An introduction to Kinetic Theory of Gases and the Boltzmann equation.

Synopsis

Kinetic equations have been used for more than a century to describe the collective behaviour of many particle systems. At the end of the XIXth century J. C. Maxwell and L. E. Boltzmann addressed independently the problem of the mathematical description of classical dilute gases, in an attempt to produce a reduced kinetic picture emerging from the microscopic laws of classical mechanics. The general idea underlying the study of kinetic equations, namely the methodology of the Kinetic Theory, is the following one. Many interesting systems in physics are composed of a large number of identical particles. We will focus on systems in which the microscopic behavior is driven by the Newton equations with pair interaction potentials. Due to the large number of particles, instead of studying the detailed microscopic description, we are interested in the collective behavior of the system, which arises on space and time scales which are much larger than the ones characterizing the microscopic dynamics. On such macroscopic scales, the systems appear much simpler, and can be usually described by integro-differential equations that provide a reduced picture for which the mathematical analysis is more feasible.

One of the most famous example of kinetic equation is the Boltzmann equation which describes the evolution of a rarefied gas, namely

$$\partial_t f + v \cdot \nabla_x f = Q(f,f) \qquad f = f(t,x,v) \tag{1}$$
$$Q(f,f)(t,x,v) = \int_{S^2} d\omega \int_{\mathbb{R}^3} dv_* B(v - v_*, \omega) \left\{ f(t,x,v') f(t,x,v'_*) - f(t,x,v) f(t,x,v_*) \right\}.$$

The goal of this course is to present some fundamental historical results on the mathematical aspects of the Boltzmann equation as well as an overview of some recent results on the qualitative analysis of the behavior of its solutions. More precisely, we will focus on the following topics.

- Scaling limits and (heuristic) derivation of the Boltzmann equation.
- Properties of the Boltzmann equation.
- Well posedness theory and description of the long-time behaviour for the solutions of the Boltzmann equation (1) which are spatially homogeneous, i.e. $f = f\,(t,v)$.
- Some insights on the mathematical theory of the Boltzmann equation in open systems: a special class of solutions to the Boltzmann equation, i.e. "homoenergetic solutions".

Prerequisites: basic knowledge of PDEs and functional analysis.