Numerics for SDEs

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Useful information

- This is not a course of probability theory;
- the focus is 100% numerical;
- lab part (in Matlab) is substantive;
- contact: raffaele.dambrosio@univaq.it;
- reference paper: D. Higham, *An Algorithmic Introduction to Numerical Simulation of Stochastic Differential Equations*, SIAM Review 43(3), 525–546 (2001);
- classical comprehensive monograph: P. E. Kloeden, E. Platen, *Numerical Solution of Stochastic Differential Equations*, Springer-Verlag (1992);
- new books:
  - R. D'Ambrosio, *Numerical approximation of differential problems*, Springer, to appear;
  - D.J. Higham, P. E. Kloeden, *An Introduction to the Numerical Simulation of Stochastic Differential Equations*, SIAM (2020).

Tentative topics

- Discretized Weiner process; simulation of stochastic integrals;
- introduction to SDEs; basics of deterministic one-step methods;
- Euler-Maruyama, Milstein methods; strong and weak convergence;
- linear stability analysis; mean-square contractivity;
- stochastic geometric numerical integration.

Stochastic differential equations

Itō problem:

$$\mathrm{d}X(t) = f(X(t))\mathrm{d}t + g(X(t))\,\mathrm{d}W(t), \quad t \geq 0,$$

$f : \mathbb{R}^d \to \mathbb{R}^d$ (drift), $g : \mathbb{R}^d \to \mathbb{R}^{d\times m}$ (diffusion),

$W(t)$ m-dimensional Wiener process.
Integral form:

$$X(t) = X(0) + \int_0^t f(X(s))ds + \underbrace{\int_0^t g(X(s))dW(s)}_{\text{Itō integral*}}.$$

*On a uniform grid $\{0 < t_1 < t_2 < \cdots \leq t_n\}$, it is defined as

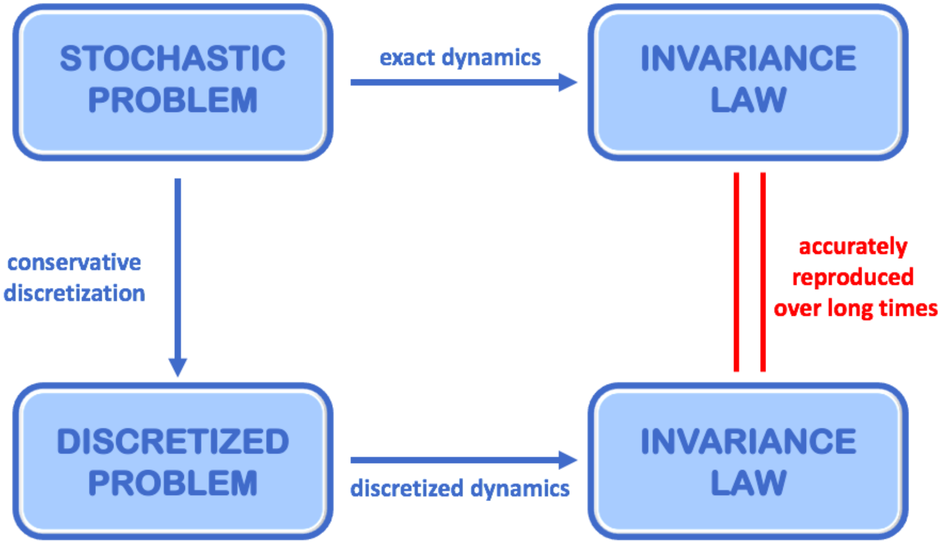
$$\lim_{n\to\infty} \sum_{j=1}^{n} g(X(t_j))(W(t_{j+1}) - W(t_j)).$$

Wiener increments are $\sqrt{h}\cdot\mathcal{N}(0,1) - \textit{distributed}$, where $h = t_{j+1} - t_j$.

Numerics for SDEs

- From Taylor to Ito-Taylor expansions (Kloeden-Platen, 1992);
- linear stability (Buckwar, Higham, wide literature from 2000 on);
- invariance of asymptotic laws in the discretization of **linear systems** (Schurz, 1999);
- long-term conservation of stationary densities for **damped linear oscillators**, with additive noise (Burrage, Lythe, 2007, 2009; D'Ambrosio, 2018, 2020; Strommen-Melbo, Higham, 2004) and for **nonlinear oscillators** (D'Ambrosio, Scalone, 2020);
- stochastic **Duffing** oscillator (Welfert, 2017);
- energy-preserving methods for **stochastic Hamiltonian problems** (Burrage 2012, 2014; Cohen, D'Ambrosio, Lang, Vilmart, 2020);
- stochastic **geometric numerical integration** (Cohen, Vilmart, from 2015 on;
- nonlinear stability, **mean-square contractivity** (Buckwar, D'Ambrosio, Di Giovacchino, from 2020).

A problem-oriented numerics



Enjoy this course!