Measurements, uncertainties and probabilistic inference/forecasting

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(A. Einstein)

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(R. Feynman)

"Probability is good sense reduced to a calculus"
(S. Laplace)

Introducing the logic of uncertainty

- No collection of formulae.

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  that can be used for a broad range of applications.
  - Avoid unneeded '*principles*'

Introducing the logic of uncertainty

- No collection of formulae.
- No collection of tests "with Russian names".
- Try to build up a consistent theory
  that can be used for a broad range of applications.
  - Avoid unneeded '*principles*'...
    whose results will *possibly* be reobtained
    as *approximations under well stated conditions*.

Please be patient

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"...today I'll learn to read,

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"... today I'll learn to read,

tomorrow to write,

Please be patient

"...today I'll learn to read,
tomorrow to write,
and the day after tomorrow
I'll do arithmetic."

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- No rush to get formulae

$\rightarrow$ If you understand the basic reasoning
you can derive many formulae by yourself'!

What is measurement?

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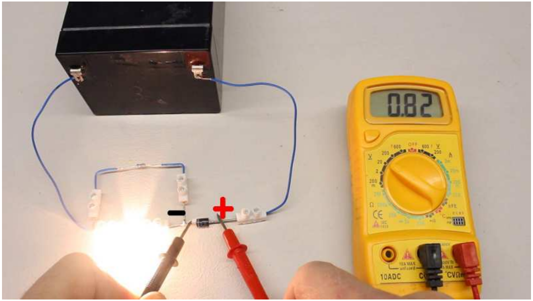

What is measurement?

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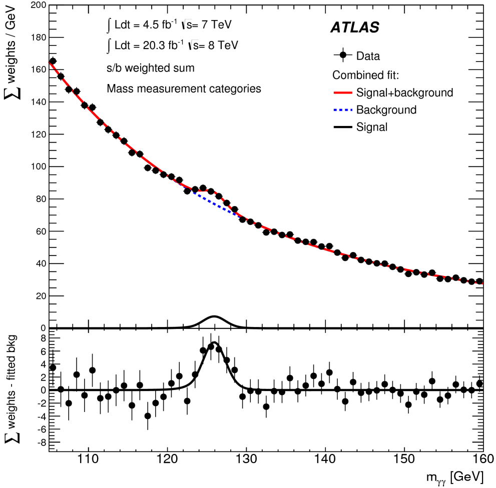
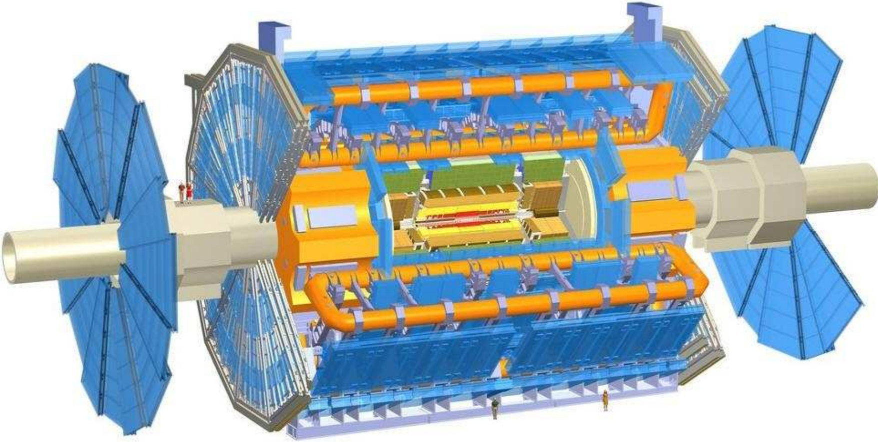
Higgs $\rightarrow \gamma\gamma$ (2012)



Two-photon *invariant mass*

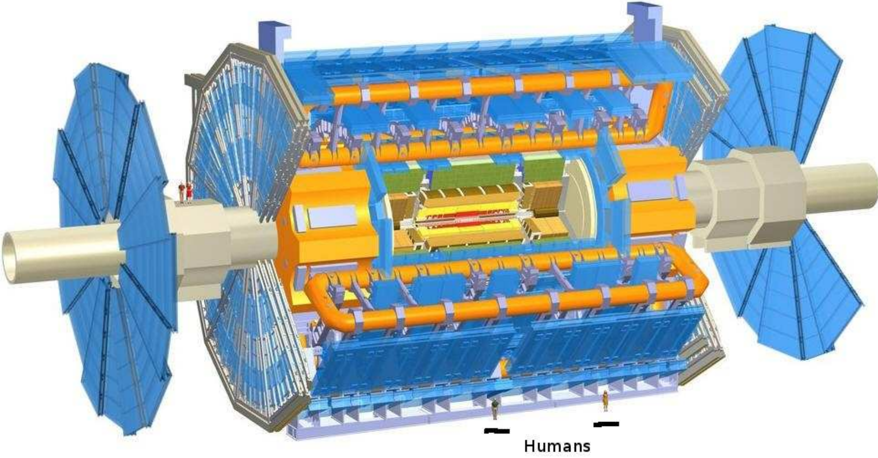
What is measurement?

ATLAS Experiment at LHC (CERN, Geneva)



What is measurement?

ATLAS Experiment at LHC [length: 46 m; ⌀ 25 m]

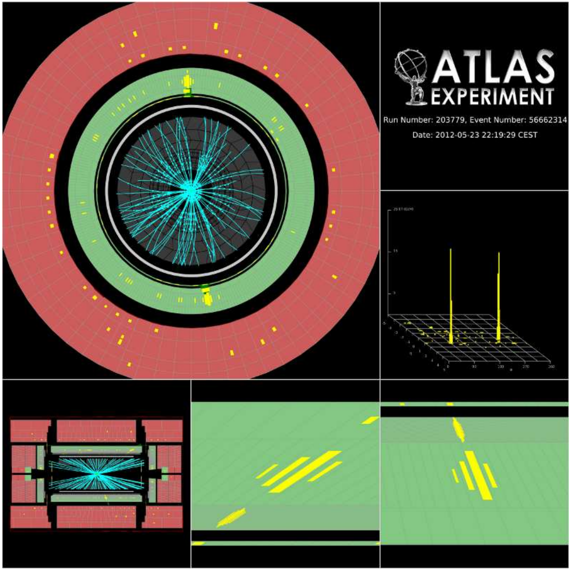


≈ 3000 km cables

≈ 7000 tonnes

≈ 100 millions electronic channels

What is measurement?



Two flashes of ‘light’ (2 γ’s) in a ‘noisy’ environment.

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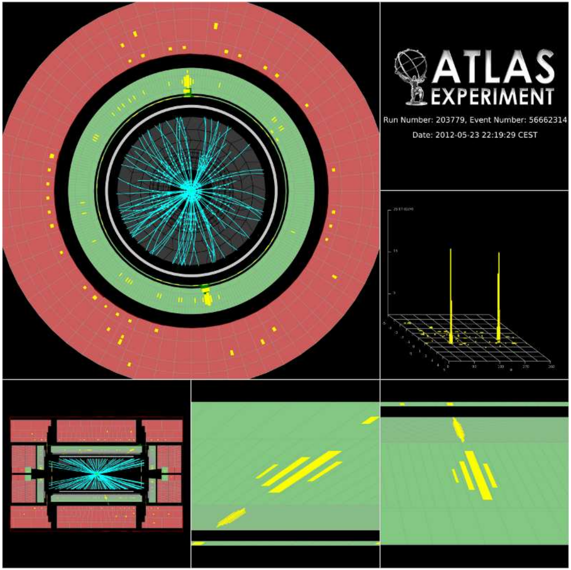

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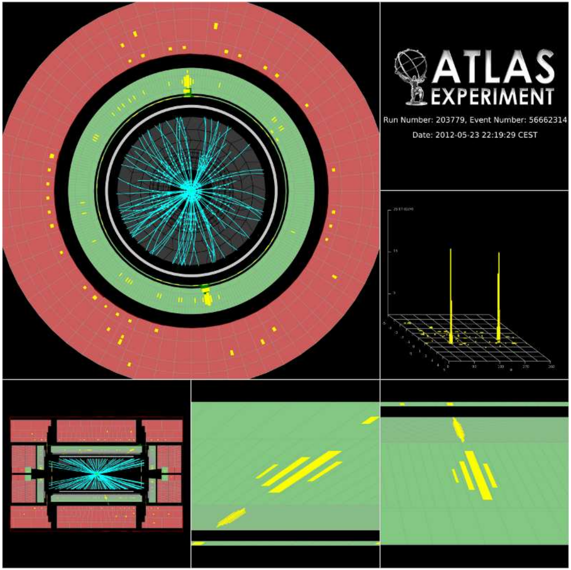

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Higgs $\rightarrow \gamma\gamma$? Probably not...

