Flavour Problems and New Physics at TeV Scale

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Introduction

The Standard Model

The SM describes the fundamental interactions among elementary particles as gauge interactions.

- Gauge Symmetry: $SU(3)_C \times SU(2)_L \times U(1)_Y$

- Fermions:

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \sim (3,2)_{1/6} \qquad u_R \sim (3,1)_{2/3} \qquad d_R \sim (3,1)_{-1/3}$$

$$l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \sim (1,2)_{-1/2} \qquad e_R \sim (1,1)_{-1}$$

The electroweak part $SU(2)_L \times U(1)_Y$ is chiral.

- A single Higgs doublet: $\phi \sim (1,2)_{1/2}$

Yukawa interactions $\mathcal{L}_{\text{Yuk}} = Y_u \tilde{\phi} \overline{q_L} u_R + Y_d \phi \overline{q_L} d_R + Y_e \phi \overline{\ell_L} e_R + \text{h.c.}$

The Standard Model

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Replication of fermion families

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- Yukawa matrices are not diagonal in weak basis.

The Standard Model

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- Spontaneous symmetry breaking:

$$\phi \sim (1,2)_{1/2}\,, \qquad \langle\phi^0\rangle = \begin{pmatrix} 0 \\ v_w \end{pmatrix} \qquad v_{\rm w} = 174 \text{ GeV}$$

$$SU(3)_C \times SU(2)_L \times U(1)_Y \to SU(3)_C \times U(1)_{em}$$

The Standard Model

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- Fermion masses: $m^{(u,d,e)ij} = Y^{ij}_{u,d,e} v_w$

- Gauge bosons masses: $m_W^2 = \dfrac{g^2 v_w^2}{2}\,, \qquad m_Z^2 = \dfrac{g^2 v_w^2}{2\cos^2\theta_w}$

The Standard Model

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- $m^{(u,d,e)ij} = Y^{ij}_{u,d,e}v_w$
- Yukawa matrices are not diagonal in weak basis.
- Weak eigenstates are not mass eigenstates:

$$\mathbf{m}_{u,d,e} = V_L^{(u,d,e)\dagger}\mathbf{m}^{(u,d,e)}V_R^{(u,d,e)}$$

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Quark mass eigenstates:

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad \begin{pmatrix} c_L \\ s_L \end{pmatrix} \quad \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$

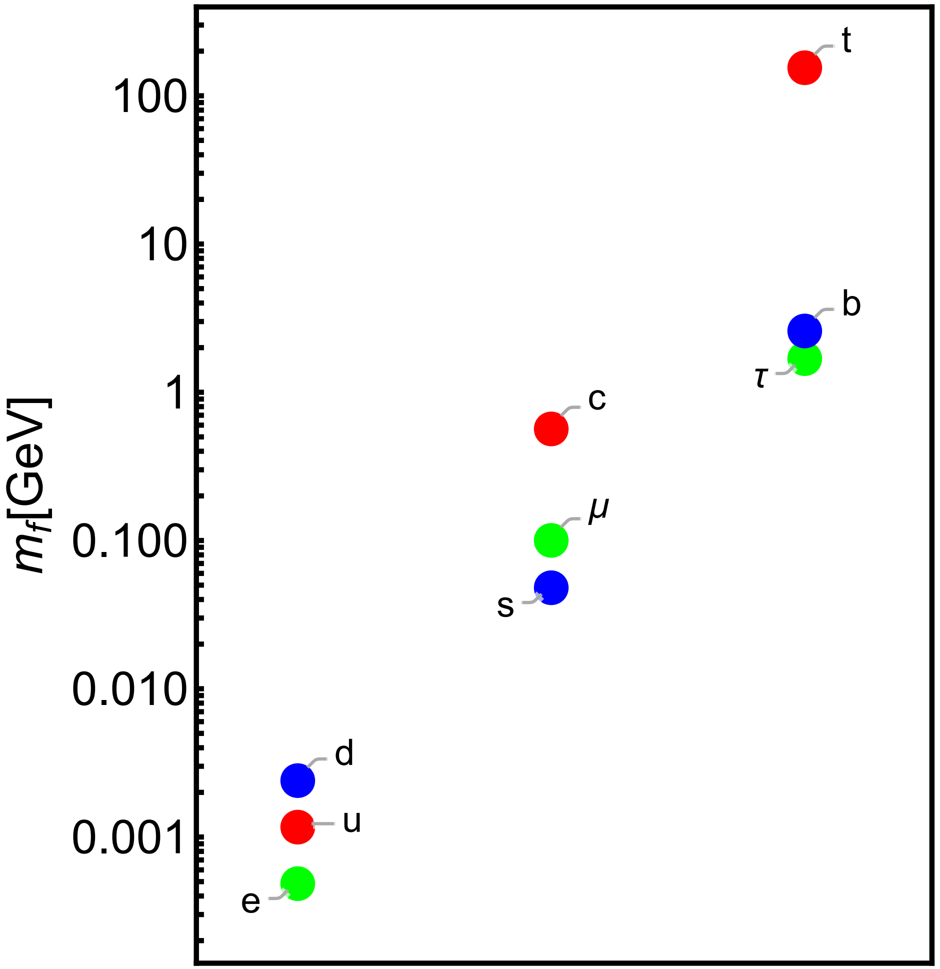
$$u_R \quad c_R \quad t_R$$

$$d_R \quad s_R \quad b_R$$

The Standard Model

- $\mathbf{m}_{u,d,e} = V_L^{(u,d,e)\dagger} \mathbf{m}^{(u,d,e)} V_R^{(u,d,e)}$
- Quark charged currents in mass basis: $\frac{g}{\sqrt{2}} \begin{pmatrix} u & c & t \end{pmatrix}_L \gamma^\mu V_L^{(u)\dagger} V_L^{(d)} \begin{pmatrix} d \\ s \\ b \end{pmatrix} W_\mu^\dagger$
- $V_{\mathrm{CKM}} = V_L^{(u)\dagger} V_L^{(d)} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$
- V_{CKM} is unitary.
- V_{CKM} is the only source of **CP violation** in the SM.

The Standard Model



- $m^{(u,d,e)ij} = Y^{ij}_{u,d,e} v_w$
- $\mathbf{m}_{u,d,e} = V_L^{(u,d,e)\dagger} \mathbf{m}^{(u,d,e)} V_R^{(u,d,e)}$
- $V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$
- Technically natural: SM tolerates Yukawa hierarchy but cannot explain it.

The Standard Model

- $m^{(u,d,e)ij} = Y^{ij}_{u,d,e} v_w$: the same transformation diagonalizes Yukawa couplings and mass matrices.
- Neutral currents: $\mathcal{L}_{\rm nc} = \frac{g}{\cos\theta_W}\left(g^f_L \overline{f_L}\gamma^\mu f_L + g^f_R \overline{f_R}\gamma^\mu f_R\right) Z_\mu$
- Yukawa couplings and photon/Z couplings (for **unitarity** of $V_L^{(u)}$ and $V_L^{(d)}$) are diagonal in mass basis.

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- Yukawa couplings and photon/Z couplings (for **unitarity** of $V_L^{(u)}$ and $V_L^{(d)}$) are diagonal in mass basis.
- **NO flavour changing neutral currents** at tree level.
- The SM is extremely successfull in describing experimental data on flavor changing and CP violating processes, and in the SM FCNC are strongly suppressed.
- Then if **new physics** shows no suppression for FCNC, its scale must be above $\sim 10^4$ TeV.

In the SM there is no explanation for…

- replication of fermion families;
- inter-family mass hierarchy (Yukawa hierarchy);
- weak mixing pattern: small angles for quarks, large angles for neutrinos;
- neutrino masses: very small (seesaw?), mass hierarchy yet unknown.
- Hierarchy problem (physics at TeV scale needed..), dark matter, dark energy, baryogenesis, ...

OUTLINE

1. Gauge family symmetries:
    - Masses and mixings from family symmetry
    - How light can (flavour) gauge bosons be?
2. Anomalies in the first row of CKM matrix: new physics at TeV scale?
    - The present situation
    - Solving CKM unitarity problem with family symmetry
3. Extra vector-like quarks
    - Are they a solution for anomalies in the first row of CKM?

1. Family symmetries

- Masses and mixings from family symmetry
- How light can (flavour) gauge bosons be?

Replication of fermion families

$$q_L^1 = \begin{pmatrix} u_L^1 \\ d_L^1 \end{pmatrix}$$
$$u_R^1$$
$$d_R^1$$
$$l_L^1 = \begin{pmatrix} \nu_L^1 \\ e_L^1 \end{pmatrix}$$
$$e_R^1$$

$$q_L^2 = \begin{pmatrix} u_L^2 \\ d_L^2 \end{pmatrix}$$
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$$e_R^1$$

$U(3)_\ell \times U(3)_e \times U(3)_q \times U(3)_u \times U(3)_d$

- In the limit of vanishing Yukawa couplings, $Y_f \to 0$, the SM acquires a maximal global chiral **family symmetry**.

- Fermions transform as triplets

$$\ell_L \sim 3_\ell \,, \quad e_R \sim 3_e$$
$$q_L \sim 3_q \quad u_R \sim 3_u$$
$$d_R \sim 3_d$$

- Yukawa interactions break the $SU(3)^5$ symmetry.

$q_L^1 = \begin{pmatrix} u_L^1 \\ d_L^1 \end{pmatrix}$ $\quad q_L^2 = \begin{pmatrix} u_L^1 \\ d_L^1 \end{pmatrix}$ $\quad q_L^3 = \begin{pmatrix} u_L^1 \\ d_L^1 \end{pmatrix}$

$\longrightarrow U(3)_q \longrightarrow$

u_R^1 $\quad u_R^2$ $\quad u_R^3$

$\longrightarrow U(3)_u \longrightarrow$

d_R^1 $\quad d_R^2$ $\quad d_R^3$

$\longrightarrow U(3)_d \longrightarrow$

$l_L^1 = \begin{pmatrix} \nu_L^1 \\ e_L^1 \end{pmatrix}$ $\quad l_L^2 = \begin{pmatrix} \nu_L^1 \\ e_L^1 \end{pmatrix}$ $\quad l_L^3 = \begin{pmatrix} \nu_L^1 \\ e_L^1 \end{pmatrix}$

$\longrightarrow U(3)_\ell \longrightarrow$

e_R^1 $\quad e_R^2$ $\quad e_R^3$

$\longrightarrow U(3)_e \longrightarrow$

Family symmetries

In the context of $SU(5)$ grand unified theory (GUT) the maximal symmetry reduces to

$$U(3)_\ell \times U(3)_e$$

$$\bar{5}_L = (\ell, d^c)_L \sim (3_\ell, 1) \qquad 10_L = (e^c, u^c, Q)_L \sim (1, \bar{3}_e)$$

Family symmetries

$$U(3)_\ell \times U(3)_e \times U(3)_q \times U(3)_u \times U(3)_d$$

- Gauge the symmetry $SU(3)^5$.
- The $U(1)$ factors remain as global symmetries.
- Fermions cannot get mass if the symmetry is unbroken.

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- Fermions cannot get mass if the symmetry is unbroken.

$\rightarrow$ Can the fermion mass hierarchy be related to the breaking pattern?

$\rightarrow$ Can the breaking scale be low enough to have flavor bosons within the experimental reach?

Family symmetries and fermion masses

$$U(3)_\ell \times U(3)_e \times U(3)_q \times U(3)_u \times U(3)_d$$

$$\ell_L \sim 3_\ell\,, \qquad e_R \sim 3_e\,, \qquad q_L \sim 3_q \qquad u_R \sim 3_u\,, \qquad d_R \sim 3_d$$

- Fermions cannot get mass if the symmetry is unbroken.
- Yukawa couplings are induced by non-zero VEVs of scalars.

Effective operators for fermion masses

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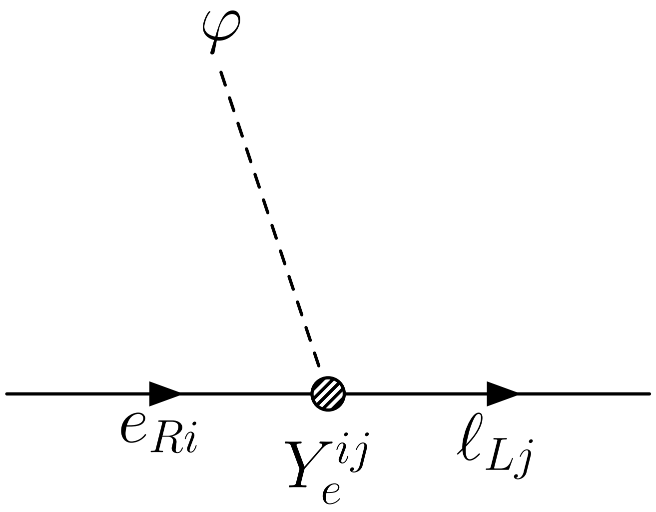
$$Y_e^{ij}\varphi\overline{l_{Lj}}e_{Ri}$$



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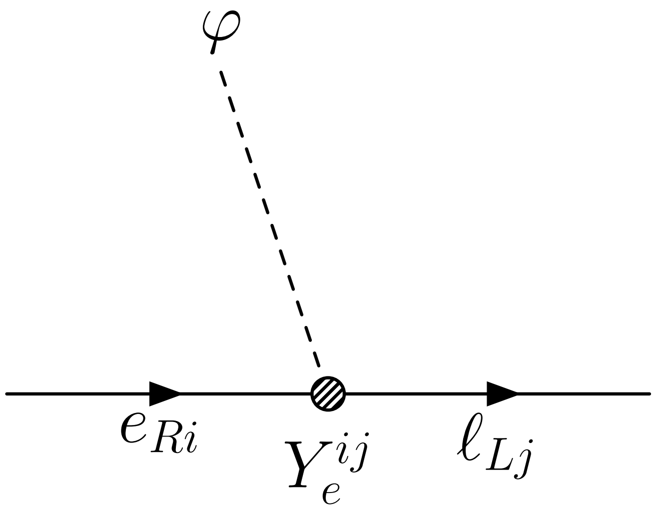
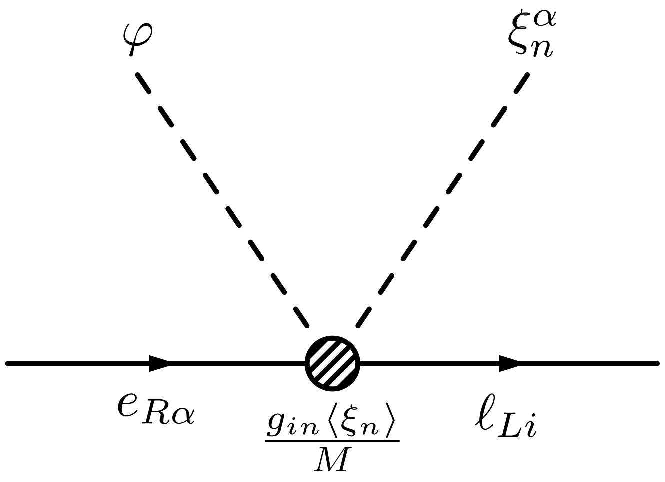
$$\frac{g_{in}\xi_n^\alpha}{M_L}\varphi\overline{\ell_{Li}}e_{R\alpha} \qquad \xi \sim 3_e$$





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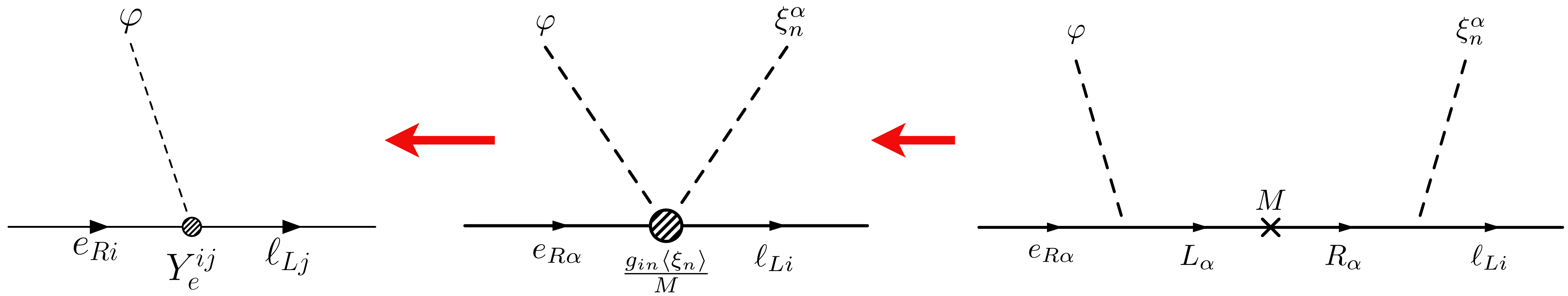

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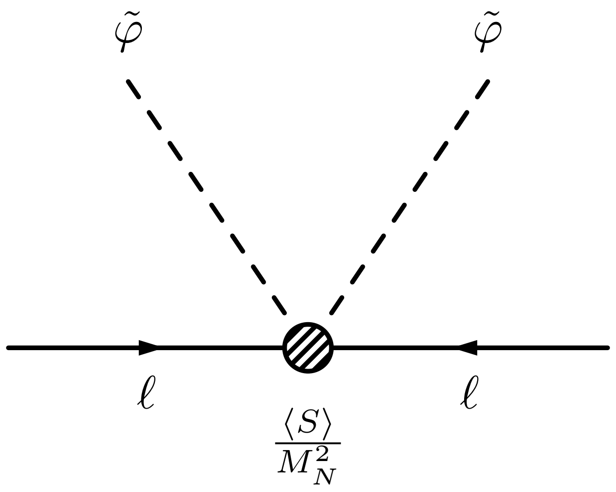

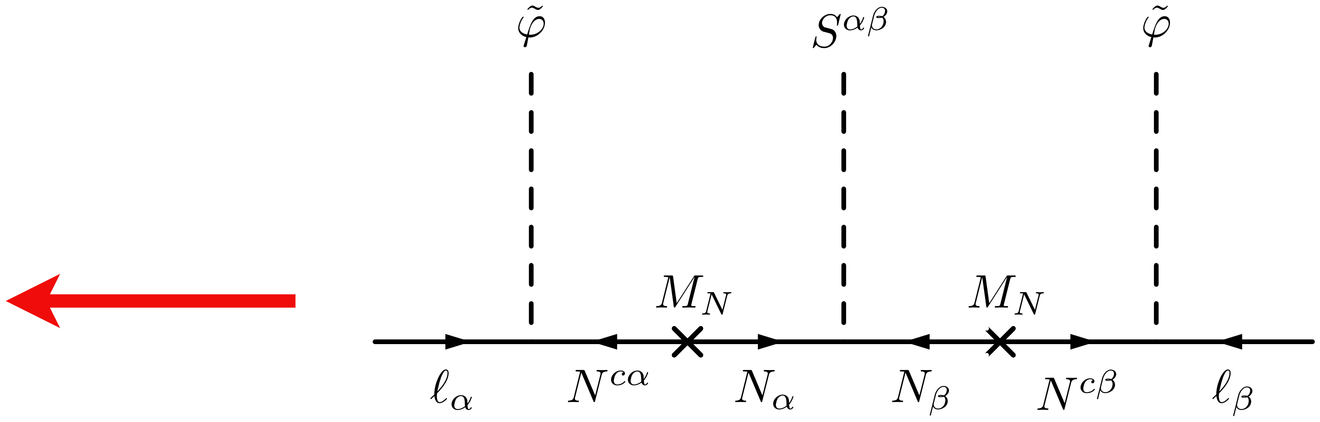

$$S_{\alpha\beta} \sim 6_l$$

Fermion masses

- The inter-family hierarchy between quark species and CKM angles can be parameterized by a small parameter $\epsilon \sim \mathbf{1/20}$:

$$m_d : m_s : m_b \sim \epsilon^2 : \epsilon : 1 \qquad m_u : m_c : m_t \sim \epsilon^4 : \epsilon^2 : 1$$

$$\sin\theta_{12} \sim \sqrt{\epsilon} \sim 4\epsilon; \quad \sin\theta_{23} \sim \epsilon; \quad \sin\theta_{13} \sim \epsilon^2$$

- Hierarchy among charged leptons can be parametrized with ϵ:

$$m_e : m_\mu : m_\tau \sim k^{-1}\epsilon^2 : k\epsilon : k$$

with $k \simeq 3$ (factor $O(1)$), or:

$$m_e : m_\mu : m_\tau \sim \tilde{\epsilon}\epsilon : \epsilon : 1$$

- The neutrino mixing angles are much larger and presumably there is no significant hierarchy between the neutrino masses.

Leptons and family symmetry

$$U(3)_\ell \times U(3)_e \times U(3)_q \times U(3)_u \times U(3)_d$$

$$\ell_L \sim 3_\ell \,, \quad e_R \sim 3_e$$

Leptons and family symmetry

$$SU(3)_\ell \times SU(3)_e$$

- Three $SU(3)_e$ triplets $\xi_i \sim 3_e$
- Three $SU(3)_\ell$ triplets $\eta_i \sim 3_\ell$

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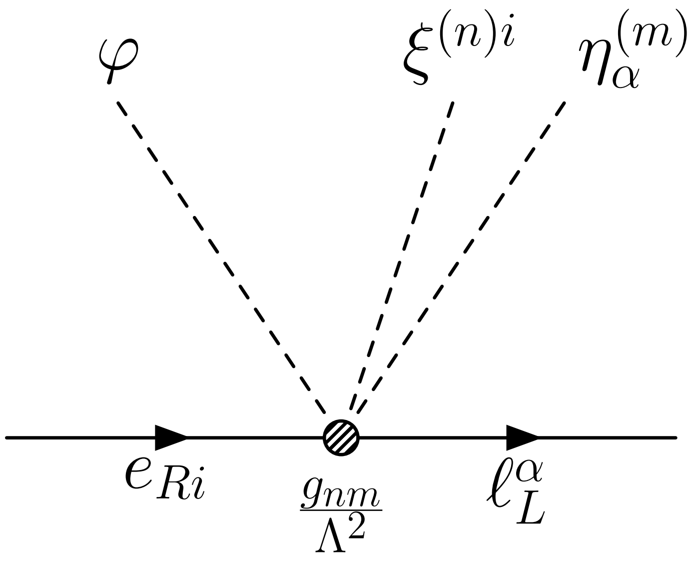
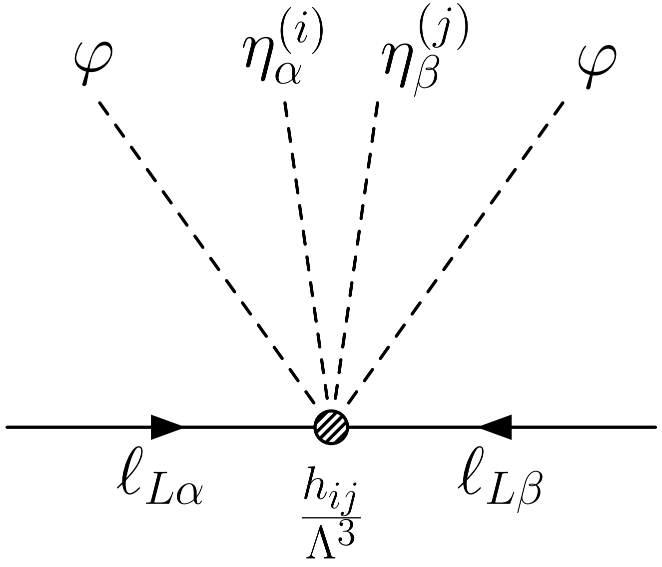
$$\frac{y_{ij}\bar{\xi}_j^\gamma \eta_{i\alpha}}{M_L^2}\varphi\overline{\ell_{L\alpha}}e_{Ri} + \frac{h_{ij}\bar{\eta}_i^\alpha \bar{\eta}_j^\beta}{M_\nu^3}\varphi\varphi\ell_{L\alpha}^T C\ell_\beta + h.c.$$





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$$SU(3)_e \xrightarrow{v_3} SU(2)_e \xrightarrow{v_2} \text{nothing}$$

$$SU(3)_\ell \xrightarrow{w_3} SU(2)_\ell \xrightarrow{w_2} \text{nothing}$$

$$\langle\eta_1\rangle = \begin{pmatrix} w_1 \\ 0 \\ 0 \end{pmatrix}, \quad \langle\eta_2\rangle = \begin{pmatrix} 0 \\ w_2 \\ 0 \end{pmatrix}, \quad \langle\eta_3\rangle = \begin{pmatrix} 0 \\ 0 \\ w_3 \end{pmatrix}$$

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$$m_\nu^{ij} = \frac{h_{ij}w_i w_j v_w^2}{M_\nu^3}, \qquad U_{\text{PMNS}}$$

