

# Homework 1

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## Policies

These homeworks are not mandatory for this course but you are highly encouraged to try to solve them and any work in this direction will be highly appreciated. You may discuss your homework problems freely with other students, but please refrain from looking at their code or writeups. Please submit your solutions using latex to write your solutions in pdf format.

## Questions

- (1) Show that if  $A$  and  $B$  are similar, i.e.  $A = X^{-1}BX$ , then  $\sigma(A) = \sigma(B)$
- (2) Let  $A = \begin{bmatrix} 1 & 1 \\ 0 & -(2\varepsilon)^{-1} \end{bmatrix}$ . Show that  $\kappa_2(A) = O(1/\varepsilon)$
- (3) Let  $\mathcal{U} = \text{Span}(U)$ ,  $U \in \mathbb{C}^{n \times p}$ . Show that  $A\mathcal{U} \subseteq \mathcal{U}$  if and only if there exists  $B$  such that  $AU = UB$
- (4) A matrix  $A$  is normal if  $AA^* = A^*A$ . Use the Schur decomposition theorem to show that for any normal matrix  $A$  there exists a unitary matrix  $U = [u_1 \cdots u_n]$  such that  $A = \lambda_1 u_1 u_1^* + \cdots + \lambda_n u_n u_n^*$
- (5) Show that for all Hermitian matrices  $A$  it holds  $\min_{x \perp \mathcal{U}_k} r_A(x) = \lambda_{k+1}(A)$ , where  $\mathcal{U}_k$  is the eigenspace of the first  $k$  eigenvalues of  $A$ .