Gran Sasso Science Institute
PhD in Astroparticle Physics: Cycle XXXIII

The importance of being «consistent» in Cosmology

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PhD Dissertation Defense – April 23, 2021

Recall of Cosmology

- ***Cosmological principle:*** Our Universe results to be homogeneous and Isotropic at sufficiently large scales.
- ***Idea:*** Homogeneous and isotropic background+ small perturbations

-Background $$ds^2 = \bar{g}_{\mu\nu}\, dx^\mu dx^\nu = -dt^2 + a(t)^2 \frac{\delta_{ij}}{(1 - x^2\chi)^2} dx^i dx^j$$

-Perturbations $$g_{\mu\nu} = \bar{g}_{\mu\nu}(t) + g^{(1)}_{\mu\nu}(t, x) + \ldots$$

$$g_{00} = -1 + 2\Psi; \qquad g_{0i} = a\partial_i F + G_i;$$

$$g_{ij} = a^2\left[(1 + 2\Phi)\delta_{ij} + \partial_{ij}E + \partial_j C_i + \partial_i C_j + h_{ij}\right];$$

-Cosmic and conformal time: $$d\tau = a(t)\, dt$$

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-Cosmic and conformal time:

$$\boxed{d\tau = a(t)\, dt}$$

Statistical properties of Cosmological Perturbations

- At the linear level we have no phase-correlation among different modes…

  Gaussianity

$$< \Phi(x_1)..\Phi(x_{2n}) >= \sum_{Perm.} \prod_{Pairs} < \Phi(x_i)\, \Phi(x_j) >,$$

$$< \Phi(x_1)..\Phi(x_{2n+1}) > = 0.$$

- At the non-linear level we get coupling among different modes…

  non-Gaussianity

$$< \Phi(x_1)..\Phi(x_{2n+1}) > \neq 0.$$

$$< \Phi(x_1)..\Phi(x_{2n}) > - < \Phi(x_1)..\Phi(x_{2n}) > \Big|_G \neq 0.$$

What observations suggest..

- The FLRW Universe is a good approximation for $k_p \sim 0.05 Mpc^{-1}$
- ***First:*** CMB observations.. an almost perfect black body..

$n_s = 0.9652$

$|f_{NL}| = 0/50$

$A_\Phi = 2.4\ 10^{-9}$

$$T_0 \sim 2.7K; \qquad \frac{\delta T}{T_0} \sim 10^{-5};$$

$$< \left(\frac{\delta T}{T}\right)^N > \quad \Rightarrow \quad N = 2: < \Phi_p \Phi_q >= (2\pi)^3 P_\Phi(q) \delta^{(3)}(q+p),$$

$$\boxed{P_\Phi = A_\Phi \left(\frac{q}{q^*}\right)^{n_s - 4}}$$

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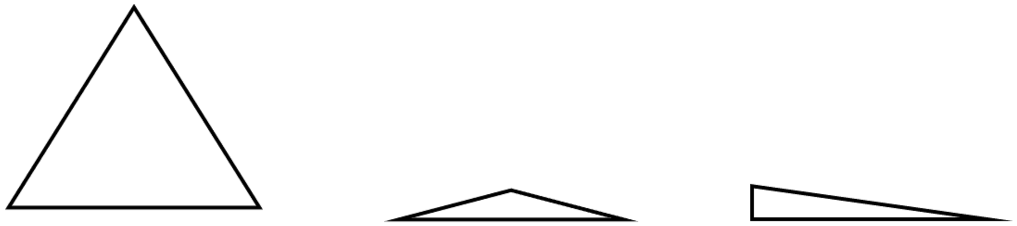
$$N = 3: < \Phi_k \Phi_p \Phi_q >= (2\pi)^3 B_\Phi(k,p,q)\ \delta^{(3)}(k+q+p)$$

$$\boxed{B_\Phi \sim f_{NL} \frac{5}{6}\left(P_\Phi(k)P_\Phi(q) + P_\Phi(p)P_\Phi(q) + P_\Phi(k)P_\Phi(p)\right)}$$



equilateral, $f_{NL} = -4 +/- 45$

folded, $f_{NL} = -26 +/- 21$ (68 %)

squeezed. $f_{NL} = 0.8 +/- 5$

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- ***Second:*** CMB and the tensor sector

$n_T = 1 \pm 0.6$

$r \leq 0.06$

$$h_{ij} = \sum_{s=-2}^{2} \varepsilon_{ij}^{(s)}\, h_k^{(s)}$$

$$\Rightarrow\ < h_p^{(s)}\, h_q^{(r)} >= (2\pi)^3 \delta_s^r\, P_h(q)\, \delta^{(3)}(q+p),$$

$$\boxed{\mathrm{r} = \frac{P_h}{P_\Phi}, \qquad P_h = A_h q^{n_T - 4}.}$$

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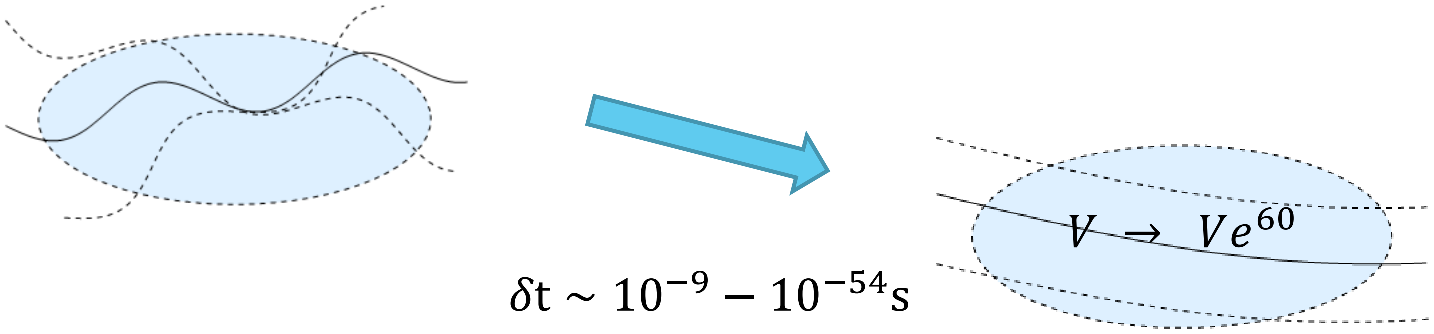
TTT	$< h_p^{(s)}\, h_q^{(r)}\, h_k^{(o)} >$	$f_{TTT} = \dfrac{B_{TTT}}{P_h(p)P_h(k)}$	**SQ:** 300 ± 200
TTS	$< h_p^{(s)}\, h_q^{(r)} \Phi_k >$	$f_{TTS} = \dfrac{B_{TTS}}{P_h(p)P_\Phi(k)}$	??
TSS	$< h_p^{(s)} \Phi_q \Phi_k >$	$f_{TSS} = \dfrac{B_{TSS}}{P_h(p)P_\Phi(k)}$	**SQ:** 90 ± 40

WHY INFLATION???

- ***Problems:*** 1) Background: The hot big bang drawbacks

  2) Perturbations: How to reproduce anisotropies with the observed properties?

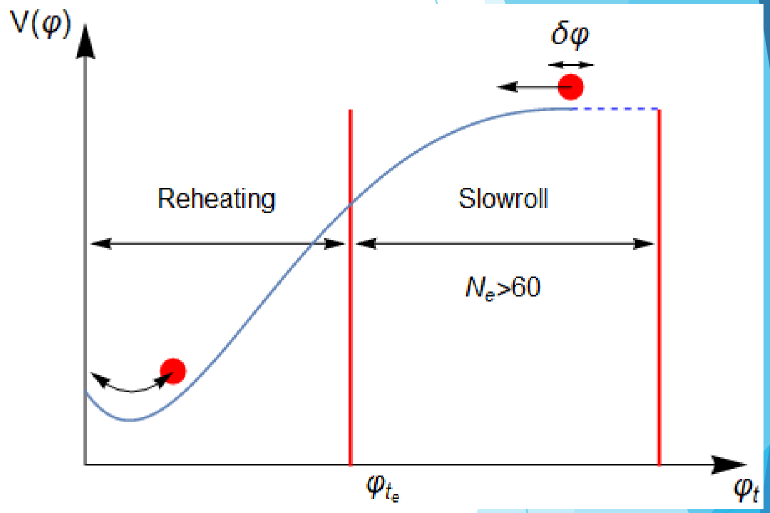
- ***Definition:*** - A period of sudden expansion



$\delta t \sim 10^{-9} - 10^{-54} s$

Single-field linear predictions

- Action: $$S = M_{pl}^2 \int dx^4 \sqrt{-g} \left[R + \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \, \partial_\nu \varphi + V(\phi) \right]$$
- SR limit: $$\epsilon = -\frac{\dot{H}}{H^2} \ll 1 \rightarrow p \approx -\rho, \qquad \eta = \frac{\dot{\epsilon}}{\epsilon H} \ll 1,$$



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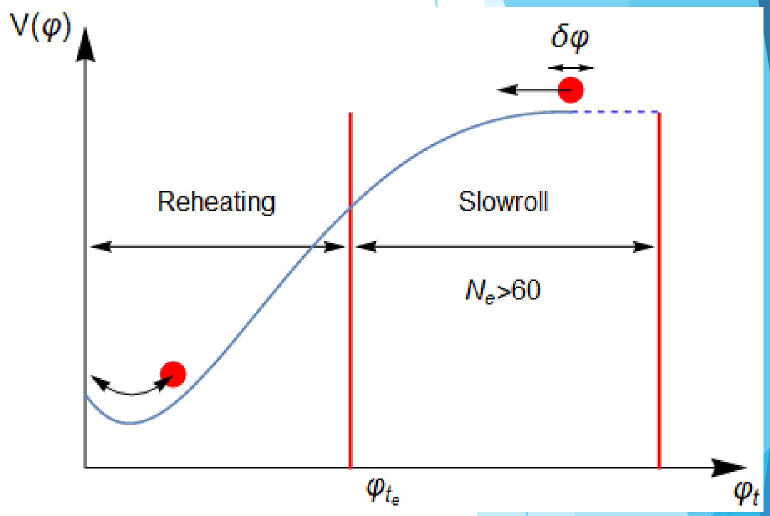
- Famous equivalent fields..

