Theoretical calculation of nuclear reactions of interest for Big Bang Nucleosynthesis

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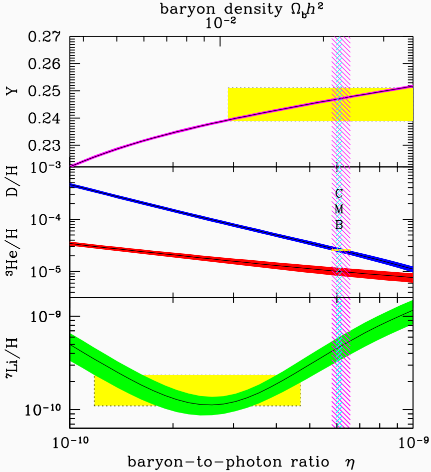
Big Bang Nucleosynthesis

- Predicts abundances of light elements
- Network of reactions (cross-sections)
  ⇒ NO free parameters
- Good agreement with Astrophysical Observations (A.O.)
- A.O. more and more accurate
  ⇒ secondary products



PDG, Phys. Rev. D **98**, 030001 (2018)

Is there a ^{6}Li problem?

- The ^{6}Li abundance in the BBN
  $^6\mathrm{Li}/^7\mathrm{Li} \sim 10^{-5}$ BBN prediction
  $^6\mathrm{Li}/^7\mathrm{Li} \sim 5 \times 10^{-3}$ measured in halo-stars [1]
  $\Rightarrow$ results under debate

- Possible solutions [2]
  - systematic errors in A.O.
  - new physics (BSM) appearing
  - incomplete knowledge of reaction cross-sections

[1] Asplund *et al.*, Astrophys J. **664**, 229 (2006)

[2] Fields, Ann. Rev. Nucl. Part. Phys. **61**, 47 (2011)

Motivations

- Main uncertainties comes from $\alpha + d \rightarrow {}^6\text{Li} + \gamma$ [1]
- Presence of non-thermal photons (BSM) $\Rightarrow {}^7\text{Be} + \gamma \rightarrow p + {}^6\text{Li}$ [2]
  $\Rightarrow$ studied with $p + {}^6\text{Li} \rightarrow {}^7\text{Be} + \gamma$
- Both the reactions studied by the LUNA Collaboration[3]
- *Why theory?*
  In the BBN energy range ($50 < E < 400$ keV) the measurements are very hard due to the Coulomb barrier

The goal is the determination of the S-factor

$$S(E) = E\, exp(2\pi\eta)\, \sigma(E)$$

$\eta = Z_1 Z_2 e^2 \frac{\mu}{\hbar^2 k}$

[1] K.M. Nollett, *et. al* Phys. Rev. C **56**, 1144 (1997)

[2] M. Kusukabe, *et al.* Phys. Rev. D **74**, 023526 (2006)

[3] M. Anders, *et al.* Phys. Rev. Lett. **113**, 042501 (2014)

Theoretical approaches

- Phenomenological approach ($p + {}^6\text{Li} \to {}^7\text{Be} + \gamma$)
  - Nucleus = system of “pointlike” clusters (${}^7\text{Be} = p + {}^6\text{Li}$)
  - “Phenomenological” interactions between clusters
  - “Model dependent” prediction
  - numerically “Fast”

- *Ab-initio* approach ($\alpha + d \to {}^6\text{Li} + \gamma$)
  - Nucleus = system of A bodies interacting among themselves and with external probes
  - Realistic nucleon-nucleon and nucleon-probe interactions
  - Exact method to solve the quantum-mechanical problem
  - “True” predictions
  - numerically “Slow”$\Rightarrow$ We limit the study to ^{6}Li

$$p + {}^6\mathrm{Li} \to {}^7\mathrm{Be} + \gamma$$

A.G. and L.E. Marcucci, Nucl. Phys. A **987**, 1 (2019)

Nuclear Physics Motivations

- Is there a low-energy resonance? [1]

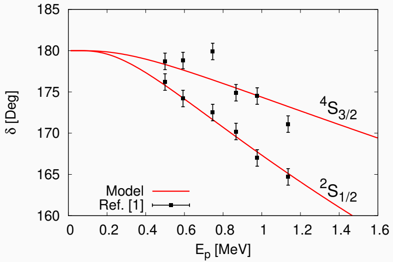
- Photon angular distributions (for LUNA)

[1] J.J. He *et al.*. Phys. Lett. B **725**, 287 (2013)

The $p + {}^6$Li system in the cluster model

- Clusters: p and ^{6}Li
- Intercluster potential (state dependent)

$$V(r) = -V_0 \exp(-a_0 r^2)$$



- Elastic scattering data+bound state properties ⇒ cluster potential parameters

	J^π	Spin	E (MeV)
GS	$3/2^-$	1/2	-5.6068
FES	$1/2^-$	1/2	-5.1767

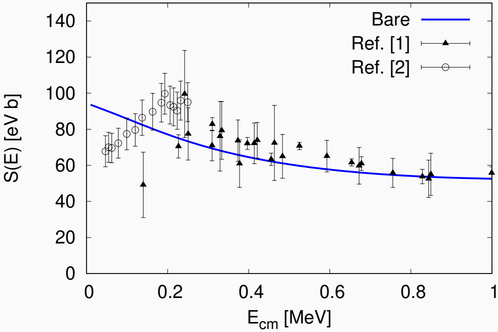
- Wave functions ⇒ prediction for radiative capture
  Dominated by E_1 transition

$$\sigma(E) \propto |\langle \psi_{^7\text{Be}} | E_1 | \psi_{p+^6\text{Li}} \rangle|^2$$

[1] S.B. Dubovichenko *et al.*, Phys. Atom. Nucl. **74**, 1013 (2011)

The S-factor

$$S(E) = E\exp(2\pi\eta)(\sigma_{3/2}(E) + \sigma_{1/2}(E))$$



Branching ratio

$$\left.\frac{\sigma_{1/2}(E)}{\sigma_{1/2}(E) + \sigma_{3/2}(E)}\right|_{th.} \simeq 33\%$$

$$\left.\frac{\sigma_{1/2}(E)}{\sigma_{1/2}(E) + \sigma_{3/2}(E)}\right|_{exp.} \simeq 39\%$$

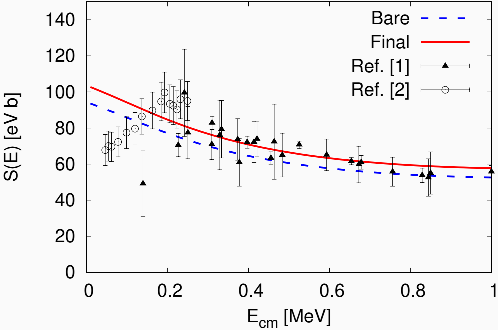
Internal structure of ^{6}Li is missing!