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$$v_1 : v_2 : v_3 = \tilde{\epsilon}\epsilon : \epsilon : 1 \longrightarrow Y_e = y_\tau \begin{pmatrix} \sim\tilde{\epsilon}\epsilon & \sim\epsilon & \sim 1 \\ \sim\tilde{\epsilon}\epsilon & \sim\epsilon & \sim 1 \\ \sim\tilde{\epsilon}\epsilon & \sim\epsilon & \sim 1 \end{pmatrix} \longrightarrow m_e\,, m_\mu\,, m_\tau$$

$$Y_e^{ij} = \frac{y_{ij}w_i v_j}{M_L^2}$$

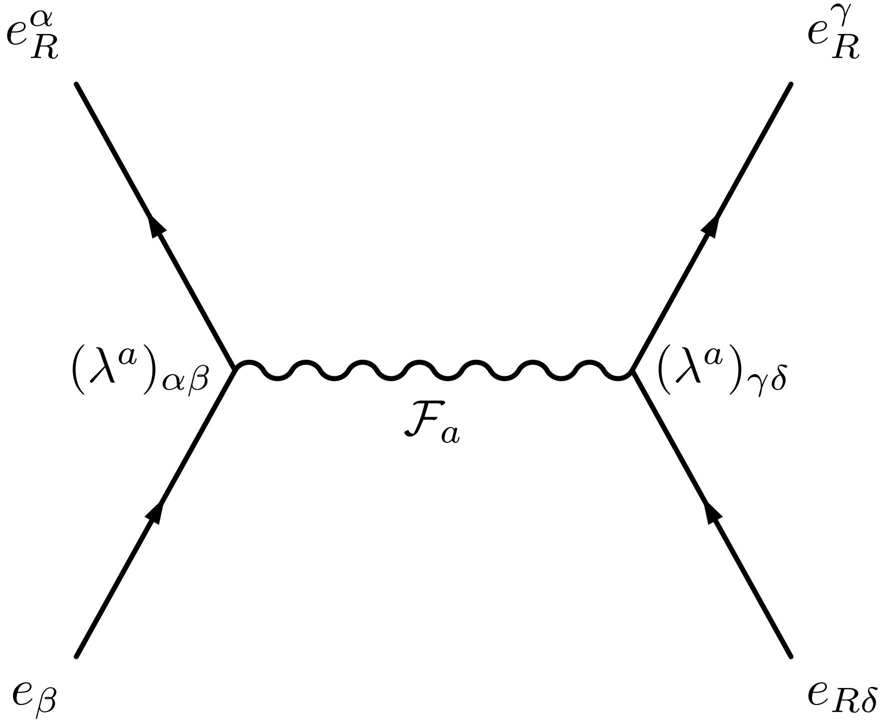
Gauge bosons

$$SU(3)_e$$

Gauge bosons

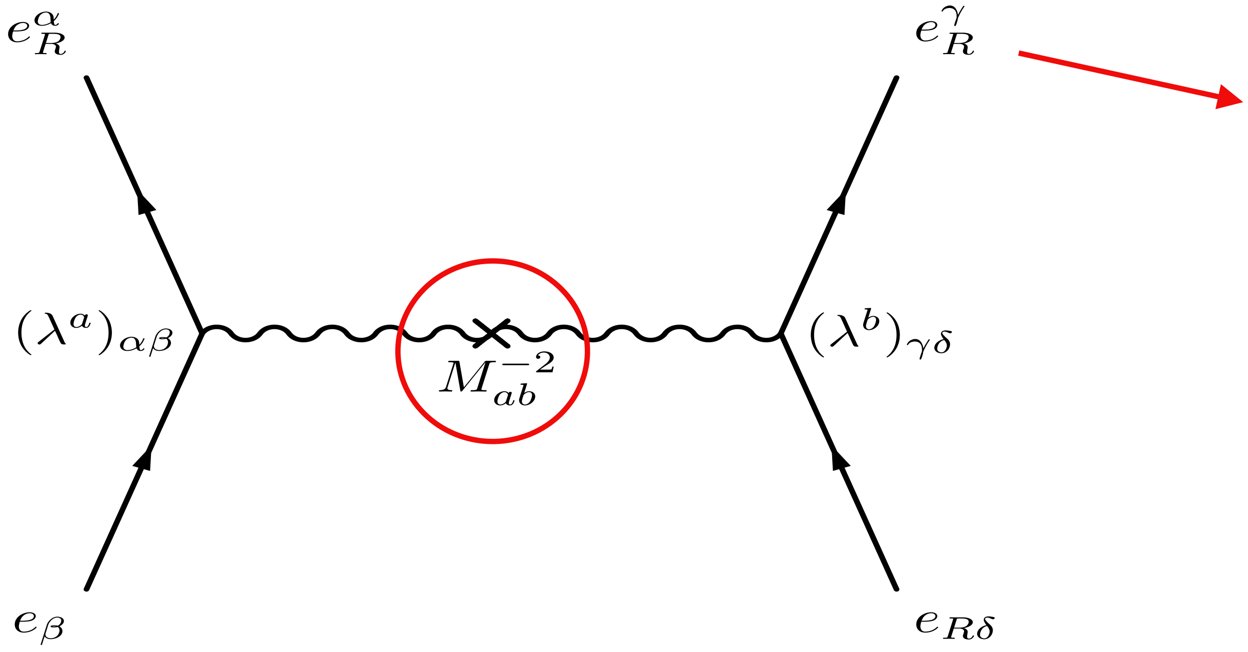
$$SU(3)_e$$

- Gauging of $SU(3)_e$, new gauge bosons, new horizontal interactions.



Gauge bosons, $SU(3)_e$

- Generically, FCNC...(in the SM only at loop level, and excellent agreement with experimental data...):



$$e_{Ri} = V^{(e)}_{R1e} e_R + V^{(e)}_{R1\mu} \mu_R + V^{(e)}_{R1\tau} \tau_R$$

- Heavy gauge bosons may suppress FCNC. Nevertheless, can they be "light"?

Gauge bosons, $SU(3)_e$

$$SU(3)_e \xrightarrow{v_3} SU(2)_e \xrightarrow{v_2} \text{nothing}$$

$$\boxed{v_1 : v_2 : v_3 = \tilde{\epsilon}\epsilon : \epsilon : 1}$$

Gauge bosons, $SU(3)_e$

$$SU(3)_e \xrightarrow{v_3} SU(2)_e \xrightarrow{v_2} \text{nothing} \qquad v_1 : v_2 : v_3 = \tilde{\epsilon}\epsilon : \epsilon : 1$$

$$\frac{1}{2}\begin{pmatrix} \mathcal{F}_3 + \frac{1}{\sqrt{3}}\mathcal{F}_8 & \mathcal{F}_1 - i\mathcal{F}_2 & \mathcal{F}_4 - i\mathcal{F}_5 \\ \mathcal{F}_1 + i\mathcal{F}_2 & -\mathcal{F}_3 + \frac{1}{\sqrt{3}}\mathcal{F}_8 & \mathcal{F}_6 - i\mathcal{F}_7 \\ \mathcal{F}_4 + i\mathcal{F}_5 & \mathcal{F}_6 + i\mathcal{F}_7 & -\frac{2}{\sqrt{3}}\mathcal{F}_8 \end{pmatrix}$$

$$M^2_{4,5,6,7} = \frac{g^2 v_3^2}{2} \qquad M^2_{1,2} = \frac{g^2 v_2^2}{2} \qquad M^2_{38} = \frac{g^2}{2}\begin{pmatrix} v_2^2 & -\frac{1}{\sqrt{3}}v_2^2 \\ -\frac{1}{\sqrt{3}}v_2^2 & \frac{1}{3}(4v_3^2 + v_2^2) \end{pmatrix}$$

Gauge bosons, $SU(3)_e$

$$SU(3)_e \xrightarrow{v_3} SU(2)_e \xrightarrow{v_2} \text{nothing} \qquad v_1 : v_2 : v_3 = \tilde{\epsilon}\epsilon : \epsilon : 1$$

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$$v_2 \gtrsim \, ?$$

Flavour changing neutral currents

In $\mathbf{SU(2)_e}$ gauge symmetry limit ($v_3 \gg v_2$):

- $SU(2)_e$ gauge bosons have equal masses;
- **no FCNC** for CUSTODIAL SYMMETRY, no matter if two families are mixed:

$$
-\frac{1}{v_2^2}\sum_{a=1}^{3}(J_a^\mu)^2 = -\frac{1}{4v_2^2}\sum_{a=1}^{3}(\overline{\mathbf{e}_R}\,\lambda_a\gamma^\mu\mathbf{e}_R)^2 =
$$

$$
= -\frac{1}{4v_2^2}\left[\begin{pmatrix} \overline{e_R} & \overline{\mu_R} & \overline{\tau_R} \end{pmatrix}\gamma_\mu V_R^{(e)\dagger}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} V_R^{(e)}\begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}\right]^2
$$

- no mixing with 3rd family $\rightarrow$ NO FCNC.
- Constraints on masses are proportional to violation of custodial symmetry $(\epsilon,\, \tilde{\epsilon})$.

Flavour changing neutral currents

- Constraints on masses are proportional to violation of custodial symmetry:

$$\mathcal{L}_{\text{fcnc}} = -\frac{1}{4v_2^2}\left[\begin{pmatrix} \overline{e_R} & \overline{\mu_R} & \overline{\tau_R} \end{pmatrix}\gamma_\mu V_R^{(e)\dagger}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} V_R^{(e)}\begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}\right]^2 +$$

$$-\frac{\epsilon^2}{4v_2^2}\left[\begin{pmatrix} \overline{e_R} & \overline{\mu_R} & \overline{\tau_R} \end{pmatrix}\gamma_\mu V_R^{(e)\dagger}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} V_R^{(e)}\begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}\right]^2 =$$

$$-\frac{1}{4v_2^2}\left[\begin{pmatrix} \overline{e_R} & \overline{\mu_R} & \overline{\tau_R} \end{pmatrix}\gamma_\mu \begin{pmatrix} 1 & \sim\tilde{\epsilon}\epsilon^2 & \sim\tilde{\epsilon}\epsilon \\ \sim\tilde{\epsilon}\epsilon^2 & 1 & \sim\epsilon \\ \sim\tilde{\epsilon}\epsilon & \sim\epsilon & \sim\epsilon^2 \end{pmatrix}\begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}\right]^2 +$$

$$-\frac{\epsilon^2}{4v_2^2}\left[\begin{pmatrix} \overline{e_R} & \overline{\mu_R} & \overline{\tau_R} \end{pmatrix}\gamma_\mu \begin{pmatrix} 1 & \sim\tilde{\epsilon} & \sim\tilde{\epsilon}\epsilon \\ \sim\tilde{\epsilon} & \sim\epsilon^2 & \sim\epsilon \\ \sim\tilde{\epsilon}\epsilon & \sim\epsilon & 1 \end{pmatrix}\begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}\right]^2$$

Experimental constraint

- **Flavour conserving operators** constrained by compositeness limits:

$$-\frac{1}{4v_2^2}\left(\overline{e_R}\gamma^\nu e_R\right)^2-\frac{1}{2v_2^2}\left(\overline{e_R}\gamma^\nu e_R\right)\left(\overline{\mu_R}\gamma_\nu\mu_R\right) \qquad \boxed{v_2>2\text{ TeV}} \quad \to v_3>35\text{ TeV}$$

- From FCNC:

LFV mode	Exp. $\Gamma_i/\Gamma_\mu(\Gamma_\tau)$	Main contribution to $\frac{\Gamma_i}{\Gamma_{\mu/\tau}}$	Predicted value of $\frac{\Gamma_i}{\Gamma_{\mu/\tau}}$
$\mu\to eee$	$<1.0\cdot10^{-12}$	$\frac{1}{8}\left(\frac{v_{\rm EW}}{v_2}\right)^4\left\lvert V_{3e}^*V_{3\mu}+V_{2e}^*V_{2\mu}\epsilon^2\right\rvert^2$	$\le 1.1\cdot10^{-13}\left(\frac{2\,\rm TeV}{v_2}\right)^4\epsilon_{20}^4\tilde\epsilon_{20}^2$
$\tau^-\to\mu^-e^+e^-$	$<1.8\cdot10^{-8}$	$\frac{1}{4}\left(\frac{v_{\rm EW}}{v_2}\right)^4\left\lvert V_{3\mu}^*V_{3\tau}\right\rvert^2\frac{\Gamma_{\rm w}}{\Gamma_\tau}$	$=6.2\cdot10^{-9}\left(\frac{2\,\rm TeV}{v_2}\right)^4\epsilon_{20}^2$
$\tau\to\mu\mu\mu$	$<2.1\cdot10^{-8}$	$\frac{1}{8}\left(\frac{v_{\rm EW}}{v_2}\right)^4\left\lvert V_{3\mu}^*V_{3\tau}\right\rvert^2\frac{\Gamma_{\rm w}}{\Gamma_\tau}$	$=3.1\cdot10^{-9}\left(\frac{2\,\rm TeV}{v_2}\right)^4\epsilon_{20}^2$
$\mu\to e\gamma$	$<4.2\cdot10^{-13}$	$\frac{3\alpha}{2\pi}\left(\frac{v_{\rm EW}}{v_2}\right)^4\left\lvert V_{3e}^*V_{3\mu}\right\rvert^2$	$=3.1\cdot10^{-15}\left(\frac{2\,\rm TeV}{v_2}\right)^4\epsilon_{20}^4\tilde\epsilon_{20}^2$
$\tau\to\mu\gamma$	$<4.4\cdot10^{-8}$	$\frac{3\alpha}{2\pi}\left(\frac{v_{\rm EW}}{v_2}\right)^4\left\lvert V_{3\mu}^*V_{3\tau}\right\rvert^2\frac{\Gamma_{\rm w}}{\Gamma_\tau}$	$=8.7\cdot10^{-11}\left(\frac{2\,\rm TeV}{v_2}\right)^4\epsilon_{20}^2$

Gauge bosons in left-handed sector, $SU(3)_\ell$

$$SU(3)_\ell \xrightarrow{w_3} SU(2)_\ell \xrightarrow{w_2} \text{nothing}$$

$$\frac{1}{2}\begin{pmatrix} \mathcal{F}_3 + \frac{1}{\sqrt{3}}\mathcal{F}_8 & \mathcal{F}_1 - i\mathcal{F}_2 & \mathcal{F}_4 - i\mathcal{F}_5 \\ \mathcal{F}_1 + i\mathcal{F}_2 & -\mathcal{F}_3 + \frac{1}{\sqrt{3}}\mathcal{F}_8 & \mathcal{F}_6 - i\mathcal{F}_7 \\ \mathcal{F}_4 + i\mathcal{F}_5 & \mathcal{F}_6 + i\mathcal{F}_7 & -\frac{2}{\sqrt{3}}\mathcal{F}_8 \end{pmatrix}$$

$$M^2_{\ell 4,5} = \frac{g^2}{2}(w_3^2 + w_1^2) \qquad M^2_{\ell 6,7} = \frac{g^2}{2}(w_3^2 + w_2^2) \qquad M^2_{\ell 38} = \frac{g^2}{2}\begin{pmatrix} w_2^2 + w_1^2 & \frac{1}{\sqrt{3}}(w_1^2 - w_2^2) \\ \frac{1}{\sqrt{3}}(w_1^2 - w_2^2) & \frac{1}{3}(4w_3^2 + w_1^2 + w_2^2) \end{pmatrix}$$

$$M^2_{\ell 1,2} = \frac{g^2}{2}(w_2^2 + w_1^2) = \frac{g^2}{2}v_{\mathcal{F}}^2$$

New interactions in left-handed sector

$$\mathcal{L}_{\rm eff}^{ee} = -\frac{G_{\mathcal{F}}}{\sqrt{2}} \sum_{a=1}^{8} \left(\overline{\boldsymbol{e}_L} \, \gamma_\mu \frac{\lambda_a}{x_a} \, \boldsymbol{e}_L \right)^2$$

Charged leptons flavour conserving interactions,
lepton flavour violating interactions

$$\mathcal{L}_{\rm eff}^{e\nu} = -\frac{2G_{\mathcal{F}}}{\sqrt{2}} \sum_{a=1}^{8} \left(\overline{\boldsymbol{e}_L} \, \gamma^\mu \frac{\lambda_a}{x_a} \, \boldsymbol{e}_L \right) \left(\overline{\boldsymbol{\nu}_L} \, \gamma_\mu \frac{\lambda_a}{x_a} \, \boldsymbol{\nu}_L \right)$$

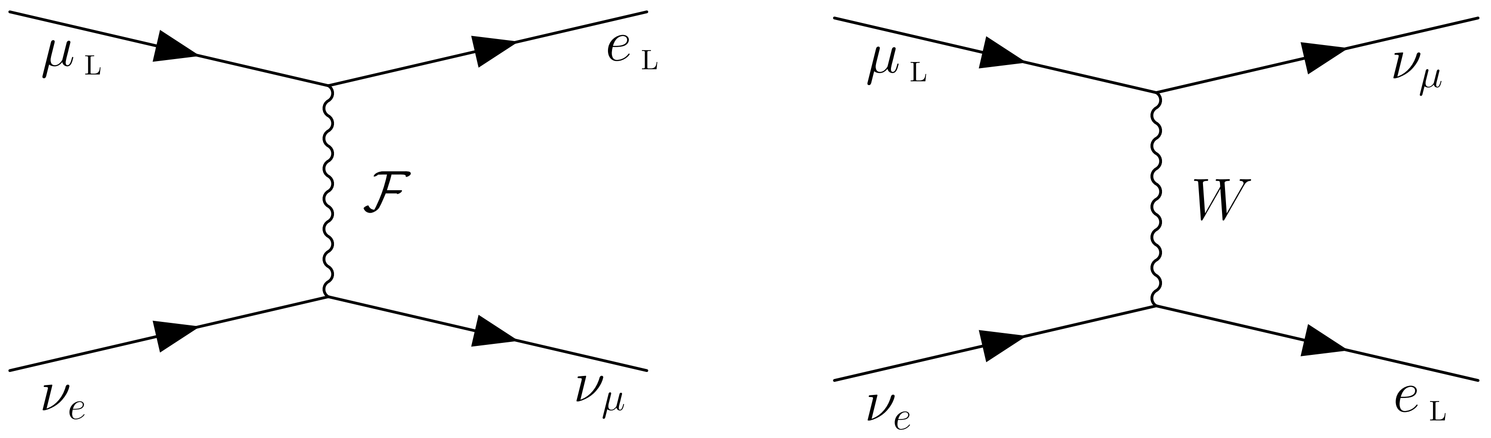
muon decay, tau decays,
non-standard neutrino interactions with leptons

$$\mathcal{L}_{\rm eff}^{\nu\nu} = -\frac{G_{\mathcal{F}}}{\sqrt{2}} \sum_{a=1}^{8} \left(\overline{\boldsymbol{\nu}_L} \, \gamma_\mu \frac{\lambda_a}{x_a} \, \boldsymbol{\nu}_L \right)^2$$

Non-standard interactions between neutrinos

Muon decay

$$\mathcal{L}_{\rm eff}^{e\nu} = -\frac{2G_{\mathcal{F}}}{\sqrt{2}} \sum_{a=1}^{8} \left(\overline{\boldsymbol{e}_L}\, \gamma^{\mu} \frac{\lambda_a}{x_a}\, \boldsymbol{e}_L \right) \left(\overline{\boldsymbol{\nu}_L}\, \gamma_{\mu} \frac{\lambda_a}{x_a}\, \boldsymbol{\nu}_L \right)$$



The sum of the diagrams gives:

$$G_\mu = G_F + G_{\mathcal{F}} = G_F(1+\delta_\mu) \qquad \boxed{G_\mu \neq G_F}$$

CKM unitarity restored if $\delta_\mu = \frac{v_w^2}{v_{\mathcal{F}}^2} \simeq 7 \cdot 10^{-4}$, corresponding to

$$\boxed{v_{\mathcal{F}} = 6\text{-}7 \text{ TeV}}$$

New interactions in left-handed sector

$$\mathcal{L}_{\rm eff}^{ee} = -\frac{G_{\mathcal{F}}}{\sqrt{2}} \sum_{a=1}^{8} \left(\overline{\boldsymbol{e}_L} \, \gamma_\mu \frac{\lambda_a}{x_a} \, \boldsymbol{e}_L \right)^2$$

Flavour conserving interactions
(compositeness limits):

$$v_{\mathcal{F}} > 3 \text{ TeV}$$

FCNC in left-handed sector

$$\mathcal{L}_{\text{eff}}^{ee} = -\frac{G_{\mathcal{F}}}{\sqrt{2}} \sum_{a=1}^{8} \left(\overline{\boldsymbol{e}_L} \, \gamma_\mu \frac{\lambda_a}{x_a} \, \boldsymbol{e}_L \right)^2$$

If $u_3 = u_2 = u_1$ (e. g. symmetry between ηs) then

- Gauge bosons have equal masses and do not mix.
- $\lambda_a \to V^\dagger \lambda_a V$ is simply a basis redetermination of the Gell-Mann matrices
- From Fierz identities for λ matrices:

$$\mathcal{L}_{eff} = -\frac{1}{4v_\ell^2} (\overline{\mathbf{e}_\mathrm{L}} \, \lambda^a \, \gamma^\mu \mathbf{e}_\mathrm{L})(\overline{\mathbf{e}_\mathrm{L}} \, \lambda^a \, \gamma_\mu \mathbf{e}_\mathrm{L}) = -\frac{1}{3v_2^2} (\overline{\mathbf{e}_\mathrm{L}} \, \mathbb{I} \, \gamma_\mu \mathbf{e}_\mathrm{L})^2$$

- **no FCNC**, the global $SO(8)_\ell$ symmetry acts as a custodial symmetry.

FCNC in left-handed sector

$$\mathcal{L}_{\text{eff}}^{ee} = -\frac{G_{\mathcal{F}}}{\sqrt{2}} \sum_{a=1}^{8} \left(\overline{\boldsymbol{e}_L}\, \gamma_\mu \frac{\lambda_a}{x_a}\, \boldsymbol{e}_L \right)^2$$

- In general case e.g. $\mu \to 3e$ decay: $(\delta_\mu = \frac{v_w^2}{v_{\mathcal{F}}^2} \simeq 7 \cdot 10^{-4})$

$$\frac{\Gamma(\mu \to ee\bar{e})}{\Gamma(\mu \to e\nu_\mu \bar{\nu}_e)} \simeq \frac{1}{8}\,(C(r)|U_{3e}^* U_{3\mu}|)^2\, \delta_\mu^2$$

$r = 2u_3^2/v_\ell^2$, $|C(r)| < 1$. $|U_{3\mu}|$ and $|U_{3e}|$ can be as large as $\sin\theta_C = V_{us}$.

- The experimental limits on other LFV effects as $\tau \to 3\mu$ are much weaker.
- $\mathbf{v}_\ell \simeq \mathbf{6}$ **TeV** is not contradicting experimental constraints.
- For $r = 1$ all LFV effects are vanishing owing to custodial symmetry.

Conclusions #1

- Hierarchy between fermion masses and mixing pattern can be obtained from the breaking pattern of gauge family symmetries.
- Gauge bosons of flavour can be as light as few **TeV** (also for quarks).
- The Fermi constant $\mathbf{G_F}$ can be different from the muon decay constant.

1. Anomalies in the first row of CKM matrix: new physics at TeV scale?

- The present situation
- Solving CKM unitarity problem with family symmetry

Present situation of CKM unitarity

$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \begin{pmatrix} u & c & t \end{pmatrix}_L \gamma^\mu V_{\rm CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix} W_\mu^\dagger + \text{h.c.}$$

- $V_{\rm CKM} = V_L^{(u)\dagger} V_L^{(d)}$ is the Cabibbo-Kobayashi-Maskawa (CKM) matrix:

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- $V_{\rm CKM}$ is unitary.
- The unitarity condition for the first row is:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

- Deviation from CKM unitarity can be a signal of new physics beyond SM.

Present situation of CKM unitarity

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

- $|V_{us}|$ can be directly determined from semileptonic $K\ell 3$ decays

$$f_+(0)|V_{us}| = 0.21654 \pm 0.00041$$

- The ratio of decay rates $K \to \mu\nu(\gamma)$ and $\pi \to \mu\nu(\gamma)$ determines:

$$|V_{us}/V_{ud}| \times (f_{K^\pm}/f_{\pi^\pm}) = 0.27599 \pm 0.00038$$

- Form factor $f_+(0)$ and decay constant ratio f_K/f_π from lattice QCD.
- $|V_{us}| = 0.2238(8)$, $\left|\frac{V_{us}}{V_{ud}}\right| = 0.2315(10)$ (PDG 2018);
- New $f_{K^\pm}/f_{\pi^\pm}$ (FLAG 2019) and $f_+(0)$ (FLAG+Fermilab Lattice & MILC).

Present situation of CKM unitarity

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

- Semileptonic kaon decays:

$$\mathbf{A}: \quad |\mathbf{V_{us}}| = \mathbf{0.22326(55)}$$

- Leptonic kaon (and pion) decays:

$$\mathbf{B}: \quad |\mathbf{V_{us}}|/|\mathbf{V_{ud}}| = \mathbf{0.23130(49)}$$

- Determination from leptonic K decay recently obtained (Di Carlo et al):

$$\mathbf{A'}: \quad \mathbf{V_{us}} = \mathbf{0.22567(42)}$$

Present situation of CKM unitarity

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

- Superallowed nuclear beta-decays ($0^+ \to 0^+$ Fermi transitions) determine:

$$G_F^2 |V_{ud}|^2 = \frac{K}{2\mathcal{F}t\,(1+\Delta_R^V)} = G_F^2 \frac{0.97147(20)}{1+\Delta_R^V}$$

- Δ_R^V are short-distance (transition independent) radiative corrections.