  Com. curvature: $R = -\Phi - H\, v^{(1)}, \qquad v = const.$

  Uniform curvature: $\zeta = -\Phi - H \frac{\delta\rho^{(1)}}{\dot{\rho}}, \qquad \rho = const.$



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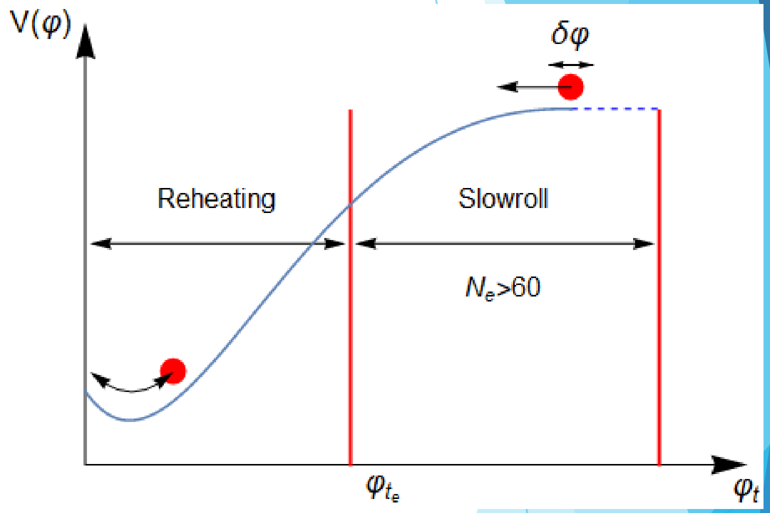


Results

- $P_R(q) \equiv P_\zeta(q) = \frac{2\,\pi^2}{q^3} \boldsymbol{\mathcal{P}}(\mathrm{q});$ $\qquad \boldsymbol{\mathcal{P}}(\mathrm{q}) = \boldsymbol{\mathcal{P}}_{SF}\, q^{n_s - 1} \begin{cases} \boldsymbol{\mathcal{P}}_{SF} = \frac{H^2}{8\pi^2 M_{pl}^2 \epsilon}, \\ n_s = 1 - 2\epsilon - \eta. \end{cases}$
- $r = 8\,\epsilon; \qquad \mathrm{n_T} = 1 - 2\,\epsilon <\sim 1$

Breaking the model degeneracy..

[a] Maldacena, 2003. Non-Gaussian features of primordial fluctuations in single-field inflationary models.
[b] Acquaviva et al. 2003. Second order cosmological perturbations from inflation.

- Plenty of models in agreement with data..

PNG Predictions:

- Single-field inflation:

A special sym. pattern

↓

Validity of the consistency relation[a,b]

$$B_R(k_1 \ll k_2 \sim k_3) = \boxed{-\frac{1}{2}(n_s - 1)} P_R(k_1) P_R(k_2)$$

$$\frac{5}{6} f_{NL}^{SQ}$$

- Something more exotic:

A different sym. Pattern

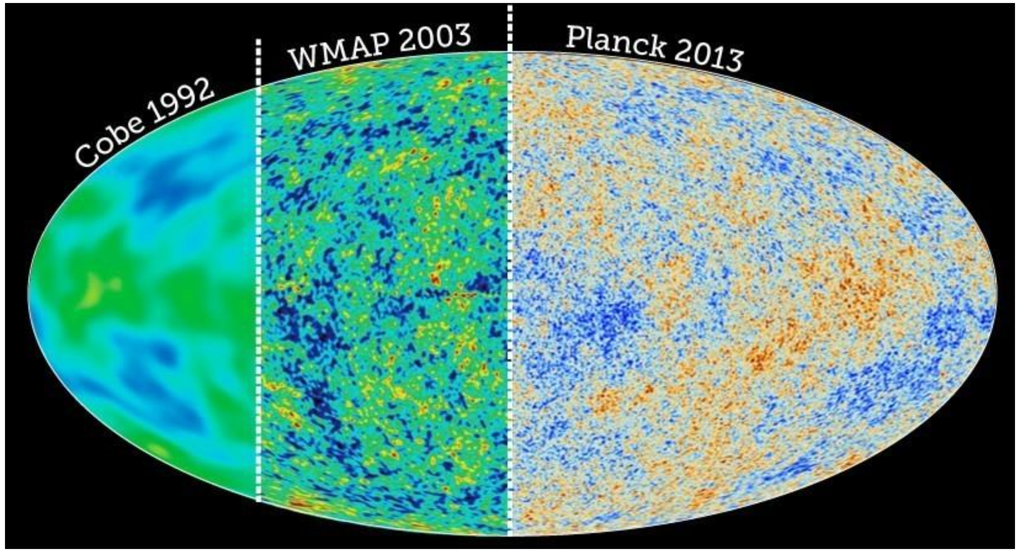
↓

Violation of the consistency relation

$$B_X(k_1 \ll k_2 \sim k_3) = \frac{5}{6} f(k_1, k_2)\, P_X(k_1) P_X(k_2)$$

PNG detection…

- $< \frac{\delta T}{T}\frac{\delta T}{T}\frac{\delta T}{T} >$ Improving CMB (LiteBIRD) …



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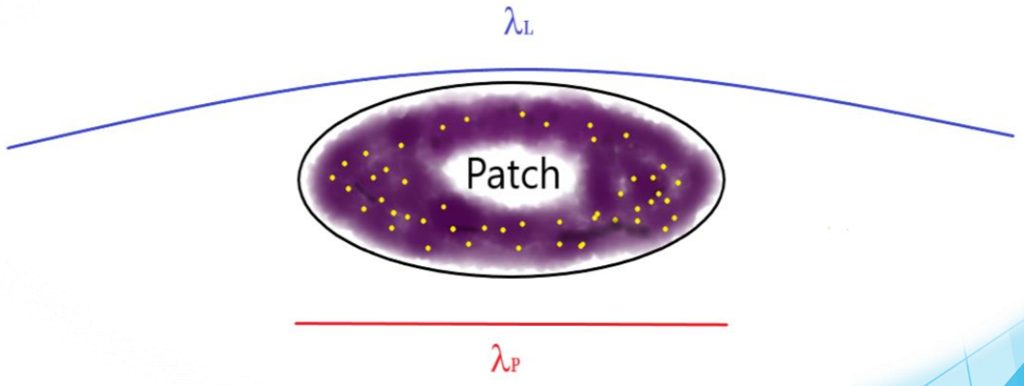
- Galaxy clustering.. The scale dependent halo bias

$$\delta g = \frac{n_g(x,z) - \bar{n}_g(z)}{\bar{n}_g(z)} \sim \mathrm{b}_0 \frac{\delta\rho}{\rho}$$



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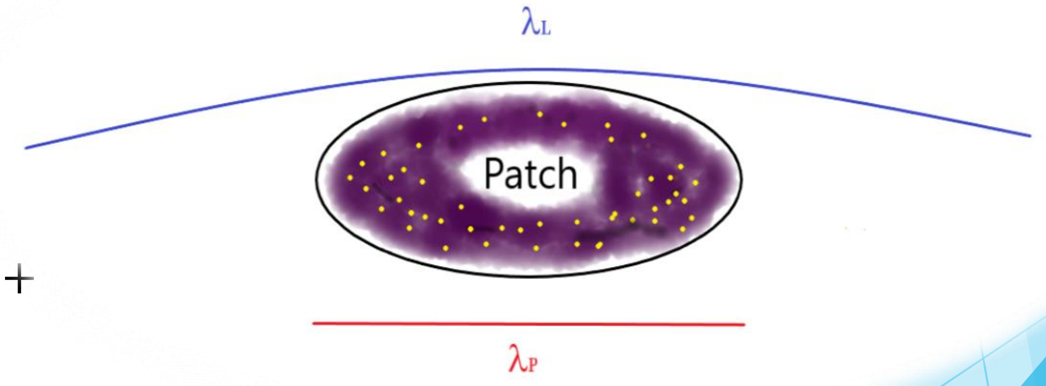
$$\delta \equiv \frac{\delta\rho}{\rho} \sim (\ldots)\left(\nabla^2\Phi - 2\,\Phi\,\nabla^2\Phi + \frac{1}{2}(\nabla\Phi)^2\right) +$$

$$\delta g \sim \left(\mathrm{b}_0 + (\ldots)\frac{f_{NL}^{sq}}{k^2}[c]\right)\delta$$