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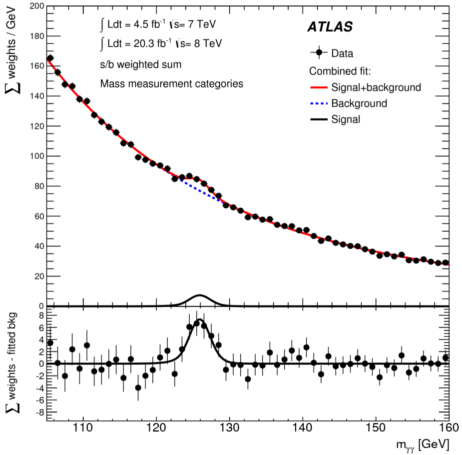

$\Rightarrow \begin{cases} \text{Mass value} \\ \text{Production rate} \end{cases}$

What is measurement?

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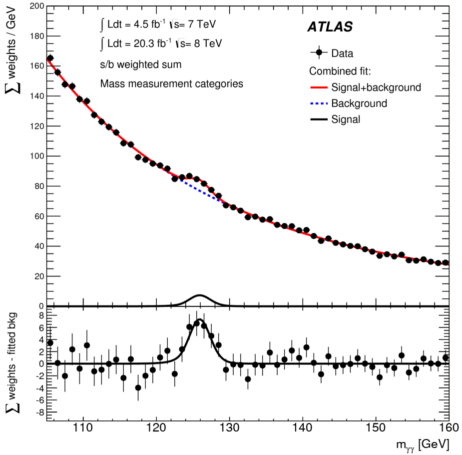

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(with uncertainties)

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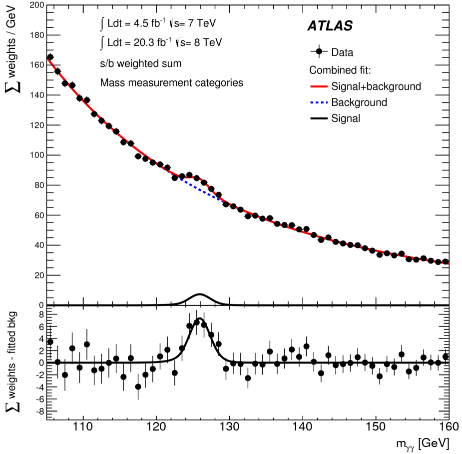

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(with uncertainties)

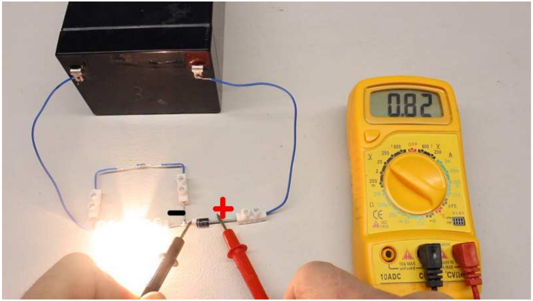
Quite indirect measurements of something we do not "see"!

Can we “see” physics quantities?

But, can we see our mass?

Can we "see" physics quantities?

. . . or a voltage?



Can we "see" physics quantities?

. . . or our blood pressure?

Can we “see” physics quantities?

Certainly not!

Can we "see" physics quantities?

Certainly not!

. . . although for some quantities we can have

a 'vivid impression' (in the David Hume's sense)

Measuring a mass on a scale

Equilibrium:

$$\color{red}{m}\color{black}{g - k}\color{blue}{\Delta x}\color{black}{\quad = \quad 0}$$

$$\color{blue}{\Delta x}\color{black}{\quad \rightarrow \quad}\color{blue}{\theta}\color{black}{\rightarrow}\color{blue}{\text{scale reading}}$$

(with 'g' *gravitational acceleration;* 'k' spring constant.)

Measuring a mass on a scale

Equilibrium:

$$
\begin{aligned}
mg - k\Delta x &= 0 \\
\Delta x &\rightarrow \theta \rightarrow \text{scale reading}
\end{aligned}
$$

(with 'g' *gravitational acceleration;* 'k' spring constant.)

From the reading to the value of the mass:

$$\text{scale reading} \xrightarrow[\textit{given } g,\ k,\ \text{“etc.”}\ldots]{} m$$

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Dependence on 'g': $$g \overset{?}{=} \frac{GM_{\oplus}}{R_{\oplus}^2}$$

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Certainly not to watch our weight

But think about it!

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Dependence on 'k':

- temperature
- non linearity
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$+$ **randomic effects:**

- stopping position of damped oscillation;
- variability of all quantities of influence (in the ISO-GUM sense);
- reading of analog scale.

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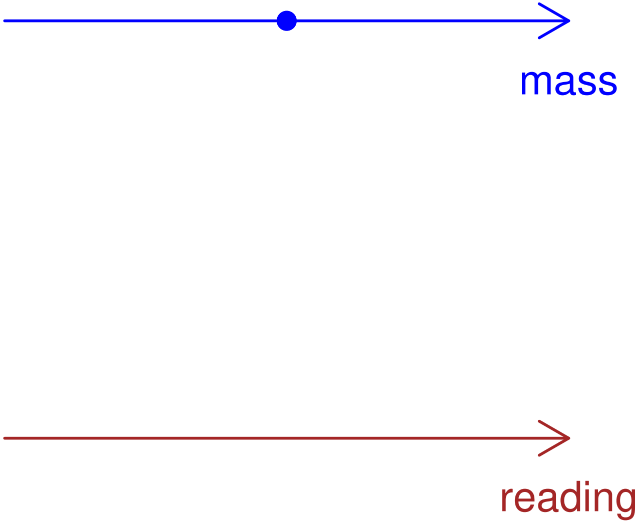
$\Rightarrow\ m$??

Mass $\longrightarrow$ Reading



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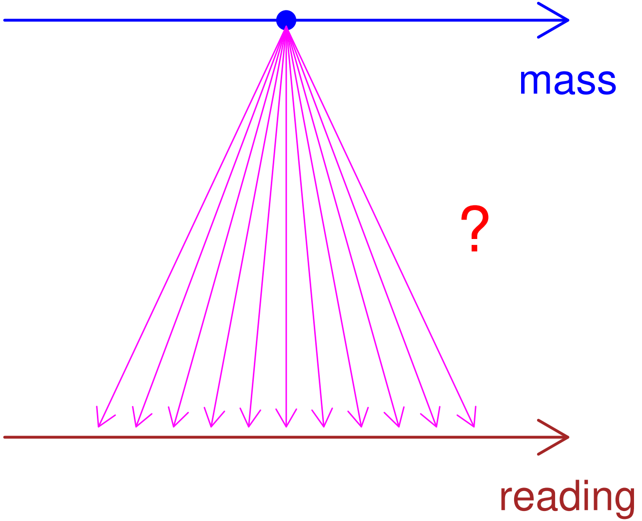

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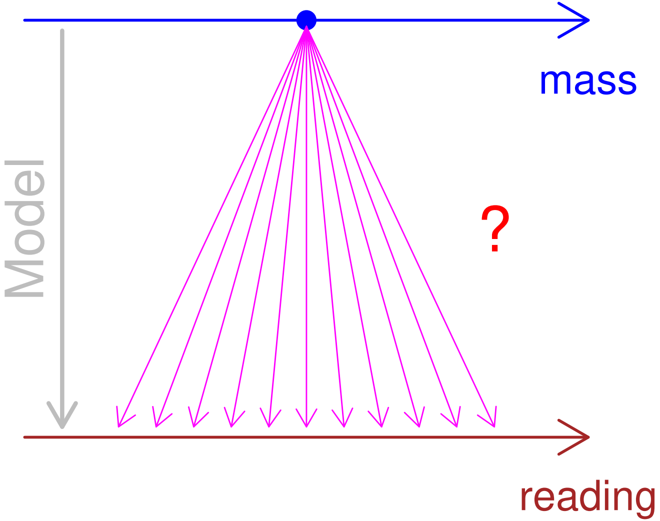