[1] S.K. Switkowski, *et al.* Nucl. Phys. A **331**, 50 (1979)

[2] J.J. He *et al.*, Phys. Lett. B **725**, 287 (2013)

The S-factor

$$S(E) = E\exp(2\pi\eta)(\mathcal{S}^2_{3/2}\sigma_{3/2}(E) + \mathcal{S}^2_{1/2}\sigma_{1/2}(E))$$



- $\mathcal{S}$ spectroscopic factor

J^π	$\mathcal{S}_J$	χ^2_0/N	$\chi^2_{\mathcal{S}}/N$
$3/2^-$	1.003	0.064	0.064
$1/2^-$	1.131	2.096	0.219

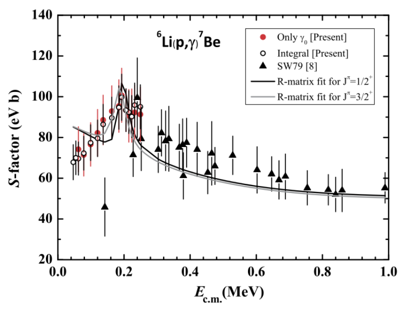
- Fitted on data of [1]
- Branching ratio well reproduced

[1] S.K. Switkowski, *et al.* Nucl. Phys. A **331**, 50 (1979)

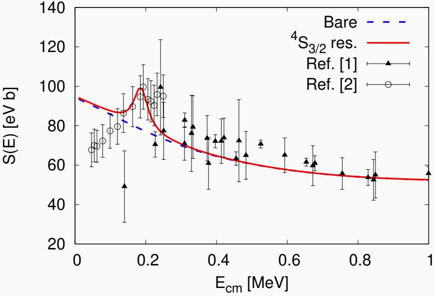
[2] J.J. He *et al.*, Phys. Lett. B **725**, 287 (2013)

The “He *et al.*” resonance

- “He *et al.*” suggested the presence of a resonance $J^\pi = (1/2, 3/2)^+$, $E_r = 195$ MeV $\Gamma_p = 50$ keV.
- Can we add the resonance in our model?



YES!

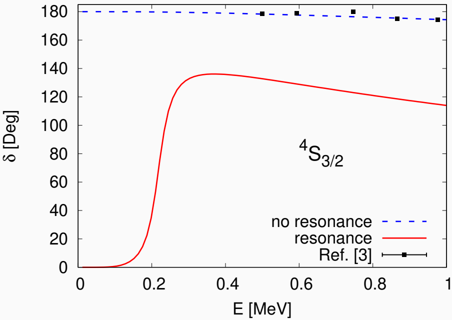


- $\sigma(E) = \mathcal{S}^2_{3/2}\sigma_{3/2}(E) + \mathcal{S}^2_{1/2}\sigma_{1/2}(E) + \mathcal{S}^2_{\text{res}}\sigma_{\text{res}}(E)$
- $\mathcal{S}_0 \simeq \mathcal{S}_1 \sim 1$
- $\mathcal{S}_{\text{res}} = 0.011 \Rightarrow$ small % of S=3/2 in ^{7}Be

BUT...

The “He *et al.*” resonance

The $^4S_{3/2}$ phase shift is NOT reproduced



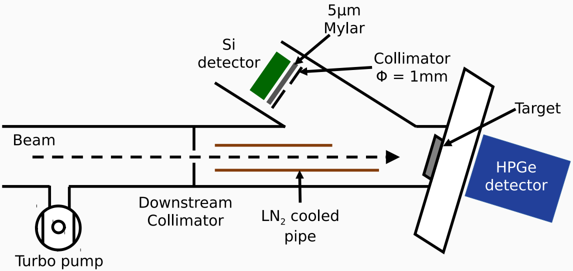
We cannot add the resonance in our model

[1] S.K. Switkowski, *et al.* Nucl. Phys. A **331**, 50 (1979)

[2] J.J. He *et al.*, Phys. Lett. B **725**, 287 (2013)

[3] S.B. Dubovichenko *et al.*, Phys. Atom. Nucl. **74**, 1013 (2011)

LUNA experimental setup



Il nuovo cimento **42C**, 116 (2019) -Courtesy of T. Chillery (LUNA Coll.)

- Not a 4π detector
- The yield ($=N_\gamma/N_p$) must be corrected by

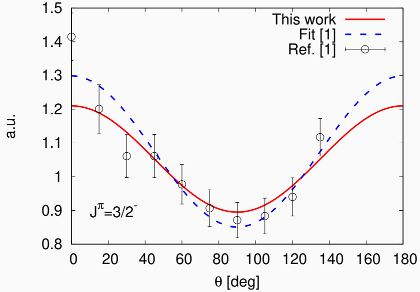
$$W(\theta, E) = \sum_{k\geq 1} a_k(E)P_k(\cos\theta)$$

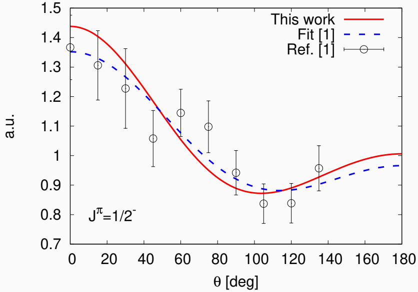
- Angle detector/beam $\theta_0 \simeq 55^\circ \Rightarrow P_2(\cos\theta_0) \simeq 0$

Photon angular distribution

$$W(\theta, E) = \sum_{k \geq 1} a_k(E) P_k(\cos\theta)$$

- a_1 and a_2 are the only significant coefficients
- Dominated by the interference of $E1$ (S-waves) and $E2$ (P-waves)



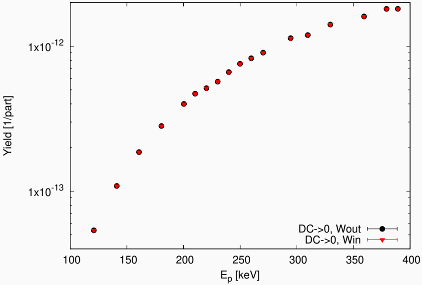


$a_1 \propto E_1(^2S_{3/2}) \times (E_2(^2P_{1/2}) - E_2(^2P_{3/2})) \sim 0$

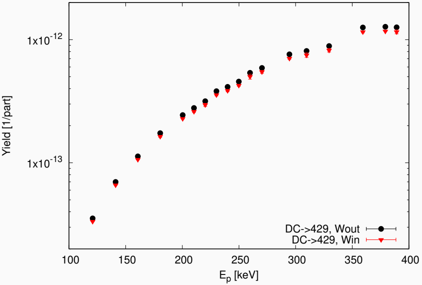
$a_1 \propto E_1(^2S_{3/2}) \times E_2(^2P_{3/2})$

[1] C.I. Tingwell, J. D. King and D.G. Sargood, Aust. J. Phys. **40**, 319 (1987) – $E_p = 0.5$ MeV

Final Yields



$J^{\pi} = 3/2^{-}$



$J^{\pi} = 1/2^{-}$

- Correction to the ground state negligible ($a_1 \sim 0$)
- Correction to the first excited state$\sim 6 - 9\%$ [1]

[1] Courtesy of R. Depalo (LUNA Coll.)