[c] Verde, Matarrese, 2009. Detectability of the effect of Inflationary non-Gaussianity on halo bias

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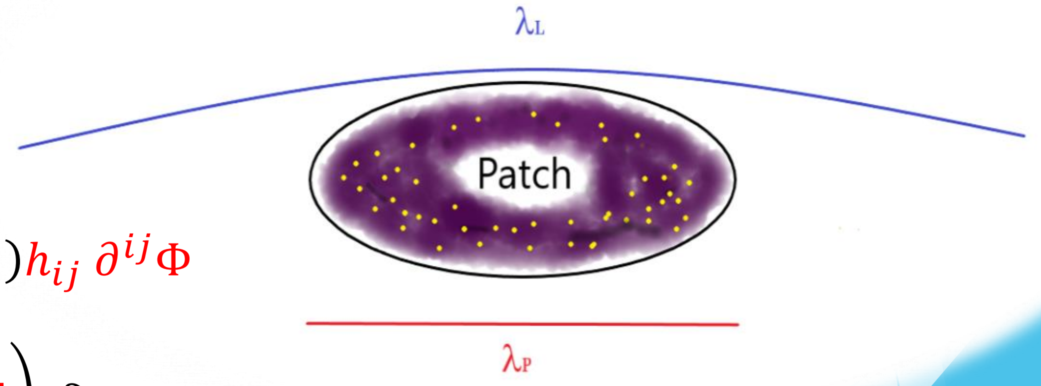
$$\delta \equiv \frac{\delta\rho}{\rho} \sim (\dots)\left(\nabla^2\Phi - 2\,\Phi\,\nabla^2\Phi + \frac{1}{2}(\nabla\Phi)^2\right) + (\dots) h_{ij}\,\partial^{ij}\Phi$$

$$\delta g \sim \left(\mathrm{b}_0 + (\dots)\frac{f_{NL}^{sq}}{k^2}[c] + (\dots) f_{TSS}^{sq}\,\varepsilon_{ij}^{(s)}\frac{k^i k^j}{k^2}[d,e]\right)\delta$$



[c] Verde, Matarrese, 2009. Detectability of the effect of Inflationary non-Gaussianity on halo bias

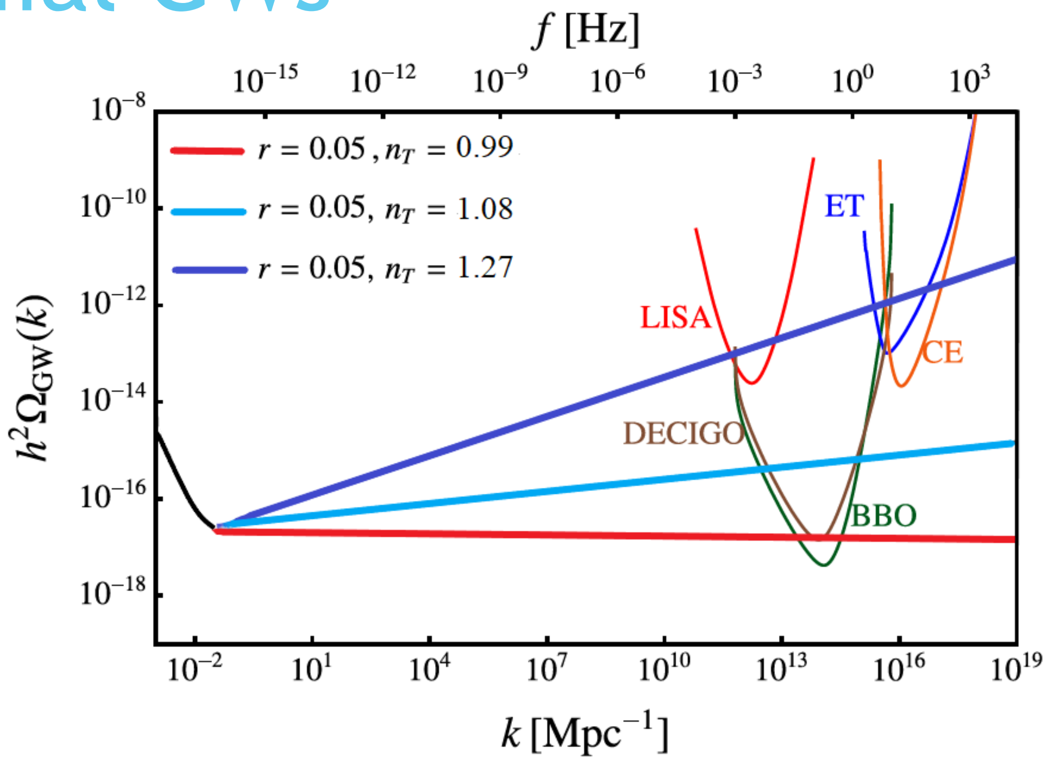
[d] Jeong, Kamionkowski 2012. Clustering Fossils frome the Early-Universe.

[e] Akhshik, 2015. Clustering fossils in Solid Inflation

PNG and primordial GWs in the future…

- Detection of GWs background [f]

$$\Omega_{GW} \sim k^{n_T - 1}$$



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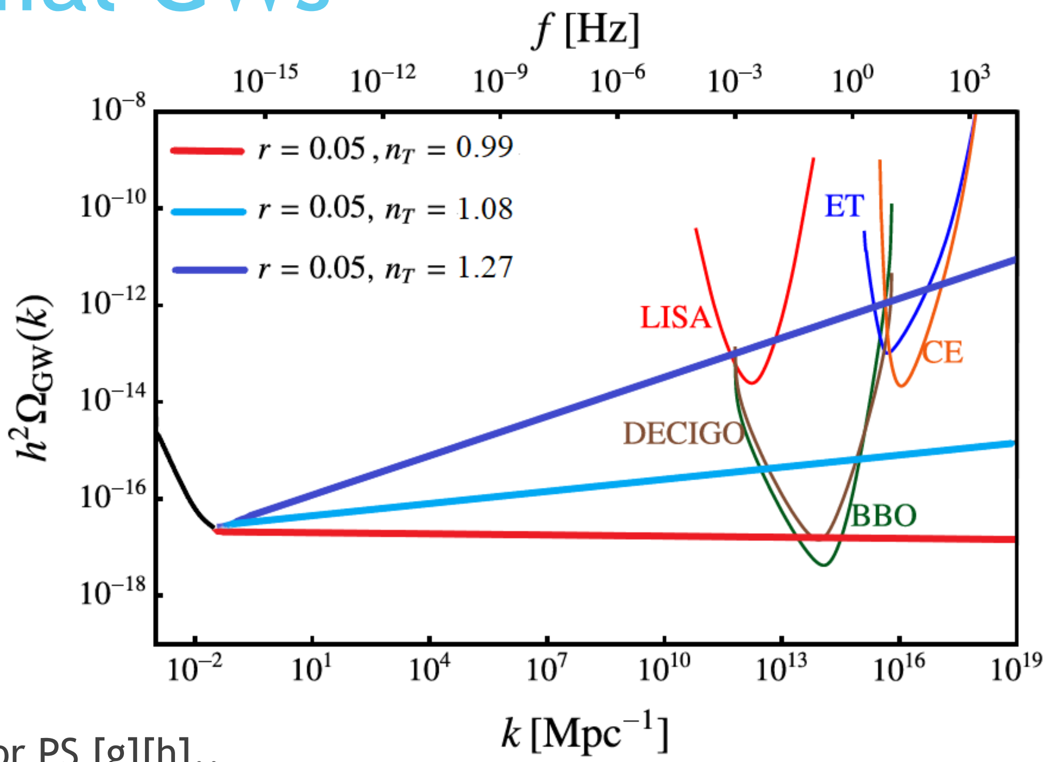

- Local non-linear corrections of tensor PS [g][h]..

$$< h^2 > \Big|_{\mathrm{Non-}Lin.} \sim [1 + (\ldots)\Phi_L\, f^{sq}_{TTS}] < \mathrm{h}^2 >$$

[f]: Bartolo et al., 2016. Probing inflation with gravitational waves.
[g]: Malhotra, Dimastrogiovani, Fasiello, Shiraishi, 2020. Cross-correlations as a Diagnostic Tool For PGWs.
[h]: Adshead, Afshordi, Dimastrogiovani, Fasiello, Lim, Tasinato, 2020. Multimessanger Cosmology: Correlating CMB and SGWB measurements.

Outline:

1- Is the consistency rel. Trivial?

2- The k=0 world

3- Outside the k=0 world

[a]= Matarrese, Pilo, Rollo, 2020,

Resilience of long modes in cosmological observables

Part I: *Single-field Inflation*

Question: is the consistency relation trivial?