- Redetermination of $\Delta_R^V = 0.02467(22)$ from (Seng et al.):

$$C_1 : \quad |\mathbf{V_{ud}}| = \frac{\mathbf{G_V^{exp}}}{\mathbf{G_F(=G_\mu)}} = \mathbf{0.97366(15)}$$

- Czarnecki et al. $\Delta_R^V = 0.02426(32)$:

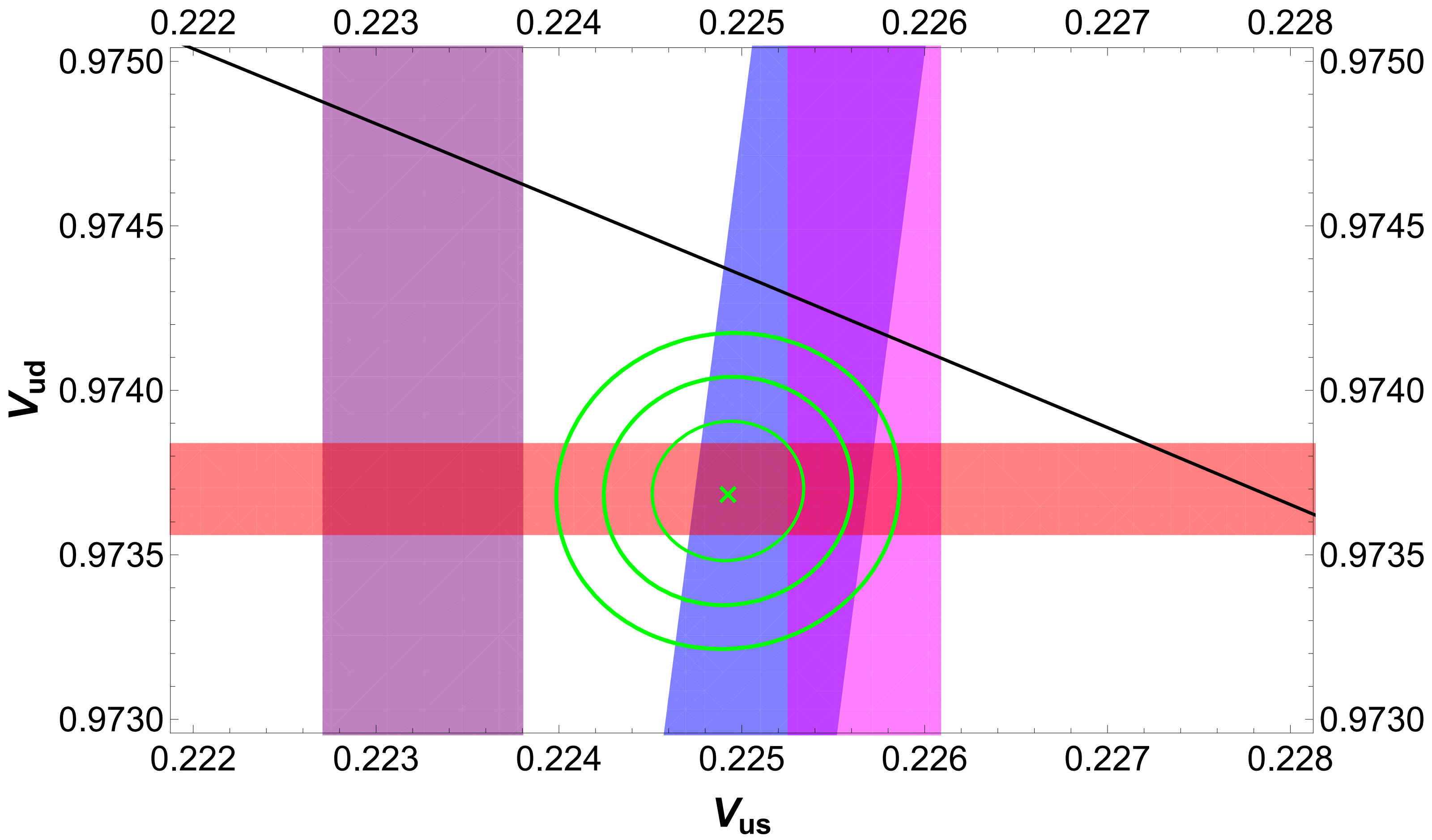
$$C_2 \quad |\mathbf{V_{ud}}| = \frac{\mathbf{G_V^{exp}}}{\mathbf{G_F(=G_\mu)}} = \mathbf{0.97389(18)}$$

- Average (without reducing uncertainty):

$$C : \quad |\mathbf{V_{ud}}| = \frac{\mathbf{G_V^{exp}}}{\mathbf{G_F(=G_\mu)}} = \mathbf{0.97376(16)}$$

Present situation of CKM unitarity

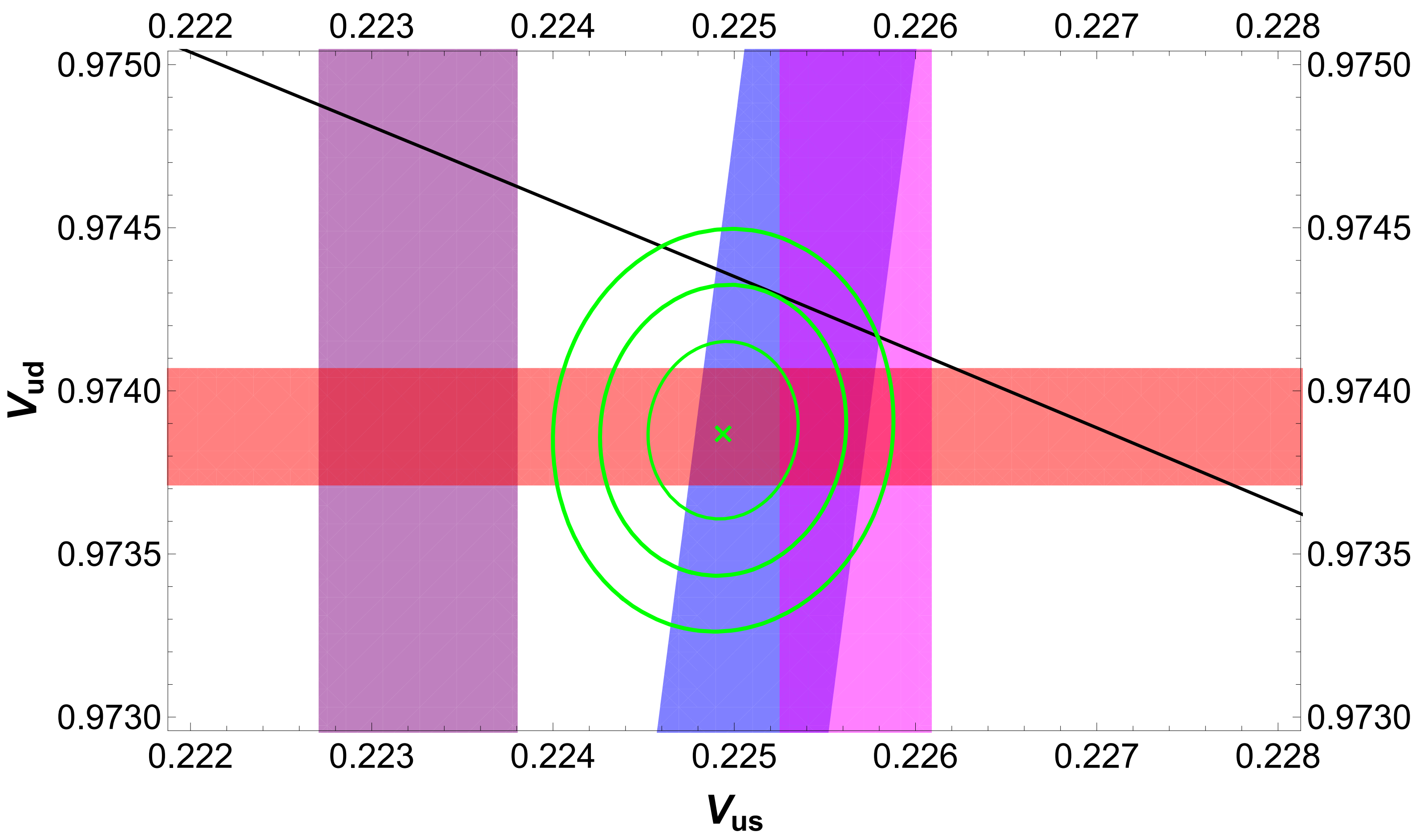
With $\mathbf{C_1}$:



3.9σ away from unitarity

Present situation of CKM unitarity

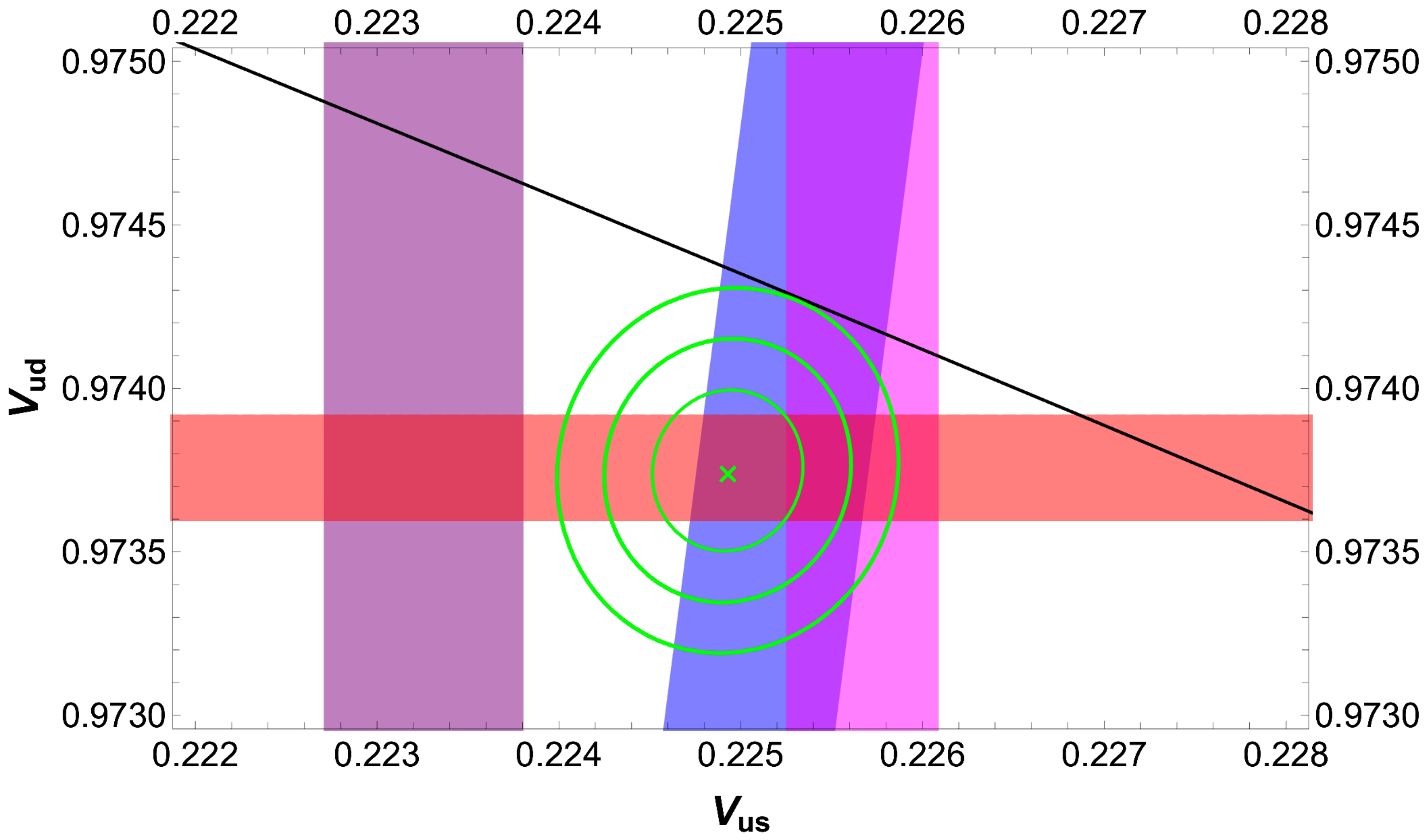
With $\mathbf{C_2}$:



2σ away from unitarity

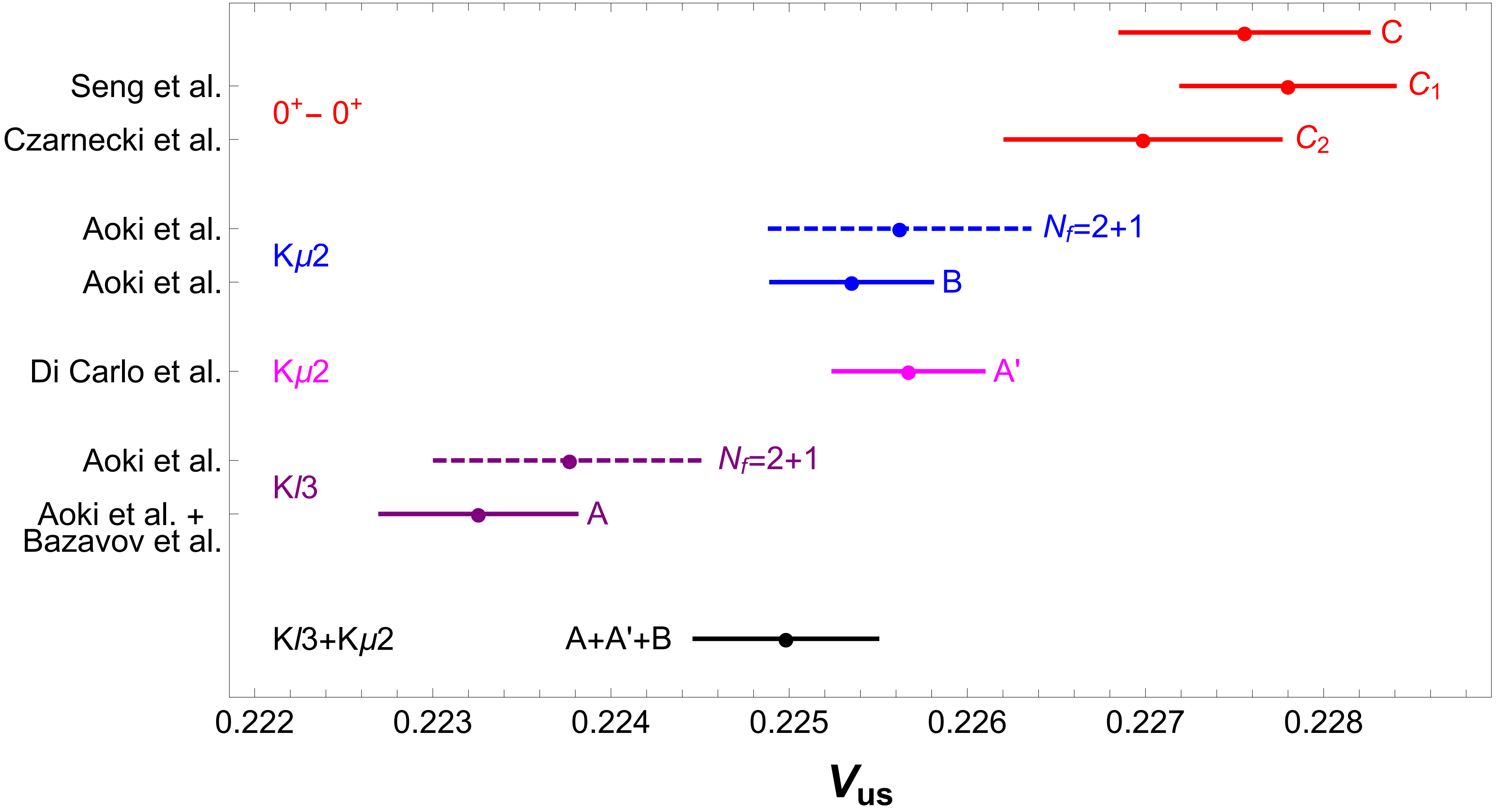
Present situation of CKM unitarity

With **C**:

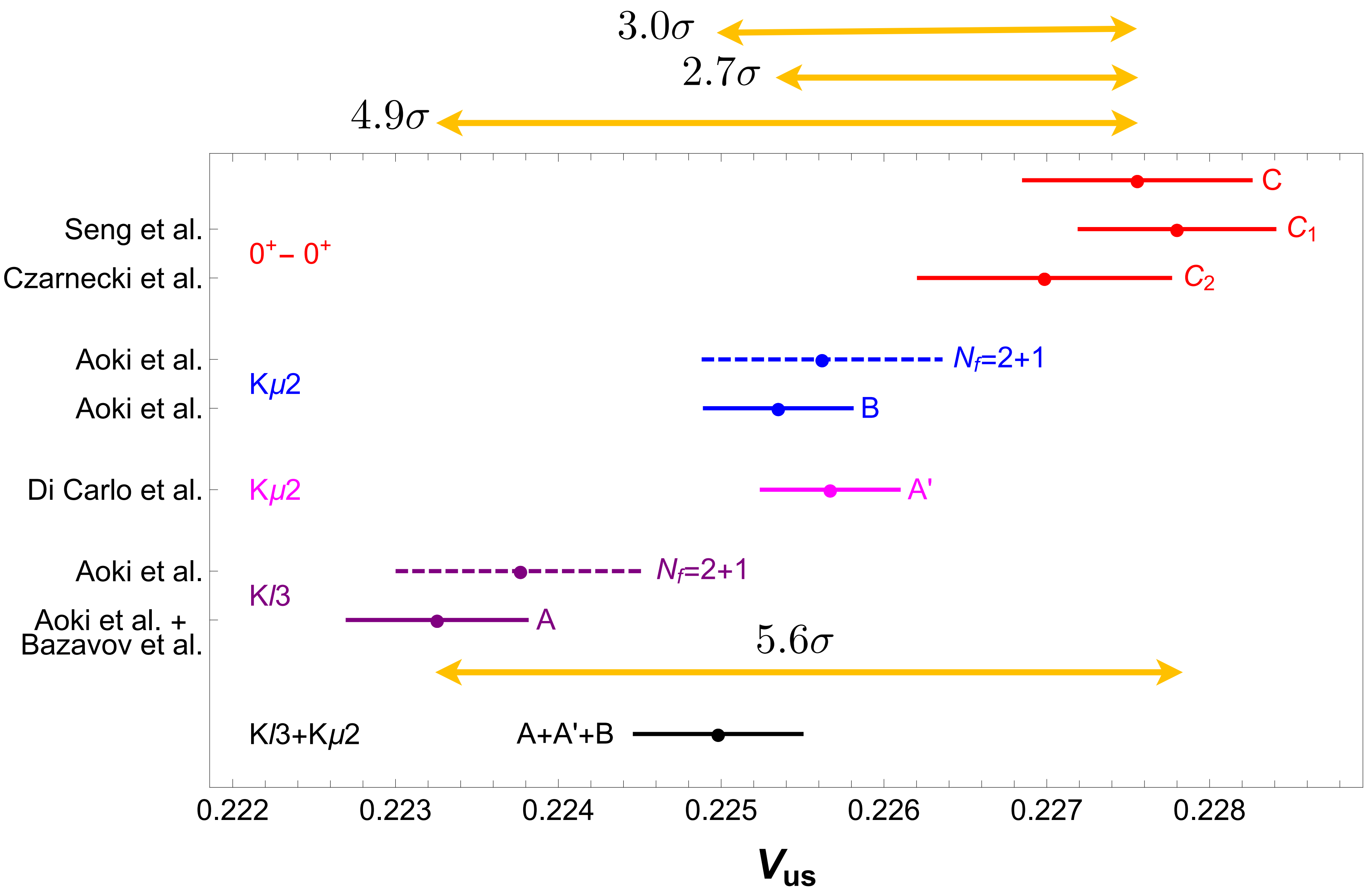


3.1σ away from unitarity

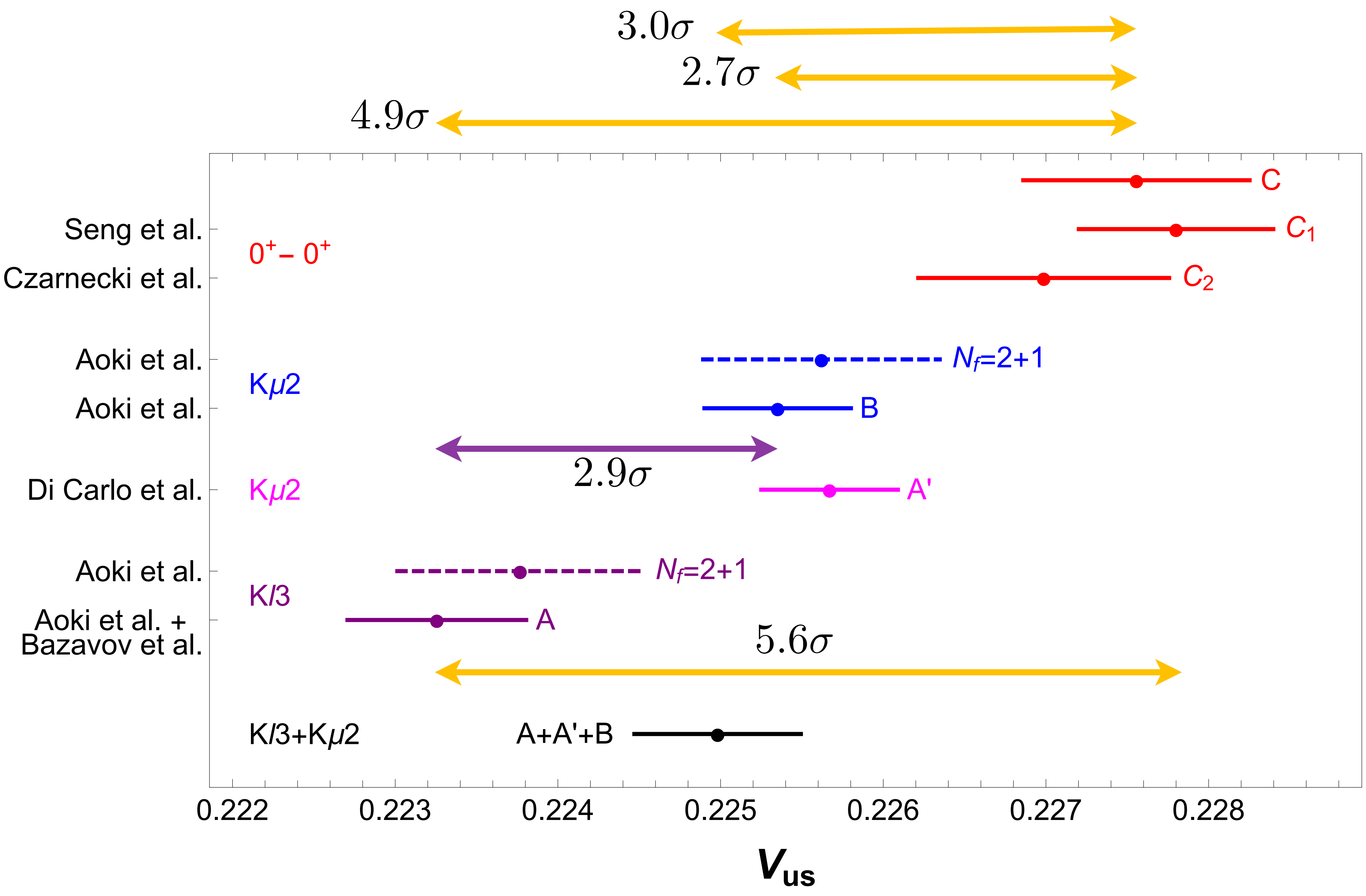
Present situation of CKM unitarity



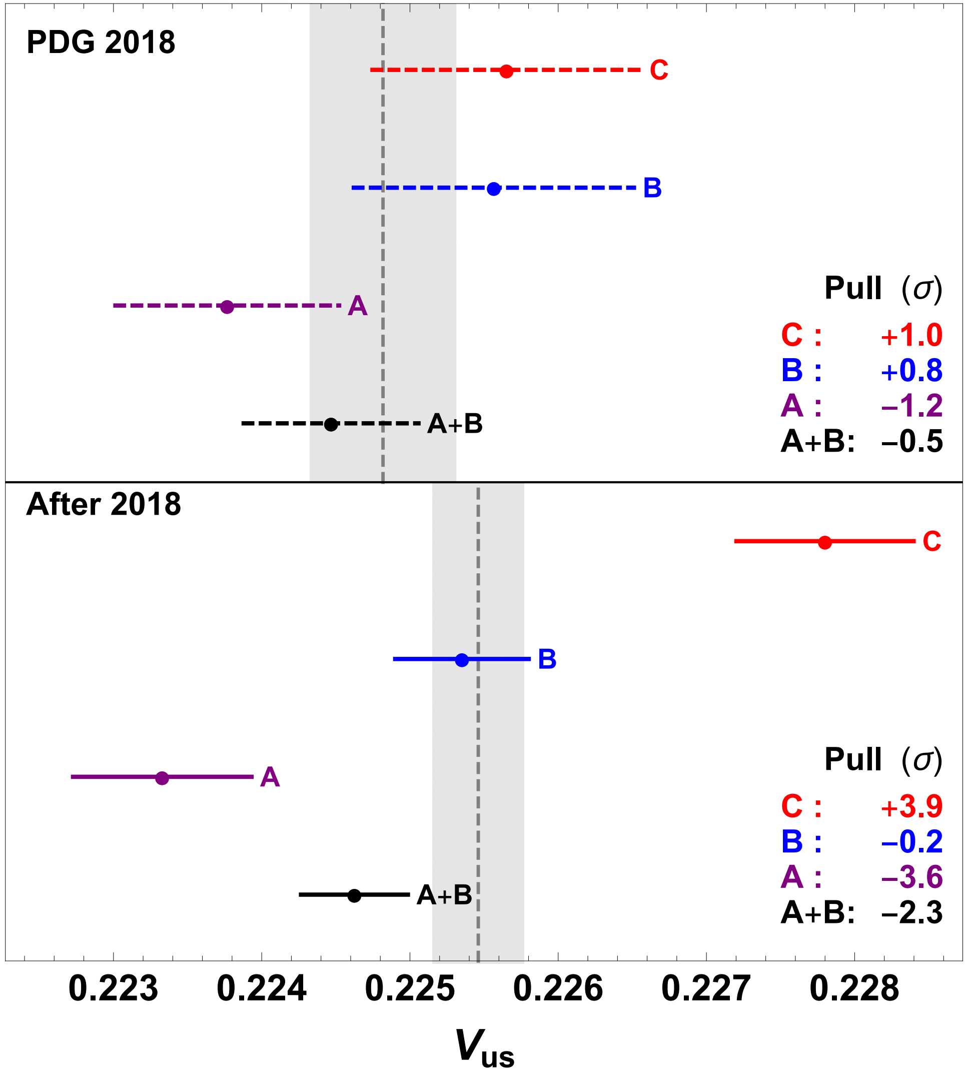
Present situation of CKM unitarity



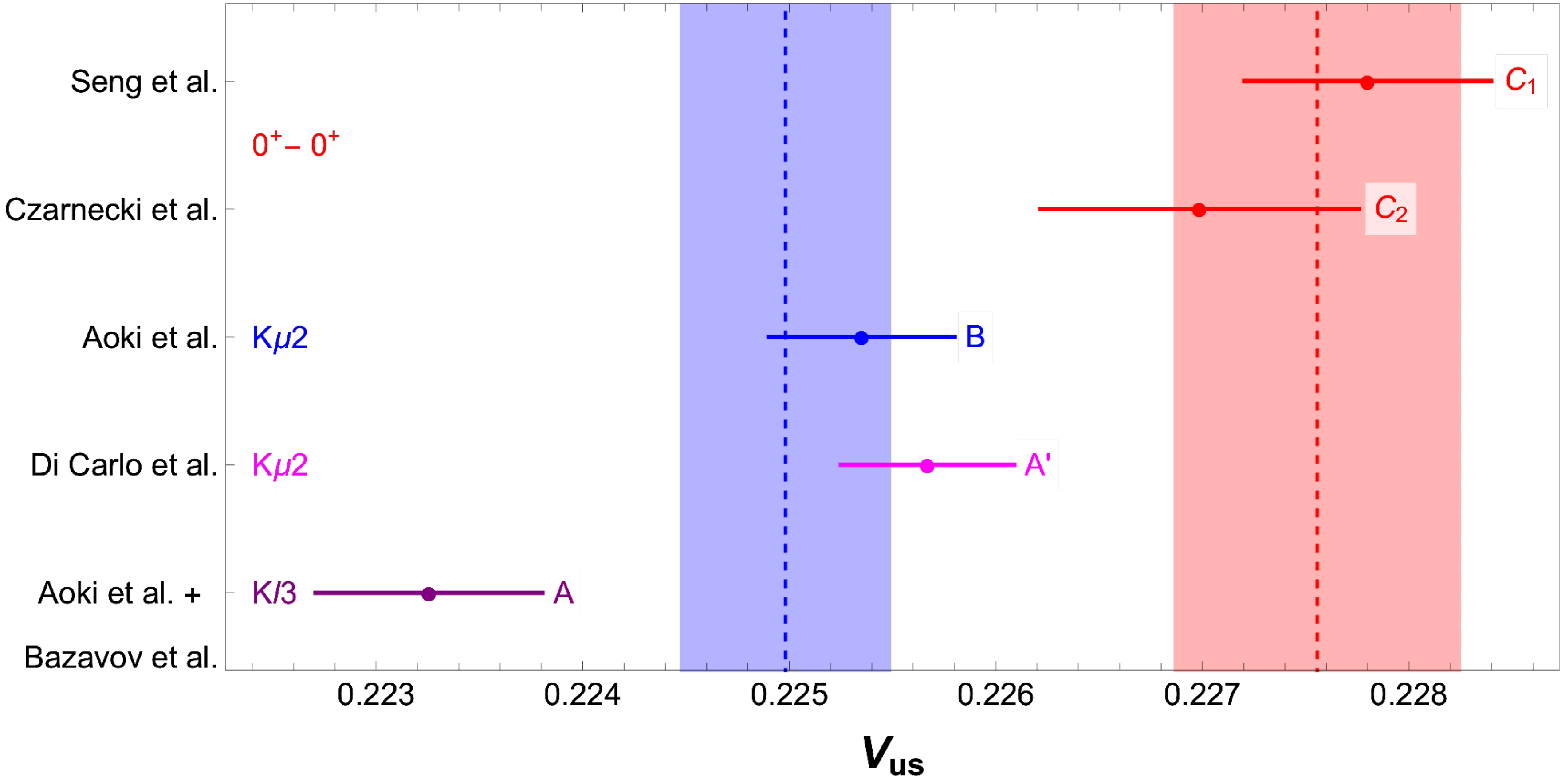
Present situation of CKM unitarity



CKM unitarity problem



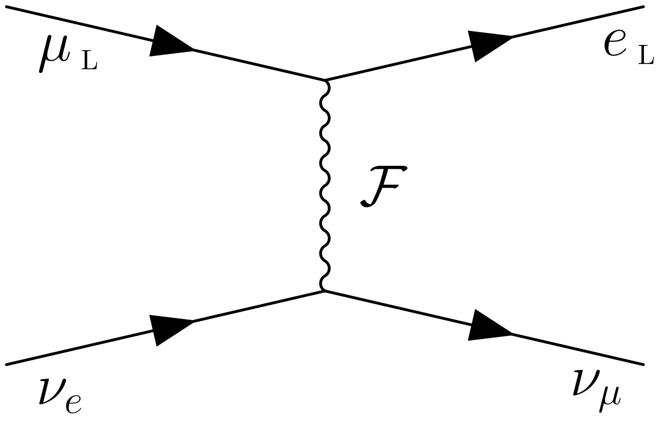
The CKM unitarity problem



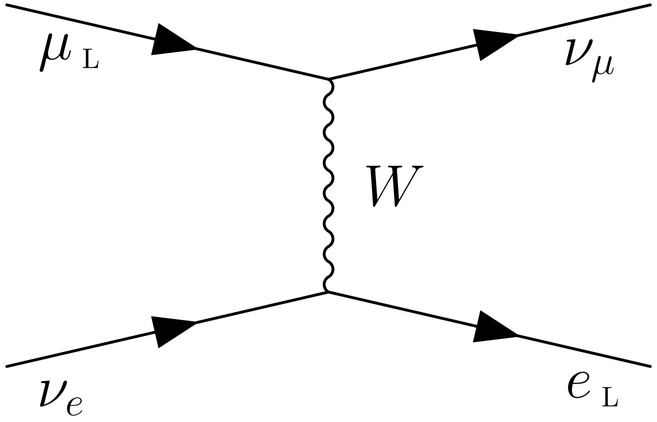
Solution #1

- Suppose the existence of **flavor changing bosons**.

$$-\frac{4G_{\mathcal{F}}}{\sqrt{2}}(\overline{e_L}\gamma^\alpha\mu_L)(\overline{\nu_\mu}\gamma_\alpha\nu_e)$$



$$-\frac{4G_{\mathrm{F}}}{\sqrt{2}}(\overline{\nu_\mu}\gamma^\alpha\mu_L)(\overline{e_L}\gamma_\alpha\nu_e)$$



- Horizontal interactions have positive interference with SM;
- After Fierz transformation, the sum of the diagrams gives the operator:

$$-\frac{4G_\mu}{\sqrt{2}}(\overline{\nu_\mu}\gamma^\alpha\mu_L)(\overline{e_L}\gamma_\alpha\nu_e) \qquad G_\mu = G_F + G_{\mathcal{F}} = G_F(1+\delta_\mu) \qquad \boxed{G_\mu \neq G_F}$$

Flavour bosons for CKM

- Different $\mathbf{G}_{\mu} = \mathbf{G}_{\mathbf{F}} + \mathbf{G}_{\mathcal{F}} = \mathbf{G}_{\mathbf{F}}(\mathbf{1} + \delta_{\mu}) = 1 + \frac{v_{\mathrm{w}}^2}{v_{\mathcal{F}}^2}$
- The values of V_{us}, V_{ud} (and corresponding errorbars) should be rescaled:

$$|V_{\mathrm{us}}| = 0.22333(60) \times (1 + \delta_{\mu})\ , \qquad |V_{\mathrm{ud}}| = 0.97370(14) \times (1 + \delta_{\mu})$$

while the ratio is not affected.

- Unitarity recovered: $\left(\frac{G_F}{G_\mu}\right)^2 (|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2) = 1 - \frac{2G_{\mathcal{F}}}{G_{\mathrm{F}}}$

Flavour bosons for CKM

- Different $\mathbf{G}_\mu = \mathbf{G_F} + \mathbf{G}_\mathcal{F} = \mathbf{G_F}(\mathbf{1} + \delta_\mu) = 1 + \frac{v_\mathrm{w}^2}{v_\mathcal{F}^2}$
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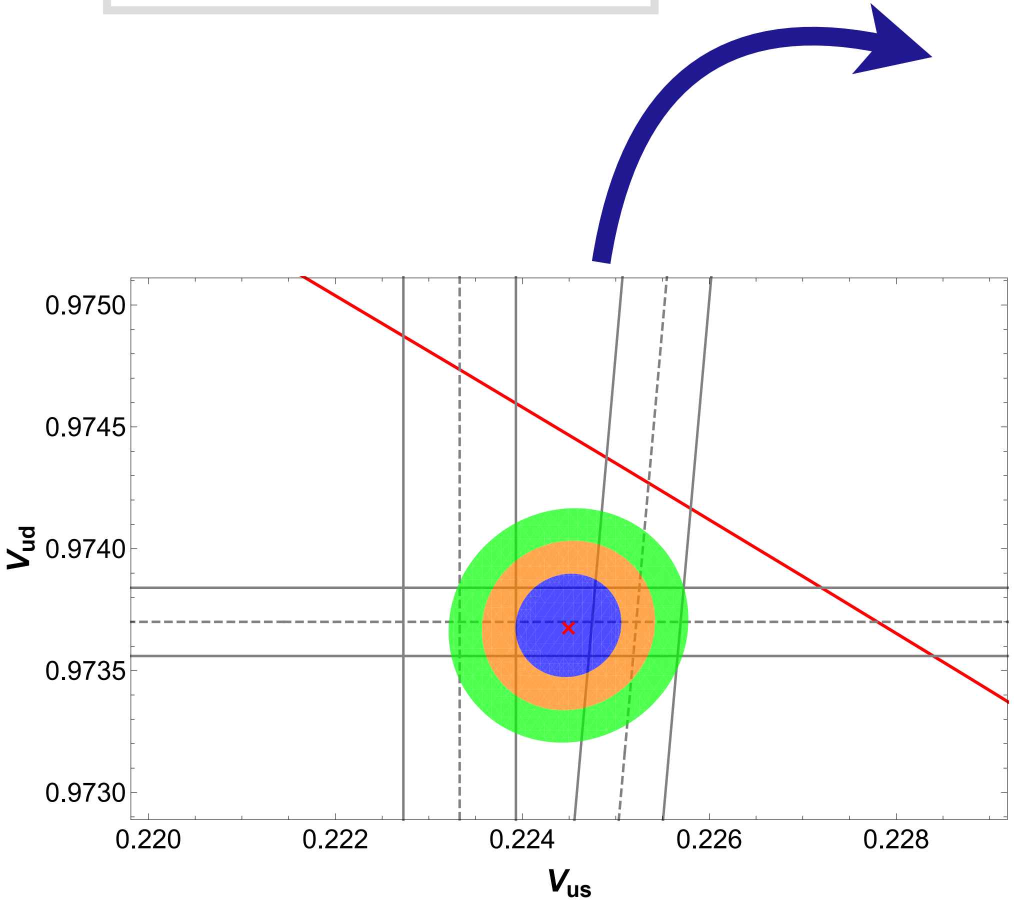
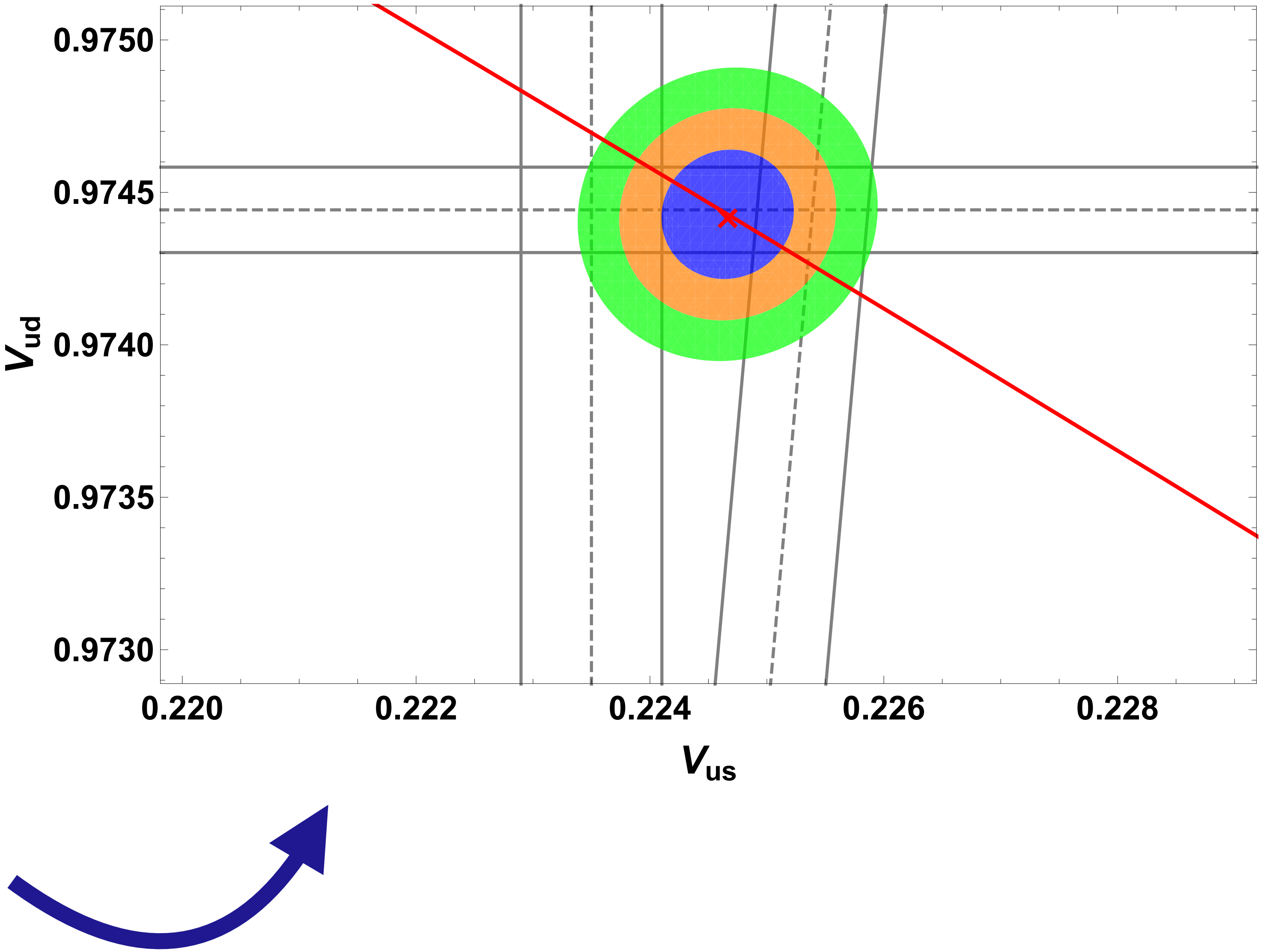
while the ratio is not affected.

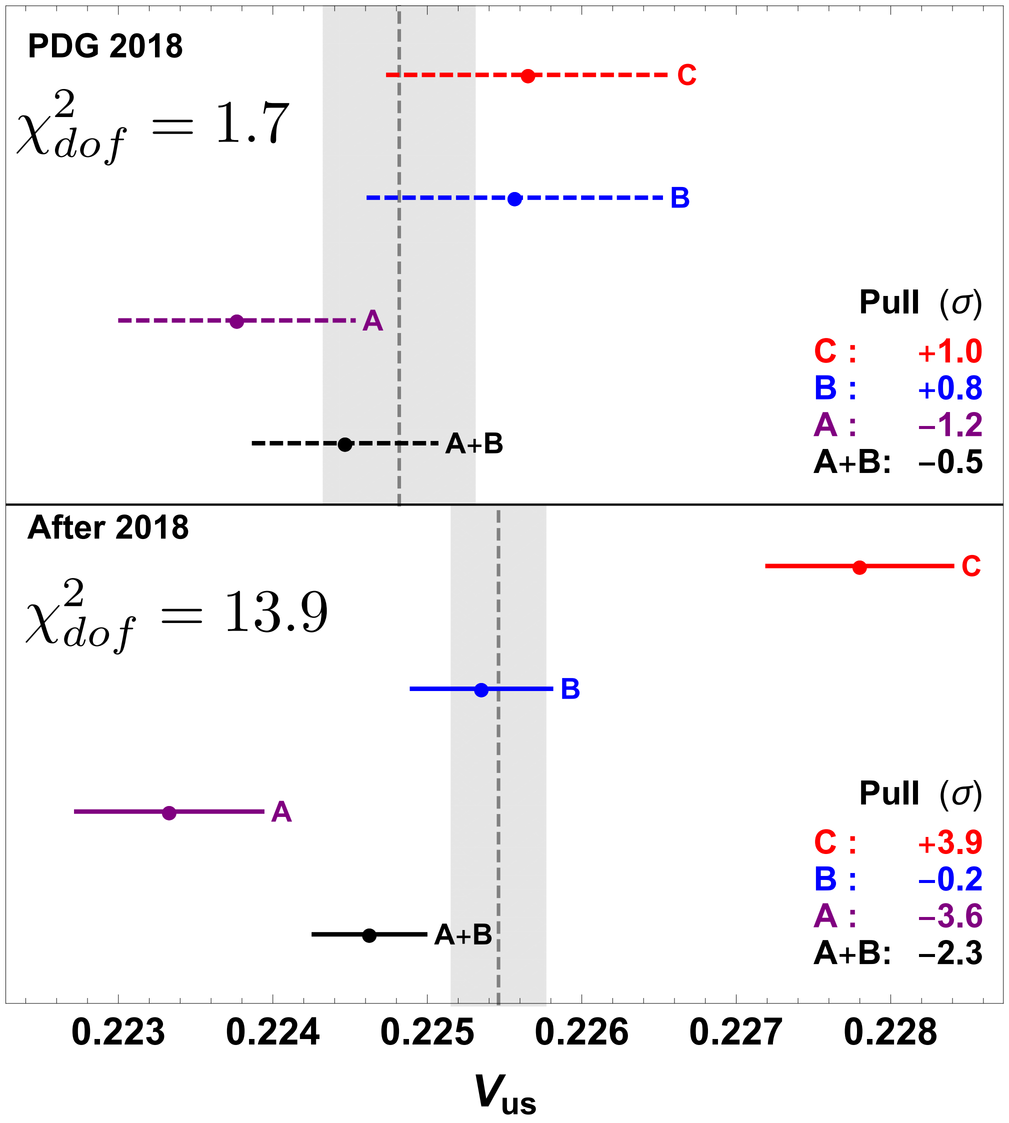
- Unitarity recovered: $\left(\frac{G_F}{G_\mu}\right)^2 (|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2) = 1 - \frac{2G_\mathcal{F}}{G_\mathrm{F}}$
- CKM unitarity is recovered ($\chi^2_{dof} = 3.0$) with $\delta_\mu = \mathbf{7.6 \cdot 10^{-4}}$, or

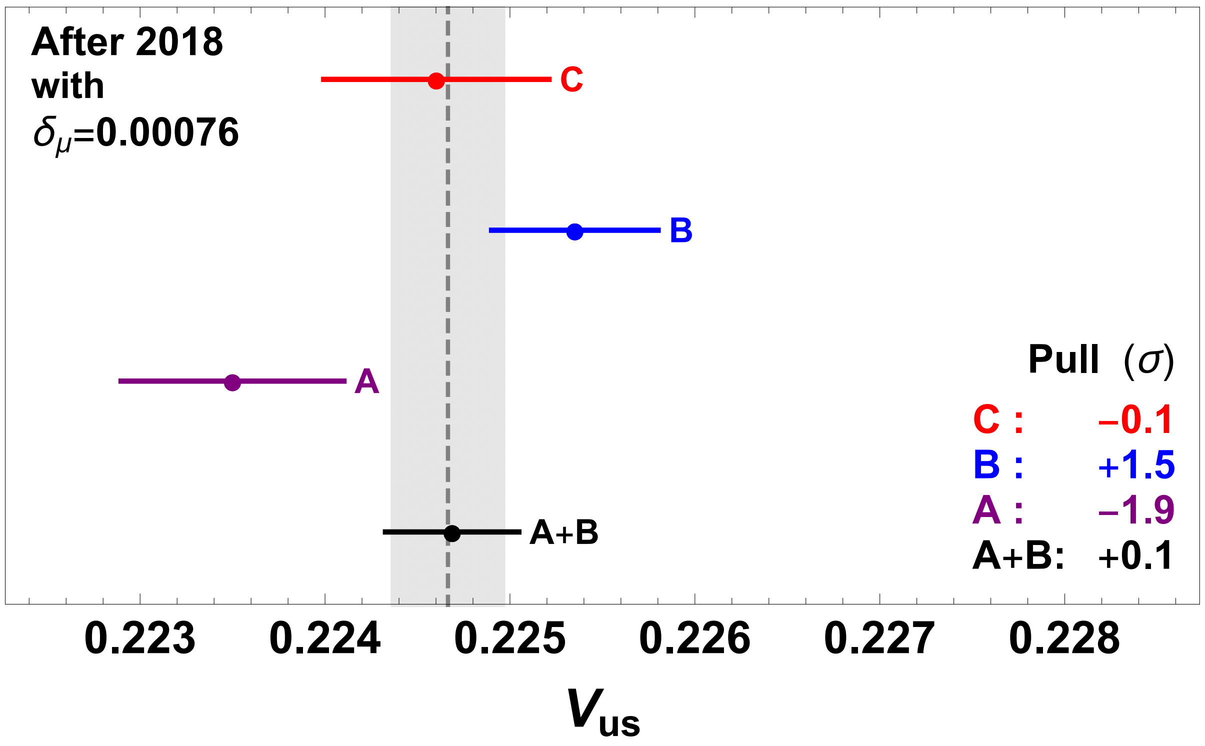
$$v_\mathcal{F} = 6\text{-}7 \text{ TeV}$$

$$\delta_\mu = 7.6 \cdot 10^{-4}$$
$$v_{\mathcal{F}} = 6.3\,\text{TeV}$$






PDG 2018
$\chi^2_{dof} = 1.7$
C
B
A
A+B
Pull (σ)
C : +1.0
B : +0.8
A : −1.2
A+B: −0.5
After 2018
$\chi^2_{dof} = 13.9$
C
B
A
A+B
Pull (σ)
C : +3.9
B : −0.2
A : −3.6
A+B: −2.3
0.223
0.224
0.225
0.226
0.227
0.228
V_{us}


After 2018
with
δ_μ=0.00076
C
B
A
A+B
Pull (σ)
C : −0.1
B : +1.5
A : −1.9
A+B: +0.1
0.223
0.224
0.225
0.226
0.227
0.228
V_{us}

Conclusions #2

- There is tension between independent determinations of the elements in the first row of CKM matrix (V_{us} and V_{ud}).

- A new effective operator in positive interference with the SM muon decay as the one generated by flavour changing gauge bosons can solve CKM unitarity problem. Then the Fermi constant would be different from the muon decay constant, $G_F = G_\mu/(1+\delta_\mu)$.

- CKM unitarity is restored with $\delta_\mu \simeq 7\times 10^{-4}$, corresponding to a breaking scale of the symmetry $SU(3)_\ell$ (and gauge boson mass) of few TeV, which was shown to be possible without contradicting experimental constraints.

3. Extra vector-like quarks

- Are they a solution for anomalies in the first row of CKM?