Conclusions I

- Calculation of the S-factor
  $\Rightarrow$ nice agreement with the data

- A resonance?
  $\Rightarrow$ not possible in the cluster model

- Photon angular distribution
  $\Rightarrow$ Important correction to first excited state yields

$$\alpha + d \rightarrow {}^6\text{Li} + \gamma$$

A.G., M. Viviani and L.E. Marcucci, arXiv:2004.05814 (2020)

Nuclear Physics Motivations

- ^{6}Li has an exotic structure
  - Weakly bound nucleus
  - Strong clusterization
- Study of electromagnetic moments
  - Small and negative electric quadrupole moment
- Asymptotic Normalization Coefficients ($\Rightarrow$ S-factor)
- Dark matter search $\Rightarrow$ CRESST Coll.

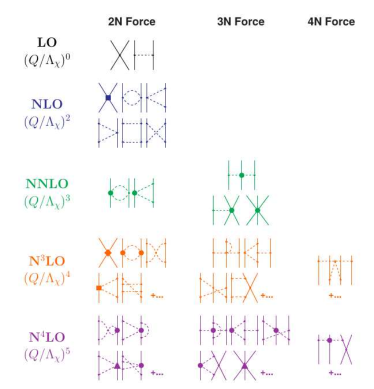
- Which nuclear potential?

- Which method to solve the Schrödinger Equation?

$$\text{QCD} \xrightarrow{\text{chiral symmetry}} \chi\text{EFT}$$

- Low Energy Theory ($\Lambda_\chi \sim 1$ GeV)
  - N, π as d.o.f.
  - high energy d.o.f. integrated out $\rightarrow$ Low Energy Constants
- Perturbative expansion ($\propto (Q/\Lambda_\chi)^\nu$)
- Various phenomena in a consistent framework

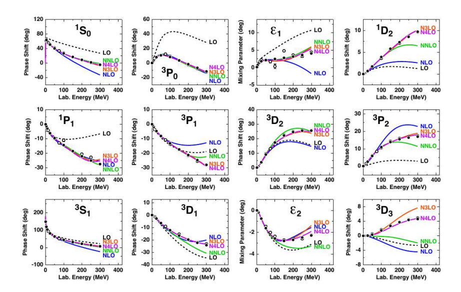
(A.G. and M. Viviani, *Time-reversal violation in light nuclei*, PRC **101**, 024004 (2020).)



D.R. Entem, *et al.* Phys. Rev. C **96**, 024004 (2017)

E. Epelbaum, *et al.* Phys. Rev. Lett. **115**, 122301 (2015)

Nuclear chiral potential



D.R. Entem, *et al.*, Phys. Rev. C **96**, 024004 (2017)

- Non-relativistic expansion
- Regularization with a cutoff ($\Lambda_C = 400 - 600$ MeV)
- LECs fitted to the *NN* experimental scattering data
- The chiral convergence must be checked *a posteriori*
- Controlled theoretical uncertainties

The Hyperspherical Harmonic method

$$H = \sum_i \frac{p_i^2}{2M} + \sum_{i<j} V(i,j) + \sum_{i<j<k} W(i,j,k) + \dots$$

Search for accurate solution of $H\Psi = E\Psi$

- Variational approach
- Expansion of Ψ on the basis of Hyperspherical Harmonic (HH) functions
- [L.E. Marcucci, J. Dohet-Eraly, L. Girlanda, A.G., A. Kievsky, and M. Viviani, Front. Phys. **8**, 69 (2020)]
- Applied for $A = 3, 4$ bound and scattering states

For $A = 6$ implemented from scratch

The HH wave function

- Jacobi vectors $\vec{\xi}_1, \ldots, \vec{\xi}_N \Rightarrow$ CoM completely decoupled
- Hyperangular variables $\rho = \sum_{k=1}^{5} (\xi_k)^2$, $\Omega = \{\hat{\xi}_i, \phi_i\}$, $\cos \phi_k = \frac{\xi_k}{\sqrt{\xi_1^2 + \ldots + \xi_k^2}}$

$$T = -\frac{\hbar^2}{m}\left(\frac{\partial^2}{\partial \rho^2} + \frac{D-1}{\rho}\frac{\partial}{\partial \rho} - \frac{L^2(\Omega)}{\rho^2}\right)$$

- Expansion on a base $\Rightarrow$ Hyperspherical Harmonics (HH)

$$L^2(\Omega)\mathcal{Y}_{[K]}(\Omega) = K(K+13)\mathcal{Y}_{[K]}(\Omega)$$

- The variational wave function

$$\psi_A = \sum_{l,[K]} a_{l,[K]} f_l(\rho)\mathcal{Y}_{[K]}(\Omega_{A-1})\Big[\chi_S \otimes \chi_T\Big],$$

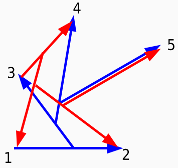
- Check convergence on K

The HH wave function

- Sum over the permutations $\Rightarrow$ antisymmetrization

- Transformation Coefficients (TC)

$$\mathcal{Y}_{[K]}(\Omega') = \sum_{[K']}^{K=K'} a_{[K],[K']}\mathcal{Y}_{[K']}(\Omega)$$



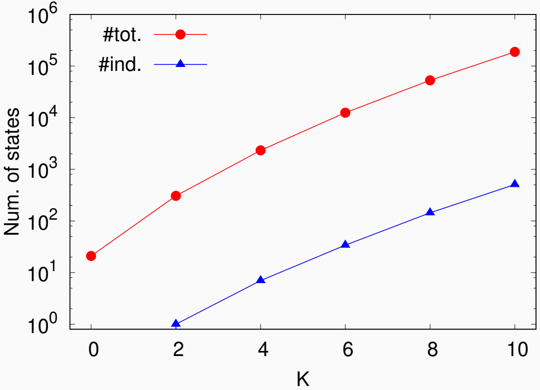
- Sum over the permutations rewritten in terms of the transformation coefficients

$$\sum_{perm} \mathcal{Y}_{[\alpha]}^{KLSTJ}(\Omega_{perm}) = \sum_{[\alpha']} a_{[\alpha],[\alpha']}^{KLSTJ}\mathcal{Y}_{[\alpha']}^{KLSTJ}(\Omega)$$

- Basis states are linearly dependent $\Rightarrow$ orthogonalization procedure

Orthonormalization

- Ground state of ^{6}Li is $J^\pi = 1^+$ (mainly $T = 0$)



$$\frac{\#tot}{\#ind} \sim cost. = 370$$

Bound state calculation

- Evaluation of the matrix elements

$$H_{\alpha,\beta} = \langle \Phi_\alpha | H | \Phi_\beta \rangle$$

- Analytical for the kinetic energy

- Potential matrix elements

$$\langle \Phi_\alpha | \sum_{i<j} V(i,j) | \Phi_\beta \rangle = \frac{A(A-1)}{2} \sum_{[\alpha'],[\beta']} a_{[\alpha],[\alpha']} a_{[\beta],[\beta']} \underbrace{V_{[\alpha'],[\beta']}(1,2)}_{\text{depend on } \mu \text{ only}}$$

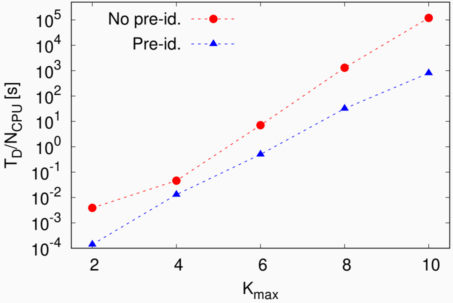
$\Rightarrow$ By using the sum over the permutations and the TC

$\alpha, \beta =$ qn of 6-bodies, $\mu =$ qn of the couple (1,2)

A new computational approach

$$\langle \Phi_\alpha | \sum_{i<j} V(i,j) | \Phi_\beta \rangle = \frac{A(A-1)}{2} \sum_\mu D_\mu^{[\alpha],[\beta]} V_\mu(1,2)$$

- $D_\mu^{[\alpha],[\beta]}$ independent on the potential model
  $\Rightarrow$ evaluated and stored only once
- Disk space used $\sim$ 100GB
- Algorithmic improvements $\Rightarrow$ pre-identification
- Fast construction of potential matrix elements



Warning!