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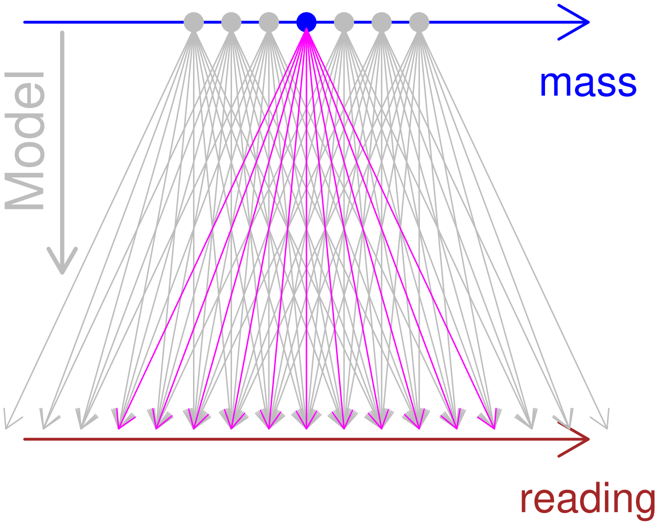
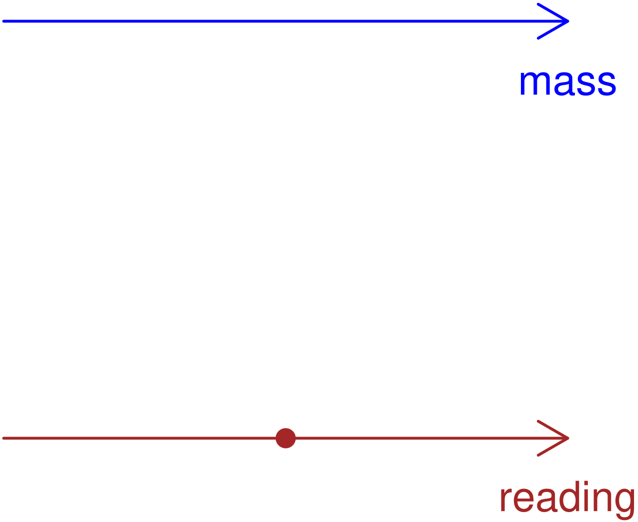


Reading $\longrightarrow$ ‘true’ mass



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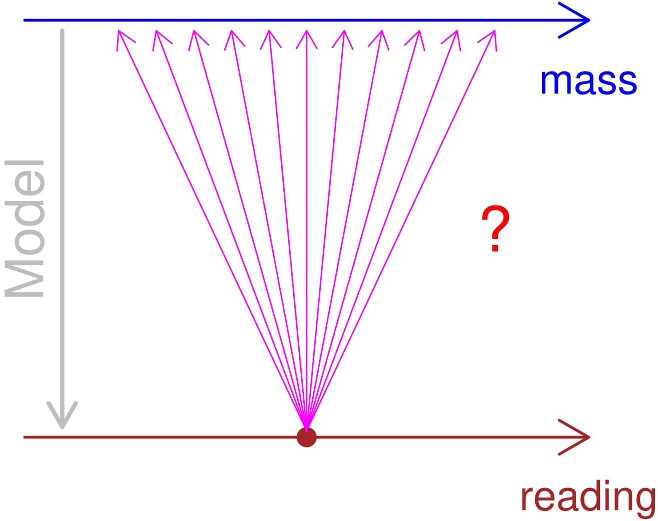

Sources of uncertainties (from ISO GUM)

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5. *personal bias in reading analogue instruments;*

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Note

- Sources not necessarily independent
- In particular, sources 1-9 may contribute to 10 (e.g. not-monitored electric fluctuations)

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Error and *uncertainty* are not synonyms!

Type A and Type B uncertainties

Type A evaluation (of uncertainty): *"method of evaluation of uncertainty by the statistical analysis of series of observations."*

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- *uncertainties assigned to reference data taken from handbooks."*

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  not necessarily

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  - a single measurement has been performed on a particular quantity;
  - previous experiments on <u>other</u> particular quantities, on the same condition, have provided a repeatability standard deviation σ_r;
  - at the best of our knowledge

$$u = \sigma_r\,.$$

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not necessarily

In general yes, but imagine the case in which

- a single measurement has been performed on a particular quantity;
- previous experiments on <u>other</u> particular quantities, on the same condition, have provided a repeatability standard deviation σ_r;
- at the best of our knowledge

$$u = \sigma_r \, .$$

$\rightarrow$ Type B uncertainty due to 'statistical effects'.

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These cases have not to be seen as "the exception that confirms the rule" [in physics exceptions falsify laws!], but as symptoms of something flawed in the reasoning, that could seriously effects also results that are not as self-evidently paradoxical as in these cases!

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In my opinion, simply the reluctance to combine linearly 10, 20 or more contributions to a global uncertainty, as the (out of fashion) 'theory' of maximum bounds would require.

$\rightarrow$ Right in most cases!

$\rightarrow$ Good sense of physicists $\Longleftrightarrow$ cultural background

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n independent measurements of the same quantity μ (with n large enough and no systematic effects, to avoid, for the moment, extra complications).

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$$P(\mu - \frac{\sigma}{\sqrt{n}} \leq \overline{X} \leq \mu + \frac{\sigma}{\sqrt{n}}) = 68\%\,,$$

(that is a probabilistic statement about $\overline{X}$: probabilistic statements about μ are not allowed by the theory).

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as we shall see later ($\rightarrow$ 'predictive distributions').

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  - 'technically' [see e.g. G. Zech, *Frequentistic and Bayesian confidence limits*, EPJdirect C12 (2002) 1]
  - ‘in terms of performance’ $\rightarrow$ ‘very strange’ that no quantities show in ‘other side’ of a 95% C.L. bound !
- Not suited to express our confidence! Simply because it was not invented for that purpose!

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> *“that technological and commercial apparatus which is known as an acceptance procedure”*
> *(Fisher, referring to Neyman’s statistical confidence method)*

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For a more argued criticism on how confidence intervals technically derive (trictly following the frequentistic prescription):
⇒ `arXiv:physics/0605140 [physics.data-an]`

Arbitrary probability inversions

How do we turn, just 'intuitively'

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We can paraphrase as

"the dog and the hunter"

The dog and the hunter

We know that a dog has a 50% probability of being 100 m from the hunter

$\Rightarrow$ if we observe the dog, what can we say about the hunter?

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Intuitive and reasonable answer:

> *"The hunter is, with 50% probability, within 100 m of the position of the dog."*

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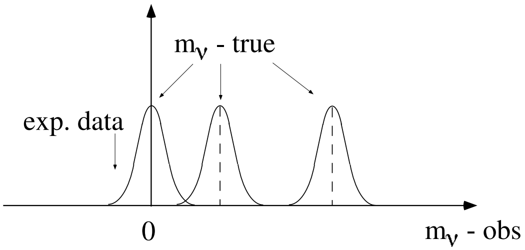
$\rightarrow$ not always valid!

Measurement at the edge of a physical region

Electron-neutrino experiment, mass resolution $\sigma = 2\,\mathrm{eV}$, independent of m_ν.



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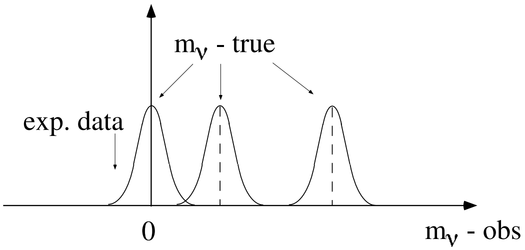

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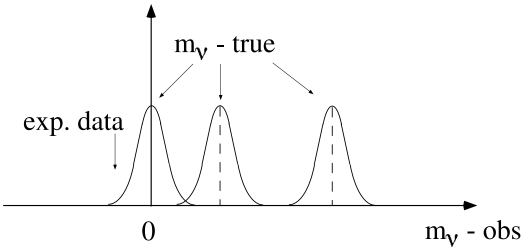

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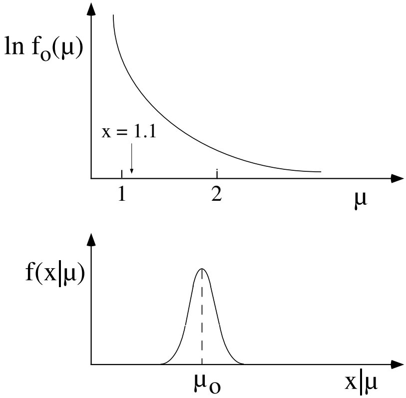
$m_\nu = -4 \pm 2$ eV ?

$P(-6 \le m_\nu/\text{eV} \le -2) = 68\%$?

$P(m_\nu \le 0 \text{ eV}) = 98\%$?

Non-flat distribution of a physical quantity

Imagine a cosmic ray particle or a bremsstrahlung γ.



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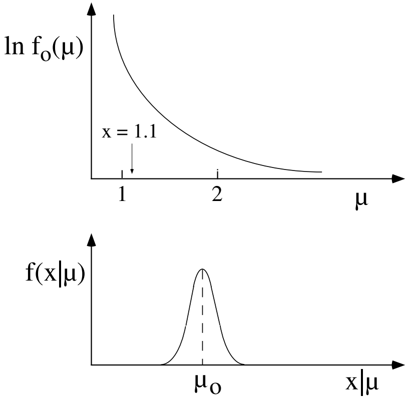

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What can we say about the true value μ that **has caused** this observation?