In literature: The consistency relation can be cancelled with a spatial diff. [b, c, d…]

[b]= Pajer, Schmidt, Zaldarriaga, 2013, The Observed Squeezed Limit of Cosmological Three-Point Functions;

[c]= Dai, Pajer, Schmidt, 2015, Conformal Fermi Coordinates;

[d]= Dai, Pajer, Schmidt, 2015, On Separate Universes;

Question: is the consistency relation trivial?

In literature: The consistency relation can be cancelled with a spatial diff. [b, c, d...]

The action is a scalar under a spatial diffeomorphism: $x_i' = x_i + \varepsilon_i(x)$

If R is a scalar $\{R'(x') = R(x)\}$ $\Longrightarrow$ $B_R'(x_1, x_2, x_3) - B_R(x_1, x_2, x_3) \equiv 0$

The aim of the discussion: to show that R is a good scalar

k=0 world

- Validity of the Cosmological principle: Universe homogeneous and isotropic at k=0;

- so(4,1) algebra: Spatial translations; Spatial rotations;

  Dilatations: $\boxed{x^i \to (1+\lambda)x^i}$

  Special conformal transformations

$$x^i \to x^i + 2b^j x_j x^i - b^i x^2$$

k=0 world

- ***FUNDAMENTAL*** : Let us apply a dilatation, we can impose a gauge redundancy!
- For instance:

Initial gauge: comoving E=v=0

$$g_{00} = -1 + 2\Psi, \qquad g_{0i} = a\,\partial_i F, \qquad \boxed{g_{ij} = a^2\left[(1-2R)\delta_{ij}\right].}$$

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Final gauge: comoving

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k=0 world

- Redundancy:

$$\Delta \mathrm{E} = \mathrm{E}'(\mathrm{x}) - E(x) = 0, \qquad \Delta v = v'(\mathrm{x}) - v(x) = 0,$$

$$\Delta g_{ij} = -2a^2\lambda\,\delta_{ij}, \qquad \boxed{\delta R_\lambda = \lambda.}$$

- Applications:

-The Weinberg Theorem ;

-The Consistency Relation.

The Weinberg Theorem [e]

- ***Hypothesis*** : At sufficiently large scales $k \to 0$, let us consider

  -No entropy perturbations $\delta\sigma = 0$ (one DoF);

  -No anisotropic stress tensor on large scales;

- ***Thesis*** : - ζ and *R* are equivalent and conserved in time

  at superhorizon scales

[e]= S. Weinberg, Adiabatic modes in Cosmology. 2003.

Demonstrating the Theorem

- Matching the k=0 world with the Adiabatic mode (physics)

$$k = 0:\ \ \Delta g \Big|_{redundant} = \text{Solution} \qquad\qquad k \neq 0:\ \ \mathrm{k^i k^j}(\Psi_\mathrm{k} - \Phi_\mathrm{k}) = 0$$

$$\Psi_\mathrm{k} \equiv \Phi_\mathrm{k}$$

$$R_{k\to 0}\Big|_{Total} = \lambda + C\frac{H}{a}; \qquad \lambda = \lim_{k\to 0} R_k = \int d^3k\ \delta^{(3)}(k)\ R_k$$

- We can extract physics from a gauge redundancy!

The Consistency Relation

[e]: Creminelli, Norena, Simonovich. Conformal consistency relation for single-field inflation. 2012 .
[f]: Hui et al. An Infinite Set of Ward Identities for Adiabatic Modes in Cosmology. 2014 .
[g]: Hui et al. Conformal Symmetries Adiabatic Modes in Cosmology. 2012 .

- Single-field: Spontaneous breaking of so(4,1) global symmetries [e][f][g]

de Sitter: so(4,1) $\rightarrow$ rotations + translations.

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- Dilatation is a symmetry non-linearly realized. This implies a Norther current and charge:

$$Q = \int dx^3 \{P_R, \delta R_\lambda\};$$

- Using Ward identities, one can extract the consistency relation[f]:

$$\lim_{\mathrm{k}\to 0} \langle R_k R_{k_1} \ldots R_{k_N} \rangle = P(k) \left[3(N-1) + \sum_{a=1}^{N} k_a \, \partial_{k_a} \right] \langle R_{k_1} \ldots R_{k_N} \rangle$$

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$$\boxed{0???}$$

- Applying a second dilatation:

$$\boxed{R_k \to R_k - \lambda = 0}$$

Outside k=0 world

- Are the CFC-like transformations purely constant dilatations?

- NO!...Take a the lambda function $\lambda_k = W^0_{k_L}(k) R_k, \qquad x^i = (1+\lambda) x^i,$

- Necessary condition to get scalar-tensor decomposition

$$\lambda\, x^i = \partial_i \epsilon \Rightarrow x^j \partial_i \lambda - x^j \partial_i \lambda = 0.$$

Outside k=0 world

- A standard gauge transformation: $\Delta g_{ij} = -a^2\left[2\lambda\delta_{ij} + x^j\partial_i\lambda + x^i\partial_j\lambda\right]$

- Basic element $$x^i\partial_j\lambda = \frac{-1}{(2\pi)^{3/2}}\int dk^3\, e^{ikx}\partial_{k^i}\left(k^j\lambda_k\right) + BT.$$

- Final result $$\Delta g_{ij} = 2a^2\frac{k^i k^j}{k}\partial_k\lambda_k$$

$$\Delta R_k = 0, \qquad \Delta E_k = \frac{-2}{k}\partial_k\lambda_k$$

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- A standard gauge transformation: $\Delta g_{ij} = -a^2\left[2\lambda\delta_{ij} + \boxed{x^j\partial_i\lambda + x^i\partial_j\lambda}\right] \longrightarrow$ 0? ?
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A gauge change!

Outside k=0 world

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Outside k=0 world

- Assuming: $\lambda_k \approx R_k = A_R\, k^m, \qquad W^0_{k_L} = \theta[(k - k_L)H^{-1}],$
- The 3-metric: $\Delta g_{ij} = 2\, m\, a^2\, \frac{k^i k^j}{k^2} R_k. \sim R_k$
- Trace and out diagonal part are of the same order!

$$x^i \partial_i \longrightarrow \cancel{O(\partial_i)}$$

People confused a gauge redundancy with a discontinuous gradient expansion..

R is a good scalar $\Rightarrow \Delta B_R = 0??$

- In [a] we give two independent demonstrations:

  -1) Using the in-in formalism;

  -2) Using field redefinitions.

$$\Rightarrow \Delta B = < R'(x)^3 > - < R(x)^3 > \equiv \mathrm{BT} = 0.$$

- Such effects are physical and observable in principle by future high-sensitivity experiments!

Future developments

- The ambiguity is quite diffused…

  - In [a] we analyzed the scalar sector only

$$x' = (\mathrm{II} + \lambda)\, x + \omega . x \quad \mathit{Spin}\text{-}2 \text{ [b,h,i]}$$

  - A more detailed study of the halo bias scale dependence [l][m]

$$\Delta b(k)\; k^2 \propto f_{NL}^{\rho} = -\frac{5}{3} + \mathrm{f}_{\mathrm{NL}} \rightarrow 0??$$

[h]: Dimastrogiovanni et al., 2015.Inflationary tensor fossils in LSS.
[i]: Adshead et al., 2020. Multimessanger Cosmology: Correlating CMB and SGWB measurements.
[l]: Cabass, Pajer, Schmidt, 2018. Imprints of oscillatory Bispectra on Galaxy Clustering.
[m]: de Putter, Dorè, Green, 2015. Is there scale-dependent bias in single-field inflation?

- “It is perhaps surprising that a physical statement follows from what is a gauge redundancy. We know the consistency relations are physical (i.e., not trivial identities) because they can be broken; explicit examples exist, such as when additional light fields are present during infation” [f]

[f]: Hui et all. An Infinite Set of Ward Identities for Adiabatic Modes in Cosmology. 2014 .

Outline:

1- The Effective field theory

2- Supersolid Inflation: linear theory

3- Primordial non-Gaussianity

[a]=Celoria, Comelli, Pilo, Rollo, 2019,
Adiabatic Media Inflation;
[b]=Celoria, Comelli, Pilo, Rollo, 2020,
Boosting Gws in Supersolid Inflation;
[c]=Celoria, Comelli, Pilo, Rollo, 2021,
Primordial NG in Supersolid Inflation.

Part II: *EFT of Inflation*

EFT of Inflation

- Single-field inflationary models: time diff. Breaking $\tau \rightarrow \tau + \pi^0(x, \tau)$, [d]

[d] Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore, 2008; EFT of Inflation.

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- Solid inflation: spatial diff. Breaking $x^i \to x^i + \pi^i(x,\tau)$, [e]

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- Let us break both : Supersolid inflation

$$\Phi^0 = \varphi^0(\tau) + \pi^0(x,\tau),$$

$$\Phi^{\mathrm{i}} = x^i + \partial^i \pi_L(x,\tau) + \pi_T^i(x,\tau),$$

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Two scalar DoF

Vector

Sym. Breaking pattern: 4 Diff. $\to$ $1.)\ \Phi^\mu \to \Phi^\mu + C^\mu$

$2.)\ \Phi^{\mathrm{l}} \to R^l_j \Phi^j$

EFT of Inflation

- Basic operators: $C^{\mathrm{AB}} = \mathrm{g}^{\mu\nu}\partial_\mu \Phi^A \partial_\nu \Phi^A, \quad B^{\mathrm{lm}} = \mathrm{g}^{\mu\nu}\partial_\mu \Phi^l \partial_\nu \Phi^m,$

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8 independent operators

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π^i-dependent

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π_0-dependent

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$(\pi_0, \pi^{\mathrm{i}})$-dependent

Fluid-Superfuilds, Solid-Supersolids..