Chiral interactions have "hard" repulsive cores

- We will use SRG evolved N^3LO500 *NN* interaction [1-2]
  - SRG evolution parameter $\Lambda = 1.2, 1.5, 1.8$ fm^{-1}
  - The Coulomb interaction is included as "bare" (not SRG evolved)
  - Unitary transformation $\Rightarrow$ induces many-body forces
- Explorative study with $NNLO_{sat}$(NN) [3]
- No 3-body forces ($\Rightarrow$ see future perspectives)
- We compute the mean values of "bare" operators

[1] S.K. Bogner, R.J. Furnstahl, and R.J. Perry, PRC **75**, 061001(R) (2007)

[2] D.R. Entem and R. Machleidt, PRC **68**, 041001(R) (2003)

[3] A. Ekström, *et al.*, PRC **91**,051301 (2015)

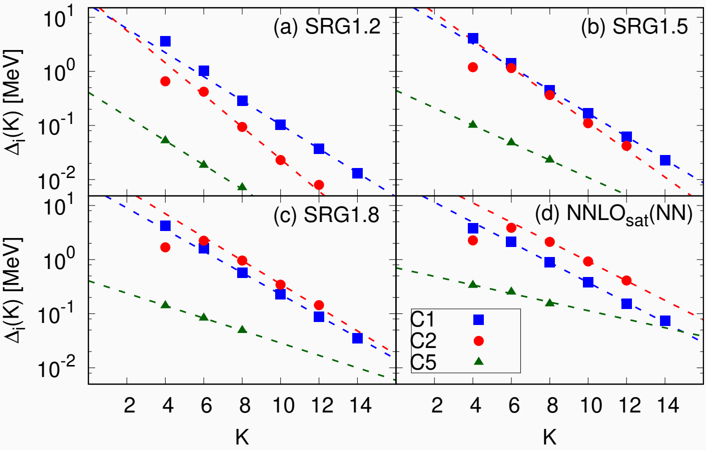
Selection of the states

- Include all the states up to a K_{max} $\Rightarrow$ FAIL!!
- NOT all the states gives the same contribution $\Rightarrow$ division in classes [1]
  - Centrifugal barrier $\Rightarrow \ell_1 + \cdots + \ell_5 \leq 4$
  - two-body correlations are more important
- The ^{6}Li ground state is a $J^\pi = 1^+$ state

wave	class	corr.	K_{max}
S	C1	two-body	14
	C3	many-body	10
D	C2	two-body	12
	C4	many-body	10
P	C5	all	8
F-G	C6	all	8

[1] M. Viviani, *et al.*, PRC **71**,024006 (2005)

Convergence of the classes



$$\Delta_i(K) = B_i(K) - B_i(K-2)$$

Extrapolation to $K = \infty$

- Fit the quantity Δ_i with [1]

$$\Delta_i(K) = A_i e^{-b_i K}(1 - e^{-2b_i})$$

- Missing energy for class

$$\Delta_i(\infty) = \sum_{K=\overline{K}+2}^{\infty} \Delta_i(K) = A_i e^{-b_i \overline{K}}$$

where $\overline{K}$ maximum K used for the class i

- Extrapolated energy

$$B(\infty) = B_{full} + \sum_i \Delta_i(\infty)$$

where B_{full} = binding energy with all the states

[1] S.K. Bogner *et al.*, NPA **801**, 21 (2008)

Final extrapolation

	"bare" Coul.		SRG Coul.*		
	B_{full}	$B(\infty)$	B_{full}	$B(\infty)$	Ref. [1]
SRG1.2	31.75	31.78(1)	31.78	31.81(1)	31.85(5)
SRG1.5	32.75	32.87(2)	32.79	32.91(2)	33.00(5)
SRG1.8	32.21	32.64(9)	32.25	32.68(9)	32.8(1)
$NNLO_{sat}$(NN)	29.77	30.71(15)			–

- All the energies are in MeV
- The errors come from the fit
- Results of Ref. [1] extrapolated from $N_{max} = 10$(NCSM)
- Experimental value $B = 31.99$ MeV

$*$= not included in the thesis

[1] E.D Jungerson, P. Navrátil and R.J. Furnstahl, PRC **83**, 034301 (2011)

Magnetic dipole moment

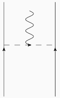
$$^{6}\text{Li} \simeq \alpha + d \Rightarrow \mu_z(^{6}\text{Li}) \simeq \mu_z(d)$$

Experiment tells us $\mu_z(^{6}\text{Li}) < \mu_z(d)$

	$\mu_z(d)$	$\mu_z(^{6}\text{Li})$
SRG1.2	0.872	0.865
SRG1.5	0.868	0.858
SRG1.8	0.865	0.852
NNLO$_{sat}$	0.860	0.845
Exp.	0.857	0.822

- Negative contribution only from the $L = 2$ $S = 1$ component
  $\Rightarrow$ NOT SUFFICIENT

Two-body currents could be necessary! [1]



[1] R. Schiavilla, *et al.*, PRC **99**, 034005 (2019)

Electric quadrupole moment

	Q [e fm2]
SRG1.2	-0.191
SRG1.5	-0.101
SRG1.8	-0.055
NNLO_{sat}(NN)	$+0.068$
Ref. [1]	$-0.066(40)$
Exp.	$-0.0806(8)$

- Experiment $\Rightarrow$ small and negative
- Large differences between the potentials

[1] CDB2k-SRG1.5 C. Forssén, E. Caurier, P. Navrátil, PRC **71**, 021303 (2009)

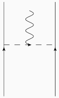
Electric quadrupole moment

Matrix elements between different waves

	$S-D$	$D-D$	$P-P$	$P-D$	remaining
SRG1.2	-0.187	-0.023	0.009	0.009	<0.001
SRG1.5	-0.102	-0.023	0.014	0.010	<0.001
SRG1.8	-0.058	-0.024	0.016	0.010	0.001
NNLO$_{sat}$(NN)	0.049	-0.018	0.023	0.011	0.003

- Direct connection with the strength of the tensor term in the potential

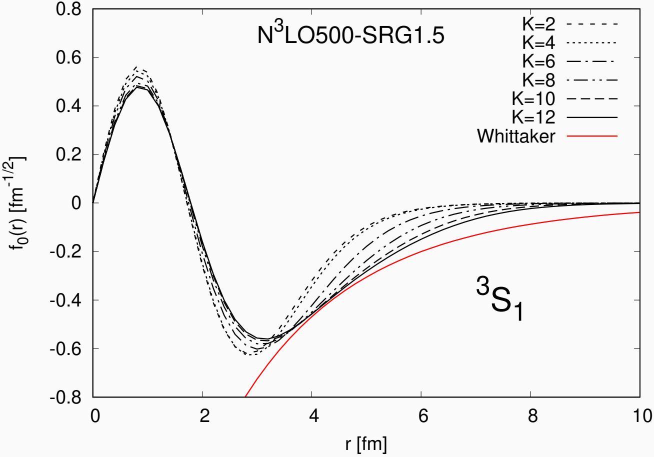
Two-body currents could be necessary!