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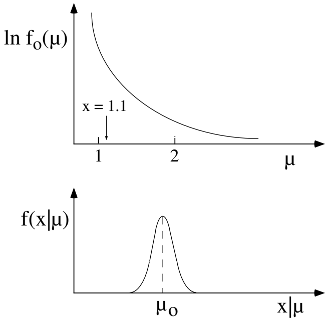

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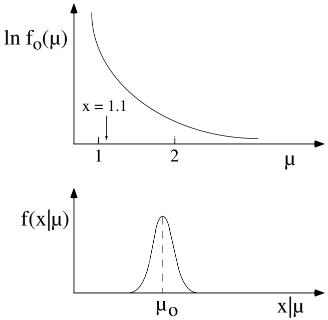

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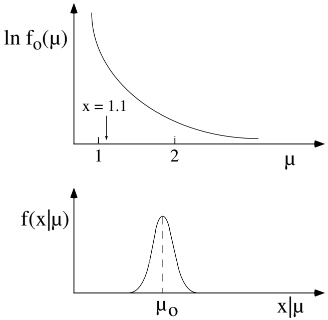

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In the jargon of the experimentalists, *"there are more migrations from left to right than from right to left"*.

Asymmetric detector response

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Summing up:
the intuitive inversion of probability

$$P(\ldots \leq \overline{X} \leq \ldots) \Longrightarrow P(\ldots \leq \mu \leq \ldots),$$

besides being theoretically unjustifiable in the frequestist approach to probability, yields results which are numerically correct only in the case of symmetric problems.

Summary about ‘standard methods’

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- hypotheses tests
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Moreover there are issues not easy to treat in that frame
[and I smile at the heroic effort to get some result :-)]

- systematic errors
- background

Implicit assumptions

We have seen clearly what are the hidden assumptions
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Curiously enough, these methods are advertised as objective because they do not need as input our scientific expectations of where the value of the quantity might lie, or of which physical hypothesis seems more reasonable!

But if we are convinced (by logic, or by the fact that neglecting that knowledge paradoxical results can be achieved) that prior expectation is relevant in inferences, we cannot accept methods which systematically neglect it and that, for that reason, solve problems different from those we are interested in!

Let's restart

Observation $\rightarrow$ value of a quantity

$$\text{scale reading} \xrightarrow[\textit{given } g,\ k,\ \text{“etc.”}\dots]{} m$$

Observations → hypotheses

This problem occurs not only "determining" *the* value of a physical quantity.

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(But logically no substantial difference.)

Human ancestral problem

???

Human ancestral problem

???

→ Chase?

→ Run away?

Observation → hypotheses

Dependence from the context

Chase o Run away?

Dependence from the context

Chase o Run away?

...or simply stay quite

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if it is a mold in a museum,

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(. . . or we are just sated tourists, with no interest in chasing, well protected inside our safari minibus)

Contemporary anthropology (and technology)

???

Effect and possible causes

Effect: car broken down

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- Causes:
  - no gasoline
  - broken pump
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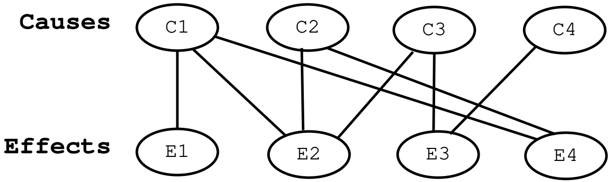
- Action: balance between probability of the several hypotheses, costs and times.

Causes $\rightarrow$ effects

The same *apparent* cause might produce several,different effects



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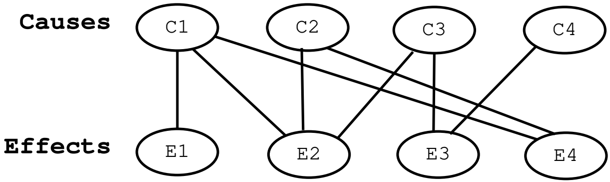

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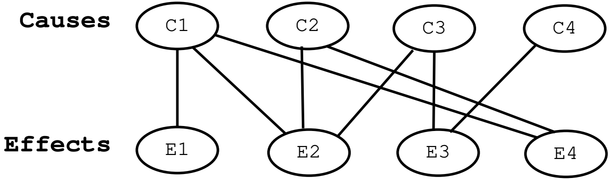

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$$E_2 \Rightarrow \{C_1,\ C_2,\ C_3\}?$$

The "essential problem" of the Sciences

"Now, these problems are classified as *probability of causes*, and are most interesting of all for their scientific applications. I play at *écarté* with a gentleman whom I know to be perfectly honest. What is the chance that he turns up the king? It is $1/8$. This is a problem of the probability of effects.

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I play with a gentleman whom I do not know. He has dealt ten times, and he has turned the king up six times. What is the chance that he is a sharper? This is a problem in the probability of causes. It may be said that it is the essential problem of the experimental method."

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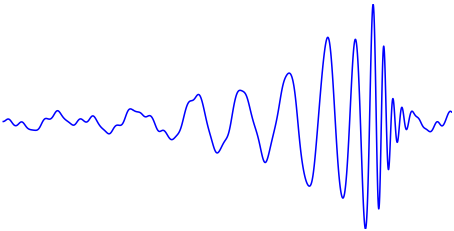
Why we (or most of us) have not been taught how to tackle this kind of problems?

Who has done this 'scribble'?



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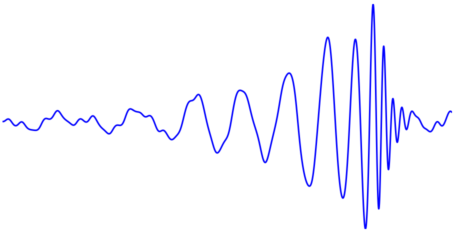

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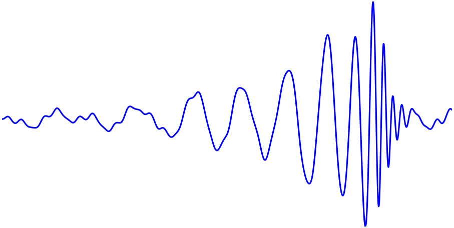

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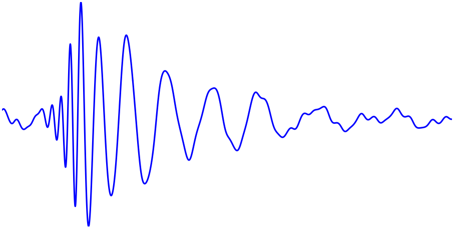

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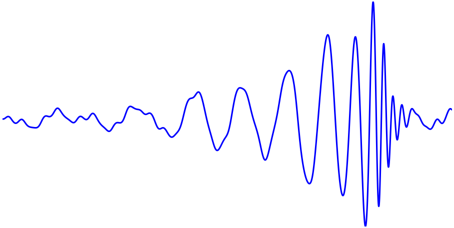
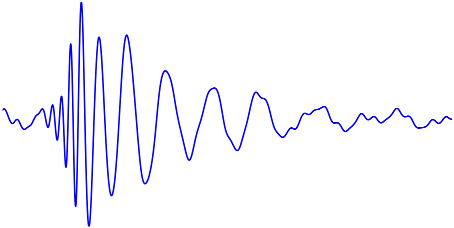




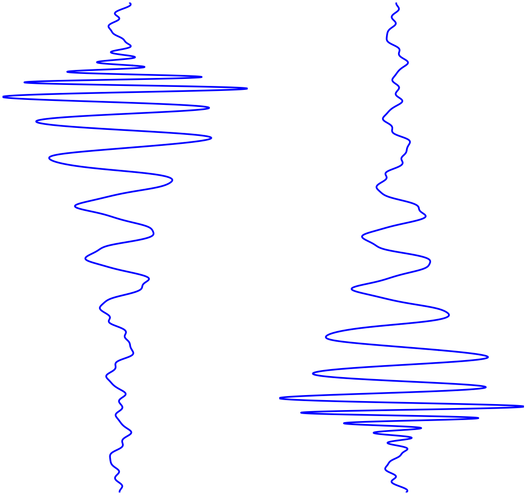
???

???

- Cardiogram?
- Signature?
- Sound?
- Earthquake?

Let's change orientation

(pure despair. . .)