- Symmetries imply a medium-classification

Fluids: $U(b, y)$, invariance under VsDiff. and $\Phi^0 \to \Phi^0 + f(\Phi^a)$,

S-Fluids: $U(b, y, \chi)$, invariance under VsDiff. (entropy prop.)

$$T_{\mathrm{ij}} = \mathrm{p}\, \mathrm{g}_{\mathrm{ij}} + (\rho + p)\, \mathrm{u}_{\mathrm{i}}\, \mathrm{u}_{\mathrm{j}}$$

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Solids: $U(b, y, \tau_Y, \tau_Z)$, the most general Lag. with only Φ^l present

S-Solids: $U(b, y, \chi, \tau_Y, \tau_Z, w_Y, w_Z)$, the most general Lag. with 2 sclar DoF (entropy prop.)

$$T_{\mathrm{ij}} = \mathrm{p}\, \mathrm{g}_{\mathrm{ij}} + (\rho + p)\, \mathrm{u}_{\mathrm{i}}\, \mathrm{u}_{\mathrm{j}} + \alpha\, \partial_{ij}\, \pi_L$$

Defining parameters in unitary gauge..

- Unitary gauge: $\Phi^0 = \varphi^0(\tau), \quad \Phi^i = x^i,$

- Extract parameters:

$$\begin{aligned}\sqrt{-g}U &\sim f_1(U, U_A)\ \mathrm{g}^{(1)} \to \ \bar{\rho}, \bar{p} \\ &\sim f_2(U, U_A, U_{AB})\ \mathrm{g}^{(2)} \to \ M_{l=0,1,2,3,4} \\ &\sim f_3(U, U_A, U_{AB}, U_{ABC})\ \mathrm{g}^{(3)} \to \ \lambda_{l=1,\ldots,10}\end{aligned}$$

- Masses parameters:

$$M_0 = \frac{\bar{\varphi}'^2 \left(U_{\chi\chi} + 2\, U_{y\chi} + U_{yy}\right)}{2a^2}\,, \qquad M_1 = -\frac{\bar{\varphi}'\, U_\chi}{a}\,,$$

$$M_2 = -\frac{4}{9}\left(U_{w_Y} + U_{w_Z} + U_{\tau_Y} + U_{\tau_Z}\right)\,,$$

$$M_3 = \frac{M_2}{3} + \frac{1}{2}\, a^{-6}\, U_{bb}\,, \qquad M_4 = \frac{\bar{\varphi}' \left[U_{b\chi} + U_{by} - a^3 \left(U_\chi + U_y\right)\right]}{2a^4}\,;$$

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$$\boxed{M_2 = -\frac{4}{9}\left(U_{w_Y} + U_{w_Z} + U_{\tau_Y} + U_{\tau_Z}\right)\,,} = 6\,(\rho + p)\, c_T^2$$

it defines the transverse speed of solids and s.solids

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Longitudinal speed: $\mathbf{c}_{\mathbf{L}}^{\mathbf{2}} = -1 + \frac{4}{3}\, c_T^2$

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Not indepenent $\quad \frac{p'}{\rho'} = c_s^2 \approx -1$

Longitudinal speed: $\mathbf{c_L^2} = -1 + \frac{4}{3}\,c_T^2$

Too many fields..

- Fundamental fields:

  \- n=const. curvature: $\boxed{\zeta_n = \frac{k^2}{3}\pi_L = -\Phi + \frac{H}{\dot{n}}\delta n}$ $\longrightarrow$ ζ, in absence of π_0 (solids)

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  - Entropy perturbations: $\delta\sigma = \boxed{-2\, a^3 \rho \frac{1}{\dot{\varphi}}(\zeta - \zeta_n)}$

$\downarrow$

Strictly related to π_0 propagation

Reheating phase

- Sizable entropy perturbations+ Anisotropic stress:

  The option of inflating and forgetting is not available!

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$$\zeta_{\mathrm{n}}|_{\mathrm{Inf}} - \zeta_{\mathrm{n}}|_{\mathrm{Rad}} = 0$$

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- What is constrained by CMB??

$$\Phi_{Rad} = -\frac{2}{3}\boxed{\zeta_n} \longrightarrow \text{Red tilt and } 10^{-9} \text{ amplitude.}$$

Supersolid Inflation

- Masses parameterization:

$$M_{0,1,2} = \begin{cases} 4\,H^2 \epsilon\, c_0^2 \\ 4\,H^2 \epsilon\, c_1^2 \\ 4\,H^2 \epsilon\, c_T^2 \end{cases}$$

- SR quadratic action

$$S = \int d\tau\, d^3k \left[\frac{1}{2}\,\pi^{t\prime} K\,\pi' + \pi'^{t} D_\pi\,\pi - \frac{1}{2}\pi^T M_\pi\,\pi\right] M_{pl}^2 H^2\, a(\tau)^4\,\epsilon, \qquad \pi = \begin{pmatrix}\pi_L \\ \pi_0\end{pmatrix}$$

$$K = \begin{pmatrix} 4\,(1 + c_1^2)\,k^2 & 0 \\ 0 & 8\,c_0^2\,\varphi^{-2} \end{pmatrix}$$

$$D_\pi = -2k^2\left(2\,c_0^2 c_b^2 + c_1^2\right)\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$M_\pi = 4\,k^2 \begin{pmatrix} k^2(c_L^2 - 2\,c_0^2\,c_b^4) & \dfrac{3}{2\,\tau}\,(1 + c_b^2)\varphi^{-1} \\ \dfrac{3}{2\,\tau}\,(1 + c_b^2)\varphi^{-1} & -c_1^2\varphi^{-2} \end{pmatrix}$$

Quantization

- Canonical fields: $$\Pi = \frac{M_{pl}}{H\,\tau^2}\,\epsilon^{\frac{1}{2}}K^{\frac{1}{2}}\begin{pmatrix}\pi_L\\ \pi_0\end{pmatrix}$$

$$\boxed{L(\pi_i, \pi_i')} \longrightarrow \boxed{L\left(\Pi_i, \Pi_i'\right)}$$

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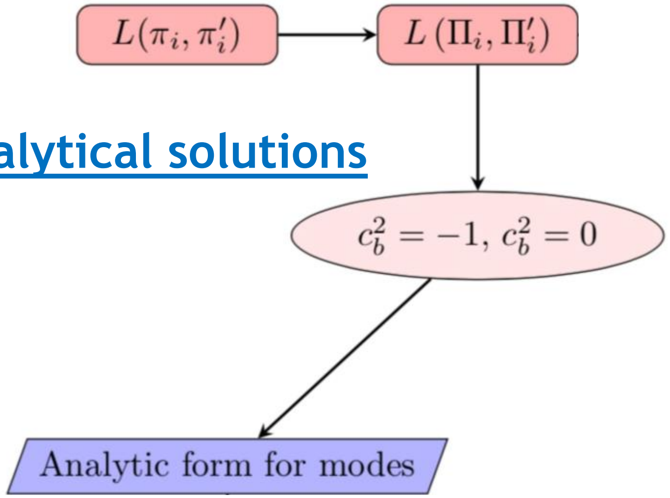

[1- Analytical solutions]