Cluster form factor $\alpha + d$

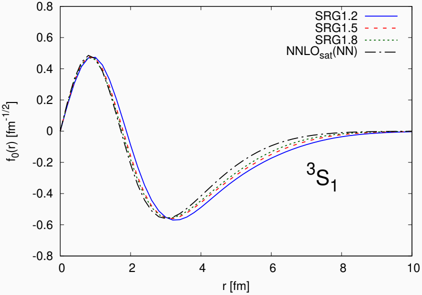
$$\frac{f_L(r)}{r} = \langle \Psi^{(L)}_{\alpha+d} | \Psi_{^6\mathrm{Li}} \rangle, \qquad |\Psi^{(L)}_{\alpha+d}\rangle = \left[(\Psi_\alpha \otimes \Psi_d)_S \, Y_L(\hat{r}) \right]_J$$

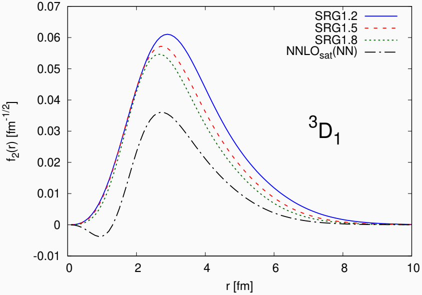
with $S = 1$, $J = 1$ and $L = 0$ ($^3S_1$) or 2 ($^3D_1$)



Cluster form factor $\alpha + d$

$$\frac{f_L(r)}{r} = \langle \Psi^{(L)}_{\alpha+d} | \Psi_{^6\text{Li}} \rangle$$





- *S*-wave - Node related to the Pauli principle
- *D*-wave - For the NNLO$_{sat}$(NN) a node appears
  $\Rightarrow$ strength of the tensor forces [1]

[1] V.I. Kukulin, *et al.* NPA **586**,151 (1995)

Spectroscopic factor

$$\mathcal{S}_L = \int_0^\infty dr\, |f_L(r)|^2$$

% of $\alpha + d$ clusterization

Method	Potential	$\mathcal{S}_0$	$\mathcal{S}_2$	$\mathcal{S}_0 + \mathcal{S}_2$
HH	SRG1.2	0.909	0.008	0.917
	SRG1.5	0.868	0.007	0.875
	SRG1.8	0.840	0.006	0.846
	NNLO_{sat}(NN)	0.805	0.002	0.807
GFMC [1]	AV18/UIX	0.82	0.021	0.84
GFMC [2]	AV18/UIX	–	–	0.87(5)
NCSM [3]	CD-B2k	0.822	0.006	0.828
Exp. [4]		–	–	0.85(4)

[1] J.L. Forest *et al.*, PRC **54**, 646 (1996)

[2] K.M. Nollet *et al.*, PRC **63**, 024003 (2001)

[3] P. Navrátil, PRC **70**, 054324 (2004)

[4] R.G.H. Robertson, PRL **47**, 1867 (1981)

Asymptotic Normalization Coefficients (ANCs)

- When $r \to \infty$

$$\Psi_{^6\mathrm{Li}}(r \to \infty) = C_0 \frac{W_{-\eta,1/2}(2kr)}{r} \Psi^{(0)}_{\alpha+d} + C_2 \frac{W_{-\eta,5/2}(2kr)}{r} \Psi^{(2)}_{\alpha+d}$$

- $W_{-\eta,L+1/2}$ Whittaker function (Sol. Coulomb Schrödinger Eq.)
- BBN reactions are peripheral (low-energies)

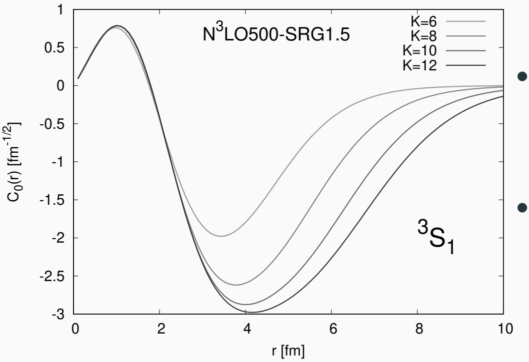
$$\sigma(E) \propto |C_0|^2$$

- Reproducing the correct magnitude of the ANCs is a key point in the description of the $\alpha + d \to {}^6\mathrm{Li} + \gamma$

$B_c = B_{^6\mathrm{Li}} - B_\alpha - B_d$, $k = \sqrt{2\mu B_c/\hbar^2}$ and $\eta = 2.88\mu/k$

Asymptotic Normalization Coefficients (ANCs)

$$C_L(r) = \frac{f_L(r)}{W_{-\eta,L+1/2}(2kr)} \xrightarrow{r\to\infty} C_L$$



- The Whittaker function depends on B_c and so on K
- ANC obtained from the "plateau" around the minimum [1]

[1] M. Viviani, *et al.*, PRC **71**,024006 (2005)

A more precise approach for the ANC

- From the Schrödinger Equation $\Rightarrow$

$$\langle \Psi^{(L)}_{\alpha+d} | H_6 | \Psi_{^6\mathrm{Li}} \rangle = \langle \Psi^{(L)}_{\alpha+d} | B_{^6\mathrm{Li}} | \Psi_{^6\mathrm{Li}} \rangle$$

- $\Rightarrow$ An equation for the cluster form factor [1-2]

$$\left[-\frac{\hbar^2}{2\mu} \left(\frac{d^2}{dr^2} - \frac{L(L+1)}{r^2} \right) + \frac{2e^2}{r} + B_c \right] f_L(r) + g_L(r) = 0$$

- Source term

$$g_L(r) = \langle \Psi^{(L)}_{\alpha+d} | \left(\sum_{i \in \alpha} \sum_{j \in d} V_{ij} - \frac{2e^2}{r} \right) | \Psi_{^6\mathrm{Li}} \rangle |_{r \text{ fixed}}$$

$\Rightarrow$ Hard to compute in this form!

$\Rightarrow$ Same matrix elements needed for scattering!