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Distrust the

Dogma of the Immaculate Observation

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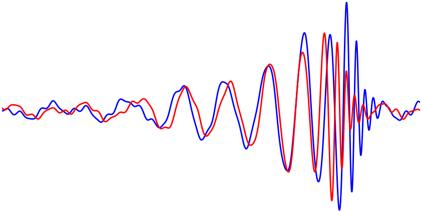

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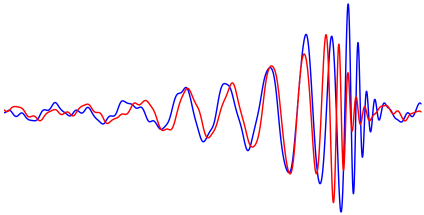

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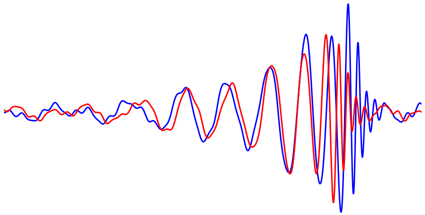

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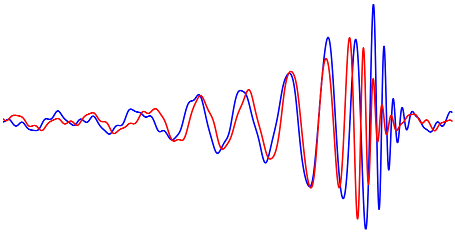
  [We tend to believe what trusted people believe]

Effect $\rightarrow$ cause



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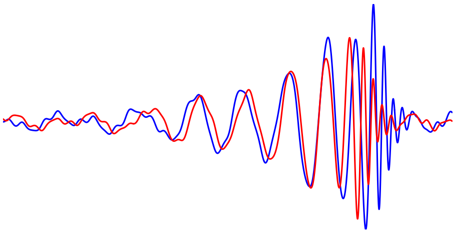

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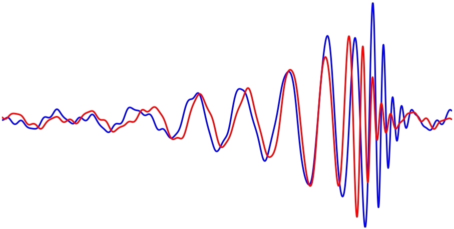

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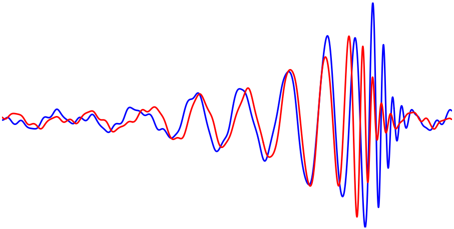

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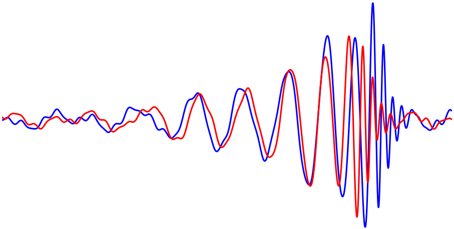

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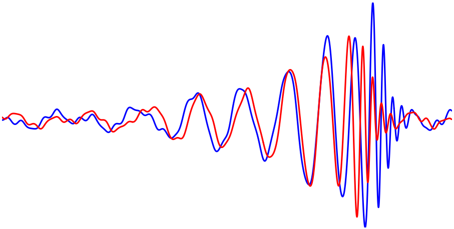

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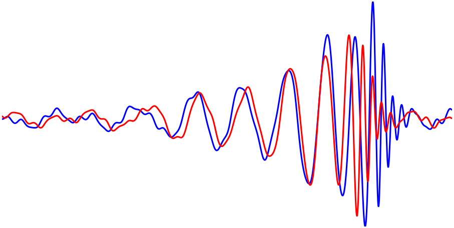

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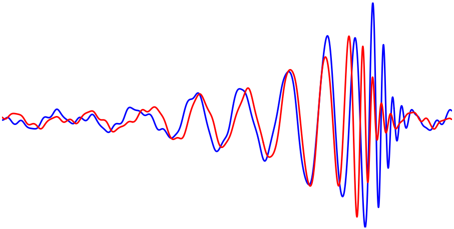

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(The last two causes are not just amenities!)

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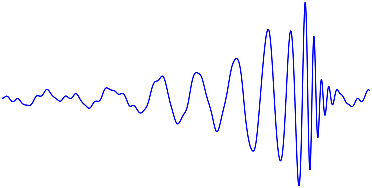
On the basis of the best knowledges about the possible causes



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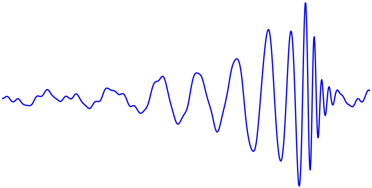

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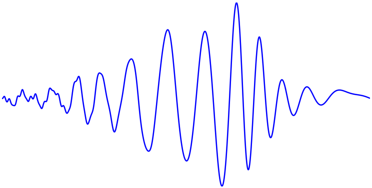

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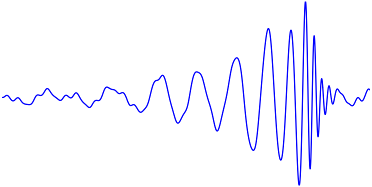

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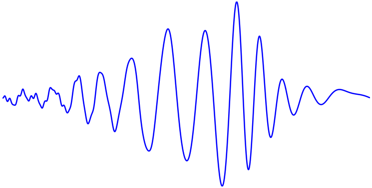

? ? ?

Perhaps more likely a local random tremble...

$\{$Effect $\rightarrow$ cause$\}$ $\rightarrow$ 'concomitant causes' $\rightarrow$ corroboration

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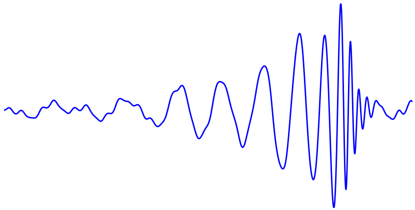
Despite of the 'sigmas'...

What is the difference between the two “scribbles”?

A)



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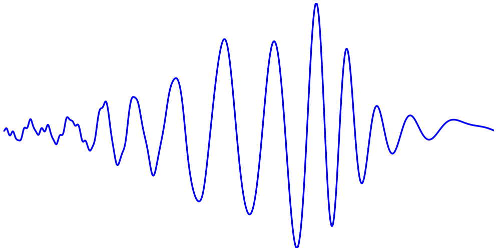

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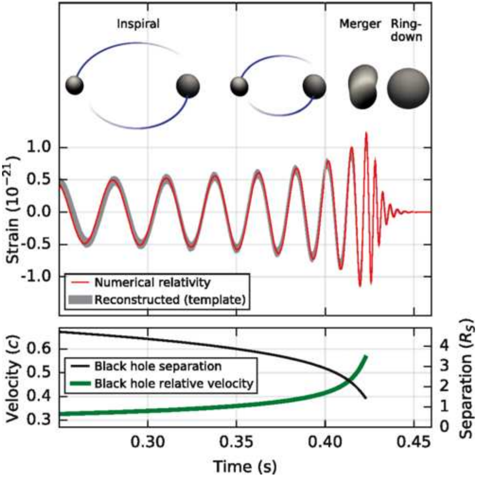

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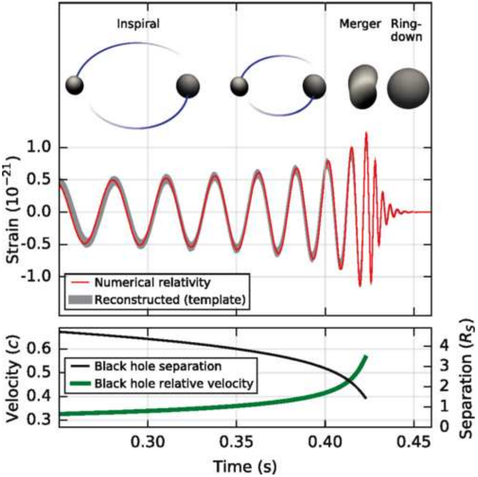

B)

NOTHING (as far as we understand it <u>now</u>. . .)

From the cosmic space down to problems of common mortals

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- two causes;
- two effects;

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- medical diagnostics helps to clarify the issues:
  - easier to reach intuitive answers
  - ... although if someone might have fallacious intuitions
    $\Rightarrow$ a formal guide helps us avoiding errors
    - $\Rightarrow$ **logics of the uncertain** (theory of probabilities)

AIDS test

An Italian citizen is selected at random
to undergo an AIDS test.
$\rightarrow$ Performance of clinical trial is not perfect, as customary:

$$\begin{aligned} P(\text{Pos}\,|\,\text{HIV}) &= 100\% \\ P(\text{Pos}\,|\,\overline{\text{HIV}}) &= 0.2\% \\ P(\text{Neg}\,|\,\overline{\text{HIV}}) &= 99.8\% \end{aligned}$$

H_1='HIV' (Infected) $\qquad E_1$ = Positive

H_2='$\overline{\text{HIV}}$' (Not infected) $\qquad E_2$ = Negative

AIDS test

An Italian citizen is selected at random
to undergo an AIDS test.
$\rightarrow$ Performance of clinical trial is not perfect, as customary:

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Result: $\Rightarrow$ Positive

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Result: $\Rightarrow$ <u>Positive</u>

Infected or not infected?