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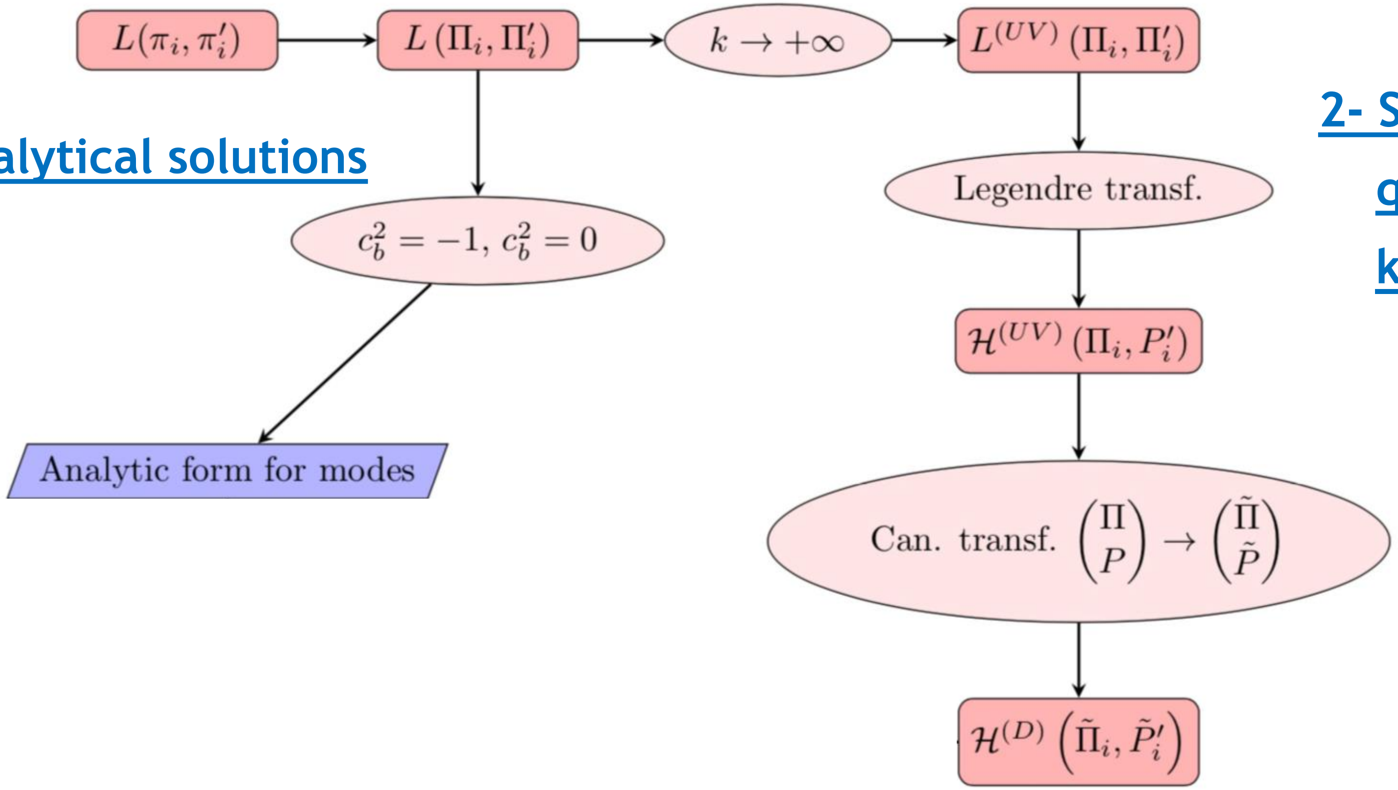

1- Analytical solutions

2- Sub-horizon quantization k t>> 1

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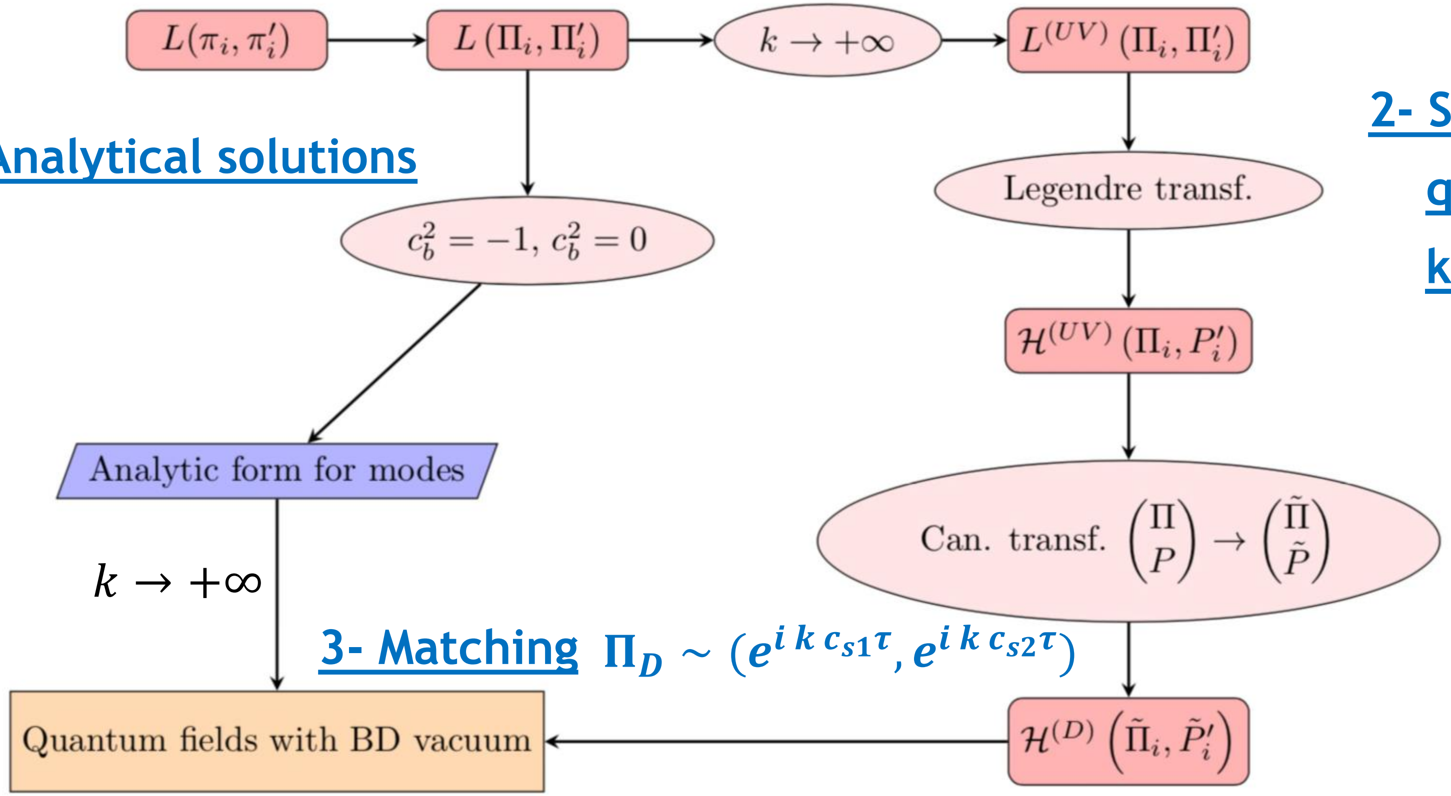

1- Analytical solutions

2- Sub-horizon quantization k t>> 1

3- Matching $\Pi_D \sim \left(e^{i\,k\,c_{s1}\tau}, e^{i\,k\,c_{s2}\tau}\right)$

Analytic solutions

- From quantization: 1) $Eigenvalues$: $c_{sl}(M_A)$; BD: $\Pi_D \sim e^{i\, c_{sl}\, k\, \tau}$ $l = 1, 2.$

  2) $Stability: 0 < c_{s2} < c_L < c_{s1}$.

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$$\Pi_L'' - 2\,k\,d\,\Pi_0' - \frac{k}{t}\lambda\,\Pi_0 + \Pi_L\left(k^2\,\lambda_L^2 - \frac{6}{\tau^2}\right) = 0$$

$$\Pi_0'' + 2\,k\,d\,\Pi_L' - \frac{k}{t}\lambda\,\Pi_\mathrm{L} + \Pi_0\left(k^2\,\lambda_0^2 - \frac{(1+3\,c_b^2)(2+3\,c_b^2)}{\tau^2}\right) = 0$$

Analytic solutions

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- Solutions at $c_b^2 = 0, -1$:

$$\Pi_L = \sum_{j=1,2} a_{\vec{k}}^{(j)} \boxed{A_L^{(j)}} \sqrt{-k\,\tau}\ \ H_{\frac{5}{2}}^{(1)}\left(-c_{sj}\,k\,\tau\right) + h.c.$$

Determined by quantization when $-k\,t \gg 1$

$$\Pi_0 = \sum_{j=1,2} a_{\vec{k}}^{(j)} \boxed{A_0^{(j)}} \sqrt{-k\,\tau}\ \ H_{\frac{3}{2}}^{(1)}\left(-c_{sj}\,k\,\tau\right) + h.c.$$

Power spectra in supersolid inflation

- Formally…

$$\boldsymbol{\mathcal{P}}_{\zeta_n} = \boldsymbol{\mathcal{P}}_{SF}\left(F_{\zeta}^{(1)}(c_b^2, c_L^2, c_{s1}^2, c_{s2}^2) + F_{\zeta}^{(2)}(c_b^2, c_L^2, c_{s1}^2, c_{s2}^2)\right)$$

$$\boldsymbol{\mathcal{P}}_{R_{\pi_0}} = \boldsymbol{\mathcal{P}}_{SF}\left(F_{R}^{(1)}(c_b^2, c_L^2, c_{s1}^2, c_{s2}^2) + F_{R}^{(2)}(c_b^2, c_L^2, c_{s1}^2, c_{s2}^2)\right)$$

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Power spectra in supersolid inflation

- Formally…

$$\mathcal{P}_{\zeta_n} = \boxed{\mathcal{P}_{SF} \boxed{\left(F_\zeta^{(1)}(c_b^2, c_L^2, c_{s1}^2, c_{s2}^2) + F_\zeta^{(2)}(c_b^2, c_L^2, c_{s1}^2, c_{s2}^2)\right)} = \hat{c}_L^{-5}} \sim 10^{-9}$$