- Correct asymptotic behavior

$$g_L(r) \xrightarrow{r \to \infty} 0 \Rightarrow f_L(r) \xrightarrow{r \to \infty} W_{-\eta, L+1/2}(2kr)$$

[1] N.K. Timofeyuk, NPA **632**,19 (1998)

[2] M. Viviani, *et al.*, PRC **71**,024006 (2005)

The “projection” method

- A cluster wave function

$$\phi_{\alpha+d} = \left\{ [\Psi_\alpha \times \Psi_d]_S \, Y_L(\hat{r}) \right\}_J f(r)$$

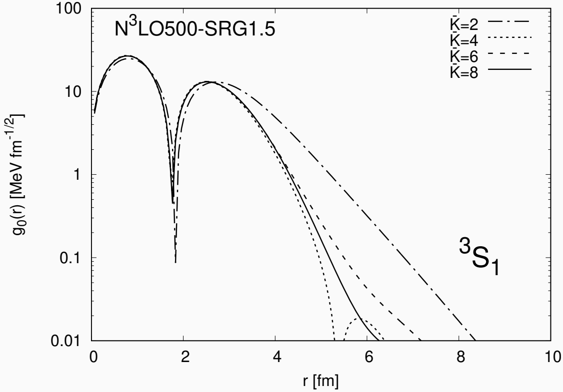
- where $f(r)$ intercluster wave function
- Expansion in term of the HH states

$$\phi_{\alpha+d} = \sum_{\bar{K}=0}^{\bar{K}_{max}} c_{[\bar{K}]}^{LSJ} \mathcal{Y}_{[\bar{K}]} \qquad c_{[\bar{K}]}^{LSJ} = \langle \mathcal{Y}_{[\bar{K}]} | \phi_{\alpha+d} \rangle$$

- The equality holds only when $\bar{K}_{max} \to \infty$
- Advantages:
  - Easy computation of the matrix elements
  - Control of the convergence $\bar{K}$

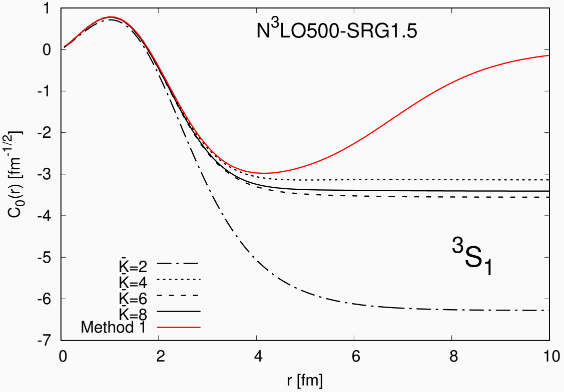
The source term

$$g_L(r) = \langle \Psi^{(L)}_{\alpha+d}(\bar{K}) | \left(\sum_{i\in\alpha} \sum_{j\in d} V_{ij} - \frac{2e^2}{r} \right) | \Psi_{^6\mathrm{Li}} \rangle$$



^{6}Li wave function computed with $K = 12$

Overlap vs. Equation



$$C_L(r) = \frac{f_L(r)}{W_{-\eta,L+1/2}(2kr)}$$

- Reproduced exactly the short range part
- Exact asymptotic behavior

^{6}Li wave function computed with $K = 12$

Results for the ANCs

	B_c [MeV]	C_0 [fm$^{-1/2}$]	C_2 [fm$^{-1/2}$]
SRG1.2	3.00(1)	−4.19(12)	0.116(18)
SRG1.5	2.46(2)	−3.44(7)	0.072(15)
SRG1.8	2.02(9)	−3.01(7)	0.047(10)
Exp.	1.4743	−2.91(9)	0.077(18)

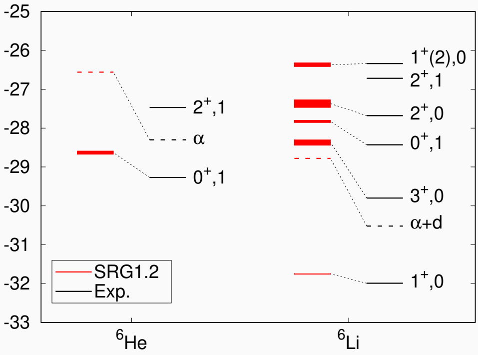
- Extrapolated values of the ANC
- Errors from the convergence in K and $\overline{K}$
- Strong dependence on B_c
- Same order of magnitude of the experiment!

Conclusions

- Extension of HH basis for $A = 6$
- Ground state of ^{6}Li
  - Good convergence for SRG and NNLO$_{sat}$(NN) potentials
  - Study of electromagnetic structure
    - Strong dependence on tensor forces
    - Signals that we need two-body currents
- $\alpha + d$ clusterization of ^{6}Li
  - Calculation of the Asymptotic Normalization Coefficients
    $\Rightarrow$ Key ingredient for S-factor
  - Test of the "projection" method
    $\Rightarrow$ First steps for the computation of scattering states

Future prospectives I (preliminary)

Extension of the calculation to the excited states of ^{6}Li and the states of ^{6}He (SRG1.2)



^{6}Li

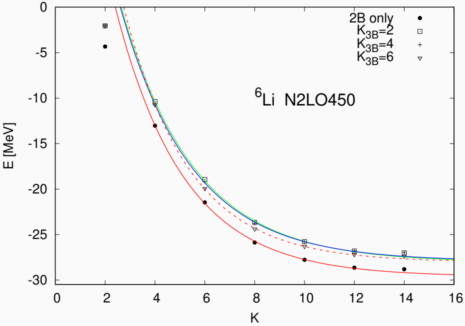
J^π, T	Exp.	SRG1.2
$1^+, 0$	-31.99	-31.78(1)
$\alpha + d$	-30.52	-28.78
$3^+, 0$	-29.80	-28.37(7)
$0^+, 1$	-28.43	-26.37(5)
$2^+, 0$	-27.86	-27.4(1)
$2^+, 1$	-26.72	–
$1_2^+, 0$	-26.34	-27.83(3)

^{6}He

J^π, T	Exp.	SRG1.2
$0^+, 1$	-29.27	-28.63(4)
α	-28.30	-26.56
$2^+, 1$	-27.47	–

- States $(2^+, 0)$ and $(3^+, 0)$ up to $K = 8$.
- States $(0^+, 1)$ up to $K = 12$.

Towards “bare” chiral interactions + 3-body forces



N2LO450 from D.R. Entem *et al.*, PRC **91**, 014002 (2015)

- Increase the basis size up to $K = 20$
  From $\sim$ 30k h to $\sim 500 - 1000$k h to compute D coefficients
- Better selection of the classes
- OpenMP $\Rightarrow$ OpenMPI
- Use of accelerators (OpenACC, CUDA,...)

Future prospectives III

Calculation of the $\alpha + d \rightarrow {}^6\text{Li} + \gamma$

- Scattering states $\Rightarrow$ use of the Kohn variational principle
- Very precise matrix elements $\Rightarrow$ “projection” method
- On going calculations...

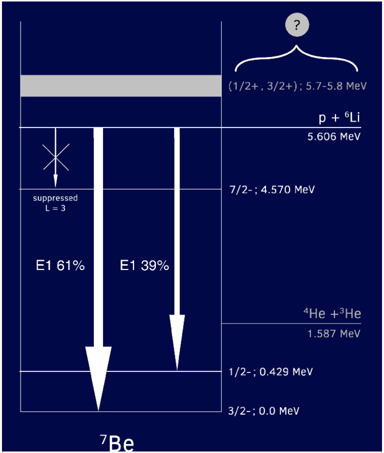
Acknowledgments

- INFN-Pisa for the hospitality and in particular the iniziativa specifica FBS
- Calculation performed on MARCONI at CINECA ($\sim$ 500.000 h)

Thank you!