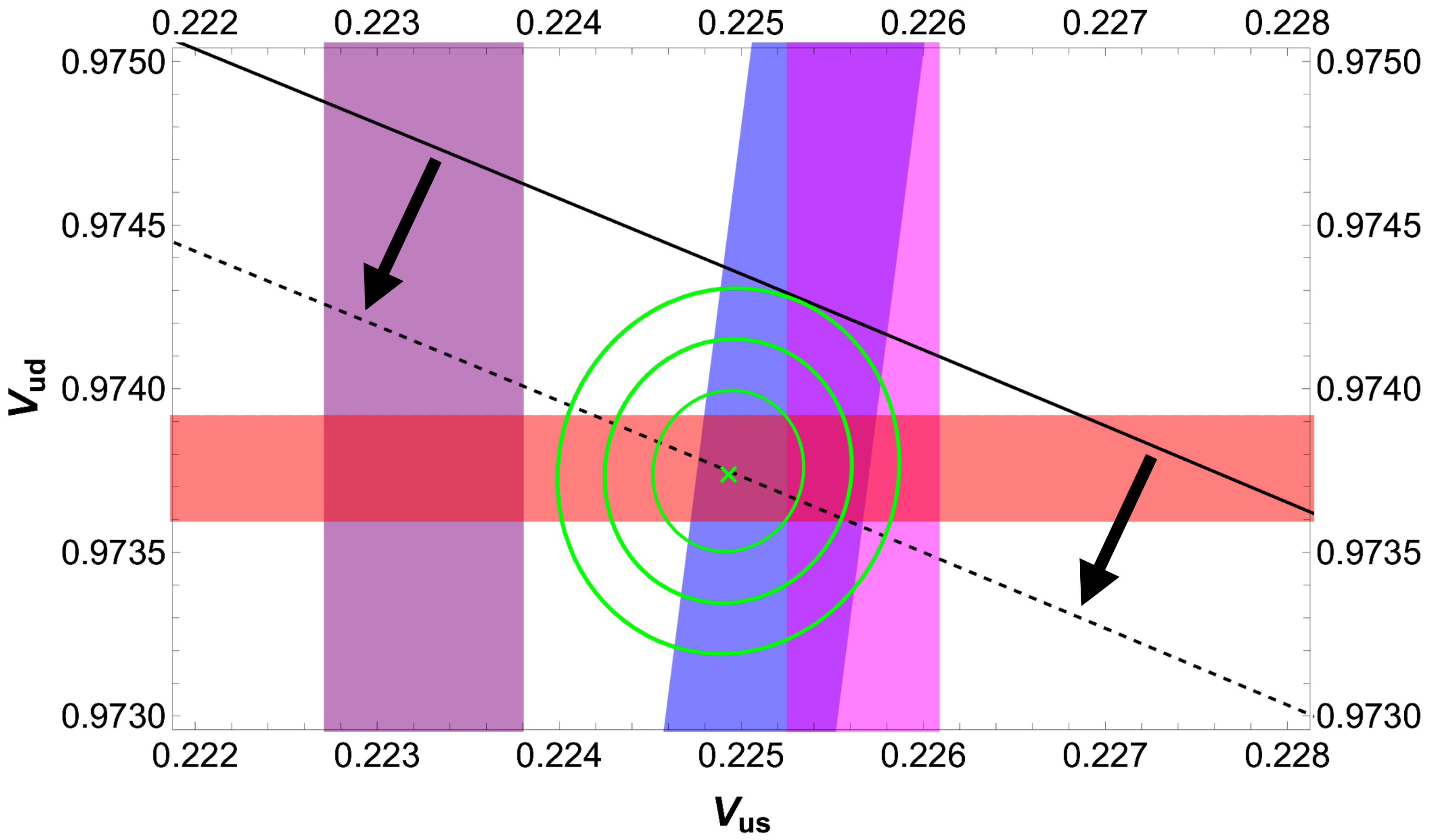
Extra vector-like quarks

- Extra down-type weak isosinglet
- Extra up-type weak isosinglet
- Extra weak isodoublet

Vector-like isosinglets

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - |V_{ub'}|^2, \qquad |V_{ub'}| = 0.035$$

- Extra down-type weak isosinglet
- Extra up-type weak isosinglets



Vector-like isosinglets

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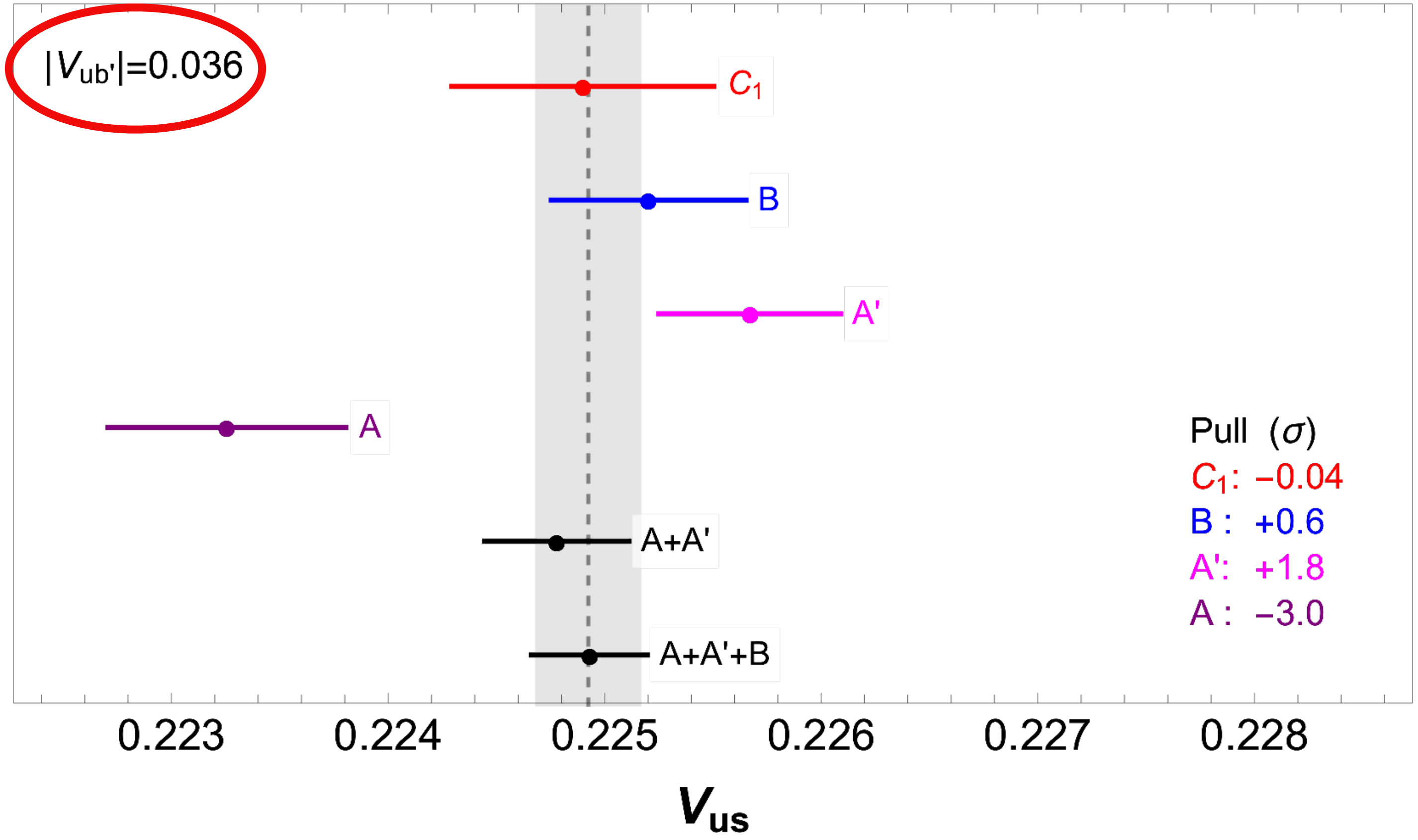


Down-type isosinglet

- Down-type vector-like species of quark $d_{4L,R}$ whose left and right components are both $SU(2)$ singlets involved in quark mixing:

$$... + h_i \phi \overline{q_{Li}} d_{4R} + M \overline{d_{4L}} b_{4R} + h.c.$$

- $\overline{d_{Li}} \mathbf{m}^{(d)}_{ij} d_{Rj} + h.c. == (\overline{q_{1L}}, \overline{q_{2L}}, \overline{q_{3L}}, \overline{d_{4L}}) \left(\begin{array}{ccc|c} & & & h_{d1} v_w \\ & \mathbf{m}^{(d)}_{3\times 3} & & h_{d2} v_w \\ & & & h_{d3} v_w \\ \hline 0 & 0 & 0 & M_4 \end{array} \right) \begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{pmatrix}$

- $V_L^{(d)\dagger} \mathbf{m}^{(d)} V_R^{(d)} = \mathbf{m}^{(d)}_{\text{diag}} = \text{diag}(m_d, m_s, m_b, M_{b'})$

$$\begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{pmatrix}_L = V_L^{(d)} \begin{pmatrix} d \\ s \\ b \\ b' \end{pmatrix}_L = \begin{pmatrix} V_{1d} & V_{1s} & V_{1b} & V_{1b'} \\ V_{2d} & V_{2s} & V_{2b} & V_{2b'} \\ V_{3d} & V_{3s} & V_{3b} & V_{3b'} \\ V_{4d} & V_{4s} & V_{4b} & V_{4b'} \end{pmatrix}_L \begin{pmatrix} d \\ s \\ b \\ b' \end{pmatrix}_L$$

Down-type isosinglet

- Charged weak currents:

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} \begin{pmatrix} \overline{u_L} & \overline{c_L} & \overline{t_L} \end{pmatrix} \gamma^\mu \tilde{V}_{\text{CKM}} \begin{pmatrix} d \\ s \\ b \\ b' \end{pmatrix}_L W_\mu^+ + \text{h.c.},$$

$$\tilde{V}_{CKM} = V_L^{(u)\dagger} \tilde{V}_L^{(d)} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} & V_{ub'} \\ V_{cd} & V_{cs} & V_{cb} & V_{cb'} \\ V_{td} & V_{ts} & V_{tb} & V_{tb'} \end{pmatrix}$$

$$\begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{pmatrix}_L = \begin{pmatrix} V_{1d} & V_{1s} & V_{1b} & V_{1b'} \\ V_{2d} & V_{2s} & V_{2b} & V_{2b'} \\ V_{3d} & V_{3s} & V_{3b} & V_{3b'} \\ V_{4d} & V_{4s} & V_{4b} & V_{4b'} \end{pmatrix}_L \begin{pmatrix} d \\ s \\ b \\ b' \end{pmatrix}_L$$

- $V_L^{(u)\dagger}$ is a unitary 3×3 matrix.
- $\tilde{V}_L^{(d)}$ and $\tilde{V}_{CKM}$ are 3×4 matrices.
- $\tilde{V}_L^{(d)}$ and $\tilde{V}_{\text{CKM}}$ are not unitary.
- $|V_{ub'}| \approx |V_{L4d}| \approx 0.035$
- Since $|V_{ub'}| \approx h_{d1} v_w / M_{b'}$, assuming $|V_{ub'}| > 0.03$ and $h_{d1} < 1$, then $M_{b'} < 6$ TeV.

Down-type isosinglet

- Weak neutral currents:

$$\mathcal{L}_{\rm nc} = \frac{g}{\cos\theta_W}\left[-\frac{1}{2}\begin{pmatrix}\overline{d_L} & \overline{s_L} & \overline{b_L} & \overline{b'_L}\end{pmatrix}\gamma^\mu \tilde{V}_L^{(d)\dagger}\tilde{V}_L^{(d)}\begin{pmatrix}d\\ s\\ b\\ b'\end{pmatrix} + \frac{1}{3}\sin^2\theta_W\left(\overline{\mathbf{d}_L}\gamma^\mu \mathbf{d}_L + \overline{\mathbf{d}_R}\gamma^\mu \mathbf{d}_R\right)\right] Z_\mu$$

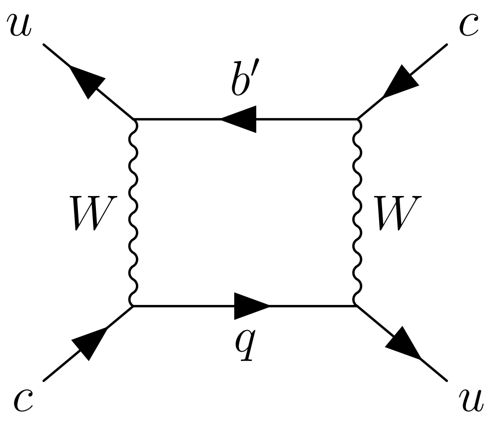
- The non-unitarity of $\tilde{V}_L^{(d)}$ is at the origin of **tree level flavour changing** couplings with Z boson (and Higgs boson), determined by the matrix:

$$\tilde{V}_L^{(d)\dagger}\tilde{V}_L^{(d)} = \begin{pmatrix} 1-|V_{4d}|^2 & -V_{4d}^*V_{4s} & -V_{4d}^*V_{4b} & -V_{4d}^*V_{4b'} \\ -V_{4s}^*V_{4d} & 1-|V_{4s}|^2 & -V_{4s}^*V_{4b} & -V_{4s}^*V_{4b'} \\ -V_{4b}^*V_{4d} & -V_{4b}^*V_{4s} & 1-|V_{4b}|^2 & -V_{4b}^*V_{4b'} \\ -V_{4b'}^*V_{4d} & -V_{4b'}^*V_{4s} & -V_{4b'}^*V_{4b} & |V_{1b'}|^2+|V_{2b'}|^2+|V_{3b'}|^2 \end{pmatrix}_L$$

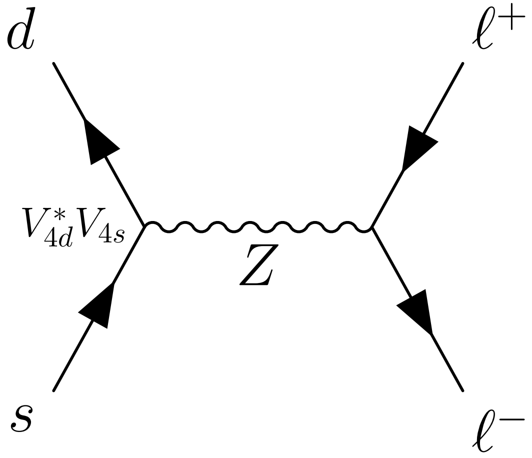
- If the 2nd and 3rd families are not mixed with the fourth, $V_{L4s} = V_{L4b} = 0$, there are not FCNC at tree level between the first three families.

Down-type isosinglet

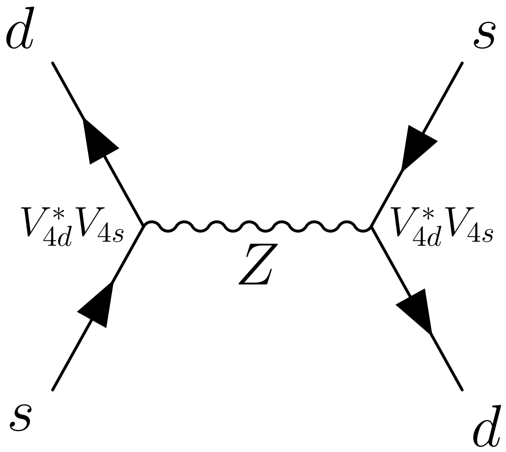
Neutral D mesons mixing:

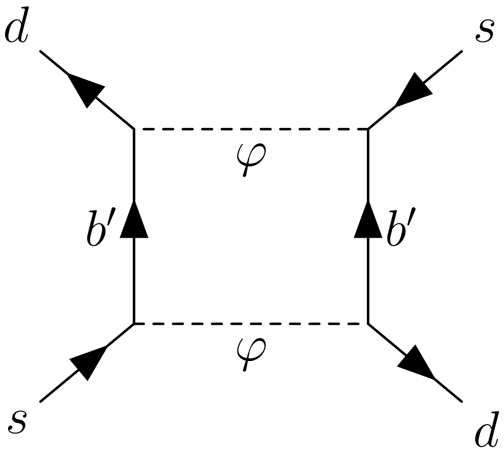


Kaon decays:



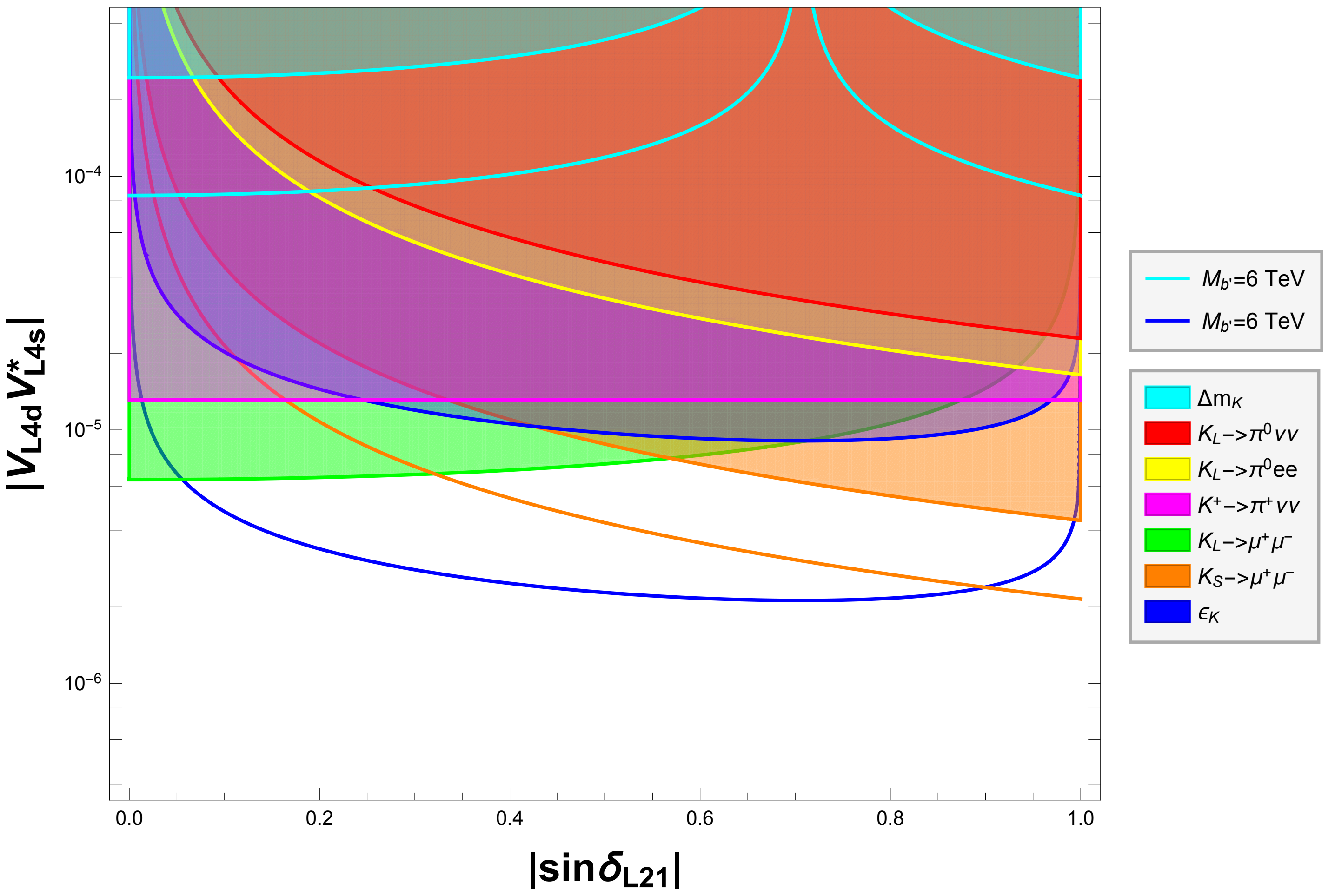
Neutral K mesons mixing:





Down-type isosinglet

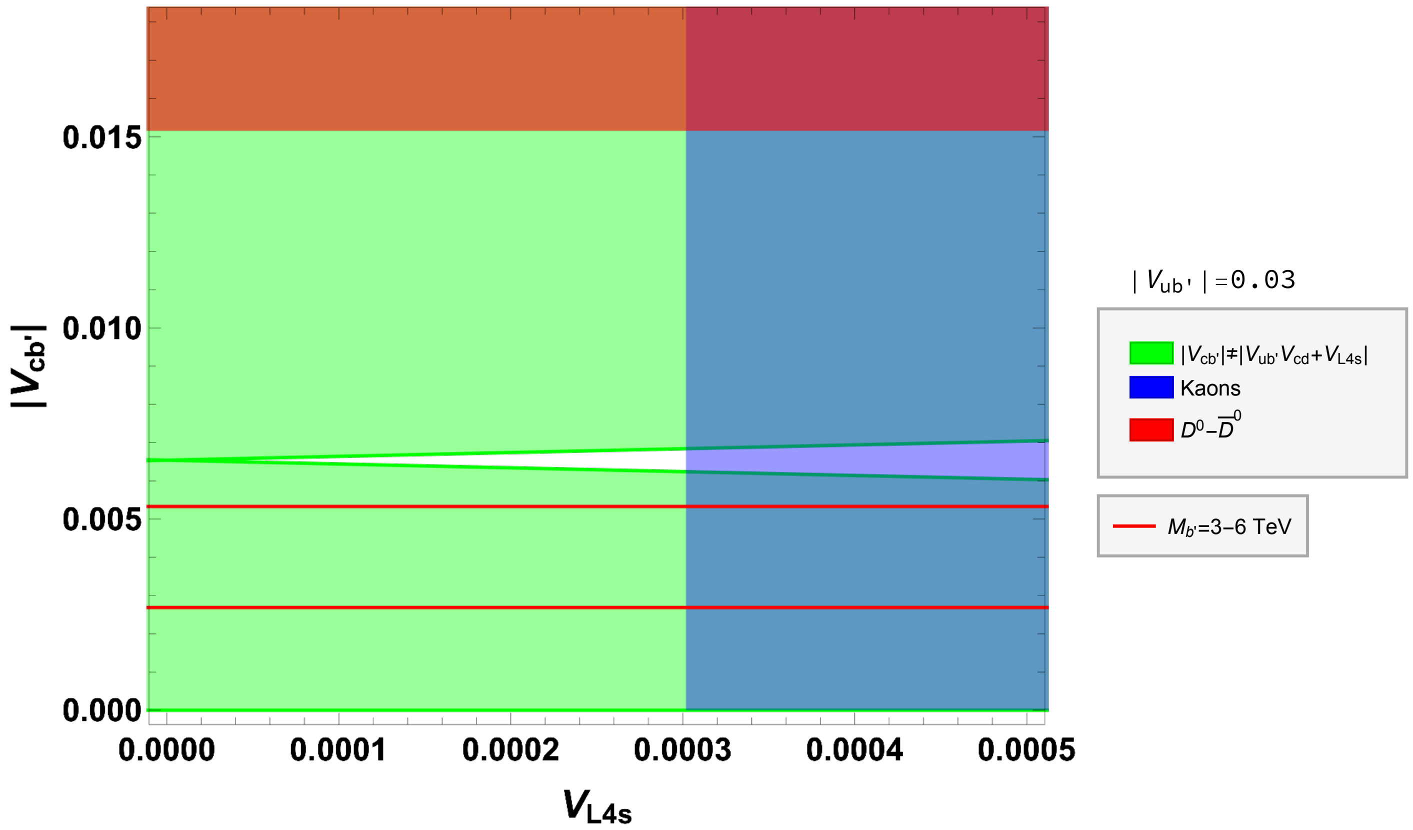
- Experimental constraints from kaon decays and neutral kaon mixing:



Down-type isosinglet

- There is little room left for $|V_{cb'}|$.
- Also $|V_{tb'}| < 4.2 \cdot 10^{-3}$ from B decays.
- The mixing of the 4th state with the second and third family should be much less than the mixing with the first family, which seems unnatural, although not excluded.

- Excluded area for parameters:



Up-type isosinglet

- Vector-like up-type quark $u_{4L,R}$ whose left and right components are both $SU(2)$ singlets involved in quark mixing:

$$... + h_i^u \tilde{\phi} \overline{q_{Li}} u_{4R} + M_{t'} \overline{u_{4L}} u_{4R} + h.c.$$

- $\tilde{V}_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \\ V_{t'd} & V_{t's} & V_{t'b} \end{pmatrix} = \tilde{V}_L^{(u)\dagger} V_L^{(d)} \;;$

- $\tilde{V}_L^{(u)}$ is the 3×4 submatrix of $V_L^{(u)}$ without the last row, $\tilde{V}_{CKM}$ is 4×3 matrix.

- $\tilde{V}_L^{(u)}$ and $\tilde{V}_{\mathrm{CKM}}$ are not unitary.