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Slow-roll corrections

- All fields are almost scale free in the range $-1 < c_b^2 < 0 \;\; (k\,t \to 0)$

$$\zeta_n^{(l)} = A_\zeta^{(l)} \boxed{(-H\,t)^{\frac{4}{3}c_T^2\epsilon}}\, k^{\frac{1}{2}(n_\zeta - 4)}, \qquad n_\zeta = 1 + 2\,\epsilon\, c_L^2 - \eta < 1$$

Violation of the W. Theorem

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Violation of the W. Theorem

- For $R_{\pi_0}^{(l)}$: enhanced in the $c_{s2} \ll 1$ when $c_b^2 \to -1$

$$R_{\pi_0} \approx \frac{A_R^{(2)}}{c_{s2}^3}\,(-H\,t)^{3(1+c_b^2)+\epsilon\,(1+3\,c_b^2)+\eta}\, k^{\frac{1}{2}(n_R-4)},$$

$$n_R = 7 + 6\,c_b^2(1+\epsilon) + \eta > 1 \Rightarrow \left(1 + c_b^2\right) > \epsilon - \frac{\eta}{6}$$

One-loop: Boosting Gws in Supersolids

- $\mathcal{P}_{R_0} \gg \mathcal{P}_{\zeta_n}$ but not transmitted to radiation.. However..

$$L_{hRR} = -\, M_{pl}^2 a^2 \, \epsilon \, G \, h_{ij} \partial_i R_{\pi_0} \partial_j R_{\pi_0}$$

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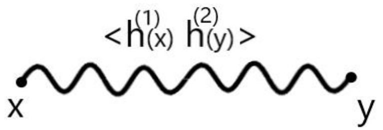
$$r \sim c_L^5 \epsilon$$



$$n_T - 1 = 2\, c_L^2 \epsilon$$

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$$\mathcal{P}_h = \boxed{\mathcal{P}_h^{(1)}} + \boxed{\mathcal{P}_h^{(2)}} \longrightarrow G^2\epsilon^2 \boxed{\frac{2\,\pi^2}{c_{s2}}\left(\mathcal{P}_{R_{\pi_0}}^{(2)}\right)^2} (-H\,t)^{12\,(1+c_b^2)-10\,\epsilon}\; k^{2(n_R-1)}$$

$$\sim \frac{\epsilon^2}{c_{s2}^{13}}\mathcal{P}_{\zeta_n}^2$$

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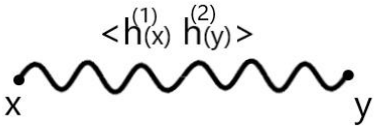

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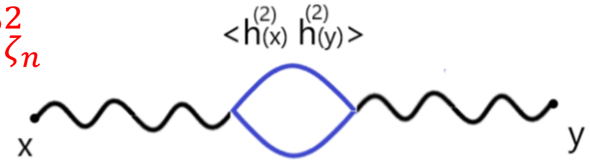

$$n_T - 1 = 12\left[\left(1 + c_b^2\right) + c_b^2\,\epsilon\right] + 2\,\eta$$

PNG in Supersolid Inflation

- In-In formalism and the Dyson formula

$$< \hat{W}(t) > = < \left(e^{-i\int^{t} dt' H_I(t')}\right)^{+} \hat{W}(t) e^{-i\int^{t} dt' H_I(t')} >$$

$$= i\int_{-\infty}^{t} dt' < 0|\left[\hat{W}(t), \boxed{H_I(t')}\right]|0 >$$

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- All possible interactions:

$$\left.\begin{aligned} \widehat{W} &= \zeta_n(x_1)\,\zeta_n(x_2)\,\zeta_n(x_3) && \Rightarrow L^{(3)}_{SSS} \\ &= h(x_1)\,\zeta_n(x_2)\,\zeta_n(x_3) && \Rightarrow L^{(3)}_{TSS} \\ &= h(x_1)\,h(x_2)\,\zeta_n(x_3) && \Rightarrow L^{(3)}_{TTS} \\ &= h(x_1)\,h(x_2)\,h(x_3) && \Rightarrow L^{(3)}_{TTT} \end{aligned}\right\} = L^{(3)}$$

PNG in Supersolid Inflation

- Typical interaction $L^{(3)} = H^2 M_{pl}^2\, X\, \epsilon\, a^{n(c_b)} D(k, k', k'')\, \xi_k \xi_{k'} \xi_{k''}$

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Depends on M_l, λ_l

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$c_b^2 = -1$: no dangerous interactions

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Spatial derivatives

PNG in Supersolid Inflation

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$$\xi = \{\zeta_n, H^{-1}\zeta_n', R_{\pi_0}, H^{-1}R_{\pi_0}{}'\}$$

- Our color code.. Let us count the number of R_{π_0} in L^3

  - **Violet:** No R_{π_0} field
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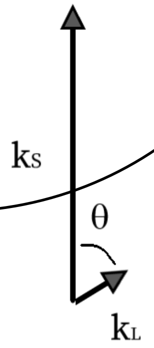

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- Three scalar SQUEEZED bispectrum:

$$f_{NL}^{(X)} = \frac{B}{\mathcal{P}_{\zeta_n}^2} = X\,[\# + \# Cos(\theta)^2] \begin{cases} c_{s2}^{0}, & c_{s2}^{-3} \\ c_{s2}^{-5}, & c_{s2}^{-6} \\ c_{s2}^{-8} & \end{cases}$$

PNG in Supersolid Inflation

- Violation of the consistency relation:

$$f_{NL}^{(SQ)} = \frac{B}{\mathcal{P}_{\zeta_n}^2} = \sum_X X\ [\# + \#Cos(\theta)^2] \begin{cases} c_{s2}^{0}, & c_{s2}^{-3} \\ c_{s2}^{-5}, & c_{s2}^{-6} \\ c_{s2}^{-8} \end{cases}$$

- First source: «solid-anisotropy» $\mathrm{T_{ij}} \sim \partial_{ij}\pi_L \sim \zeta_n$

- Second source: «non-adiabaticity» $< f_{NL}^{(SQ)} >_\theta \neq n_s - 1.$

[d]: Bordin, Creminelli, Mirbabayi and Norena, 2017. Solid Consistency

Gravitational PNG (squeezed)

- Purely tensor correlators

$$f_{TTT} = \frac{B_{TTT}}{P_h(k_L)P_h(k_s)} \sim V\, \epsilon\, [3\,\gamma_e - 7 + 3\log(-2\,k_s t)]\,\sin(\theta)^2$$

Gravitational PNG (squeezed)

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$$f_{TTS}^{(B)} \sim B[3\,\gamma_e - 7 + 3\log(-2\,k_s t)]\,\epsilon\,\frac{\boldsymbol{\mathcal{P}}_{\zeta_n R_{\pi_0}}}{\boldsymbol{\mathcal{P}}_{\zeta_n}} \sim c_{s2}^{-3}$$

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- One graviton and two scalars:

$$f_{TSS} = \frac{B_{TSS}}{P_h(k_L)P_{\zeta_n}(k_S)} = f_{TSS}^{(V)} + f_{TSS}^{(B)} + f_{TSS}^{(G)}$$

$$f_{TSS}^{(G)} \sim G\,\epsilon\, c_{s2}\sin(\theta)^2 \left(\frac{\boldsymbol{\mathcal{P}}_{\zeta_n R_{\pi_0}}}{\boldsymbol{\mathcal{P}}_{\zeta_n}}\right)^2 \sim c_{s2}^{-5}$$

Phenomenolgy

- Two main scenarios

  -1.) Not considering too small c_{s2} values

  - SSS sector under control with all the colors..