Spare

The $p + {}^6$Li radiative capture details



- Long wavelength approximation $\Rightarrow$ spin conservation
- Multipole analysis

^{7}Be	$p(1/2+) + {}^6$Li$(1+)$		
$S = 1/2$	$S = 1/2$		
$L = 0$	$L = 0$	$L = 1$	$L = 2$
${}^2P_{3/2}$	$E1$	$E2$	$E1$
${}^2P_{1/2}$	$E1$	$E2$	$E1$

- The ${}^2S_{1/2}$, ${}^2D_{3/2}$, ${}^2D_{5/2}$ waves dominate the cross section

The S-factor details

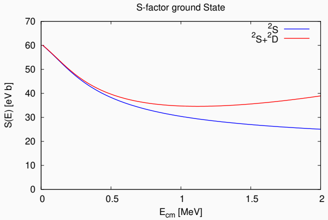
$$S(E) = E \exp(2\pi\eta) \sum_{J_f} \sigma_{J_f}(E)$$

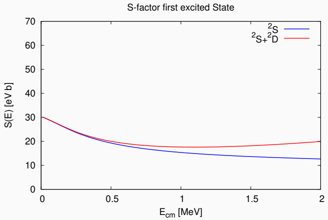
$$\sigma_{J_f}(E) = \sigma_0(E) \sum_{\Lambda \geq 1} \sum_{LSJ} \left(|E_\Lambda^{LSJ}|^2 + |M_\Lambda^{LSJ}|^2 \right)$$

which in our case reduced to

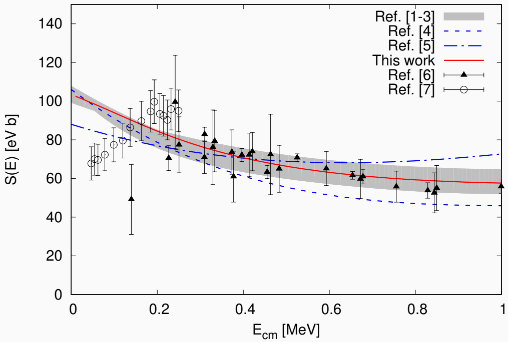
$$\sigma_{3/2}(E) \simeq \sigma_0(E) \left(|E_1^{0\frac{1}{2}\frac{1}{2}}|^2 + |E_1^{2\frac{1}{2}\frac{3}{2}}|^2 + |E_1^{2\frac{1}{2}\frac{5}{2}}|^2 \right)$$

$$\sigma_{1/2}(E) \simeq \sigma_0(E) \left(|E_1^{0\frac{1}{2}\frac{1}{2}}|^2 + |E_1^{2\frac{1}{2}\frac{3}{2}}|^2 \right)$$





A theoretical error band



$$S(0) = 104\,\text{eV b}$$

$$S_{\text{err}}(0) = 103.5 \pm 4.5\,\text{eV b}$$

Phenomenological

[1]J.T. Huang *et al.* , At. Data Nucl. Data Tables **96**, 824 (2010)

[2] S.B. Dubovichenko *et al.*, Phys. Atom. Nucl. **74**, 1013 (2011)

[3] F.C. Barker, Aust. J. Phys. **33**, 159 (1980)

Semi-phenomenological

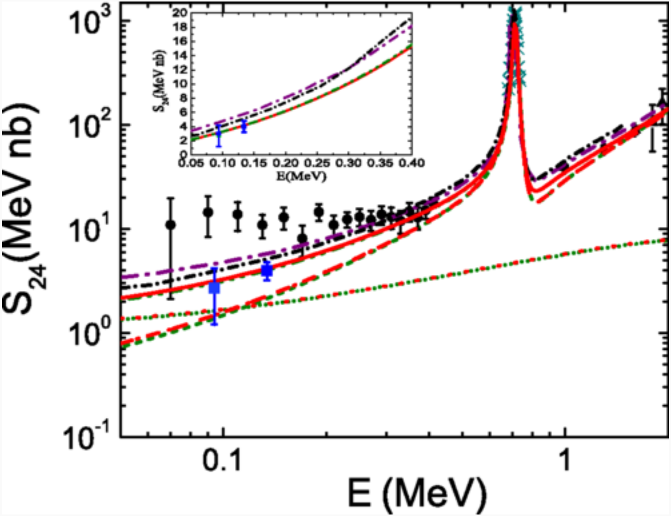
[4] K. Arai *et al.*, Nucl. Phys. A **699**, 963 (2002)

[5] G.X. Dong *et al.*, J. Phys. G: Nucl. Part. Phys. 44, 045201 (2017)

Data

[6] S.K. Switkowski, *et al.* Nucl. Phys. A **331**, 50 (1979)

[7] J.J. He *et al.*, Phys. Lett. B **725**, 287 (2013)



A.M. Mukhamedzhanov *et al.*, PRC **93**, 045805 (2016)

A new computational approach

$$\langle \Phi_\alpha | V(1,2) | \Phi_\beta \rangle = \sum_{[\alpha'],[\beta']} a_{[\alpha],[\alpha']} a_{[\beta],[\beta']} \underbrace{V_{[\alpha'],[\beta']}(1,2)}_{\text{depend on } \mu \text{ only}}$$

$\alpha, \beta =$ qn of 6-bodies, $\mu =$ qn of the couple (1,2)

To be notice that

1. The sum over $[\alpha'], [\beta']$ can run over millions of states
2. The potential involves only the particles (1,2) and so it depends on a small set μ of qn
3. The sum over the qn which do not involve the couple (1,2) is independent of the particular potential model

qn= quantum numbers

A new computational approach

Therefore, we can rewrite

$$\langle \Phi_\alpha | V(1,2) | \Phi_\beta \rangle = \sum_\mu D_\mu^{[\alpha],[\beta]} V_\mu(1,2)$$

- $D_\mu^{[\alpha],[\beta]}$ independent on the potential model
  $\Rightarrow$ evaluated and stored only once
- Small number of combinations μ
  - small disc space required for saving $\sim$ 100GB
  - fast construction of potential matrix elements
- Extensible to 3N forces (in progress...)