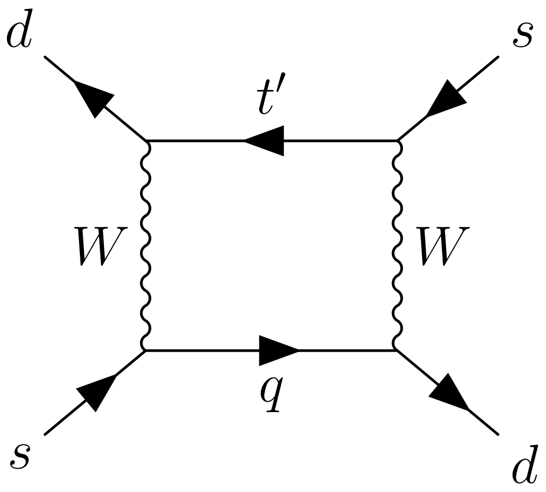
- $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - |V_{L4u}|^2 \,, \qquad |V_{L4u}| \approx 0.035$

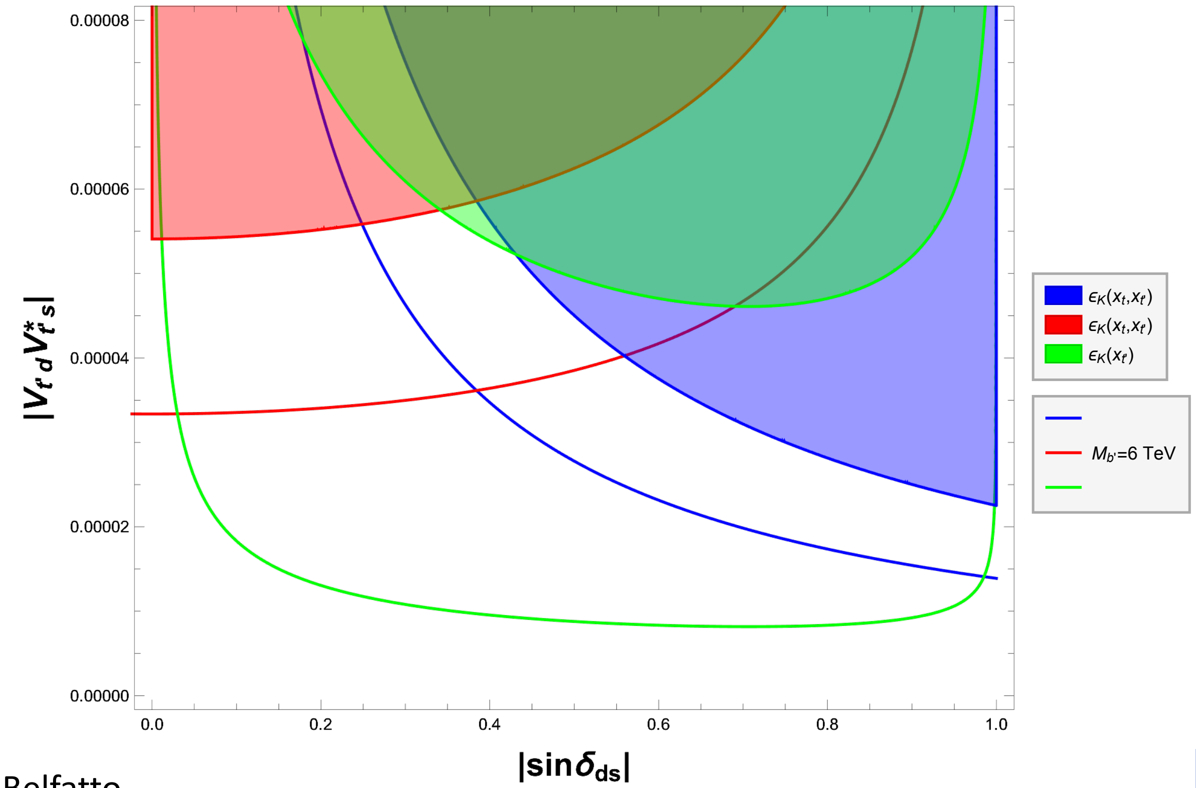
Up-type isosinglet

- The non-unitarity of $\tilde{V}_L^{(u)}$ gives flavour changing couplings of quarks with Z boson (and Higgs). The weak neutral currents Lagrangian is:

$$\mathcal{L}_{\rm nc} = \frac{g}{\cos\theta_W}\left[\frac{1}{2}\begin{pmatrix}\overline{u_L} & \overline{c_L} & \overline{t_L} & \overline{t'_L}\end{pmatrix}\gamma^\mu \tilde{V}_L^{(u)\dagger}\tilde{V}_L^{(u)}\begin{pmatrix}u\\ c\\ t\\ t'\end{pmatrix}_L - \frac{2}{3}\sin^2\theta_W\left(\overline{\mathbf{u}_L}\gamma^\mu\mathbf{u}_L + \overline{\mathbf{u}_R}\gamma^\mu\mathbf{u}_R\right)\right] Z_\mu$$

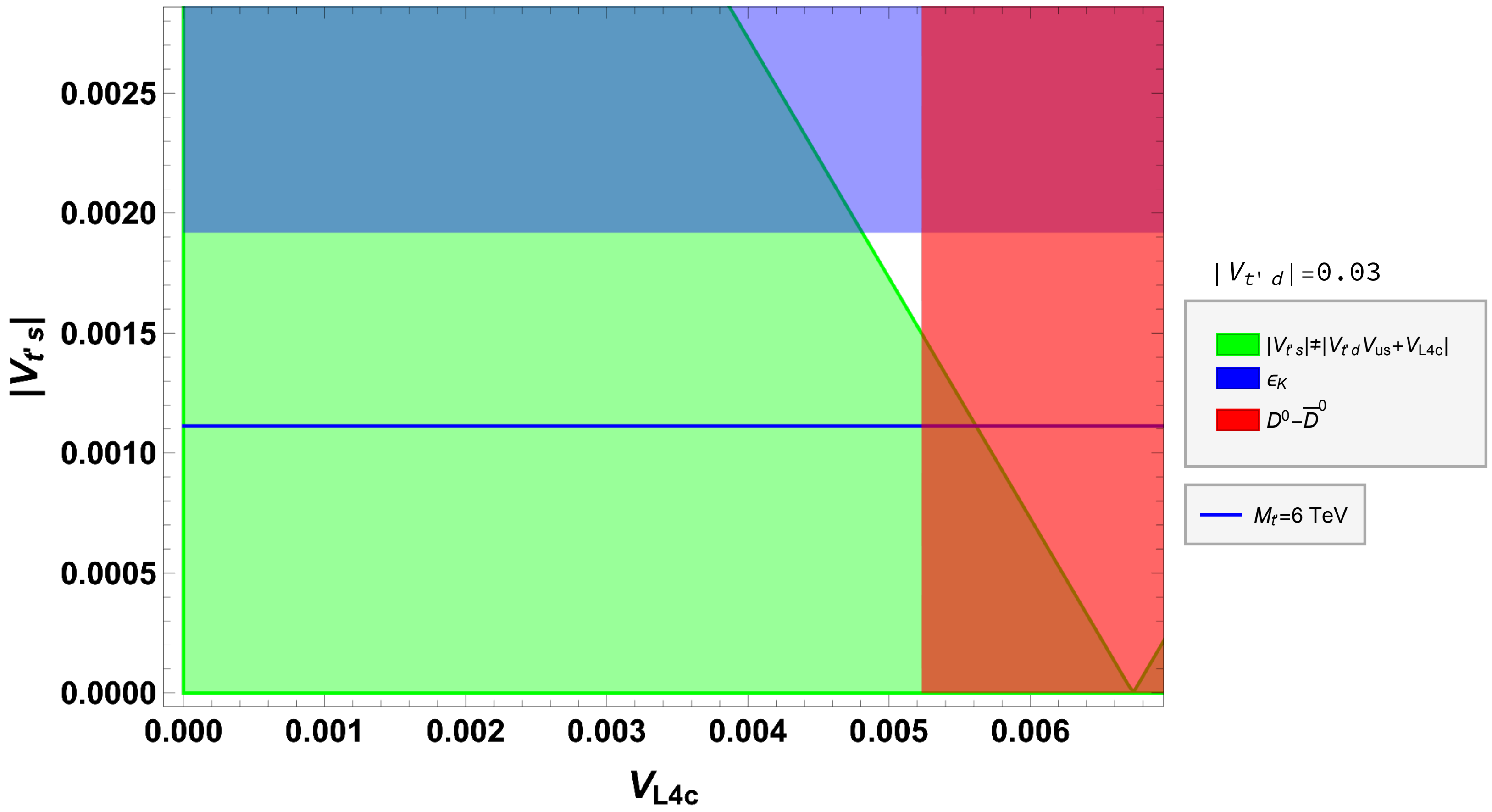
Constraints from K mesons mixing:





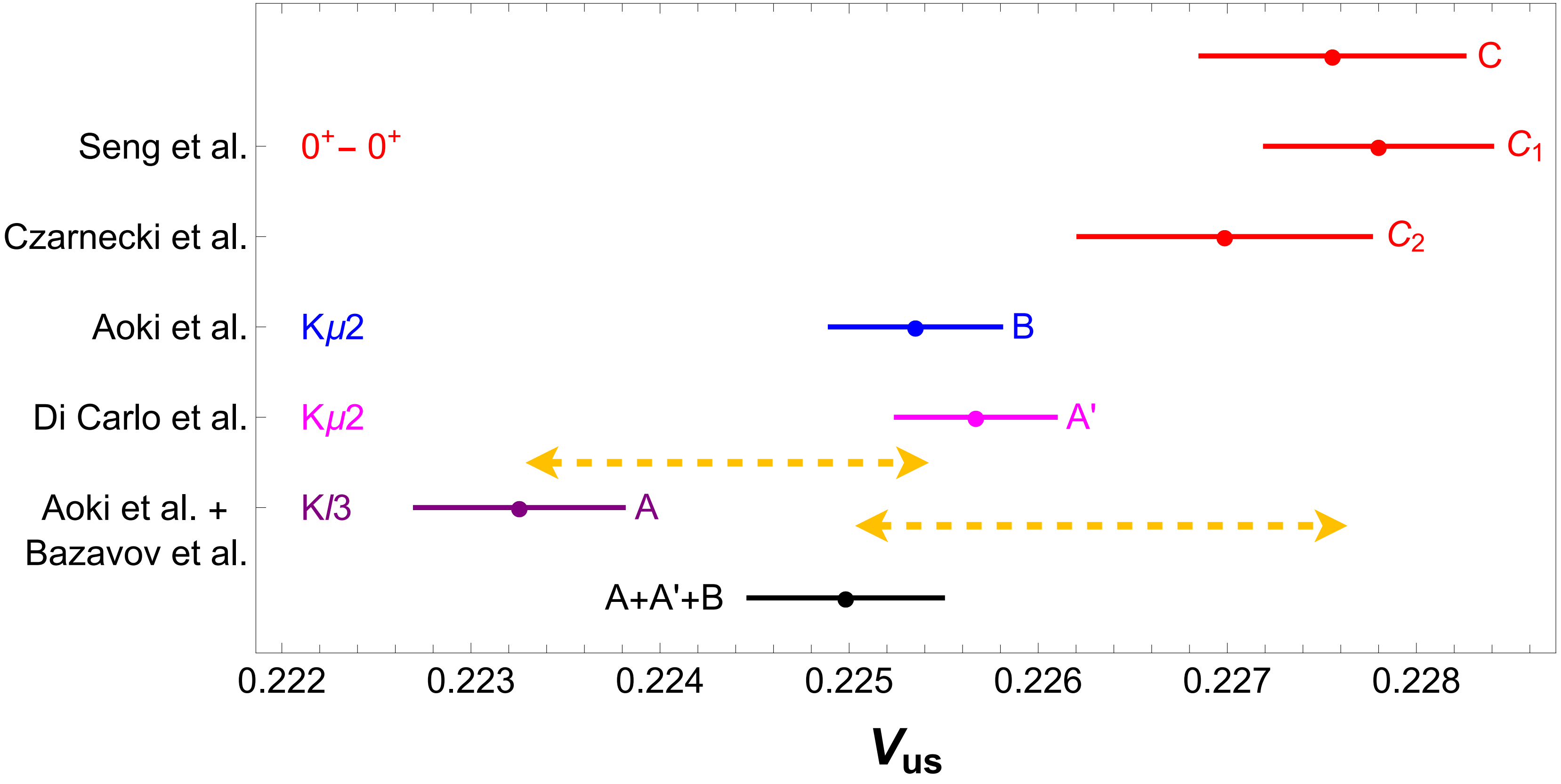
Up-type isosinglet

- Excluded area for parameters:



Vectorlike weak isodoublet

- Extra weak isodoublet: trying to cure both the *gaps*…



Vectorlike weak isodoublet

- Vectorlike extra $SU(2)$-doublet:

$$q_{4\,L,R} = \begin{pmatrix} u_4 \\ d_4 \end{pmatrix}_{L,R}$$

$$\overline{d_{Li}}\mathbf{m}^{(d)}_{ij} d_{Rj} + h.c. = \begin{pmatrix} \overline{q_{1L}} & \overline{q_{2L}} & \overline{q_{3L}} & \overline{q_{4L}} \end{pmatrix} \begin{pmatrix} & \mathbf{y}^{(d)}_{3\times 3} v_w & & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ y^d_{41} v_w & y^d_{42} v_w & y^d_{43} v_w & M_4 \end{pmatrix} \begin{pmatrix} d_{R1} \\ d_{R2} \\ d_{R3} \\ q_{R4} \end{pmatrix}$$

$$V_L^{(d)\dagger}\mathbf{m}^{(d)} V_R^{(d)} = \mathbf{m}^{(d)}_{\text{diag}} = \text{diag}(m_d\,, m_s\,, m_b\,, M_q)$$

$$V_L^{(u)\dagger}\mathbf{m}^{(u)} V_R^{(u)} = \mathbf{m}^{(u)}_{\text{diag}} = \text{diag}(m_u\,, m_c\,, m_t\,, M_q)$$

- $V^{(d,u)}_{L,R}$ are unitary 4×4 matrices:

$$V_L^{(d)} = \begin{pmatrix} V_{L\,1d} & V_{L\,1s} & V_{L\,1b} & V_{L\,1b'} \\ V_{L\,2d} & V_{L\,2s} & V_{L\,2b} & V_{L\,2b'} \\ V_{L\,3d} & V_{L\,3s} & V_{L\,3b} & V_{L\,3b'} \\ V_{L\,4d} & V_{L\,4s} & V_{L\,4b} & V_{L\,4b'} \end{pmatrix}\,, \quad V_L^{(u)} = \begin{pmatrix} V_{L\,1u} & V_{L\,1c} & V_{L\,1t} & V_{L\,1t'} \\ V_{L\,2u} & V_{L\,2c} & V_{L\,2t} & V_{L\,2t'} \\ V_{L\,3u} & V_{L\,3c} & V_{L\,3t} & V_{L\,3t'} \\ V_{L\,4u} & V_{L\,4c} & V_{L\,4t} & V_{L\,4t'} \end{pmatrix}$$

Vectorlike weak isodoublet

- The charged-current lagrangian is changed in:

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} \sum_{i=1}^{4} (\overline{u_{Li}} \gamma^\mu d_{Li}) W_\mu^+ + \frac{g}{\sqrt{2}} \overline{u_{4R}} \gamma^\mu d_{4R} W_\mu =$$

$$= \frac{g}{\sqrt{2}} \begin{pmatrix} \overline{u_L} & \overline{c_L} & \overline{t_L} & \overline{t'_L} \end{pmatrix} \gamma^\mu V_{\mathrm{CKM,L}} \begin{pmatrix} d_L \\ s_L \\ b_L \\ b'_L \end{pmatrix} W_\mu^+ + \frac{g}{\sqrt{2}} \begin{pmatrix} \overline{u_R} & \overline{c_R} & \overline{t_R} & \overline{t'_R} \end{pmatrix} \gamma^\mu V_{\mathrm{CKM,R}} \begin{pmatrix} d_R \\ s_R \\ b_R \\ b'_R \end{pmatrix} W_\mu^+$$

- $V_{\mathrm{CKM,L}} = V_L^{(u)\dagger} V_L^{(d)}$ is a 4×4 unitary matrix.

- Weak charged currents involve also right currents and the mixing matrix $V_{\mathrm{CKM,R}}$ is not unitary:

$$V_{\mathrm{CKM,R}} = V_R^{(u)\dagger} \mathrm{diag}(0,0,0,1) V_R^{(d)} = \begin{pmatrix} V^*_{R\,4u} V_{R\,4d} & V^*_{R\,4u} V_{R\,4s} & V^*_{R\,4u} V_{R\,4b} & V^*_{R\,4u} V_{R\,4b'} \\ V^*_{R\,4c} V_{R\,4d} & V^*_{R\,4c} V_{R\,4s} & V^*_{R\,4c} V_{R\,4b} & V^*_{R\,4c} V_{R\,4b'} \\ V^*_{R\,4t} V_{R\,4d} & V^*_{R\,4t} V_{R\,4s} & V^*_{R\,4t} V_{R\,4b} & V^*_{R\,4t} V_{R\,4b'} \\ V^*_{R\,4t'} V_{R\,4d} & V^*_{R\,4t'} V_{R\,4s} & V^*_{R\,4t'} V_{R\,4b} & V^*_{R\,4t'} V_{R\,4b'} \end{pmatrix}$$

Vectorlike weak isodoublet

- Couplings of u-quark with down quarks (which should be the determination of the first row of SM CKM matrix) become:

$$\frac{g}{\sqrt{2}}(\overline{u_L}\gamma^\mu V_{L\,ud} d_L + \overline{u_R} V_{R\,ud}\gamma^\mu d_R)\, W_\mu + \frac{g}{\sqrt{2}}(\overline{u_L}\gamma^\mu V_{L\,us} s_L + \overline{u_R} V_{R\,us}\gamma^\mu s_R)\, W_\mu + \text{h.c.}$$

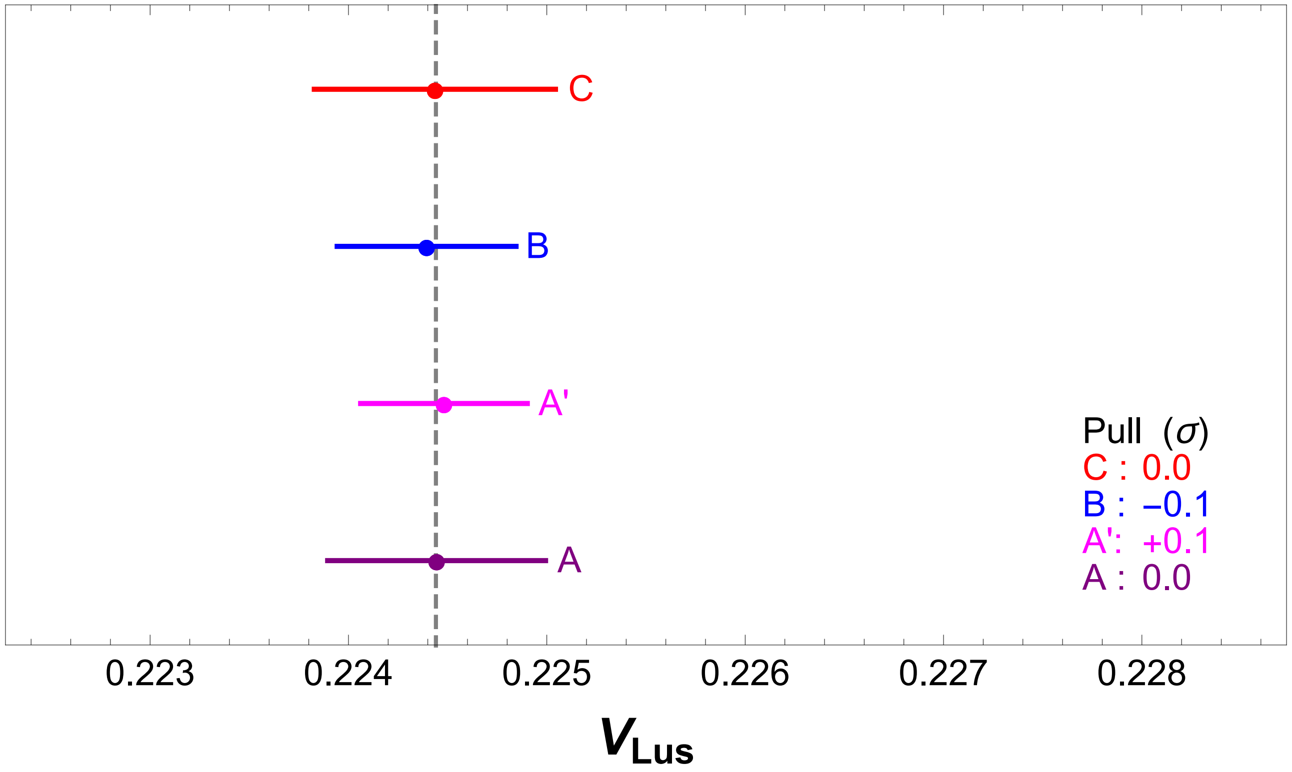
- Then, in this scenario, we are determining vector and axial coupling:

$$\begin{aligned}
\text{semileptonic K decay} \quad & A: \quad |V_{L\,us} + V_{R\,us}| = 0.22326(55) \\
\text{leptonic K decay} \quad & A': \quad |V_{L\,us} - V_{R\,us}| = 0.22567(42) \\
\text{leptonic K decay} \quad & B: \quad \frac{|V_{L\,us} - V_{R\,us}|}{|V_{L\,ud} - V_{R\,ud}|} = 0.23130(50) \\
\text{superallowed beta decays} \quad & C_1: \quad |\hat{V}_{ud}| = |V_{L\,ud} + V_{R\,ud}| = 0.97370(14)
\end{aligned}$$

Vectorlike weak isodoublet

- With:

$$V_{L\,us} = 0.22444 \qquad V_{L\,ud} = 0.97370$$
$$V_{R\,us} = -0.0012 \qquad V_{R\,ud} = -0.0008$$



Vectorlike weak isodoublet

- However also in this scenario flavour changing neutral currents appear at tree level.

$$\mathcal{L}_{\text{fcnc}} = \frac{1}{2}\frac{g}{\cos\theta_W} Z^\mu \begin{pmatrix} \overline{u_R} & \overline{c_R} & \overline{t_R} & \overline{t'_R} \end{pmatrix} \gamma^\mu V_R^{(u)\dagger} \text{diag}(0,0,0,1) V_R^{(u)} \begin{pmatrix} u_R \\ c_R \\ t_R \\ t'_R \end{pmatrix} +$$

$$- \frac{1}{2}\frac{g}{\cos\theta_W} Z^\mu \begin{pmatrix} \overline{d_R} & \overline{s_R} & \overline{b_R} & \overline{b'_R} \end{pmatrix} \gamma^\mu V_R^{(d)\dagger} \text{diag}(0,0,0,1) V_R^{(d)} \begin{pmatrix} d_R \\ s_R \\ b_R \\ b'_R \end{pmatrix}$$

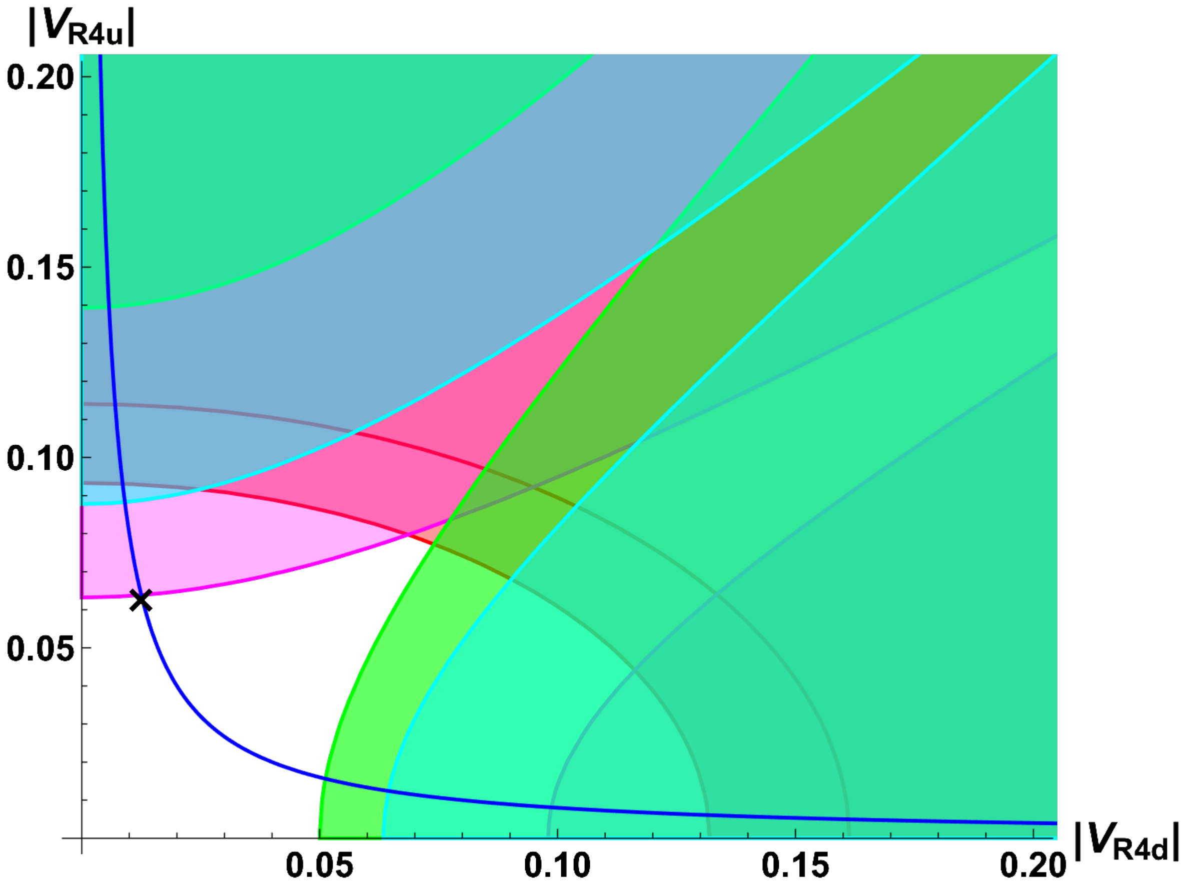
where

$$V_R^{(u)\dagger} \text{diag}(0,0,0,1) V_R^{(u)} = \begin{pmatrix} |V_{R\,4u}|^2 & V^*_{R\,4u} V_{R\,4c} & V^*_{R\,4u} V_{R\,4t} & V^*_{R\,4u} V_{R\,4t'} \\ V^*_{R\,4c} V_{R\,4u} & |V_{R\,4c}|^2 & V^*_{R\,4c} V_{R\,4t} & V^*_{R\,4c} V_{R\,4t'} \\ V^*_{R\,4t} V_{R\,4u} & V^*_{R\,4t} V_{R\,4c} & |V_{R\,4t}|^2 & V^*_{R\,4t} V_{R\,4t'} \\ V^*_{R\,4t'} V_{R\,4u} & V^*_{R\,4t'} V_{R\,4c} & V^*_{R\,4t'} V_{R\,4t} & |V_{R\,4t'}|^2 \end{pmatrix}$$

