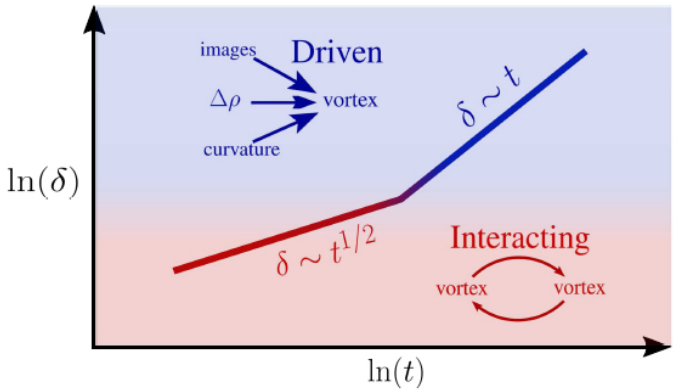


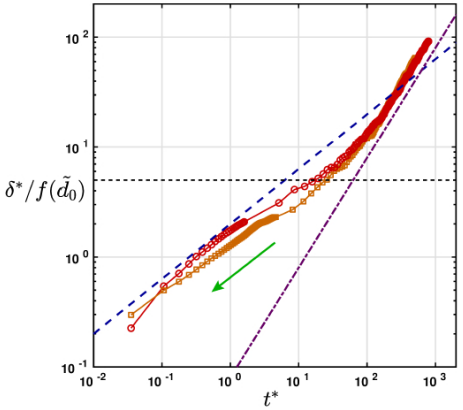
Experimental observations of quantum vortex reconnections:

- Bewley & al (PNAS 2008) in superfluid helium
- Serafini & al (Phys Rev X 2017) in atomic BECs

Vortex reconnections

$\delta(t)$ = minimum distance between reconnecting vortex lines





From experiments and simulations we predict and find two scaling laws: $\delta(t) \sim t^{1/2}$ and $\delta(t) \sim t$

Crossover determined by balance between:
(1) interaction of reconnecting vortex strands
(2) individual driving of the vortices
(curvature, density gradients, boundaries/images)

(Galantucci, Parker, Baggaley & CFB, submitted)

Conclusions: Quantum turbulence

- New physics, and new light onto an old problem
- There seems to be two kinds of quantum turbulence:
  Kolmogorov turbulence (the skeleton of classical turbulence ?)
  Vinen turbulence (turbulence without a cascade ?)
- **Universality** of vortex reconnections:
  Scaling laws of minimum vortex separation vs time

Reviews of quantum turbulence

In helium:	CFB, Skrbek & Sreenivasan, PNAS 2015
In atomic BECs:	Tsatsos, CFB & al, Phys Reports 2016

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