Gran Sasso Quantum Meetings @GSSI:
FROM MANY PARTICLE SYSTEMS TO QUANTUM FLUIDS

28 November - 1 December 2018

GS
SI

`Courses`

Universal dynamics for the logarithmic Schrödinger equation

Rémi Carles
CNRS & Université de Rennes 1

We consider the nonlinear Schrödinger equation with a logarithmic nonlinearity, whose sign is such that no non-trivial stationary solution exists. Explicit computations show that in the case of Gaussian initial data, the presence of the nonlinearity affects the large time behaviour of the solution, on at least three aspects. The dispersion is faster than usual by a logarithmic factor in time. The positive Sobolev norms of the solution grow logarithmically in time. Finally, after rescaling in space by the dispersion rate, the modulus of the solution converges to a universal Gaussian profile (whose variance is independent of the initial variance). In the case of general initial data, we show that these properties remain, up to weakening the third point (weak convergence instead of strong convergence).

In the first lecture, we present the model, some of its properties, and state the main results. In the second lecture, we analyze in details the Cauchy problem, and explicit computations in the Gaussian case. In the final lecture, we prove the main result.

Bosonic mean-field limits and the quantum de Finetti theorem

Nicolas Rougerie
CNRS & Université Grenoble-Alpes

We shall be concerned with the mean-field approximation for equilibrium states of N-body systems in statistical mechanics, i.e. the justification of effective models based on statistical independence assumptions. In particular we want to rigorously derive Hartree and/or Gross-Pitaevskii equations used to model Bose-Einstein condensates. Our main tool is a structure result going under de name of "quantum de Finetti theorem", describing the large N limits of admissible states for these systems. This relies on the symmetry under exchange of particles, due to their indiscernability. Thus we approach the mean-field approximation using as little information on the specifics of the Hamiltonian at hand as possible.

Lecture notes for (a very extended version of) the course may be found at
https://arxiv.org/abs/1506.05263, or (book in french, the pdf may be downloaded for free)
https://spartacus-idh.com/978-2-36693-012-2.html

Kinetic Equations describing Wave Turbulence

Juan J. L. Velàzquez
University of Bonn

In this series of lectures I will describe some mathematical results for a kinetic equation which is known as the equation for Wave Turbulence or Weak Turbulence. This equation describes the transfer of energy between different frequencies in many wave systems, like the water waves of the ocean, nonlinear optics and others. From the mathematical point of view the Wave Turbulence equation has some analogies with the classical Boltzmann equation, but a major difference is that the collision terms are described by means of cubic instead of quadratic terms. This is due to the fact that the collisions involve three instead of two particles. Wave Turbulence equations admit in many cases power law solutions which describe the transfer of energy from large to small wavelengths in a manner that resembles the transfer of energy in the classical theory of turbulence in fluid mechanics. Another relevant feature of the WaveTurbulence kinetic equation is that it might yield blow-up in finite time in the sense that bounded initial data might become unbounded in finite time. In spite of that, it is possible to define global solutions in the sense of measures. The blow-up can then be interpreted as the onset of a coherent wave oscillating with a single frequency which contains a positive fraction of the total number of waves of the system. This coherent wave is usually referred in several applications as a condensate.

In these lectures the specific kinetic equation arising in the statistical description of the waves given by the nonlinear Schrödinger equations will be considered. The well-posedness properties of the resulting kinetic equation, as well as its associated power law solutions describing energy fluxes, blow-up in finite time and the onset of condensates will be presented.

Invited Talks

Quantum turbulence

Carlo Barenghi
Newcastle University

Quantum mechanics constrains any rotational motion of quantum fluids (e.g. superfluid helium and atomic Bose-Einstein condensates) to vortex lines of quantised circulation which represent topological defects of the order parameter. Turbulence in quantum fluids (or "quantum turbulence") therefore takes the form of a disordered tangle of vortex lines. Recent experimental and theoretical work has highlighted important analogies between quantum turbulence and turbulence in ordinary (classical) fluids, possibly shedding some light into the nature of turbulence itself. However regimes have been identified in which quantum turbulence and classical turbulence differ sharply, opening issues about the cross-over between these behaviours. The aim of this lecture is to introduce and review the new challenges which emerge from these results at the interface of classical and quantum physics.