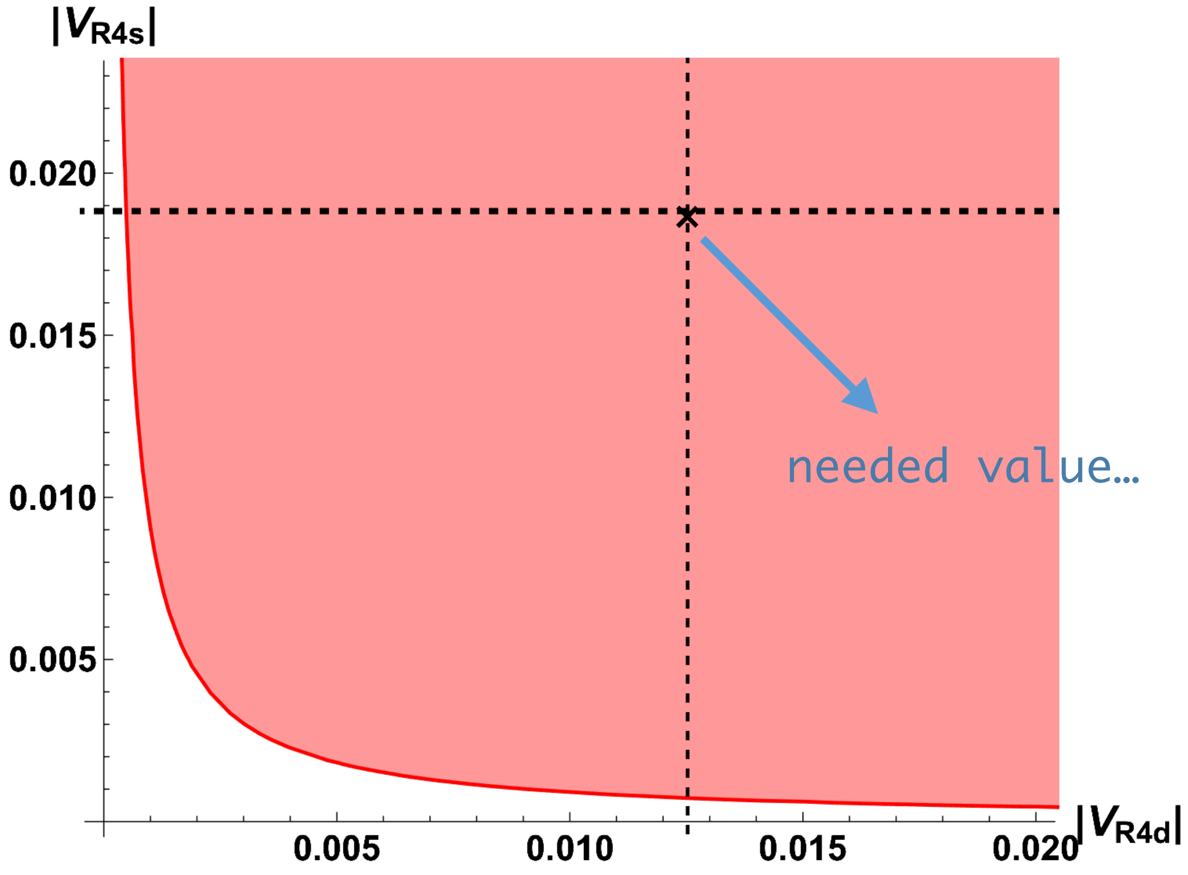
$$V_R^{(d)\dagger} \text{diag}(0,0,0,1) V_R^{(d)} = \begin{pmatrix} |V_{R\,4d}|^2 & V^*_{R\,4d} V_{R\,4s} & V^*_{R\,4d} V_{R\,4b} & V^*_{R\,4d} V_{R\,4b'} \\ V^*_{R\,4s} V_{R\,4d} & |V_{R\,4s}|^2 & V^*_{R\,4s} V_{R\,4b} & V^*_{R\,4s} V_{R\,4b'} \\ V^*_{R\,4b} V_{R\,4d} & V^*_{R\,4b} V_{R\,4s} & |V_{R\,4b}|^2 & V^*_{R\,4b} V_{R\,4b'} \\ V^*_{R\,4b'} V_{R\,4d} & V^*_{R\,4b'} V_{R\,4s} & V^*_{R\,4b'} V_{R\,4b} & |V_{R\,4b'}|^2 \end{pmatrix}$$

Vectorlike weak isodoublet

Low energies EW quantities and Z physics



Area excluded by flavor changing K decays



Possible solutions

- There can be two or more vector-like doublets or a vector-like isodoublet with a down-type or up-type isosinglet, with some zero-couplings:

$$\overline{q_{Li}}\mathbf{m}^{(d)}_{ij} d_{Rj} = \begin{pmatrix} \overline{q_{1L}} & \overline{q_{2L}} & \overline{q_{3L}} & \overline{q_{4L}} & \overline{d_{5L}} \end{pmatrix} \begin{pmatrix} & & & 0 & y^d_{15} \\ & \mathbf{y}^{(d)}_{3\times 3} v_w & & 0 & 0 \\ & & & 0 & 0 \\ 0 & y^d_{42} v_w & 0 & M_4 & 0 \\ 0 & 0 & 0 & 0 & M^d_5 \end{pmatrix} \begin{pmatrix} d_{R1} \\ d_{R2} \\ d_{R3} \\ q_{R4} \\ d_{R5} \end{pmatrix}$$

$$\overline{q_{Li}}\mathbf{m}^{(u)}_{ij} u_{Rj} = \begin{pmatrix} \overline{q_{1L}} & \overline{q_{2L}} & \overline{q_{3L}} & \overline{q_{4L}} \end{pmatrix} \begin{pmatrix} & & & 0 \\ & \mathbf{y}^{(u)}_{3\times 3} v_w & & 0 \\ & & & 0 \\ y^u_{41} v_w & 0 & 0 & M_4 \end{pmatrix} \begin{pmatrix} u_{R1} \\ u_{R2} \\ u_{R3} \\ q_{R4} \end{pmatrix}$$

- The non-zero couplings y^d_{42}, y^u_{41} can cancel the discrepancy from kaon physics without introducing flavor changing. The singlet can fix the discrepancy with the determination from superallowed beta decays.

Conclusions #3

- Large mixing of extra species with SM fermions is required for solving CKM unitarity problem.
- Vectorlike quarks cannot restore unitarity without fine tuning.
- If they exist, their mass should be no more than about 6 TeV.

Summary

1. Gauge family symmetries:
   - Natural explanation for masses and mixings of fermions from the spontaneous breaking pattern of the symmetry.
   - Flavour gauge bosons can be as light as TeV without contradicting experimental constraints, due to custodial properties.
2. Anomalies in the first row of CKM matrix: new physics at TeV scale?
   - There is tension between independent determinations of the CKM matrix elements of the first row.
   - Flavour gauge bosons mediated interactions can restore unitarity of CKM.
   - G_F can be different from muon decay constant without contradicting experimental data, rather providing a testable solution for the CKM unitarity problem.
3. Extra vector-like quarks
   - A quite large mixing with SM fermions is needed to restore unitarity, comparable to V_{cb}. The mixing with the first family should be ten times larger than the one with heavier families (seems unnatural but not excluded).
   - Their mass should be no more than about 6 TeV.
   - Flavour changing can be avoided with more isodoublets or isodoublet and singlet by setting to zero some couplings with SM families.

Summary

- Inter-family gauge symmetry can give a natural explanation to the origin of mass hierarchy and mixing pattern of fermions, as a consequence of spontaneous breaking pattern of this symmetry.
- Flavour gauge bosons can be as light as TeV without contradicting experimental constraints, due to custodial properties.
- Flavour gauge bosons mediated interactions can then be a solution for the CKM unitarity problem.
- Gauge bosons acting in the quark sector can be in the reach of LHC for direct detection. Moreover some LFV processes, as e.g. $\tau \to 3\mu$, can have widths close to present experimental limits and can be within the reach of future high precision experiments.
- In order to cancel $SU(3)^3$ anomalies an interesting possibility is to introduce mirror matter, which is also a candidate for dark matter. The flavor gauge bosons would be messengers between the two sectors and a portal for direct detection. They can mediate new phenomena such as muonium disappearance (conversion into mirror muonium), kaon disappearance, pion disappearance.
- There is tension between independent determinations of the elements in the first row of CKM matrix, which can be a signal for a violation of CKM unitarity.
- A new effective operator in positive interference with the SM muon decay as the one generated by flavour changing gauge bosons can restore unitarity of CKM. Then G_μ can be different from the Fermi constant G_F without contradicting experimental data, rather providing a testable solution for the CKM unitarity problem, with $G_F = G_\mu/(1+\delta_\mu)$, $\delta_\mu \simeq 7 \times 10^{-4}$ corresponding to a scale of the new interactions of 6-7 TeV.
- A quite large mixing with SM fermions is needed in order to recover unitarity. For example, the mixing of an extra down-type vectorlike quark with the first family should be $|V_{ub'}| \approx 0.04$, which is comparable to $|V_{cb}|$ and ten times larger than $|V_{ub}|$.
- The mixing of the 4th state with the second and third family should be much less than the mixing with the first family, which seems unnatural, although not excluded.
- The existence of two or more vector-like doublets or a vector-like isodoublet with down-type or up-type isosinglet can yield a solution to the problem of CKM unitarity by setting to zero some couplings of extra species with SM families.
- In order to obtain a large mixing of vectorlike quarks with ordinary quarks, the mass scale of these particles should be of few TeV. Further search at LHC can falsify or discover the vectorlike quarks.
- Concluding, in the TeV range there may exist a new physics related to the fermion flavour which can be revealed in future experiments at the energy and precision frontiers.

Backup

Quarks and family symmetries

$U(3)_q \times U(3)_d \times U(3)_u$ with gauge factors $SU(3)_q \times SU(3)_d \times SU(3)_u$

- Quark masses:

$$\frac{y^d_{ij}}{M_d^2}\,\eta^q_{i\alpha}\overline{\xi}^{d\gamma}_j\,\phi\,\overline{q_{L\alpha}}d_{R\gamma} + \frac{y^u_{ij}}{M_u^2}\,\eta^q_{i\alpha}\overline{\xi}^{u\gamma}_j\,\tilde{\phi}\,\overline{q_{L\alpha}}u_{R\gamma} + \text{h.c.}$$

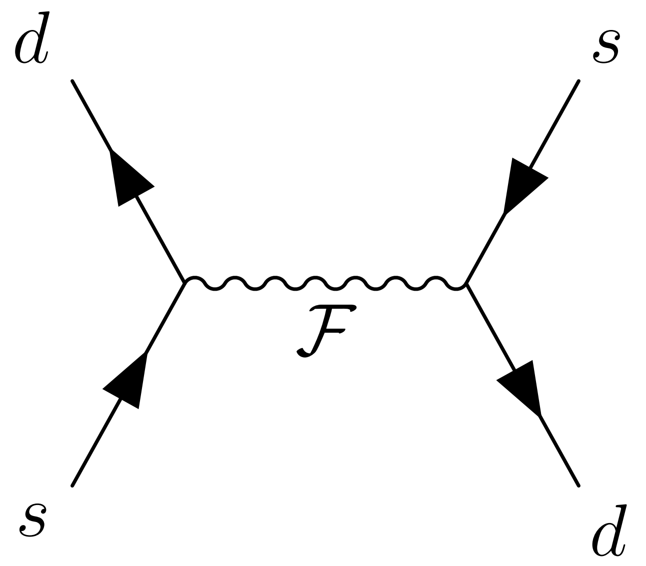
- mass hierarchy is related with hierarchies in breaking of $SU(3)_q \times SU(3)_d \times SU(3)_u$ gauge symmetry:

$$m_b : m_s : m_d = 1 : \epsilon_d\epsilon_q : \epsilon_d\tilde{\epsilon}_d\epsilon_q\tilde{\epsilon}_q$$
$$m_t : m_c : m_u = 1 : \epsilon_u\epsilon_q : \epsilon_u\tilde{\epsilon}_u\epsilon_q\tilde{\epsilon}_q$$

Quarks and family symmetries

- $K^0 - \bar{K}^0$ oscillation is induced by:

$$-\frac{1}{4v_{d2}^2}\left[(V_{3d}V_{3s}^*)^2 + \epsilon_d^2(V_{2d}V_{2s}^*)^2\right](\overline{s_R}\gamma^\mu d_R)^2$$



- Since $|V_{2d}V_{2s}^*| \sim \tilde{\epsilon}_d$, $|V_{3d}V_{3s}^*| \sim \epsilon_d^2\tilde{\epsilon}_d$, K^0 mixing is suppressed by $\epsilon_d^2\tilde{\epsilon}_d^2 \ll 1$.
- New contribution can be constrained to be less than the SM contribution. By taking $\epsilon_d\tilde{\epsilon}_d \sim 10^{-2}$, the mass scale $v_{d2} \sim 7\,\text{TeV}$ is compatible with the constraint from the neutral kaons mass difference.
- As regards the imaginary part contributing to ϵ_K, with the same choice $\epsilon_d\tilde{\epsilon}_d \sim 10^{-2}$, $v_{d2} \sim 7\,\text{TeV}$ is still allowed if the phase of $V_{2d}V_{2s}^*$ is $O(0.1)$.

VEVS

Can the hierarchy of the VEVs of ξs be natural? The generic potential is:

$$V(\xi) = \lambda_n(|\xi_n|^2 - \frac{\mu_n^2}{2\lambda_n})^2 + \lambda_{klnm}\xi_k^\dagger\xi_l\xi_n^\dagger\xi_m + (\mu\xi_1\xi_2\xi_3 + \text{h.c.})$$

- The dimensional constant μ can be arbitrarily small since if $\mu \to 0$ the Lagrangian acquires global $U(1)_e$ symmetry.
- $v_2/v_3 \sim m_\mu/m_\tau$ (one order of magnitude) can emerge from a natural fluctuation of mass terms μ_n^2 and coupling constants λ.
- Small v_1 is naturally obtained when the third flavon ξ_1 has positive mass squared. Then for $\mu \neq 0$ non-zero VEV $\langle\xi_1\rangle$ is induced:

$$v_1 = \frac{\mu v_2 v_3}{\mu_1^2}$$

- Taking μ small enough, say $\mu < v_2$, one can naturally get $v_1 \ll v_2$. **The hierarchy of the VEVs of ξs can be natural**.

Flavons and LFV

- Also flavons can mediate the LFV processes.
- Lepton Yukawa couplings with the flavon fields ξ_n:

$$h_{in}\, \xi_n^\alpha \overline{\ell_{Li}} e_{R\alpha} \qquad h_{in} = \frac{g_{in} v_{\mathrm{w}}}{M}$$

  which are generically flavor-changing.

- ξ_2, with mass $\mu_2 \sim v_2$, induces the effective operator:

$$-\frac{h_{32}h_{22}}{\mu_2^2}\,(\overline{\tau}\mu)\,(\overline{\mu}\mu)\,, \qquad \frac{h_{32}h_{22}}{\mu_2^2} \simeq \frac{m_\mu^2}{v_2^4}$$

  For $v_2 > 2$ TeV, the width of $\tau \to 3\mu$ decay induced by this operator is more than 12 orders of magnitude below the experimental limit.

- The width of $\mu \to 3e$ decay induced by analogous operator mediated by flavon ξ_1 is also suppressed by orders of magnitude.

Effective operators for fermion masses

$$\frac{\bar{\xi}^i \bar{\xi}^j}{M_u^2} \tilde{\varphi} \overline{q_L}_i u_{Rj} + \frac{\bar{\xi}^i \eta_\alpha}{M_d^2} \varphi \overline{q_L}_i d_R^\alpha + \frac{\bar{\xi}^i \eta_\alpha}{M_L^2} \varphi \overline{\ell_{L\alpha}} e_{Ri} + \frac{\bar{\eta}^\alpha \bar{\eta}^\beta}{M_\nu^3} \varphi \varphi \ell_{L\alpha}^T C \ell_\beta + h.c.$$

- Three $SU(3)_e$ triplets $\xi_i \sim 3_e$

$$\langle \xi_1 \rangle = \begin{pmatrix} v_1 \\ 0 \\ 0 \end{pmatrix}, \quad \langle \xi_2 \rangle = \begin{pmatrix} 0 \\ v_2 \\ 0 \end{pmatrix}, \quad \langle \xi_3 \rangle = \begin{pmatrix} 0 \\ 0 \\ v_3 \end{pmatrix}$$

$$SU(3)_e \xrightarrow{v_3} SU(2)_e \xrightarrow{v_2} \text{nothing}$$

$$v_3 : v_2 : v_1 = \tilde{\epsilon}\epsilon : \epsilon : 1 \qquad \epsilon = \frac{1}{20}$$

- Three $SU(3)_\ell$ triplets $\eta_i \sim 3_\ell$

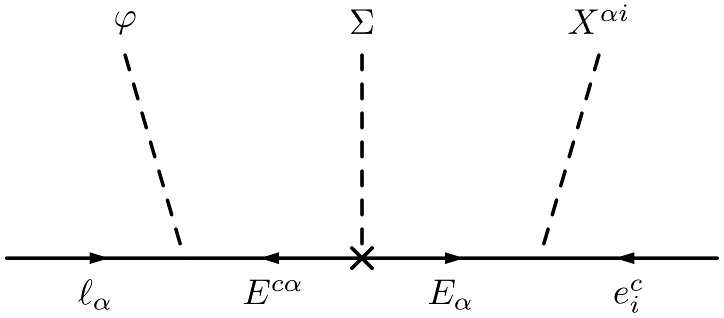
$$\langle \eta_1 \rangle = \begin{pmatrix} w_1 \\ 0 \\ 0 \end{pmatrix}, \quad \langle \eta_2 \rangle = \begin{pmatrix} 0 \\ w_2 \\ 0 \end{pmatrix}, \quad \langle \eta_3 \rangle = \begin{pmatrix} 0 \\ 0 \\ w_3 \end{pmatrix}$$

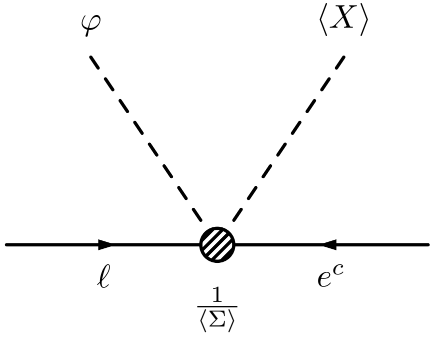
$$SU(3)_l \xrightarrow{u_3} SU(2)_l \xrightarrow{u_2} \text{nothing}$$

Then mass ratios, $V_{\rm CKM}$, U_{PMNS} are obtained.

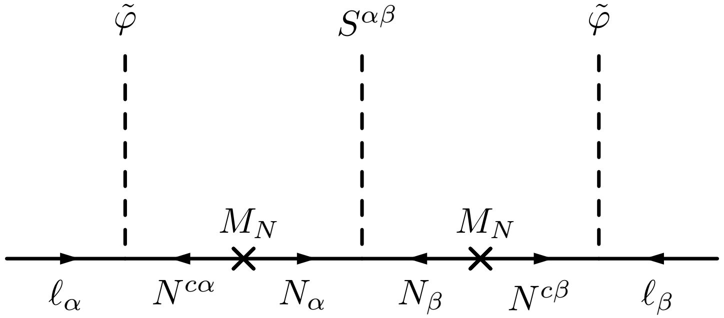
Effective operators for fermion masses

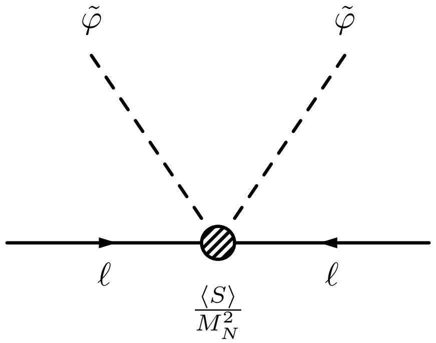
$$X_e^{\beta k} \sim (\bar{3}_l, \bar{3}_e) \qquad X_d^{i\alpha} \sim (\bar{3}_q, \bar{3}_d) \qquad X_u^{ij} \sim (\bar{3}_q, \bar{3}_u)$$





$$S_{\alpha\beta} \sim 6_l$$





$$\mathcal{L} = \frac{\lambda_u}{M_U} X_u^{ij} \bar{\phi} \bar{u}_j q_i + \frac{\lambda_d}{M_D} X_d^{i\alpha} \phi \bar{d}_\alpha q_i + \frac{\lambda_e}{M_E} X_e^{\beta k} \phi \bar{e}_k l_\beta + \lambda_\nu \mathcal{M}^{-1}_{\alpha\beta} \bar{\phi} \bar{\phi} l_\alpha l_\beta + h.c.$$

SSB

$$SU(3)_l \xrightarrow{u_3} SU(2)_l \xrightarrow{u_2} \text{nothing}$$

$$\eta^{\alpha}_{(n)} \sim \bar{3}_l \qquad \langle \eta_{(3)} \rangle = \begin{pmatrix} 0 \\ 0 \\ u_3 \end{pmatrix} \quad \langle \eta_{(2)} \rangle = \begin{pmatrix} 0 \\ u_2 \\ 0 \end{pmatrix} \quad \langle \eta_{(1)} \rangle = \begin{pmatrix} u_1 \\ 0 \\ 0 \end{pmatrix}$$

$$SU(3)_e \xrightarrow{v_3} SU(2)_e \xrightarrow{v_2} \text{nothing} \qquad v_3 : v_2 : v_1 = \epsilon^2 : \epsilon : 1 \qquad \epsilon = \frac{1}{20}$$

$$\xi^{i}_{(n)} \sim \bar{3}_e \qquad \langle \xi_{(3)} \rangle = \begin{pmatrix} 0 \\ 0 \\ v_3 \end{pmatrix} \quad \langle \xi_{(2)} \rangle = \begin{pmatrix} 0 \\ v_2 \\ 0 \end{pmatrix} \quad \langle \xi_{(1)} \rangle = \begin{pmatrix} v_1 \\ 0 \\ 0 \end{pmatrix}$$

$$X_e^{\beta k} \sim (\bar{3}_\ell, \bar{3}_e) \qquad S_{\alpha\beta} \sim (6_\ell, 1)$$

- inducing VEV to X_e and S via couplings

$$\mu_X^* \xi_{(l)} X^\dagger \eta_{(n)} + \text{h.c.} \qquad \mu_S \eta_n S \eta_l + \text{h.c.}$$

$$\langle X_e^{\beta k} \rangle \sim \frac{-\mu_X^* \langle \xi^i_{(n)} \eta^\alpha_{(l)} \rangle}{M_X^2} \qquad \langle S^{*\beta\alpha} \rangle \sim \frac{-\mu_S \langle \eta^\alpha_{(n)} \eta^\beta_{(l)} \rangle}{M_S^2}$$

Charged leptons masses

$$SU(3)_l \times SU(3)_e$$

$$SU(3)_l \xrightarrow{u_3} SU(2)_l \xrightarrow{u_2} \text{nothing} \qquad u_3 : u_2 : u_1 = \epsilon_L^2 : \epsilon_L : 1 \qquad \epsilon_L \simeq \frac{1}{2} - \frac{1}{3}$$

$$SU(3)_e \xrightarrow{v_3} SU(2)_e \xrightarrow{v_2} \text{nothing} \qquad v_3 : v_2 : v_1 = \epsilon^2 : \epsilon : 1 \qquad \epsilon = \frac{1}{20}$$

From Yukawa coupling:

$$\frac{\lambda_e}{M_E} X_e^{\beta k} \phi \bar{e}_k l_\beta \longrightarrow \qquad m^{(e)\dagger} \propto \langle X_e \rangle \simeq \begin{pmatrix} O(\epsilon^2) & 0 & 0 \\ O(\epsilon^2) & O(\epsilon) & 0 \\ O(\epsilon^2) & O(\epsilon) & O(1) \end{pmatrix} \cdot \frac{|\mu_X| v_3 u_3}{M_X^2}$$

$$V_{\rm L}^{(e)\dagger} m^{(e)} V_{\rm R}^{(e)} = m_{\rm diag}^{(e)} \qquad \overline{\mathbf{e}_{\rm R}} \mathbf{m}^{(\mathbf{e})\dagger} \mathbf{e}_{\rm L} = (\overline{\mathbf{e}_{\rm R}} V_{\rm R}^{(e)})(V_{\rm R}^{(e)\dagger} \mathbf{m}^{(\mathbf{e})\dagger} V_{\rm L}^{(e)})(V_{\rm L}^{(e)\dagger} \mathbf{e}_{\rm L})$$