The HH basis advantages

- Completely antisymmetrized

- Orthogonalization $\Rightarrow$ relatively small basis
  Full calculation $N_{HH} \sim 7000$
  Hamiltonian dimension $\sim 110000 \times 110000$

- Easy computation of the matrix elements

- No need to save the matrix elements only the D coefficients

- $\sim$ 3 hours for constructing and diagonalize the Hamiltonian
  $\Rightarrow$ easy to test various potentials

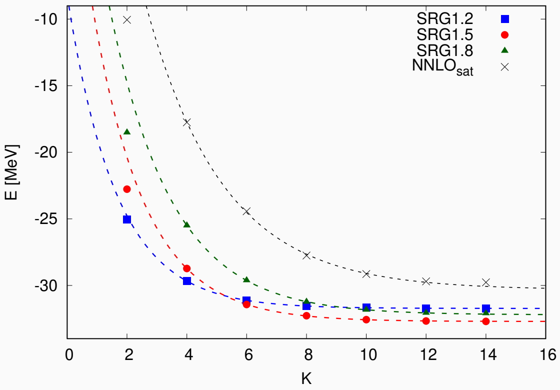
Validation

- $J^{\pi} = 1^{+}$ only $LST = 010$ Volkov potential [1]

K	This work	Ref. [2]
2	-61.142	-61.142
4	-62.015	-62.015
6	-63.377	-63.377
8	-64.437	-64.437
10	-65.354	-65.354
12	-65.884	-65.886

[1] A. Volkov, Nucl. Phys. 74, 33 (1965)

[2] M. Gattobigio *et al.*, PRC **83**, 024001 (2011)

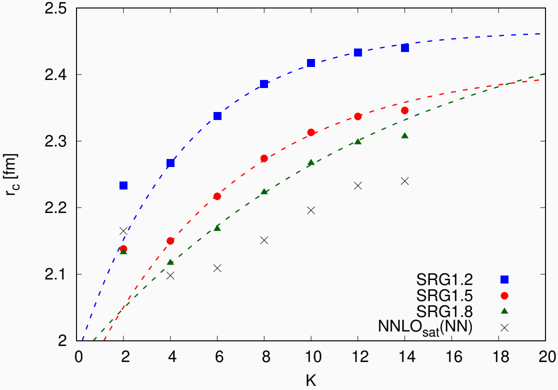


- Exponential behavior [1]

$$E(K) = E(\infty) + Ae^{-bK}$$

[1] S.K. Bogner *et al.*, NPA **801**, 21 (2008)

Charge radius



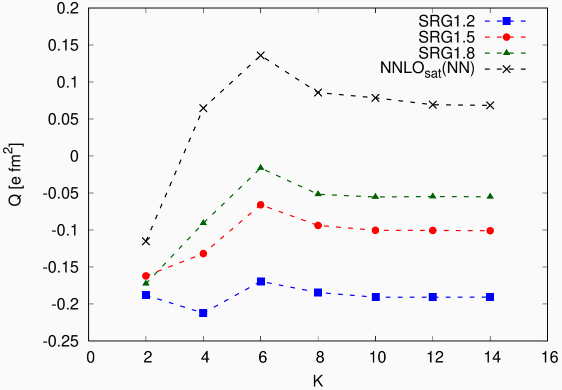
	$r_c(\infty)$ [fm]
SRG1.2	2.47(1)
SRG1.5	2.42(2)
SRG1.8	2.52(10)
Ref. [1]	2.40(6)
Exp.	2.540(28)

- Extrapolation $r_c(K) = r_c(\infty) + Ae^{-bK}$

[1] CDB2k-SRG1.5 C. Forssén, E. Caurier, P. Navrátil, PRC **71**, 021303 (2009)

		SRG1.2			SRG1.5		
i	K_{iM}	$\Delta_i(K_{iM})$	b_i	$(\Delta B)_i$	$\Delta_i(K_{iM})$	b_i	$(\Delta B)_i$
1	14	0.013	0.51	0.007(0)	0.023	0.49	0.014(0)
2	12	0.008	0.68	0.003(1)	0.042	0.58	0.019(0)
3	10	0.015	0.37	0.014(7)	0.022	0.32	0.024(12)
4	10	0.008	0.60	0.004(2)	0.022	0.49	0.013(6)
5	8	0.007	0.52	0.004(0)	0.023	0.37	0.021(0)
6	8	0.004	0.44	0.003(1)	0.018	0.26	0.026(13)
$(\Delta B)_T$				0.034(7)			0.117(19)
		SRG1.8			NNLO$_{sat}$		
i	K_{iM}	$\Delta_i(K_{iM})$	b_i	$(\Delta B)_i$	$\Delta_i(K_{iM})$	b_i	$(\Delta B)_i$
1	14	0.035	0.46	0.023(0)	0.074	0.43	0.05(0)
2	12	0.144	0.50	0.084(11)	0.411	0.42	0.32(1)
3	10	0.024	0.30	0.029(15)	0.031	0.17	0.07(4)
4	10	0.045	0.38	0.039(20)	0.093	0.25	0.14(7)
5	8	0.049	0.26	0.070(1)	0.153	0.18	0.35(14)
6	8	0.048	0.11	0.19(9)	0.112	–	–
$(\Delta B)_T$				0.43(9)			0.93(20)

Electric quadrupole moment

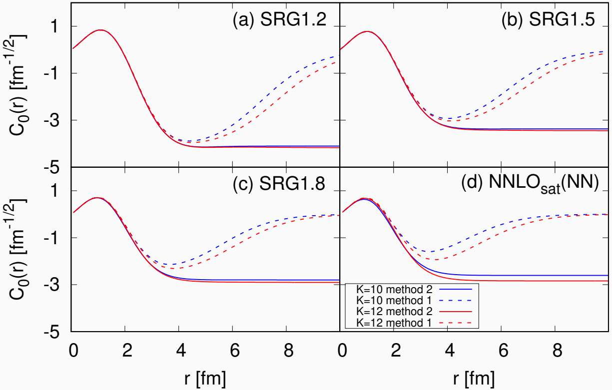


	Q [e fm2]
SRG1.2	-0.191
SRG1.5	-0.101
SRG1.8	-0.055
NNLO$_{sat}$(NN)	$+0.068$
Ref. [1]	$-0.066(40)$
Exp.	$-0.0806(8)$

- Large cancellations between different K

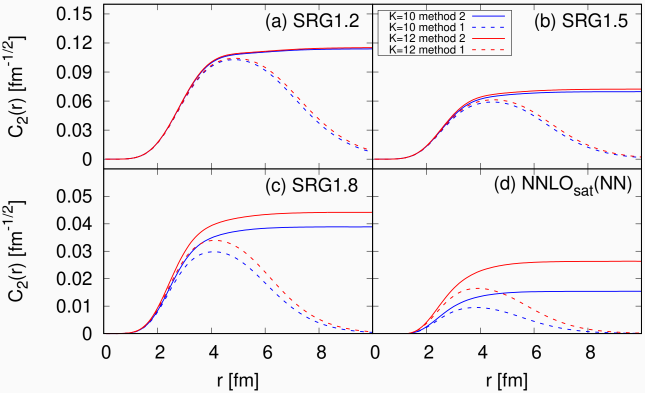
[1] CDB2k-SRG1.5 C. Forssén, E. Caurier, P. Navrátil, PRC **71**, 021303 (2009)

Function $C_0(r)$ (S wave)



- $\bar{K} = 8$ in the "projection" of the cluster function
- Better convergence of ^{6}Li wf $\Rightarrow$ better agreement with Method I and II

Function $C_2(r)$ (D wave)



- $\bar{K} = 8$ in the “projection” of the cluster function
- Better convergence of ^{6}Li wf $\Rightarrow$ better agreement with Method I and II

Direct dark matter search

- Spin dependent dark matter direct search
  - Cristals of ^{7}Li
  - ^{6}Li contaminations
- Rate of events $R \propto \langle S_{p/n} \rangle^2$
  - $S_{p/n}$ spin operator
  - Usually computed using the shell model (not possible for ^{6}Li)

	$\langle S_p \rangle (= \langle S_n \rangle)$
SRG1.2	0.479(1)
SRG1.5	0.472(2)
SRG1.8	0.464(3)