AIDS test: how to interpret the result?

Being $P(\text{Pos} \,|\, \overline{\text{HIV}}) = 0.2\%$ and having observed 'Positive', can we say?

- "It is practically impossible that the person is not infected, since it was practically impossible that a non infected person would result positive"

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(We will learn in the sequel how to evaluate it correctly)

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...from which bad decisions might result!

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The previous statements, although dealing with probabilistic issues, **are not ground** on probability theory

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... and in these issues intuition can be fallacious!

$\Rightarrow$ A sound formal guidance can rescue us

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Pay attention not to arbitrary revert conditional probabilities:

In general $P(A \mid B) \neq P(B \mid A)$

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$\Rightarrow$ Prosecutor's fallacy

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A practical example:

- I shut a picture with my faithful pocket camera.

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- Extract a number 'a random' integer between 1 and 1 billion (indeed pseudo-random, but it is the same for the purpose)

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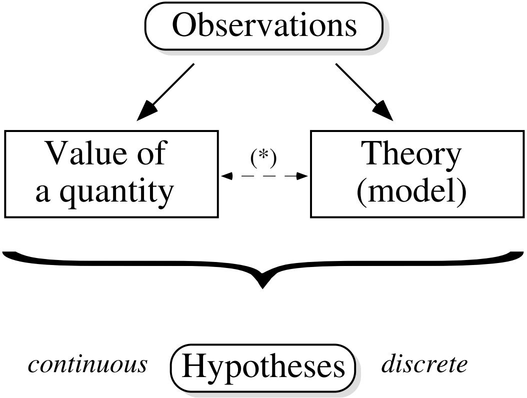
Learning from data



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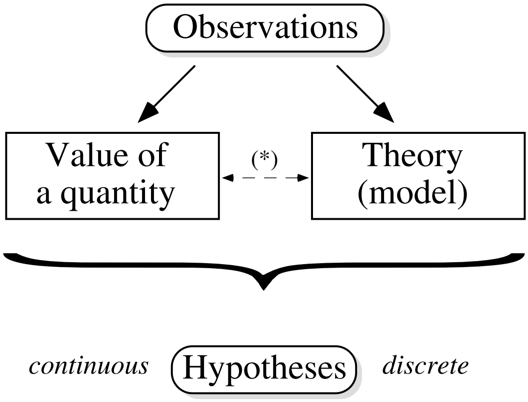
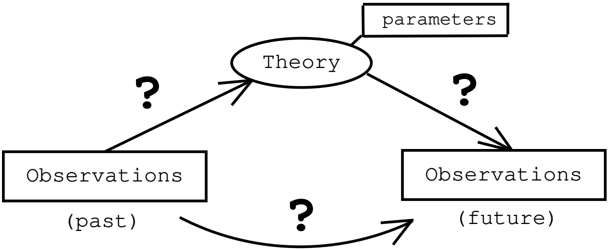


(*) A quantity might be meaningful only within a theory/model

From past to future



Our task:

- Describe/understand the 'physical world'

  $\Rightarrow$ inference of laws ('models') and their parameters $[\Theta]$

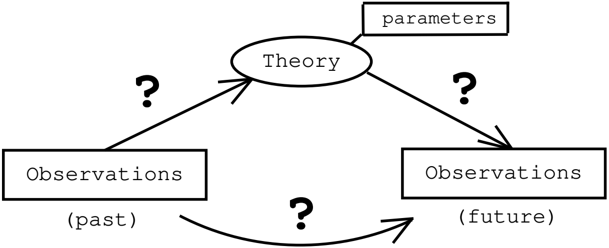
  $\Rightarrow$ $[\Theta \,|\, X_{past}]$

- Predict observations $[X]$

  $\Rightarrow$ forecasting

  $\Rightarrow$ $[X_{future} \,|\, \Theta]$ $\quad\rightarrow\quad$ $[X_{future} \,|\, X_{past}]$

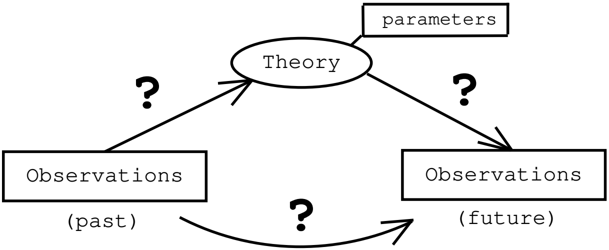
From past to future



Process

- neither automatic
- nor purely contemplative
  - → 'scientific method'
  - → planned experiments ('actions') ⇒ decision.

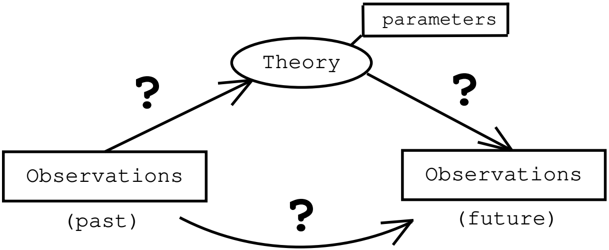
From past to future



$\Rightarrow$ The role of theories/models

- a theory and its parameters are the 'distillate' of all our knowledge about the 'universe' of interest;
- empirical analogical thinking is <u>in most cases not usable</u>:
  - A theory can predict effects never observed before
  - Example: shooting a bullet

From past to future



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"La cognizione d'un solo effetto acquistata per le sue cause ci apre l'intelletto a 'ntendere ed assicurarci d'altri effetti senza bisogno di ricorrere alle esperienze" (Galileo)

Model thinking

"The scientific method is based on repeated experiments"
(or some like that)

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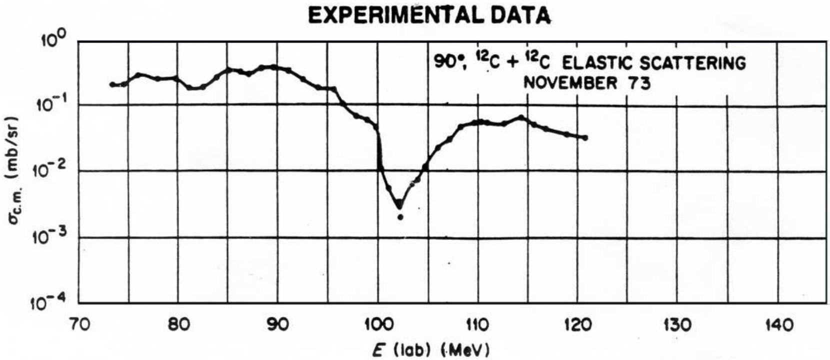
What really matters is to have a Model which links parameters to observations
But remind that "all models are wrong, some are useful"…

Inferential-predictive process



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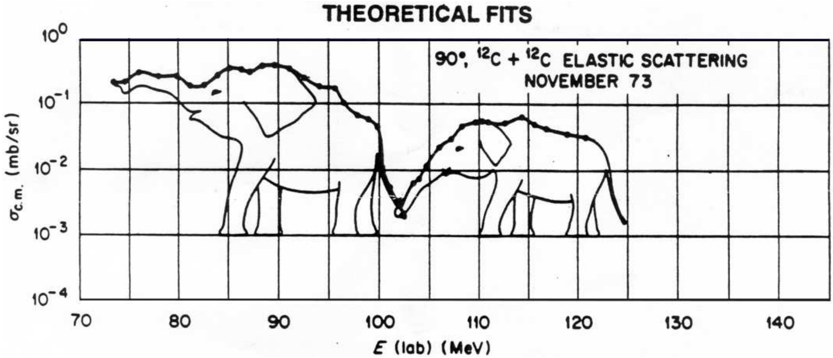