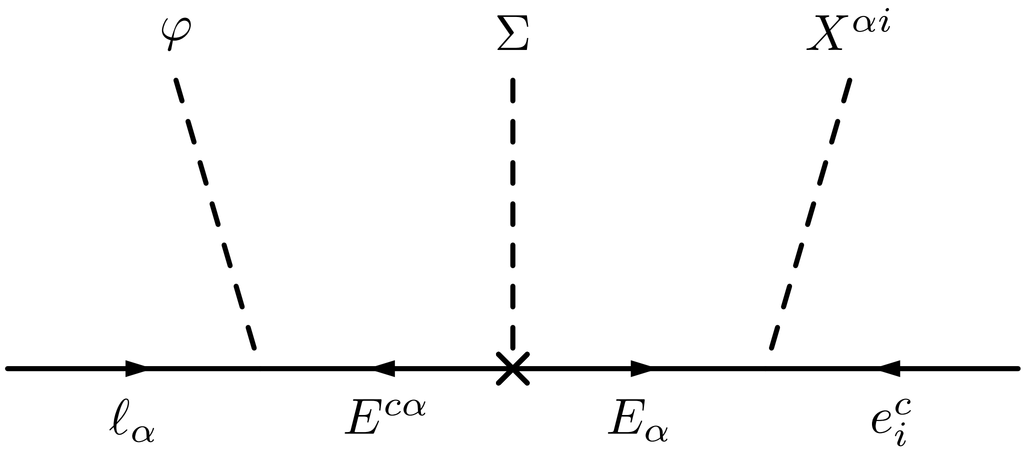
$$\begin{pmatrix} e^1 \\ e^2 \\ e^3 \end{pmatrix}_{\rm R} = V_R^{(e)} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_{\rm R} = \begin{pmatrix} V_{1e} & V_{1\mu} & V_{1\tau} \\ V_{2e} & V_{2\mu} & V_{2\tau} \\ V_{3e} & V_{3\mu} & V_{3\tau} \end{pmatrix}_{\rm R}^{(e)} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_{\rm R}$$

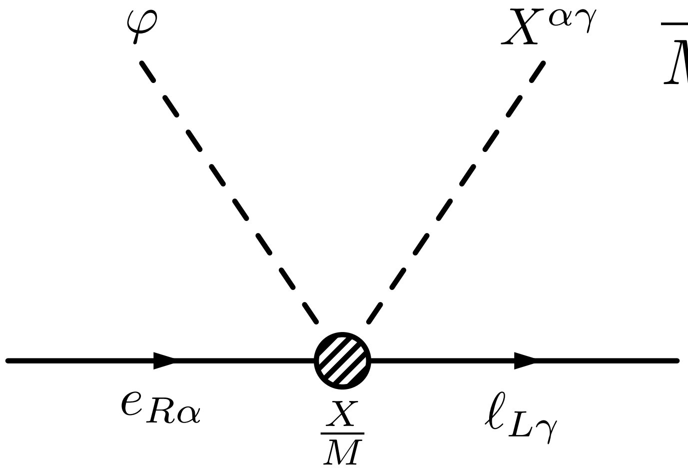
Effective operators for fermion masses

$$U(3)_\ell \times U(3)_e \times U(3)_q \times U(3)_u \times U(3)_d$$

$$\ell_L \sim 3_\ell\,, \qquad e_R \sim 3_e\,, \qquad q_L \sim 3_q \qquad u_R \sim 3_u\,, \qquad d_R \sim 3_d$$

- Fermions cannot get mass if the symmetry is unbroken.
- Yukawa couplings are induced by non-zero VEVs of scalars.





$$\frac{X}{M}_e \varphi \overline{l_L} e_R$$

$$X_e^{\beta k} \sim (\bar{3}_l, \bar{3}_e) \qquad X_d^{i\alpha} \sim (\bar{3}_q, \bar{3}_d) \qquad X_u^{ij} \sim (\bar{3}_q, \bar{3}_u)$$

Effective operators for fermion masses

$$U(3)_\ell \times U(3)_e \times U(3)_q \times U(3)_u \times U(3)_d$$

$$\ell_L \sim 3_\ell\,, \qquad e_R \sim 3_e\,, \qquad q_L \sim 3_q \qquad u_R \sim 3_u\,, \qquad d_R \sim 3_d$$

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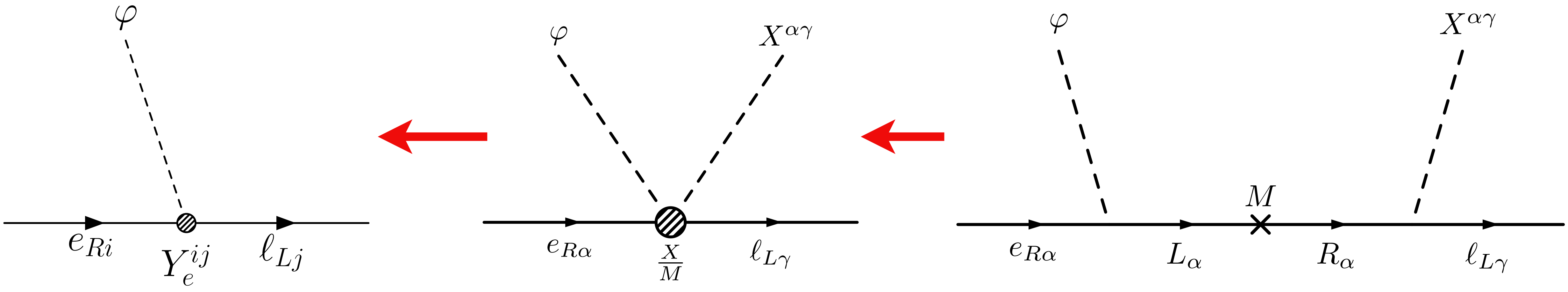

Effective operators for fermion masses

$$U(3)_\ell \times U(3)_e \times U(3)_q \times U(3)_u \times U(3)_d$$

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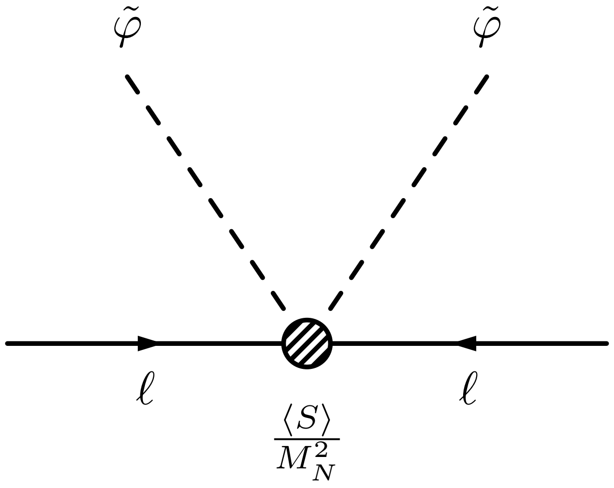
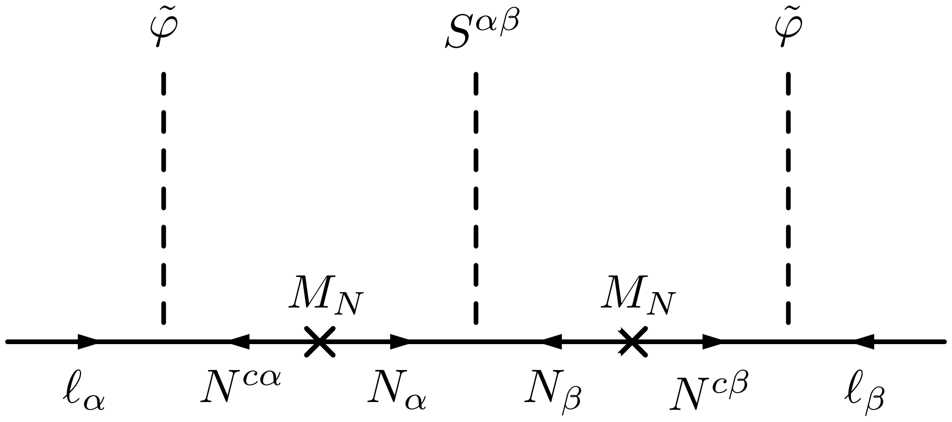




$$S_{\alpha\beta} \sim 6_l$$

FCNC in SU(5)

- In the context of GUT theories new gauge interactions require the mass scale of gauge bosons to be much higher than few TeV, also in $SU(5)$ scenario.
- Tree level contribution to K mesons decays $K^- \to \pi^- \mu^- e^+$, $K_L^0 \to e^\pm \mu^\mp$.
- Mass scales of gauge bosons are constrained to be higher than 100 TeV and 300 TeV respectively.

Triangle anomalies

$$SU(3)_\ell \times SU(3)_e \times SU(3)_Q \times SU(3)_u \times SU(3)_d$$

$$\ell_L \sim 3_\ell,\ e_R \sim 3_e,\ Q_L \sim 3_Q,\ u_R \sim 3_u,\ d_R \sim 3_d$$

- In order to cancel $SU(3)^3$ anomalies for each triplet another triplet (SM singlet) with opposite chirality is needed.
- An interesting possibility is to introduce the mirror twins with opposite chirality and analogous representation of mirror SM gauge symmetry $SU(3)' \times SU(2)' \times U(1)'$:

$$\ell'_R \sim 3_\ell,\ e'_L \sim 3_e,\ Q'_R \sim 3_Q,\ u'_L \sim 3_u,\ d'_L \sim 3_d$$

- Couplings with flavons:

$$\frac{g_{in}\xi_n^\alpha}{M}(\phi\overline{\ell_{Li}}e_{R\alpha} + \phi'\overline{\ell'_{Ri}}e'_{L\alpha}) + h.c.$$

Triangle anomalies 2

- As an example, for $SU(3)_e$, mixed triangle anomaly $U(1) \times SU(3)_e^2$ must be cancelled. New leptons

$$\mathcal{E}_{L\alpha} \sim (1, -2, 3_e; X), \qquad \mathcal{E}_{Ri} \sim (1, -2, 1; X)$$

and for mirror parity

$$\mathcal{E}'_{R\alpha} \sim (1, -2', 3_e; X), \qquad \mathcal{E}'_{Li} \sim (1, -2', 1; X)$$

cancel the mixed triangle

$$U(1) \times SU(3)_e^2,\ U(1)_X \times SU(3)_e^2,\ U(1) \times U(1)_X^2,\ U(1)_X \times U(1)^2$$

- Masses from Yukawa couplings

$$y_{in} \xi_n^\alpha \overline{\mathcal{E}_{Ri}} \mathcal{E}_{L\alpha} + y_{in} \xi_n^\alpha \overline{\mathcal{E}_{Li}}' \mathcal{E}'_{R\alpha} + h.c.$$

- The lightest has mass $O(100)$ GeV. If $U(1)_X$ is unbroken, then it is stable. Current experimental lower limit on charged new leptons is 102.6 GeV.

Cosmological implications

- Mirror matter is a viable candidate for light dark matter dominantly consisting of mirror helium and hydrogen atoms.
- The flavor gauge bosons are messengers between the two sectors and so a portal for direct detection.
- $T'/T < 0.2 \div 0.3$ from CMB and large scale structures.
- Freeze-out temperature of horizontal interactions between the two sectors should not exceed $T_d \simeq (v_2/2)^{\frac{4}{3}} \times 130$ MeV. Or $v'_{\rm EW} \gg v_{\rm EW}$.
- For neutrinos

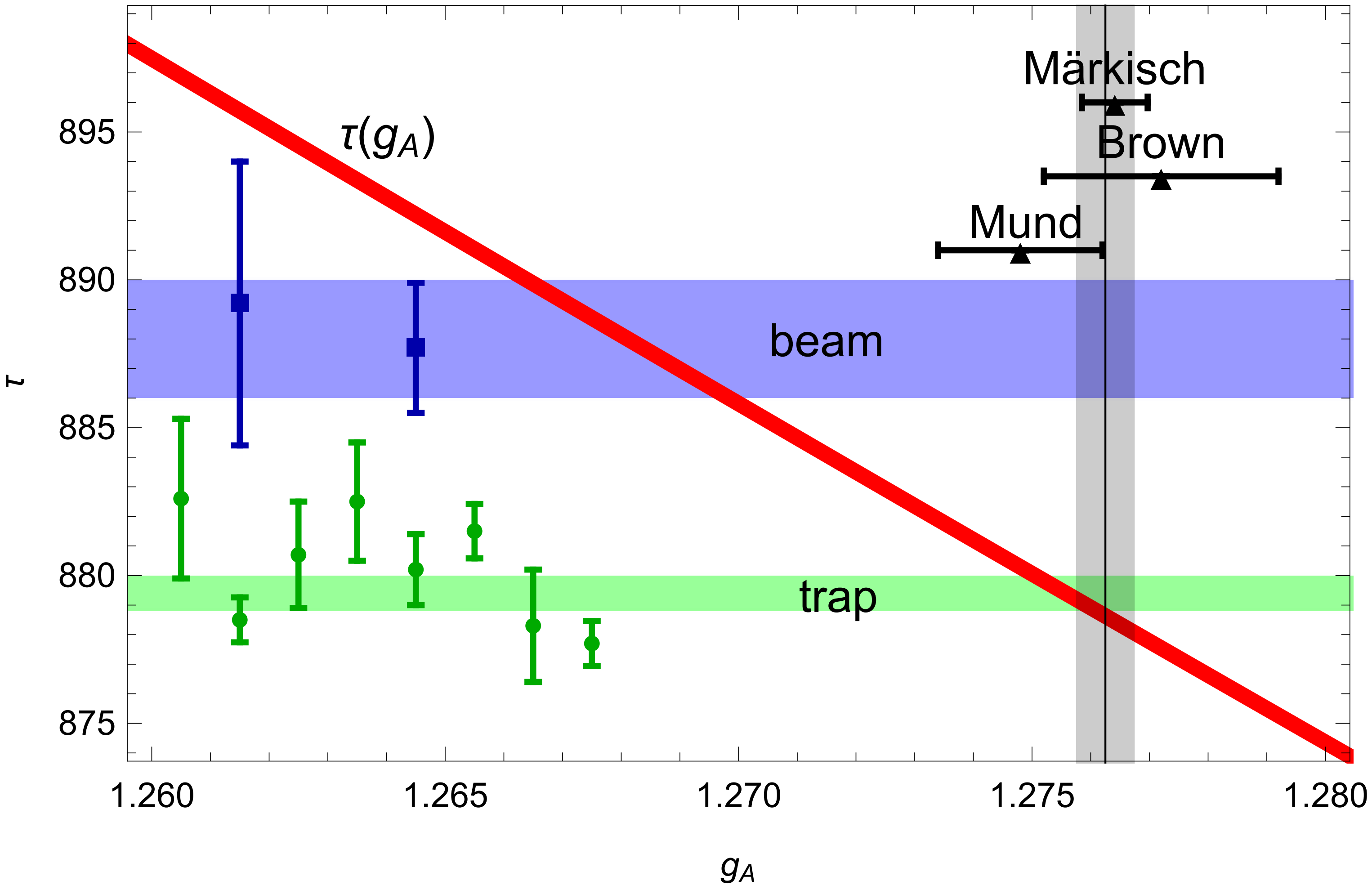
$$\frac{Y_\nu^{ij}}{\mathcal{M}}(\phi\phi l_{Li}^T C l_{Lj} + \phi'\phi' l'^T_{Ri} C l'_{Rj}) + \frac{\tilde{Y}_\nu^{ij}}{\mathcal{M}}\phi\phi' \overline{l_{Li}} l'_{Rj} + h.c.$$

the last operator gives COLEPTOGENESIS.

The CKM unitarity problem

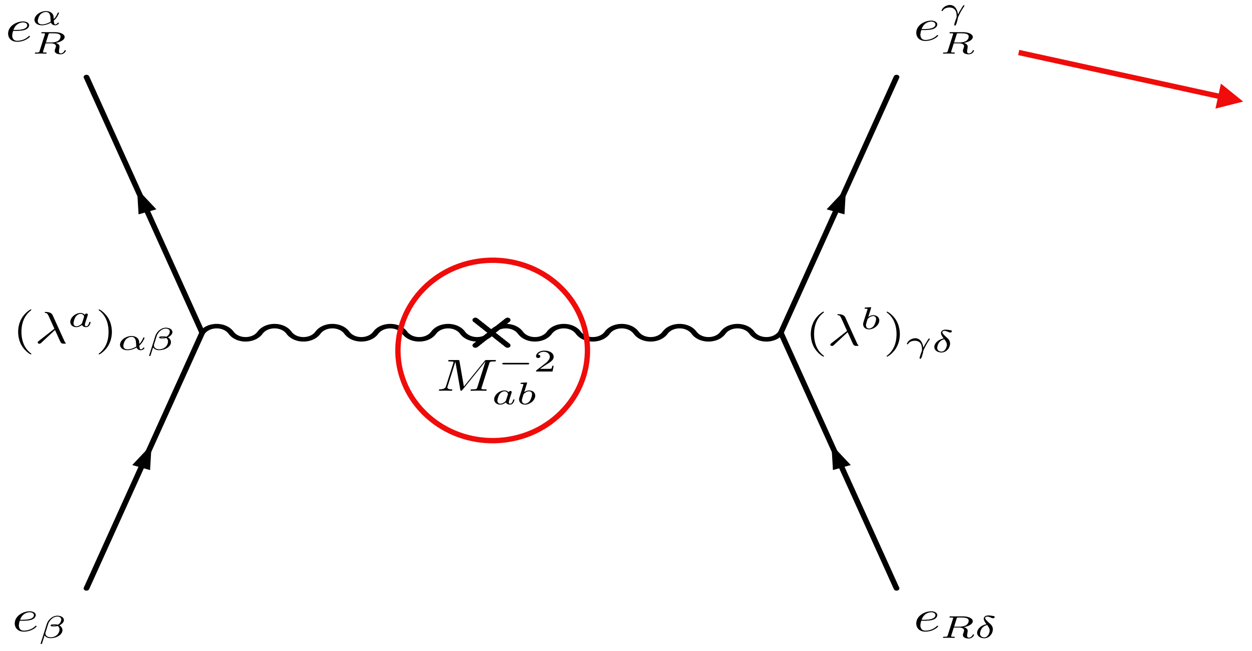
	CKM [PDG]	CKM [post 2018]	CKM+ b'	CKM+ $\mathcal{F}$
C	0.2257(9)	0.22780(60)	0.22443(61)	0.22460(61)
B	0.2256(10)	0.22535(45)	0.22518(45)	0.22535(45)
A	0.2238(8)	0.22333(60)	0.22333(60)	0.22350(60)
$\overline{A+B}$	0.2245(6)	0.22463(36)	0.22452(36)	0.22469(36)
$\overline{A+B+C}$	0.2248(5)	0.22546(31)	0.22449(31)	0.22467(31)
	$\chi^2 = 3.4$	$\chi^2 = 27.7$ †	$\chi^2 = 6.1$	$\chi^2 = 6.1$
$\lvert V_{us}\rvert$	0.2248(7)	0.2255(12) †	0.2245(5)	0.2247(5)
$\lvert V_{ud}\rvert$	0.97440(16)	0.97424(27) †	0.97369(12)	0.97443(12)

CKM and neutron lifetime problem



Gauge bosons, $SU(3)_e$

- Generically, FCNC...(in the SM only at loop level, and excellent agreement with experimental data...):



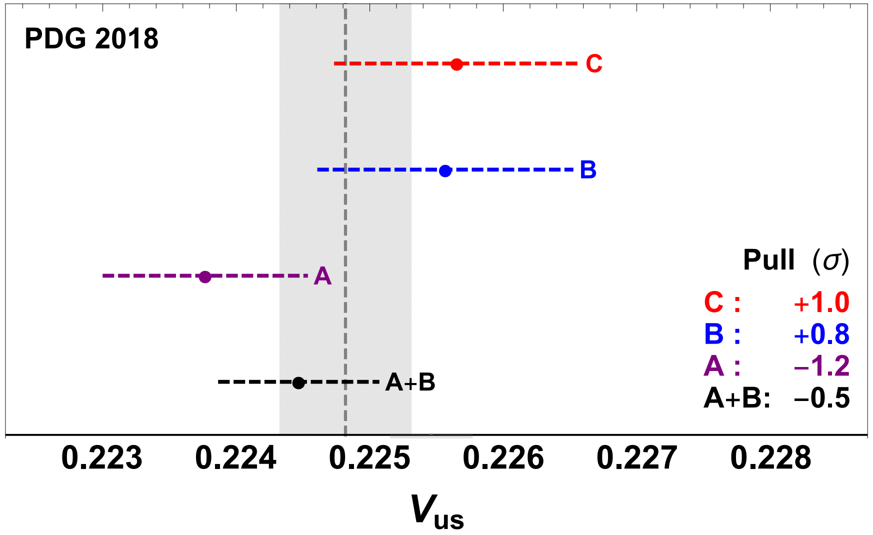
$$e_{Ri} = V^{(e)}_{R1e} e_R + V^{(e)}_{R1\mu} \mu_R + V^{(e)}_{R1\tau} \tau_R$$

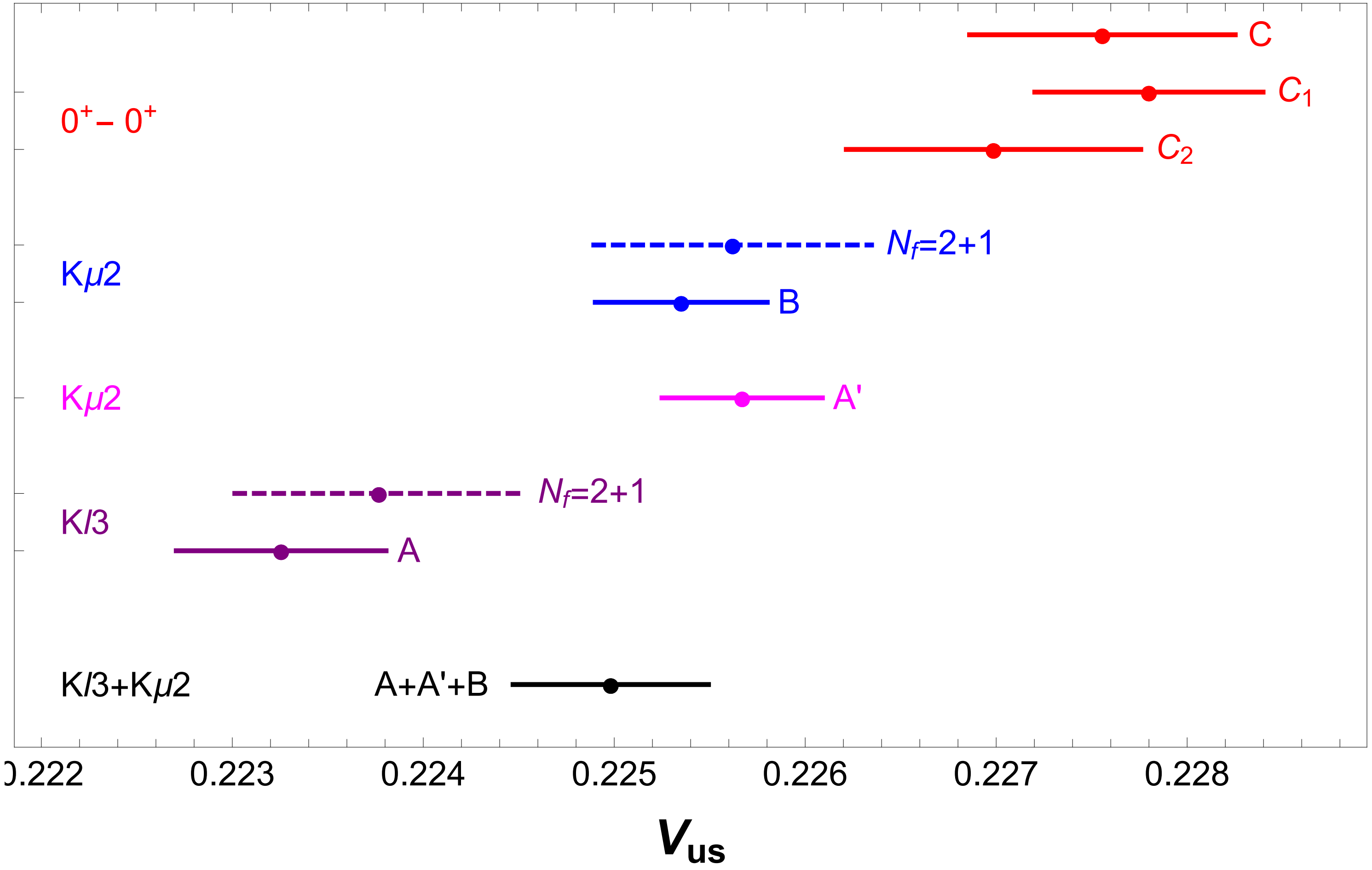
$$\mathcal{L}_{\text{eff}} = -\frac{g^2}{2} J_a^\mu \,(M^2)^{-1}_{ab}\, J_{b\mu}$$

$$J_{a\mu} = \frac{1}{2}\begin{pmatrix} \overline{e_R} & \overline{\mu_R} & \overline{\tau_R} \end{pmatrix} \gamma_\mu V_R^{(e)\dagger} \lambda_a V_R^{(e)} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}$$

- Heavy gauge bosons may suppress FCNC. Nevertheless, can they be "light"?

Present situation of CKM unitarity





Solution #2

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - |V_{ub'}|^2 , \qquad |V_{ub'}| = 0.04$$