  - TTS and TSS will receive a «mitigated» boost

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  - SSS sector under control with all the colors..
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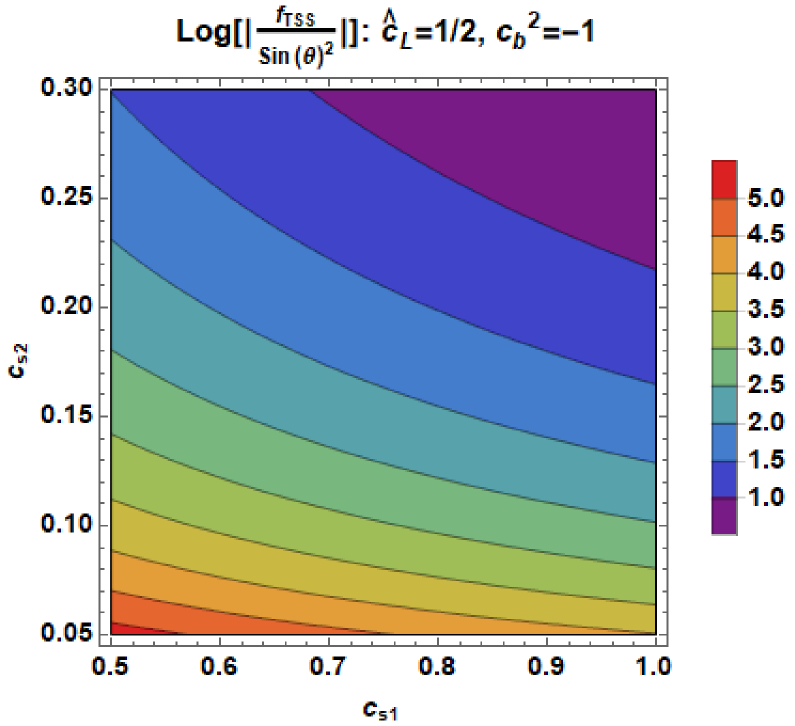
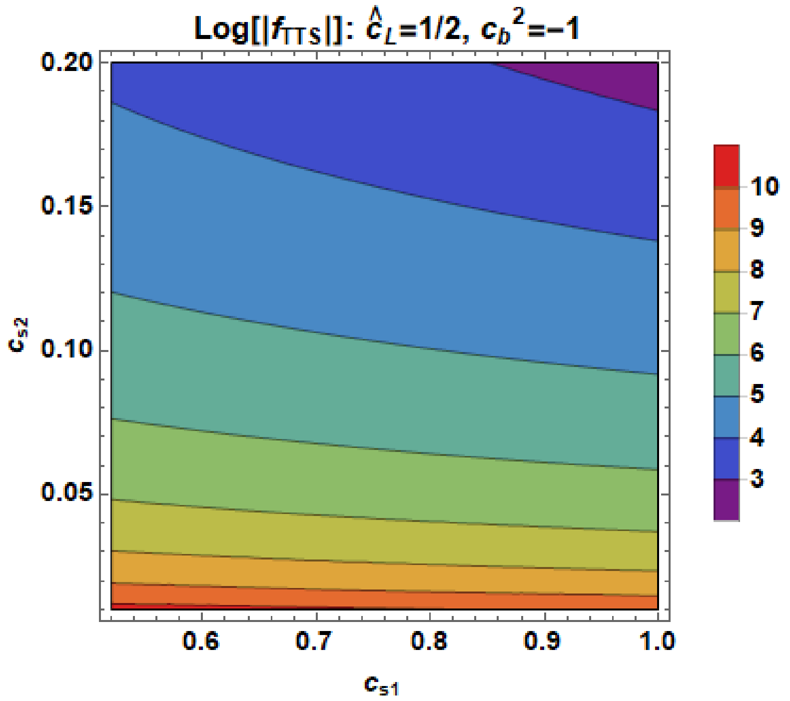
  -2.) Considering small c_{s2} values (sizable oneloop correction)

  - Can we take SSS under control with a red contribution? Yes!
  - TTS and TSS will receive a «very sizable» boost!

Phenomenolgy

- TTS, TSS





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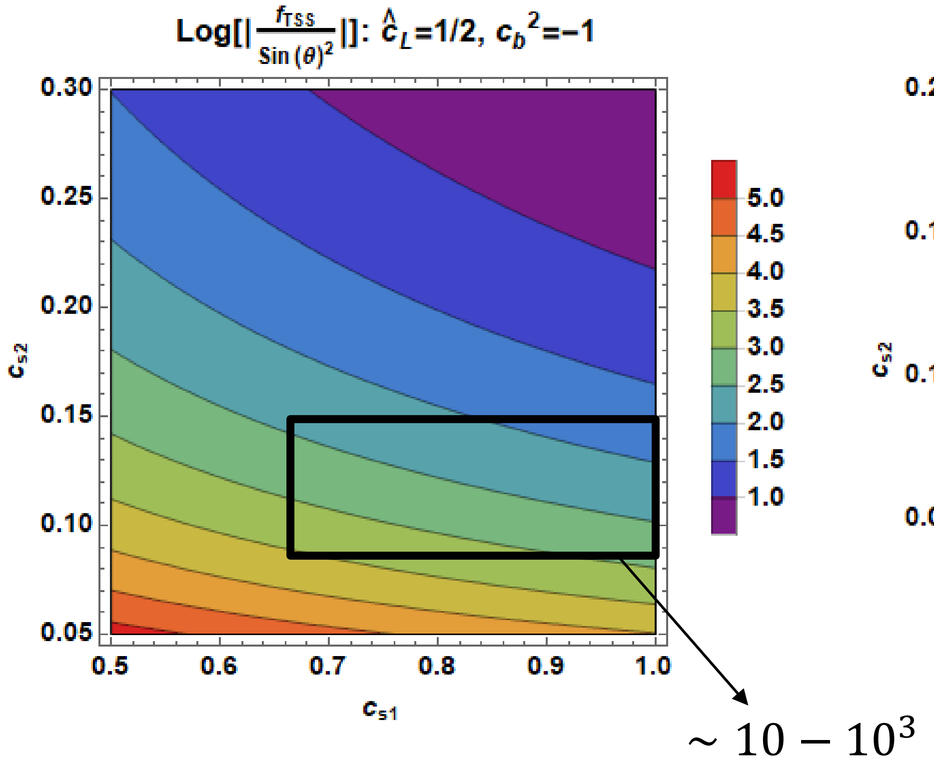
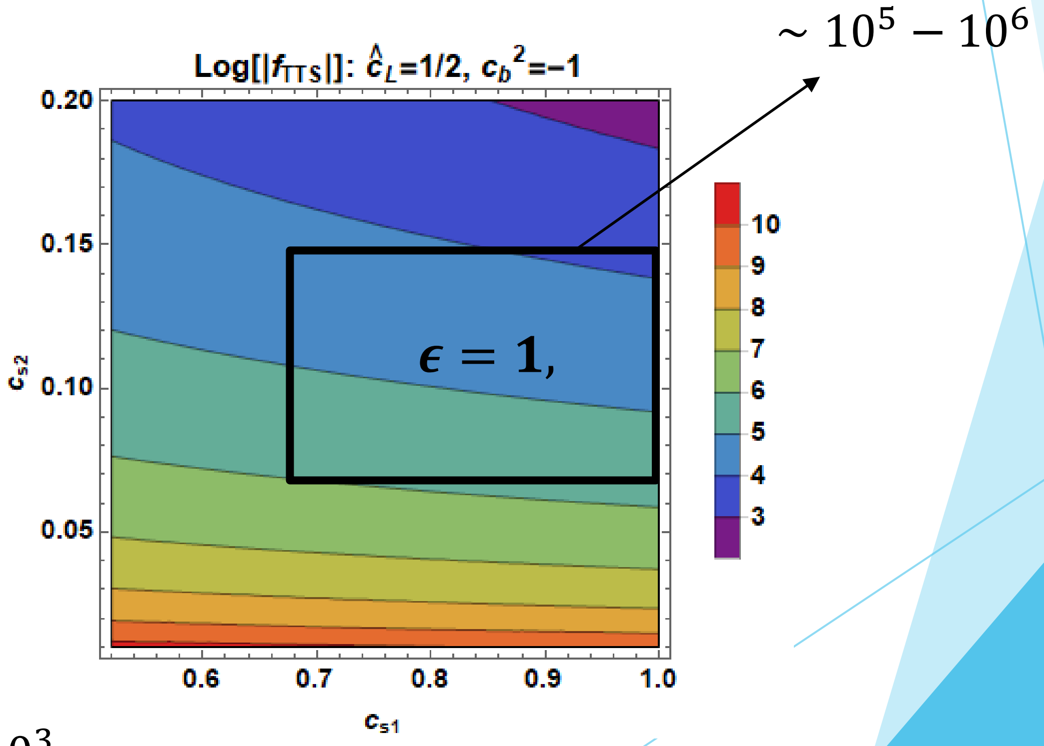
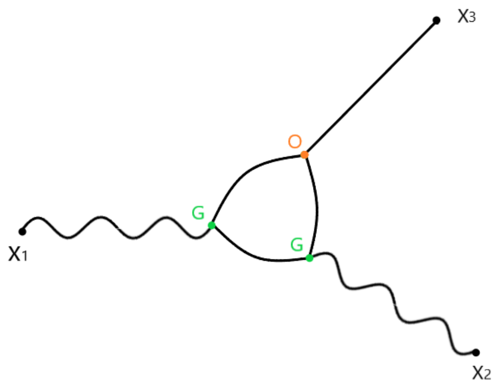
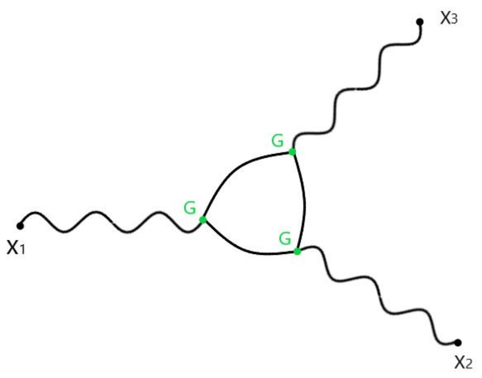




Future developments

- We studied tree-level NG… However, we know that TSS generates one-loops





Future developments

- We studied tree-level NG… However, we know that TSS generates one-loops: TTT and TTS corrections;

- A maximized TSS sector $\Rightarrow$ Tensor fossils in LSS;

- We have the GWs initial conditions let us propagate and study SGWB…

$$< h^2 > \sim < \mathrm{h}^2 >_{\mathrm{One-loop}} + < \mathrm{h}^2 >_{\mathrm{Linear}} [1 + (\ldots)\, \mathrm{f_{TTS}}\, \mathrm{R}_{\pi_0}(\mathrm{x})]$$

- The same EFT in different contexts..

Thank you!