Quantum fluids of light: from superfluid light towards fractional quantum Hall liquids

Iacopo Carusotto
INO-CNR BEC Center & University of Trento

In this talk I will review the recent advances in the theoretical and experimental study of quantum fluids of light, that is assemblies of photons confined in suitable material platforms that display collective many-body effects as a result of the interactions mediated by the optical nonlinearity of the underlying material. I will start with a survey of early results on Bose-Einstein condensation and superfluid hydrodynamical effects with an emphasis on those non-equilibrium features that are characteristic of these driven-dissipative optical systems. Open questions related to the nature of the phase transition towards a superfluid state will be addressed. I will then move towards the on-going challenge of using quantum fluids of light to generate strongly correlated states of matter, such as Mott insulators or fractional quantum Hall fluids in a completely new non-equilibrium framework. The main challenges and the potential of optical systems in this direction will be highlighted.

Semi and Quasi-Classical approximation of ground state energy for bosonic systems.

Marco Falconi
University of Tübingen

In this talk I would like to explain how semiclassical methods can be used to provide effective characterizations of the ground state energy for interacting quantum systems, consisting of many particles possibly interacting with bosonic fields. For such systems, the leading order of the energy can be described in terms of a “classical” variational problem, involving either an effective energy functional or the bottom of the spectrum of a suitable effective operator. Based on joint collaborations with Z. Ammari, M. Correggi, and M. Olivieri

---

On some coupled PDE-ODE systems in fluid dynamics

Evelyne Miot-Desecures
CNRS & Université Grenoble-Alpes

In this talk we will present existence and uniqueness issues for two coupled PDE-ODE systems. The common frame is that they arise as the asymptotical dynamics of a regular, incompressible 2D flow interacting with a point with highly concentrated vorticity (point vortex), or a rigid body contracting to a moving massive particle. This is joint work with Christophe Lacave.

Bogoliubov Theory at Positive Temperatures

Marcin Napiórkowski
University of Warsaw

I shall discuss the homogeneous Bose gas at positive temperatures within Bogoliubov theory. The theory arises by restricting the Hilbert space to quasi-free states. I will introduce the free energy functional and discuss the existence of equilibrium states, phase diagram and critical temperature. This is joint work with Robin Reuvers and Jan Philip Solovej.

Writhe and Twist Helicity of Quantum Vortex Systems

Renzo Ricca
University of Milano-Bicocca

Here we show how kinetic helicity, a fundamental invariant of classical fluid dynamics, can be equally defined for quantum vortices under the Gross-Pitaevskii equation and we discuss how the integral definition of helicity admits decomposition in terms of writhe and twist contributions [1]. We use this decomposition to study the evolution of writhe and twist under direct numerical simulation of the GPE evolution of a Hopf link and the following cascade process produced by a series of reconnections [2]. Since for quantum systems helicity remains always zero, and under anti-parallel reconnection writhe remains conserved across each reconnection event [3], twist must always balance changes in writhe. We show how this naturally leads to a continuous reduction of writhe and twist (of opposite sign) towards small-scale unknotted, unlinked planar rings.

[1] Moffatt, H.K. & Ricca, R.L. (1992) Helicity and the Clugreanu invariant. *Proc. R. Soc. A* **439**, 411-429.

[2] Zuccher, S. & Ricca, R.L. (2017) Relaxation of twist helicity in the cascade process of linked quantum vortices. *Phys. Rev. E* **95**, 053109.

[3] Laing, C.E., Ricca, R.L. & Sumners, DeW.L. (2015) Conservation of writhe helicity under anti-parallel reconnection. *Nature Sci. Rep.* **5**, 9224.