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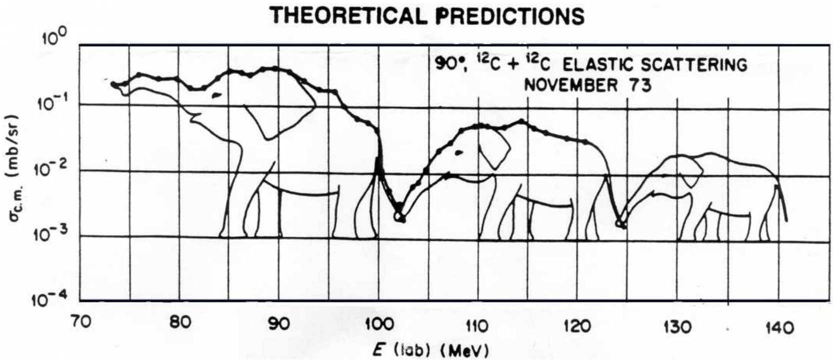
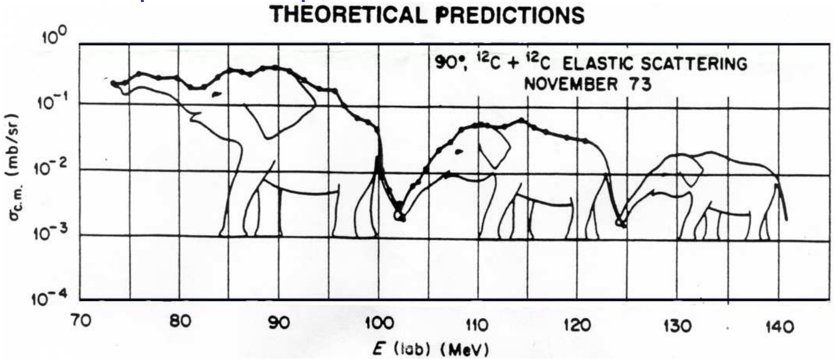


(S. Raman, *Science with a smile*)

Inferential-predictive process



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Even if the (*ad hoc*) model fits perfectly the data,
we do not believe the predictions
because we don't trust the model!

[Many 'good' models are *ad hoc* models!]

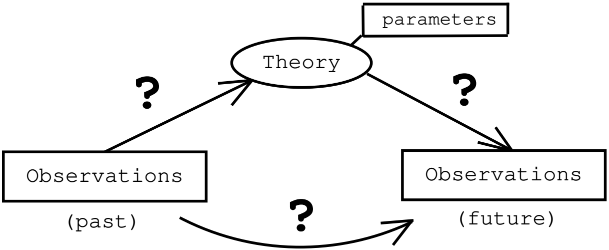
2011 IgNobel prize in Mathematics

- D. Martin of USA (who predicted the world would end in 1954)
- P. Robertson of USA (who predicted the world would end in 1982)
- E. Clare Prophet of the USA (who predicted the world would end in 1990)
- L.J. Rim of KOREA (who predicted the world would end in 1992)
- C. Mwerinde of UGANDA (who predicted the world would end in 1999)
- H. Camping of the USA (who predicted the world would end on September 6, 1994 and later predicted that the world will end on October 21, 2011)

2011 IgNobel prize in Mathematics

"For teaching the world to be careful when making mathematical assumptions and calculations"

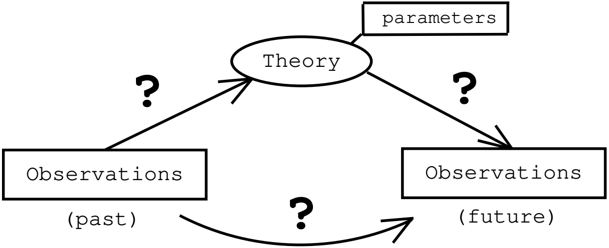
Uncertainty



⇒ Uncertainty:

1. Given the past observations, in general we are not sure about the parameters of the model (and/or the model itself)
2. Even if we were sure about theory and parameters, there could be internal ("noise", variables out of our control) or external effects (initial/boundary conditions, 'errors', etc) that make the forecasting uncertain.

Uncertainty

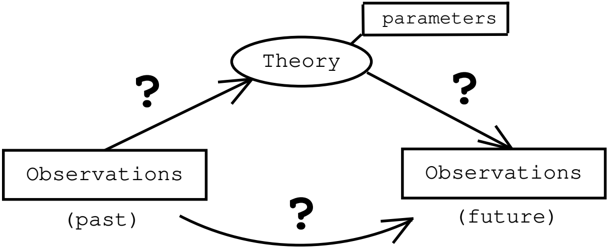


⇒ Uncertainty:

- No certainties, only probabilities

- $P(\Theta \,|\, X_{past})$
- $P(X_{future} \,|\, \Theta)$
- $P(X_{future} \,|\, X_{past})$

Deep source of uncertainty



Uncertainty:

Theory	— ? ⟶	Future observations
Past observations	— ? ⟶	Theory
Theory	— ? ⟶	Future observations

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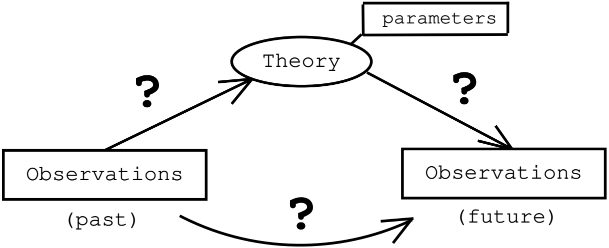


Uncertainty:

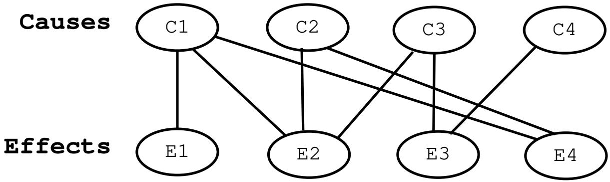
Theory	— ? $\longrightarrow$	Future observations
Past observations	— ? $\longrightarrow$	Theory
Theory	— ? $\longrightarrow$	Future observations

$\Longrightarrow$ **Uncertainty about causal connections**

CAUSE $\Longleftrightarrow$ **EFFECT**

Causes $\rightarrow$ effects

The same *apparent* cause might produce several,different effects



Given an observed effect, we are not sure about the exact cause that has produced it.

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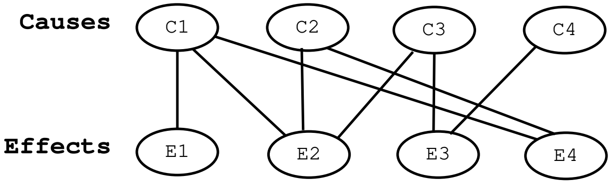

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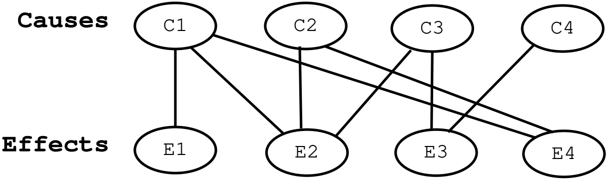
Causes $\rightarrow$ effects

The same *apparent* cause might produce several,different effects



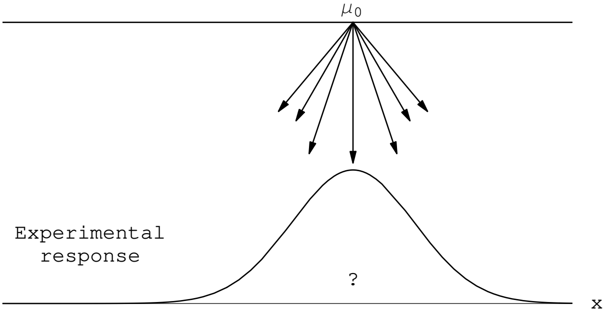
Given an observed effect, we are not sure about the exact cause that has produced it.

$$E_2 \Rightarrow \{C_1,\ C_2,\ C_3\}?$$

$\rightarrow$ Probability of causes

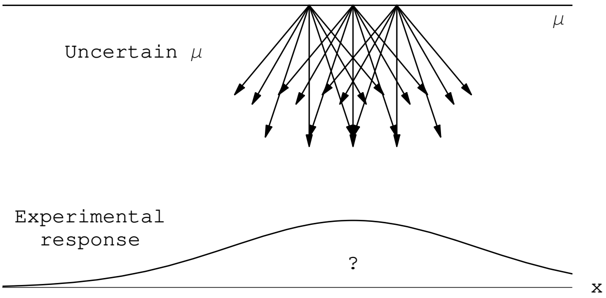
"the essential problem of the experimental method"

From 'true value' to observations



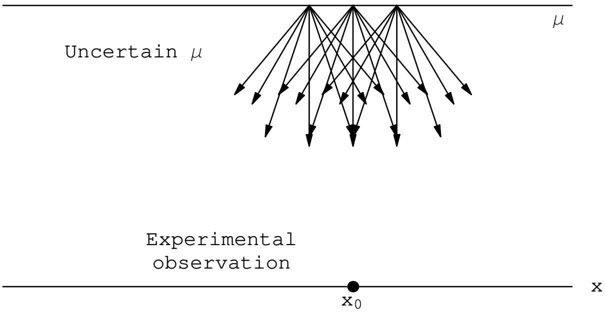
Given μ (exactly known) we are uncertain about x

From ‘true value’ to observations



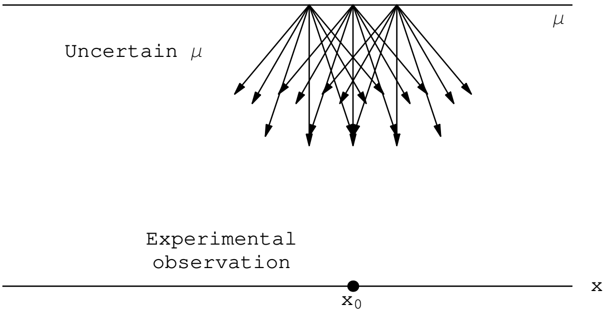
Uncertainty about μ makes us more uncertain about x

. . . and back: Inferring a true value



The observed data is <u>certain</u>: $\rightarrow$ 'true value' uncertain.

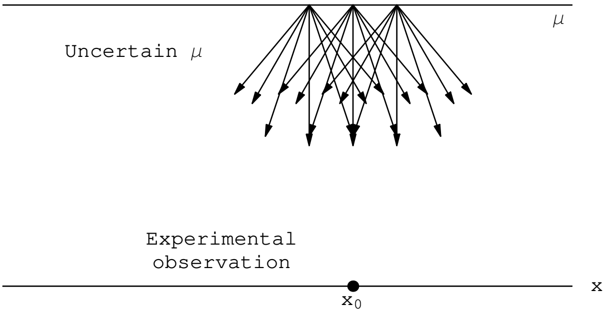
. . . and back: Inferring a true value



The observed data is certain: $\rightarrow$ 'true value' uncertain.

"data uncertainty" ?

…and back: Inferring a true value



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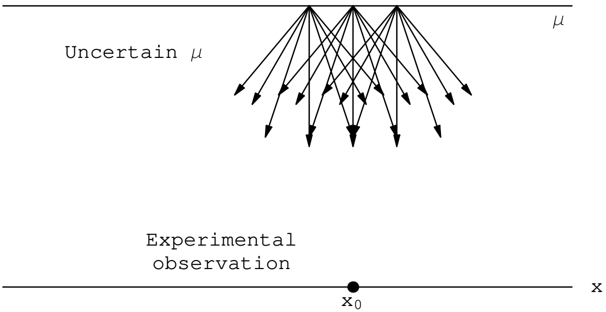
“data uncertainty” ? Data corrupted?

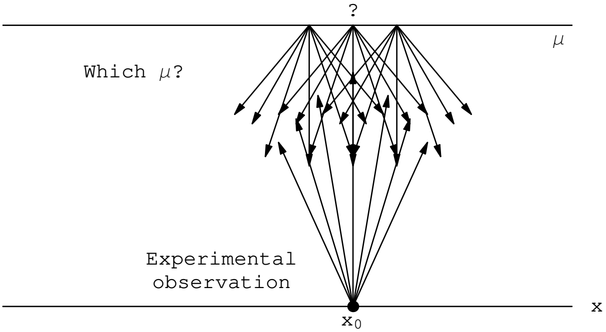
...and back: Inferring a true value



The observed data is <u>certain</u>: $\rightarrow$ 'true value' uncertain.

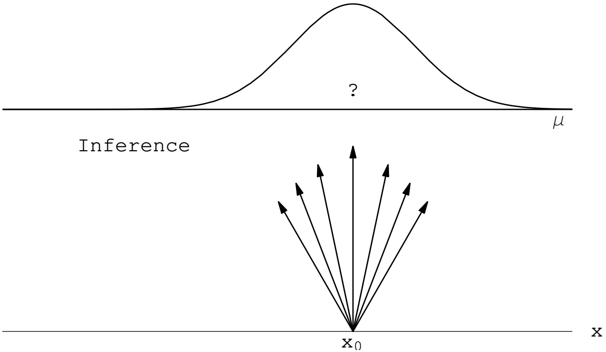
"data uncertainty" ? Data corrupted?
Even if the data were corrupted, the <u>data</u> were the corrupted data**!!**...

. . . and back: Inferring a true value



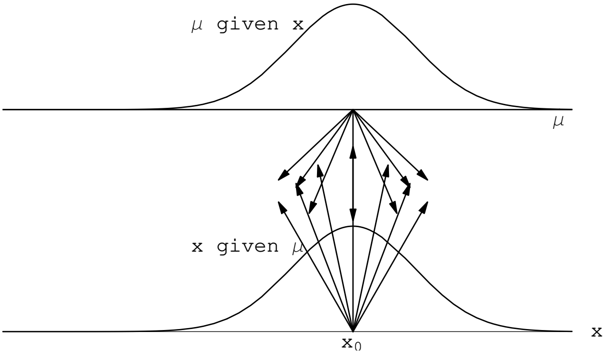
Where does the observed value of x comes from?

. . . and back: Inferring a true value



We are now uncertain about μ, given x.

. . . and back: Inferring a true value



Note the symmetry in reasoning.

A very simple experiment

Let's make an experiment

A very simple experiment

Let's make an experiment

- Here
- Now

A very simple experiment

Let's make an experiment

- Here
- Now

For simplicity

- μ can assume only six possibilities:

$$0, 1, \ldots, 5$$

- x is binary:

$$0, 1$$

$$[\,(1,2);\ \text{Black/White};\ \text{Yes/Not};\ \ldots\,]$$

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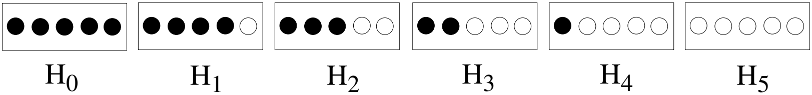
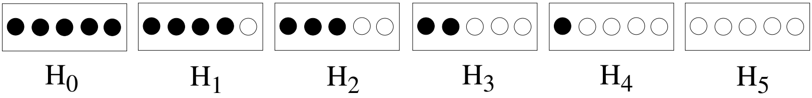
$\Rightarrow$ Later we shall make μ continuous.

Which box? Which ball?



Let us take randomly one of the boxes.

Which box? Which ball?



Let us take randomly one of the boxes.

We are in a state of uncertainty concerning several *events*, the most important of which correspond to the following questions:

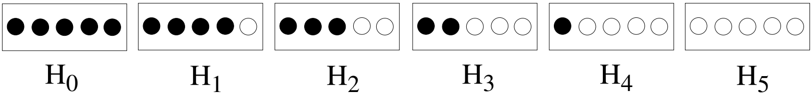
(a) Which box have we chosen, H_0, H_1, ..., H_5?

(b) If we extract randomly a ball from the chosen box, will we observe a white ($E_W \equiv E_1$) or black ($E_B \equiv E_2$) ball?

Our certainties:

$$\begin{aligned} \cup_{j=0}^{5} H_j &= \Omega \\ \cup_{i=1}^{2} E_i &= \Omega\,. \end{aligned}$$

Which box? Which ball?



Let us take randomly one of the boxes.

- What happens after we have extracted one ball and looked its color?
  - Intuitively feel *how to* roughly *change* our opinion about
    - the possible cause
    - a future observation

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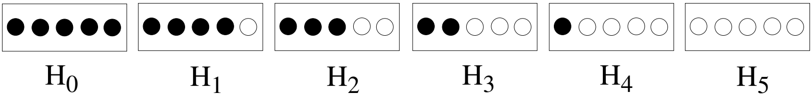

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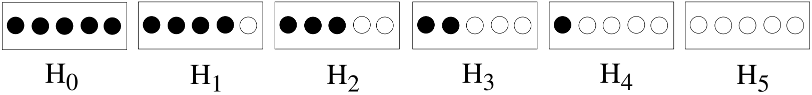

Let us take randomly one of the boxes.

- What happens after we have extracted one ball and looked its color?
  - Intuitively feel *how to* roughly *change* our opinion about
    - the possible cause
    - a future observation
  - Can we do it *quantitatively*, in an 'objective way'?
- And after a sequence of extractions?

The toy inferential experiment

The aim of the experiment will be to guess the content of the box without looking inside it, only extracting a ball, record its color and reintroducing in the box

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This toy experiment is conceptually very close to what we do in the pure and applied sciences

- ⇒ try to guess what we cannot see (the electron mass, a magnetic field, etc)

  . . . from what we can see (somehow) with our senses.

The rule of the game is that we are not allowed to watch inside the box! (As we cannot open and electron and read its properties, unlike we read the MAC address of a PC interface.)

Where is probability?

We all agree that the experimental results change

- the probabilities of the box compositions;
- the probabilities of a future outcomes,

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Where is probability?

Certainly not *in* the box!

The End