Gibbs measures of nonlinear Schrödinger equations as limits of many-body quantum states in dimension $d \leq 3$

Vedran Sohinger
University of Warwick

Gibbs measures of nonlinear Schrödinger equations are a fundamental object used to study low-regularity solutions with random initial data. In the dispersive PDE community, this point of view was pioneered by Bourgain in the 1990s. We prove that Gibbs measures of nonlinear Schrödinger equations arise as high-temperature limits of appropriately modified thermal states in many-body quantum mechanics. We consider bounded defocusing interaction potentials and work either on the d-dimensional torus or on R^d with a confining potential. The analogous problem for d=1 and in higher dimensions with smooth non translation-invariant interactions was previously studied by Lewin, Nam, and Rougerie by means of variational techniques.

Contributed Talk Session

15:30 - 15:50	L. Boßmann	Derivation of the one dimensional Gross-Pitaevskii equation for strongly confined 3d bosons
15:55 - 16:15	E. de Amorim	On the Hartree limit for the ground state of a bosonic atom without Born-Oppenheimer approximation
16:20 - 16:40	M. Olivieri	Microscopic derivation for time-dependent point interactions in ionization models
16:45 - 17:05	L. Tentarelli	Pointwise NLS in dimension two
17:10 - 17:30	Coffee break	
17:30 - 17:50	D.T. Nguyen	Many-body blow-up of bosons stars
17:55 - 18:15	R. Obermeyer	On a dissipative Gross-Pitaevskii-type model for Exciton-Polariton Condensates
18:20 - 18:40	H. Zheng	Global existence, stability and scattering of solutions to one-dimensional quantum hydrodynamic equations
18:45 - 19:05	D. Kim	Asymptotic behavior and stability problem for the Schrödinger-Lohe model

Derivation of the one dimensional Gross-Pitaevskii equation for strongly confined 3d bosons

Lea Boßmann
Universität Tübingen

We consider the dynamics of N interacting bosons initially exhibiting Bose-Einstein condensation. Due to an external trapping potential, the bosons are strongly confined in two spatial directions, with the transverse extension of the trap being of order ε. The non-negative interaction potential is scaled such that its scattering length is positive and of order $(N/\varepsilon^2)^{-1}$, the range of the interaction scales as $(N/\varepsilon^2)^{-\beta}$ for $\beta \in (0,1]$. We prove that in the simultaneous limit $N \to \infty$ and $\varepsilon \to 0$, the condensation is preserved by the dynamics and the time evolution is asymptotically described by a nonlinear Schrödinger equation in one dimension. The strength of the nonlinearity depends on the interaction and on the shape of the confining potential. For $\beta = 1$, the effective equation is a physically relevant one-dimensional Gross-Pitaevskii equation, where the coupling parameter contains the scattering length of the unscaled interaction. For our analysis, we adapt an approach by Pickl to the problem with dimensional reduction. Joint work with Stefan Teufel, based on *arXiv:1803.11011* and *arXiv:1803.11026*.

On the Hartree limit for the ground state of a bosonic atom without Born-Oppenheimer approximation

Erik de Amorim
Rutgers University

The goal of this work is to establish the Hartree theory for the ground state of a large atom with “bosonic electrons” in the limit of an infinite number of these, without invoking the Born-Oppenheimer approximation of an infinitely massive nucleus. A permutation-symmetric system of relative coordinates in the center-of-mass frame is developed which facilitates the study of the quantum ground state energy of such many-electron atoms with finite-mass nucleus (which applies equally well to bosonic and fermionic electrons). In the new coordinates the intrinsic Hamiltonian has an equivalent many-body interpretation of N electrons electrically attracted to a multiple of the empirical mean of their own positions.

Asymptotic behaivor and stability problem for the Schrödinger-Lohe model

Dohyun Kim
Seoul National University

We present asymptotic behavior and stability problem for the Schrödinger-Lohe (S-L) system which was first introduced as a possible phenomenological model exhibiting quantum synchronization. We present several sufficient frameworks leading to the emergent behavior of the S-L system. More precisely, we show that there are only two possible asymptotic states: completely synchronized state or bi-polar state. Furthermore, we provide the standing wave solutions for the S-L model with the harmonic potential and discuss the stability for standing wave solutions.

Many-body blow-up of bosons stars

Dinh-Thi Nguyen
LMU Munich

We study ground states of a system of N identical bosons in $\mathbb{R}^3$, described by the Hamiltonian

$$H_N = \sum_{i=1}^{N} \left(\sqrt{-\Delta_{x_i} + m^2} + V(x_i) \right) - \frac{a}{N-1} \sum_{1 \le i < j \le N} |x_i - x_j|^{-1}.$$

acting on Hilbert space $\bigotimes_{\text{sym}}^N L^2(\mathbb{R}^3)$. Here the parameter $m > 0$ is the mass of particles, $a > 0$ describes the strength of the attractive interaction, and $V \ge 0$ is an external potential. We are interested in the behavior of the ground state energy per particle of H_N and the corresponding ground state when $N \to \infty$ and $a := a_N$ tends to a^* (Chandrasekhar limit) from below. We first study blow-up behavior of ground state energy as well as of ground states when a tends to a^* in the effective model: Hartree theory.

On a Dissipative Gross-Pitaevskii-type model for Exciton-Polariton condensates

Ryan Obermeyer
University of Illinois at Chicago

We study a generalized dissipative Gross-Pitaevskii-type model arising in the description of exciton-polariton condensates. We derive rigorous existence and uniqueness results for this model posed on the one dimensional torus and derive various a-priori bounds on its solution. Then, we analyze in detail the long time behavior of spatially homogenous solutions and their respective steady states. In addition, we will present numerical simulations in the case of more general initial data. We also study the corresponding adiabatic regime which results in a single damped-driven Gross-Pitaveskii equation and compare its dynamics to the one of the full coupled system. This is joint work with my advisor, Christof Sparber, as well as Paolo Antonelli (GSSI), Peter Markowich (KAUST), and Jesus Sierra (KAUST), to be submitted in the near future.

Microscopic derivation for time-dependent point interactions in ionization models

Marco Olivieri
Sapienza, Università di Roma

We show how ionization models, consisting of a moving particle influenced by a point interaction modulated by a time-evolving coefficient, can be derived as effective models obtained in quasi-classical limit from the polaron model. The approximation by the scale limit has a physical justification considering time-dependent, squeezed coherent states with high intensity of the field, and it can be proved that particles observables, evolving w.r.t. microscopic dynamics, weak converge to time-evolved observables according to the effective dynamics. From a joint work with R. Carlone, M. Correggi and M. Falconi.

Pointwise NLS in dimension two

Lorenzo Tentarelli
Sapienza, Università di Roma

We discuss some recent results on the twodimensional nonlinear Schrödinger equation with pointwise nonlinearity. We start by presenting local well-posedness of the associated Cauchy problem, as well as mass and energy conservation along the flow. Then, we show that in the repulsive case solutions are global-in-time, whereas in the attractive case one can exhibit a class of initial data that give rise to blow-up phenomena. Finally, we exhibit the family of the standing waves of the problem. The talk is based on three works in collaboration with R. Adami, R. Carlone, M. Correggi and A. Fiorenza.

Global existence, stability and scattering of solutions to one-dimensional quantum hydrodynamic equations

Hao Zheng
GSSI

In this talk we consider the Cauchy problem for the one-dimensional quantum hydrodynamic (QHD) system, namely the compressible Euler equations with a quantum correction term. We consider finite energy initial data with certain higher order bounds, which imply the square integrability of the chemical potential. These type of models have been extensively used to investigate superfluidity, superconductivity and recently in the modelling of semiconductor devices. The main result is the global existence of weak solutions to the Cauchy problem. Our approach does not require the initial data to be given through a wave function as in previous results. Nevertheless the higher order bounds allow to construct a related wave function. We prove such higher order quantities are bounded for any finite time. Furthermore, those a priori estimates allow us to prove the stability of weak solutions to the QHD system. Last, we present some scattering properties of solutions to the QHD system, which can be proved in purely hydrodynamic way.